Games of No Chance

1. Choose a positive integer $n$. At each turn one of the players writes a positive integer that does not exceed $n$, the rule being that the player cannot write a divisor of a number already written. The player who cannot continue loses. Show that player one has a winning strategy.

2. In Determinant Tic-Tac-Toe, Player 1 enters a 1 in an empty $3 \times 3$ matrix. Player 0 counters with a 0 in a vacant position, and play continues in turn until the $3 \times 3$ matrix is completed with five 1’s and four 0’s. Player 0 wins if the determinant is 0 and player 1 wins otherwise. Assuming both players pursue optimal strategies, who will win and how?

3. In (a version of) the game of Nim, two players start with a pile of $n$ stones. On each turn, a player removes 1, 2, or 3 stones from the pile. The player to take the last stone wins. For which $n$ does player 1 have a winning strategy?

4. Let $k$ and $n$ be integers with $1 \leq k < n$. Alice and Bob play a game with $k$ pegs in a line of $n$ holes. At the beginning of the game, the pegs occupy the $k$ leftmost holes. A legal move consists of moving a single peg to any vacant hole that is further to the right. The players alternate moves, with Alice playing first. The game ends when the pegs are in the $k$ rightmost holes, so whoever is next to play cannot move, and therefore loses. For what values of $n$ and $k$ does Alice have a winning strategy?

5. An integer $n$, unknown to you, has been randomly chosen in the interval $[1, 2002]$ with uniform probability. Your objective is to select $n$ in an odd number of guesses. After each incorrect guess, you are informed whether $n$ is higher or lower, and you must guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than $2/3$. 