IOWA STATE UNIVERSITY
DEPARTMENT OF MATHEMATICS

COURSE ADVERTISEMENT - FALL 2021

MATH 495
Topics Course: Computational Algebraic Geometry
(3 credits)

Instructor: Dr. Jason McCullough
Email: jmccullo@iastate.edu

Brief course description: Given a single polynomial equation in one variable, say $x^3-4x = 0$, we can often solve for x by factoring. Here $x^3-4x = x(x+2)(x-2) = 0$ and the solutions are $x = 0, \pm 2$. Given a system of linear equations in several variables, such as

$$3x + 4y = 2, \qquad 5x - 7y = -1,$$

we can find a solution by row-reducing the appropriate matrix of coefficients. So how do we solve a system of non-linear polynomial equations in multiple variables like:

$$x^2 + 2y^2 = 3, \qquad x^2 + xy + y^2 = 3?$$

Geometrically this is equivalent to asking for the coordinates of the intersection of two ellipses corresponding to the two equations. This course will cover an algorithmic approach to solving these kinds of questions via Groebner bases. We will also study ideals and polynomial rings, the ideal variety correspondence in algebraic geometry, elimination theory, Hilbert's Nullstellensatz, and other topics as time allows. Students will learn how to use computer algebra software to solve complicated systems of polynomial equations. For example, a computation shows the system of equations above is equivalent to the following (better?) system:

$$y^3 - 3y = 0, \qquad xy - y^2 = 0, \qquad x^2 + 2y^2 = 3.$$

Can you find the solution set now?

Prerequisites: Math 317 (Theory of Linear Algebra) or equivalent

Textbook: *Ideals, Varieties, and Algorithms* by Cox, Little, and O'Shea
(Students have free digital access to this book through the ISU library.)