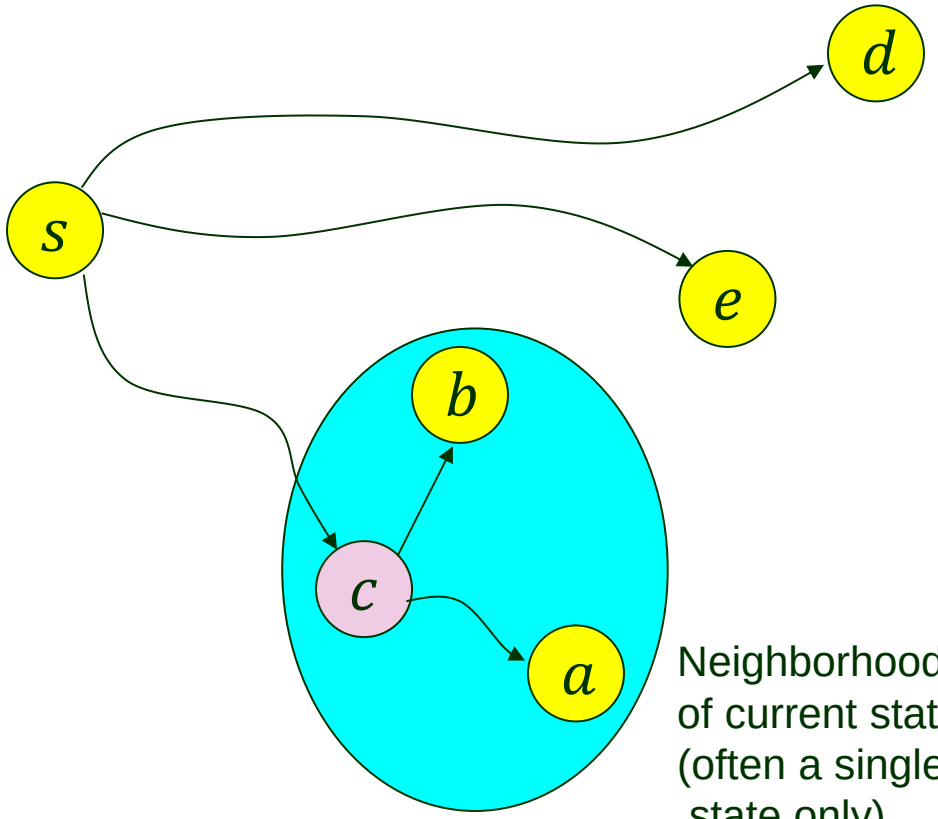
Local Search

Evaluate and modify one or more *current states* rather than systematically exploring paths from an initial state.



Neighborhood of current states (often a single state only)

* Figures are from the textbook site (or drawn by the instructor).

Local Search

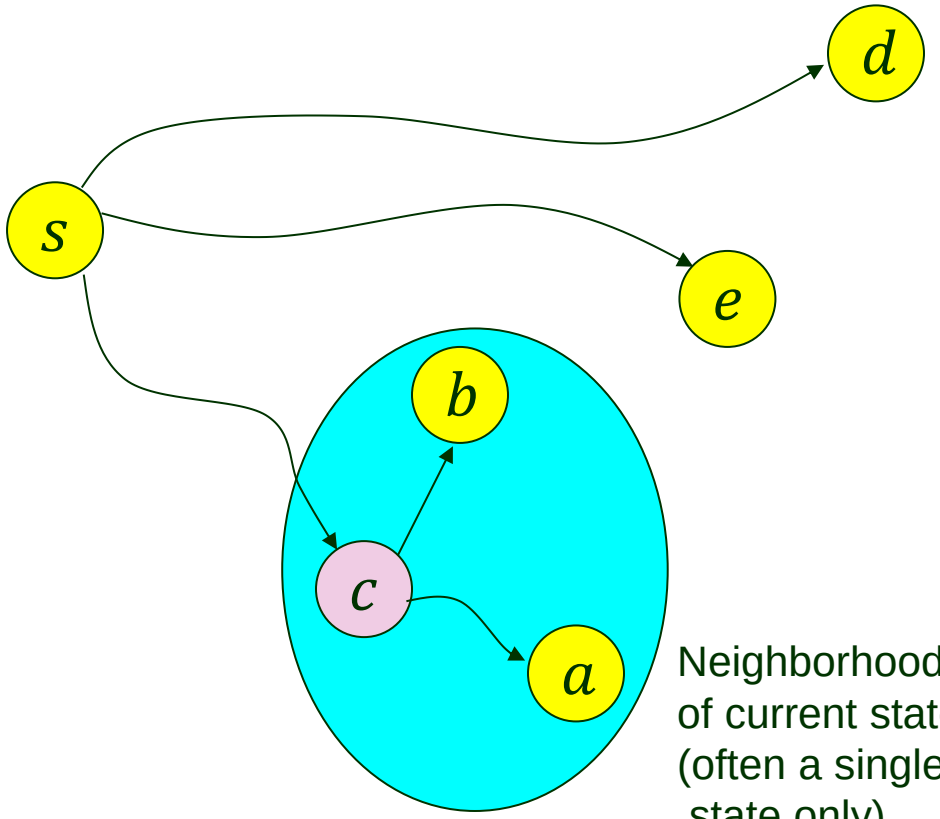
Evaluate and modify one or more *current states* rather than systematically exploring paths from an initial state.

Outline

I. Hill climbing

II. Simulated annealing

III. Genetic algorithms



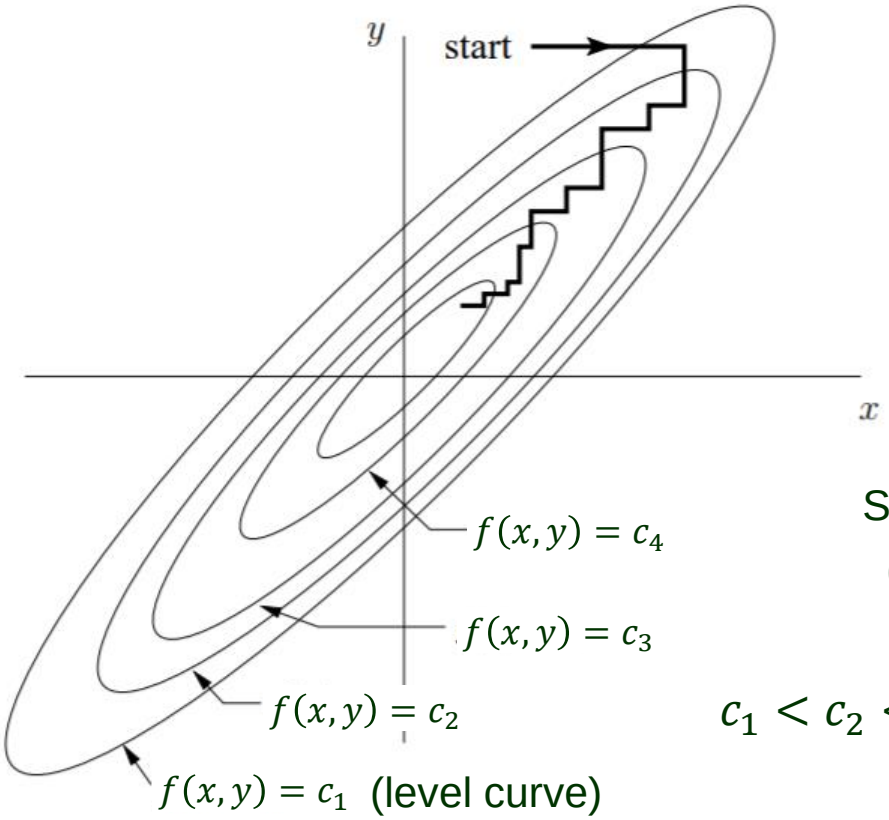
* Figures are from the textbook site (or drawn by the instructor).

Advantages of Local Search

- Use of very little memory.
- Finding good solutions in state spaces *intractable* for a systematic search.
- Useful in pure optimization (e.g., gradient-based ascent/descent methods)

Advantages of Local Search

- Use of very little memory.
- Finding good solutions in state spaces *intractable* for a systematic search.
- Useful in pure optimization (e.g., gradient-based ascent/descent methods)

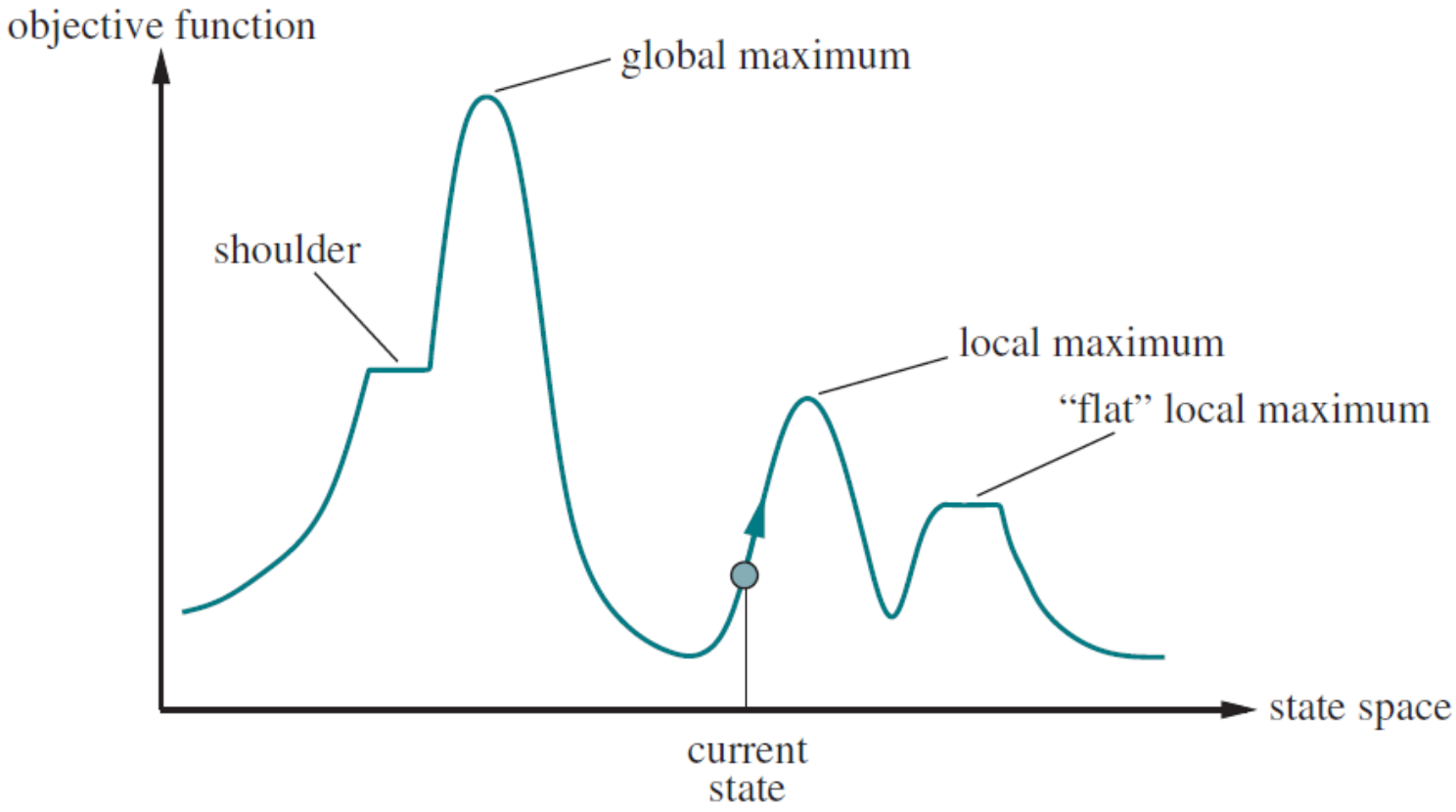


Maximize $f(x, y)$

Steepest ascent
(along $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$)

$c_1 < c_2 < c_3 < c_4$

State Space Landscape



I. Hill Climbing

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    current ← problem.INITIAL
    while true do
        neighbor ← a highest-valued successor state of current
        if VALUE(neighbor) ≤ VALUE(current) then return current
        current ← neighbor
```

I. Hill Climbing

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    current ← problem.INITIAL
    while true do
        neighbor ← a highest-valued successor state of current   // random choice
        if VALUE(neighbor) ≤ VALUE(current) then return current  // to break a tile
        current ← neighbor
```

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

8-queens problem

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current*
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

// random choice
// to break a tile

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current*
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

// random choice
// to break a tile

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current*
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

// random choice
// to break a tile

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s)$ = # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.
- $h(s) = 12$ for the best successor.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s) =$ # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.
- $h(s) = 12$ for the best successor.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s)$ = # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.
- $h(s) = 12$ for the best successor.

  8-way tie!.

I. Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  current ← *problem*.INITIAL
  while *true* **do**
    neighbor ← a highest-valued successor state of *current* // random choice // to break a tile
    if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
    current ← *neighbor*

$h = 5 + 3 + 6 + 3 = 17$

8-queens problem

- $h(s)$ = # pairs of queens attacking each other, directly or indirectly, in the state s.
- Successor is a state generated from relocating a queen in the same column.
- $h(s) = 12$ for the best successor.

  8-way tie!.

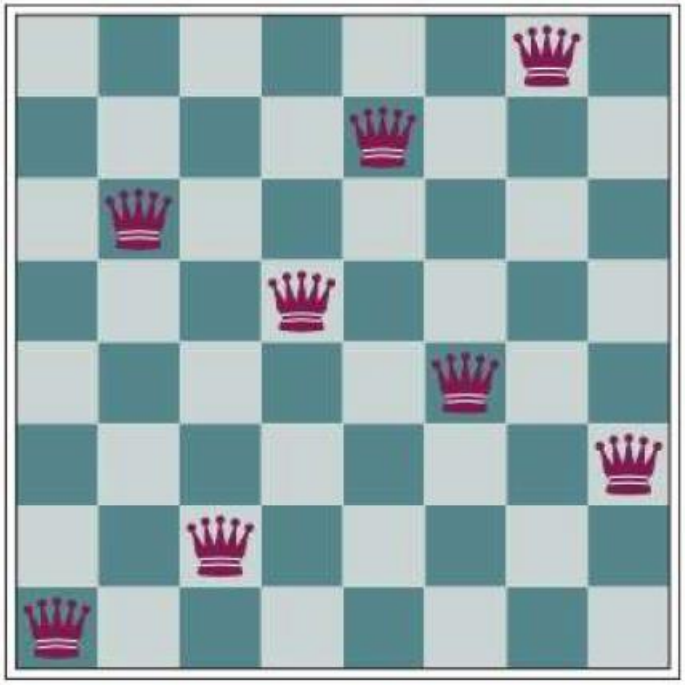
  Hill climbing randomly picks one.

Efficiency?


18 12 14 13 13 12 14 14
14 16 13 15 12 14 12 16
14 12 18 13 15 12 14 14
15 14 14 13 16 13 16
14 17 15 14 16 16
17 16 18 15 15
18 14 15 15 14 16
14 14 13 17 12 14 12 18

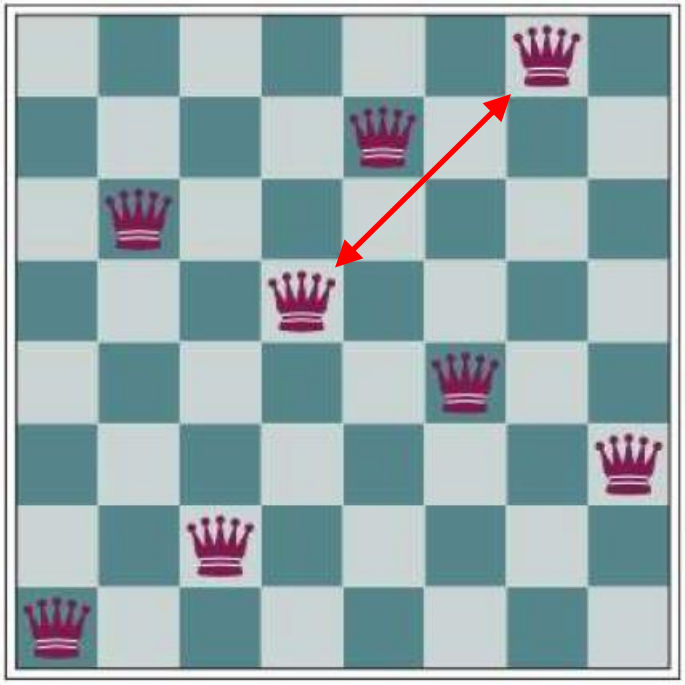
Efficiency?

5 moves



Efficiency?

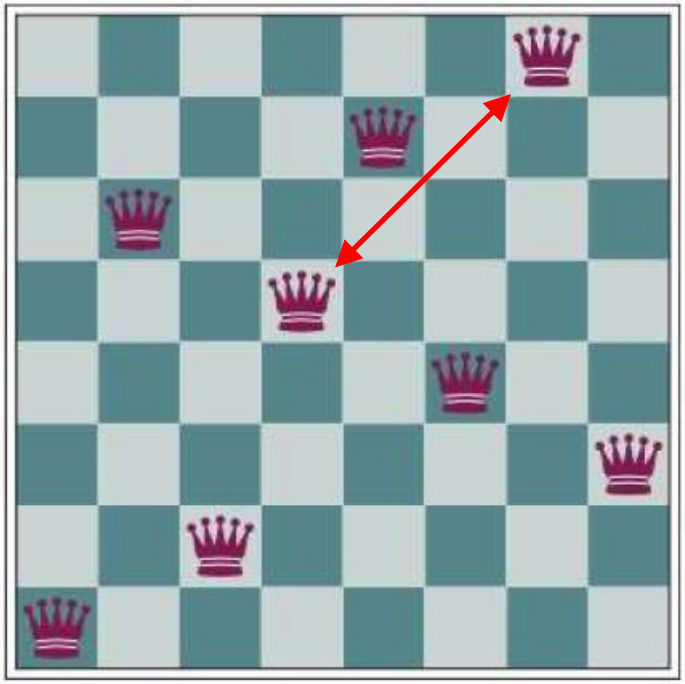
5 moves



$h = 1$

Efficiency?

5 moves



$h = 1$

- Hill climbing can make rapid progress toward a solution.

Drawback of Hill Climbing (1)

Hill climbing terminates when a peak is reached with no neighbor having a higher value.

Drawback of Hill Climbing (1)

Hill climbing terminates when a peak is reached with no neighbor having a higher value.

- ♠ Local maximum

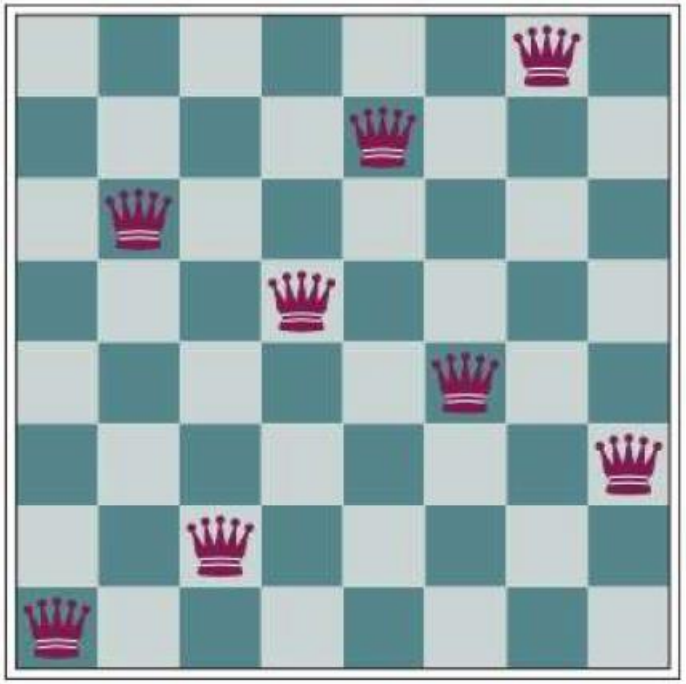
  Not the global maximum.

Drawback of Hill Climbing (1)

Hill climbing terminates when a peak is reached with no neighbor having a higher value.

♠ Local maximum

Not the global maximum.

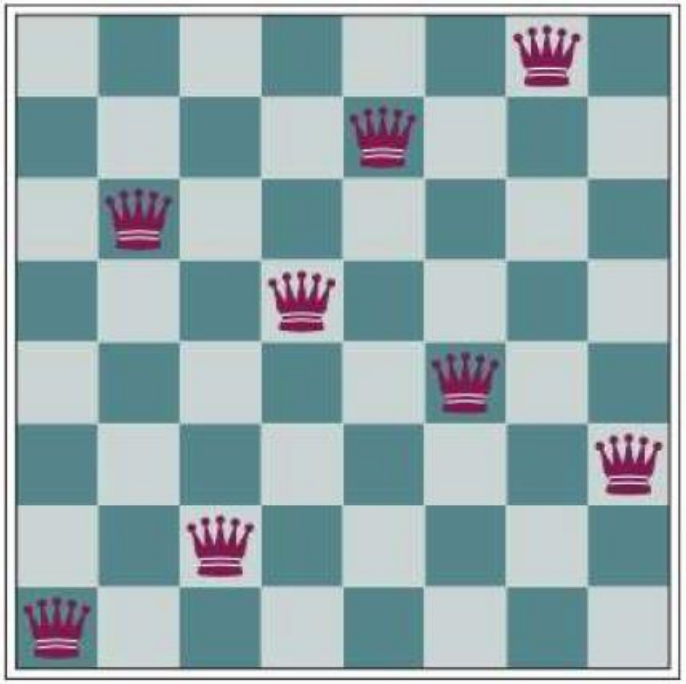


Drawback of Hill Climbing (1)

Hill climbing terminates when a peak is reached with no neighbor having a higher value.

♠ Local maximum

Not the global maximum.



Every move of one queen introduces more conflicts.

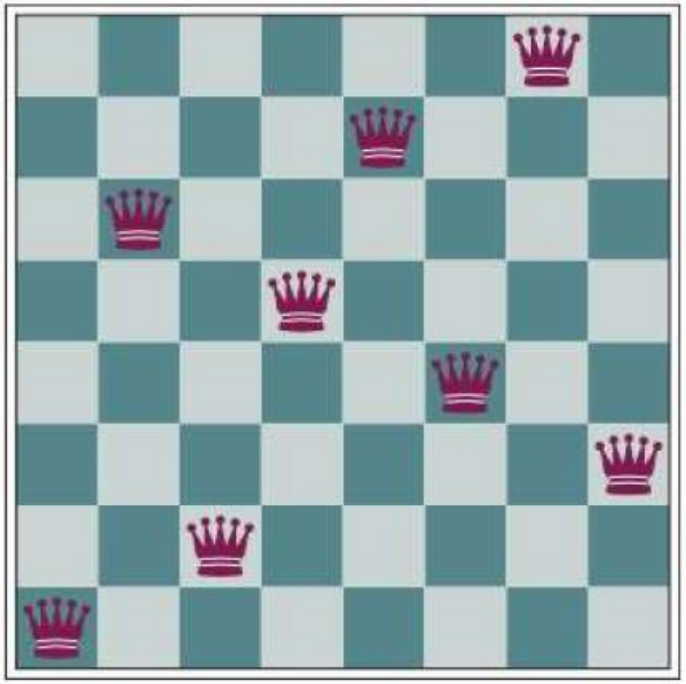
Drawback of Hill Climbing (1)

Hill climbing terminates when a peak is reached with no neighbor having a higher value.

♠ Local maximum

Not the global maximum.

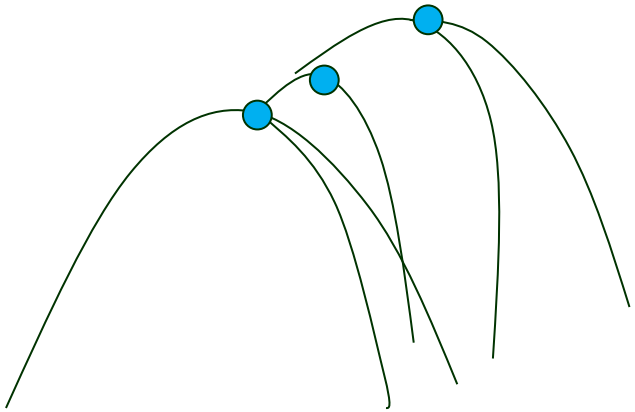
Hill climbing in the vicinity of a local maximum will be drawn toward it and then get stuck there.



Every move of one queen introduces more conflicts.

Drawback of Hill Climbing (2)

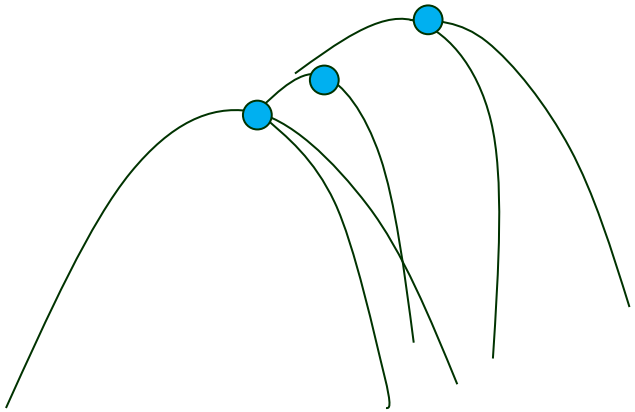
- Ridge: A sequence of local maxima difficult to navigate.



At each local maximum, all available actions are downhill.

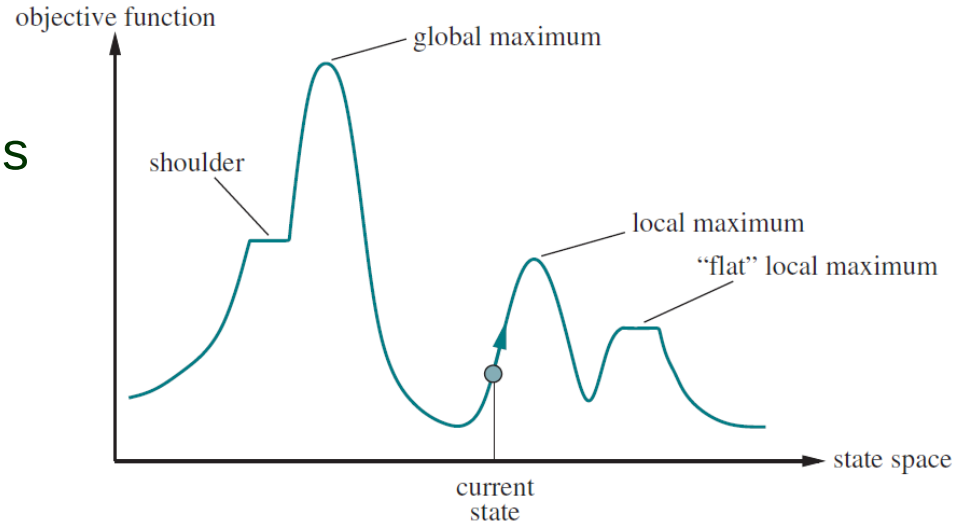
Drawback of Hill Climbing (2)

- Ridge: A sequence of local maxima difficult to navigate.



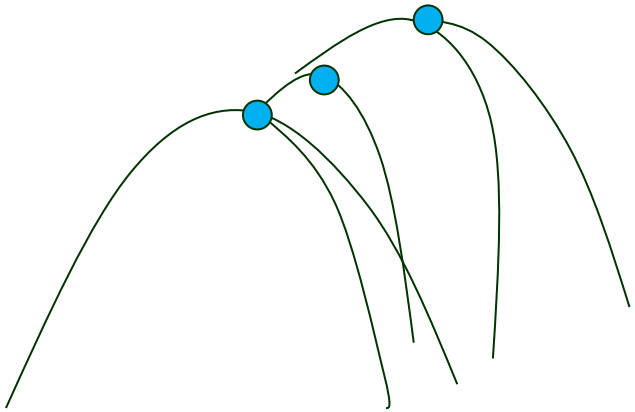
At each local maximum, all available actions are downhill.

- Plateaus: No uphill actions exist.



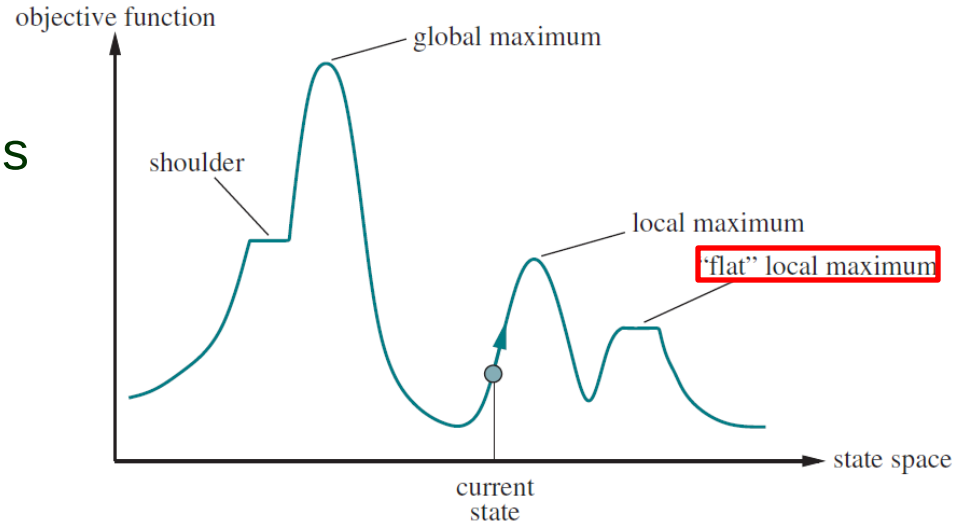
Drawback of Hill Climbing (2)

- Ridge: A sequence of local maxima difficult to navigate.



At each local maximum, all available actions are downhill.

- Plateaus: No uphill actions exist.



Variations of Hill Climbing

- Stochastic hill climbing
  - Random selection among the uphill moves.
  - Probability of selection varying with steepness..

Variations of Hill Climbing

- Stochastic hill climbing
  - Random selection among the uphill moves.
  - Probability of selection varying with steepness..
- First-choice hill climbing
  - Random generation of successors until a better (than the current) one is found.
  - Useful when many successors exist and/or the objective function is costly to evaluate.

Variations of Hill Climbing

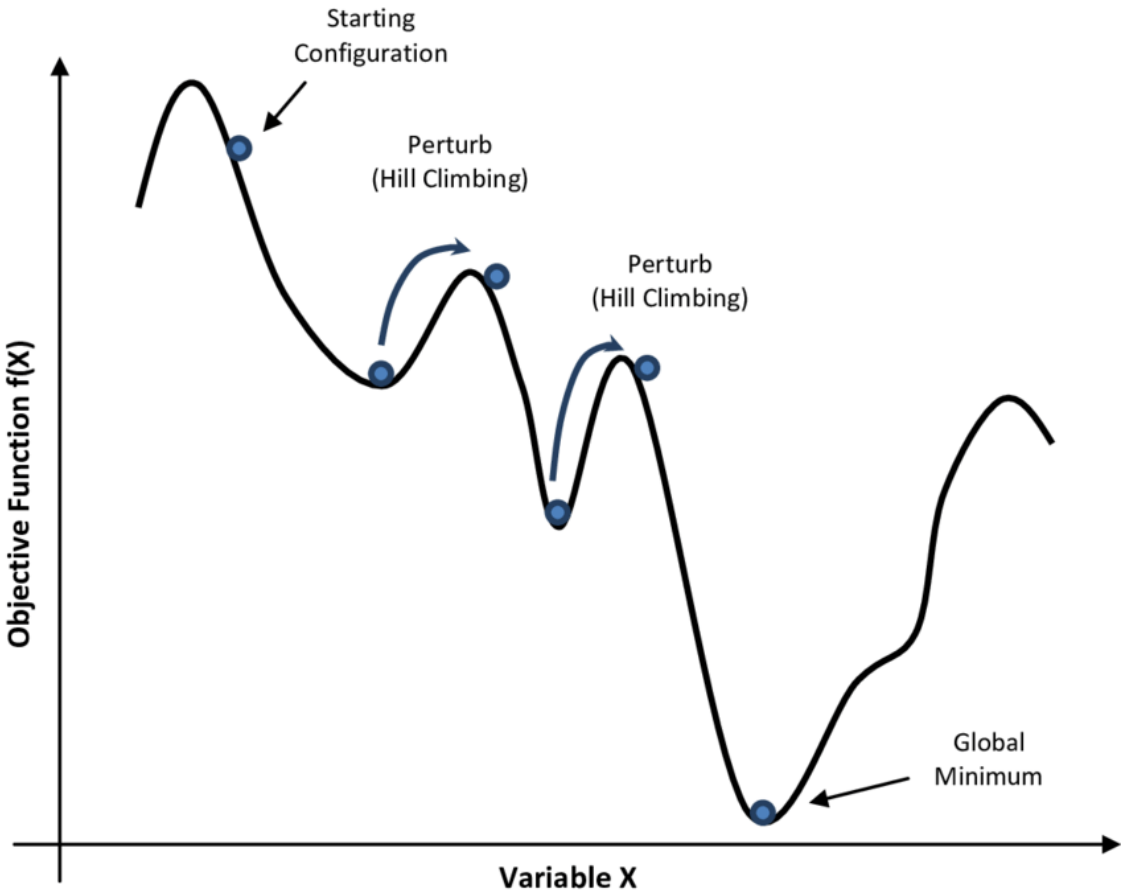
- Stochastic hill climbing
  - Random selection among the uphill moves.
  - Probability of selection varying with steepness..
- First-choice hill climbing
  - Random generation of successors until a better (than the current) one is found.
  - Useful when many successors exist and/or the objective function is costly to evaluate.
- Random restart hill climbing
  - Restart search from random initial state.

II. Simulated Annealing

Annealing: Heat a metal to a high temperature and then gradually cool it, allowing the material to reach a low-energy crystalline state so it is hardened.

II. Simulated Annealing

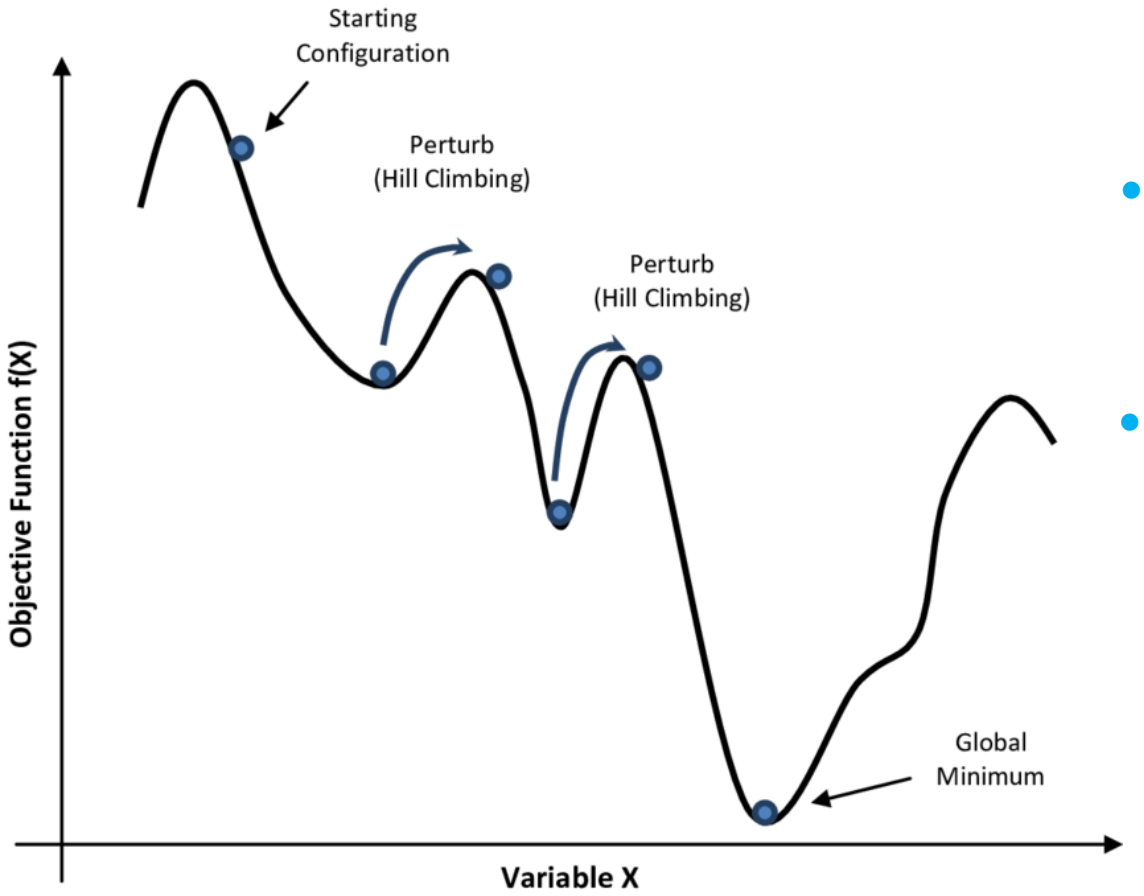
Annealing: Heat a metal to a high temperature and then gradually cool it, allowing the material to reach a low-energy crystalline state so it is hardened.



* from http://www.cs.us.es/~fsancho/?e=206

II. Simulated Annealing

Annealing: Heat a metal to a high temperature and then gradually cool it, allowing the material to reach a low-energy crystalline state so it is hardened.



- Start by shaking hard (i.e., at high temperature).
- Gradually reduce the intensity of shaking (i.e., lower the temperature).

* from http://www.cs.us.es/~fsancho/?e=206

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state

  $current \leftarrow problem$.INITIAL

  for $t = 1$ **to** ∞ **do**

Temperature → $T \leftarrow schedule(t)$

  if $T = 0$ **then return** $current$ // solution

  $next \leftarrow$ a randomly selected successor of $current$

Badness Is $-\Delta E$. → $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)

  if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .

  else $current \leftarrow next$ only with probability $e^{-\Delta E/T}$

Minimization

$\lim_{t \to \infty} T = 0$

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state

$current \leftarrow problem.$INITIAL

Minimization

for $t = 1$ **to** ∞ **do**

Temperature → $T \leftarrow schedule(t)$

$\lim_{t \to \infty} T = 0$

if $T = 0$ **then return** $current$ // solution

$next \leftarrow$ a randomly selected successor of $current$

Badness Is $-\Delta E$. → $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)

if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .

else $current \leftarrow next$ only with probability ~~$e^{-\Delta E/T}$~~ $e^{\Delta E/T}$ // $\Delta E \leq 0$

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** $current$ // solution
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .
    else $current \leftarrow next$ only with probability ~~$e^{-\Delta E/T}$~~ $e^{\Delta E/T}$ // $\Delta E \leq 0$

Minimization

Temperature → $T \leftarrow schedule(t)$

$\lim_{t \to \infty} T = 0$

Badness Is $-\Delta E$. → $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)

- Accept the next state if it is an improvement ($\Delta E > 0$).

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem.$INITIAL
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** $current$ // solution
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .
    else $current \leftarrow next$ only with probability ~~$e^{-\Delta E/T}$~~ $e^{\Delta E/T}$ // $\Delta E \leq 0$

Minimization

Temperature → $T \leftarrow schedule(t)$

$\lim_{t \to \infty} T = 0$

Badness Is $-\Delta E$. → $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)

- Accept the next state if it is an improvement ($\Delta E > 0$).
- Otherwise, accept it with a probability that decreases exponentially
  - as the badness $-\Delta E$ of the move increases ($\frac{\Delta E}{T} < 0$ decreases), and
  - as the "temperature" goes down ($\frac{\Delta E}{T} < 0$ decreases).

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** $current$ // solution
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .
    else $current \leftarrow next$ only with probability ~~$e^{-\Delta E/T}$~~ $e^{\Delta E/T}$ // $\Delta E \leq 0$

Minimization

Temperature →

$\lim_{t \to \infty} T = 0$

Badness Is $-\Delta E$. →

- Accept the next state if it is an improvement ($\Delta E > 0$).
- Otherwise, accept it with a probability that decreases exponentially
  - as the badness $-\Delta E$ of the move increases ($\frac{\Delta E}{T} < 0$ decreases), and
  - as the "temperature" goes down ($\frac{\Delta E}{T} < 0$ decreases).

Bad moves are more tolerated at the start when T is high and become less likely as T decreases.

Simulated Annealing Algorithm

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** $current$ // solution
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$ // the next state is better; choose it. .
    else $current \leftarrow next$ only with probability ~~$e^{-\Delta E/T}$~~ $e^{\Delta E/T}$ // $\Delta E \leq 0$

Minimization

Temperature

$\lim_{t \to \infty} T = 0$

Badness Is $-\Delta E$.

- Accept the next state if it is an improvement ($\Delta E > 0$).
- Otherwise, accept it with a probability that decreases exponentially
  - as the badness $-\Delta E$ of the move increases ($\frac{\Delta E}{T} < 0$ decreases), and
  - as the "temperature" goes down ($\frac{\Delta E}{T} < 0$ decreases).

Bad moves are more tolerated at the start when T is high and become less likely as T decreases.

- Escape local minima by allowing bad moves.

More About SA

- $T \rightarrow 0$ slowly enough

More About SA

- $T \rightarrow 0$ slowly enough

A property of Boltzmann distribution $e^{\Delta E/T}$ guarantees the global minimum with probability $\rightarrow 1$.

More About SA

- $T \to 0$ slowly enough

A property of Boltzmann distribution $e^{\Delta E/T}$ guarantees the global minimum with probability $\to 1$.

- Commonly used $T \leftarrow cT$ with constant $c < 1$ and close to 1 at each step.

More About SA

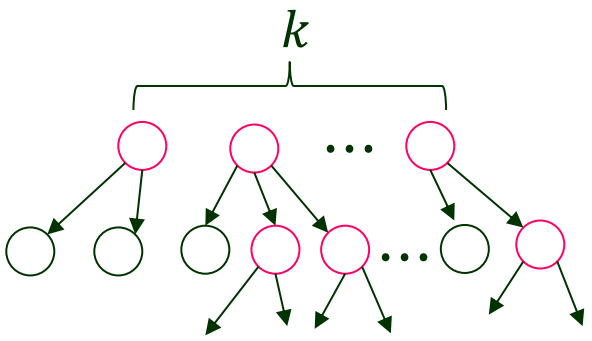
- $T \rightarrow 0$ slowly enough

A property of Boltzmann distribution $e^{\Delta E/T}$ guarantees the global minimum with probability $\rightarrow 1$.

- Commonly used $T \leftarrow cT$ with constant $c < 1$ and close to 1 at each step.
- Applied to many problems:
  - VLSL layout
  - factory scheduling
  - aircraft trajectory planning
  - NP-hard optimization (i.e., the traveling salesman problem)
  - large-scale stochastic optimization tasks

Local Beam Search

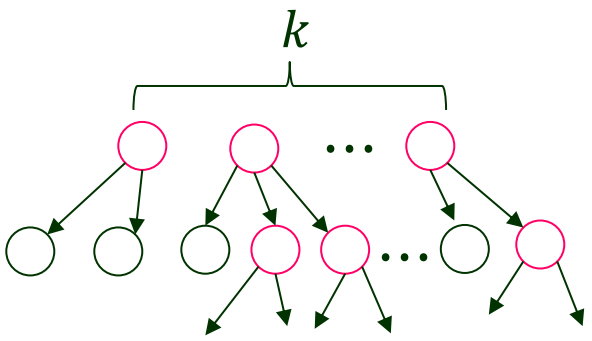
Keep track of k states rather than one.



1. Start with k randomly generated states.
2. Generate all their successors.
3. Stop if any successor is a goal.
4. Otherwise, keep the k best successors and go back to step 2.

Local Beam Search

Keep track of k states rather than one.

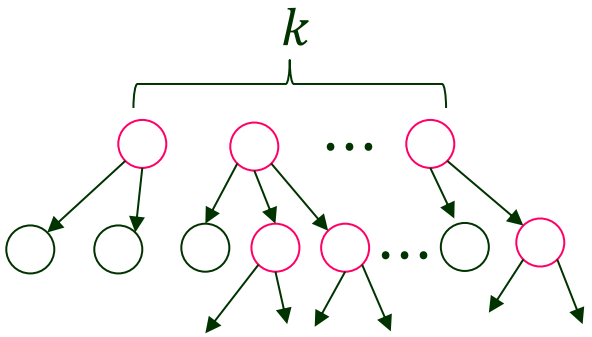


1. Start with k randomly generated states.
2. Generate all their successors.
3. Stop if any successor is a goal.
4. Otherwise, keep the k best successors and go back to step 2.

♣ May suffer from a lack of diversity among the k states.

Local Beam Search

Keep track of k states rather than one.



1. Start with k randomly generated states.
2. Generate all their successors.
3. Stop if any successor is a goal.
4. Otherwise, keep the k best successors and go back to step 2.

♣ May suffer from a lack of diversity among the k states.

Solution: stochastic beam search which chooses successors with probabilities proportional to their values.

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every ρ parents to form an offspring (typically $\rho = 2$).

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every ρ parents to form an offspring (typically $\rho = 2$).

Crossover: Split each of the parent strings and recombine the parts to form two children.

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every ρ parents to form an offspring (typically $\rho = 2$).

Crossover: Split each of the parent strings and recombine the parts to form two children.

Mutation: Randomly change the bits of an offspring.

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every ρ parents to form an offspring (typically $\rho = 2$).

Crossover: Split each of the parent strings and recombine the parts to form two children.

Mutation: Randomly change the bits of an offspring.

Culling: All individuals below a threshold are discarded from the population.

III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of k randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every ρ parents to form an offspring (typically $\rho = 2$).

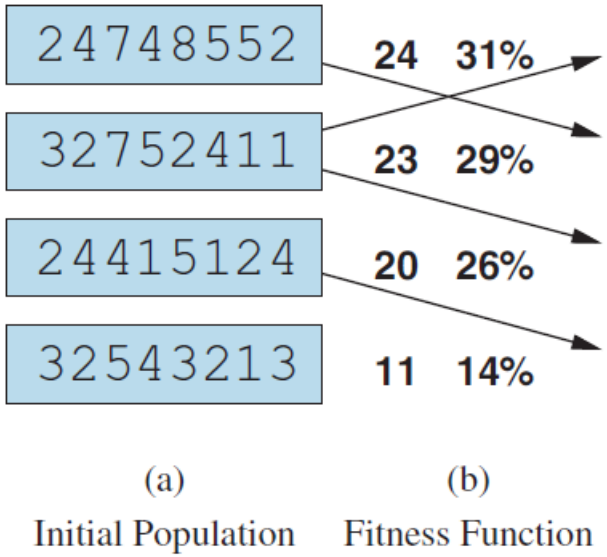
   Crossover: Split each of the parent strings and recombine the parts to form two children.

   Mutation: Randomly change the bits of an offspring.

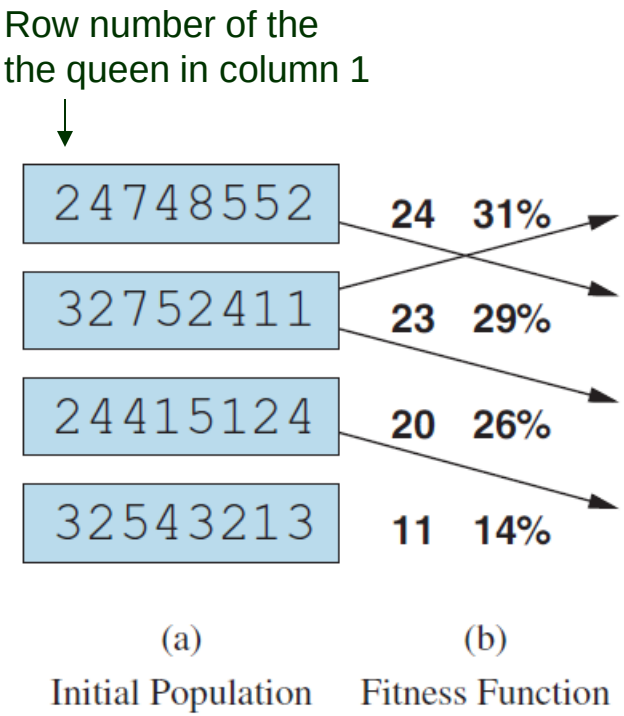
   Culling: All individuals below a threshold are discarded from the population.

4. Go back to step 2 and repeat until *sufficiently fit* states are discovered (in which case the best one is chosen as a solution).

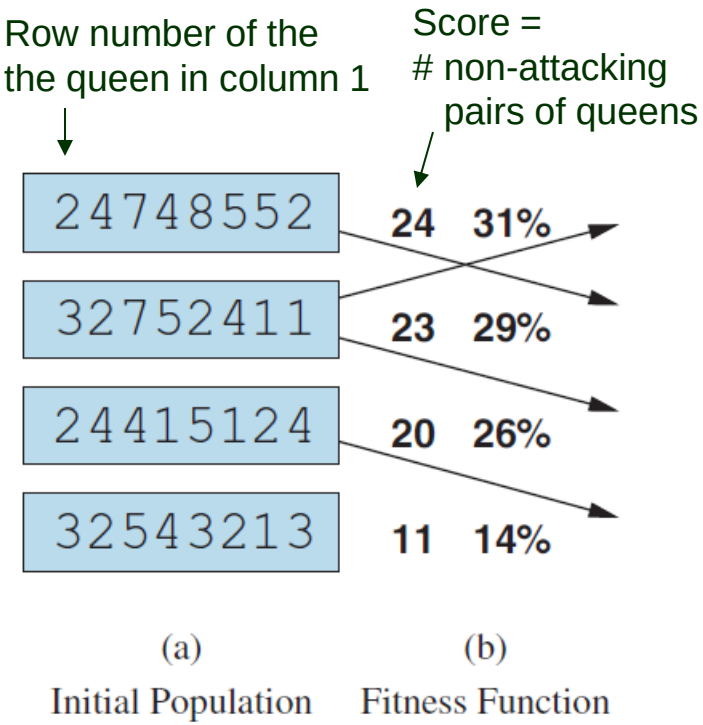
Genetic Algorithm on 8-Queen



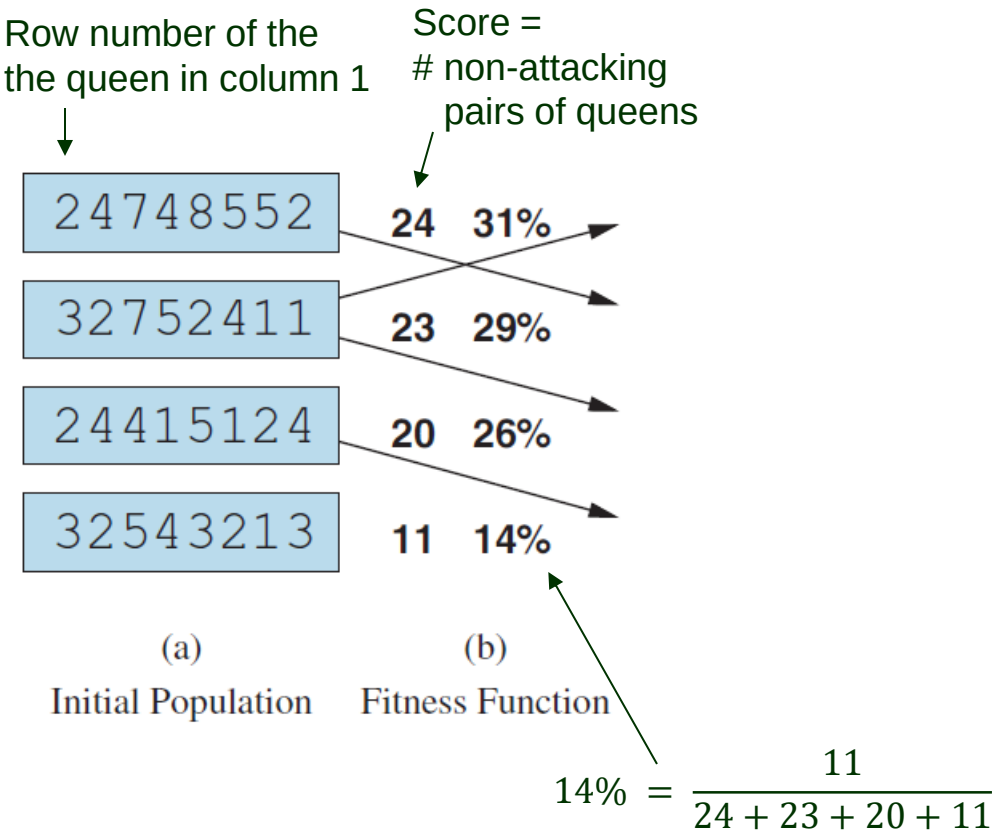
Genetic Algorithm on 8-Queen



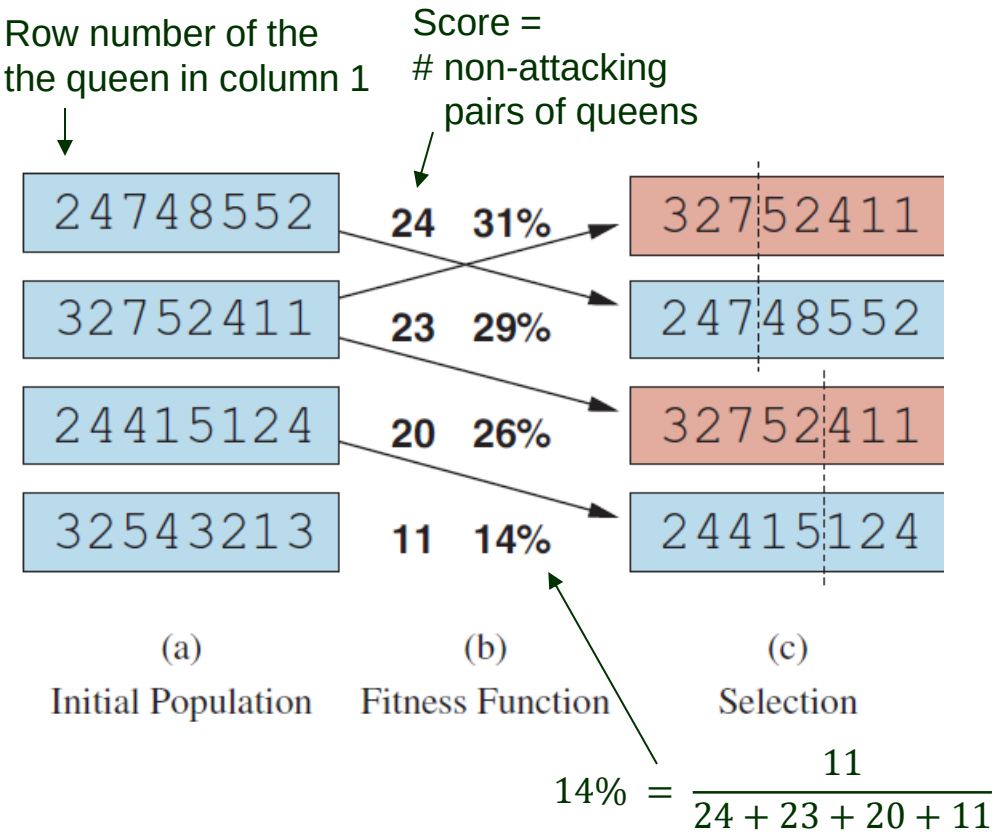
Genetic Algorithm on 8-Queen



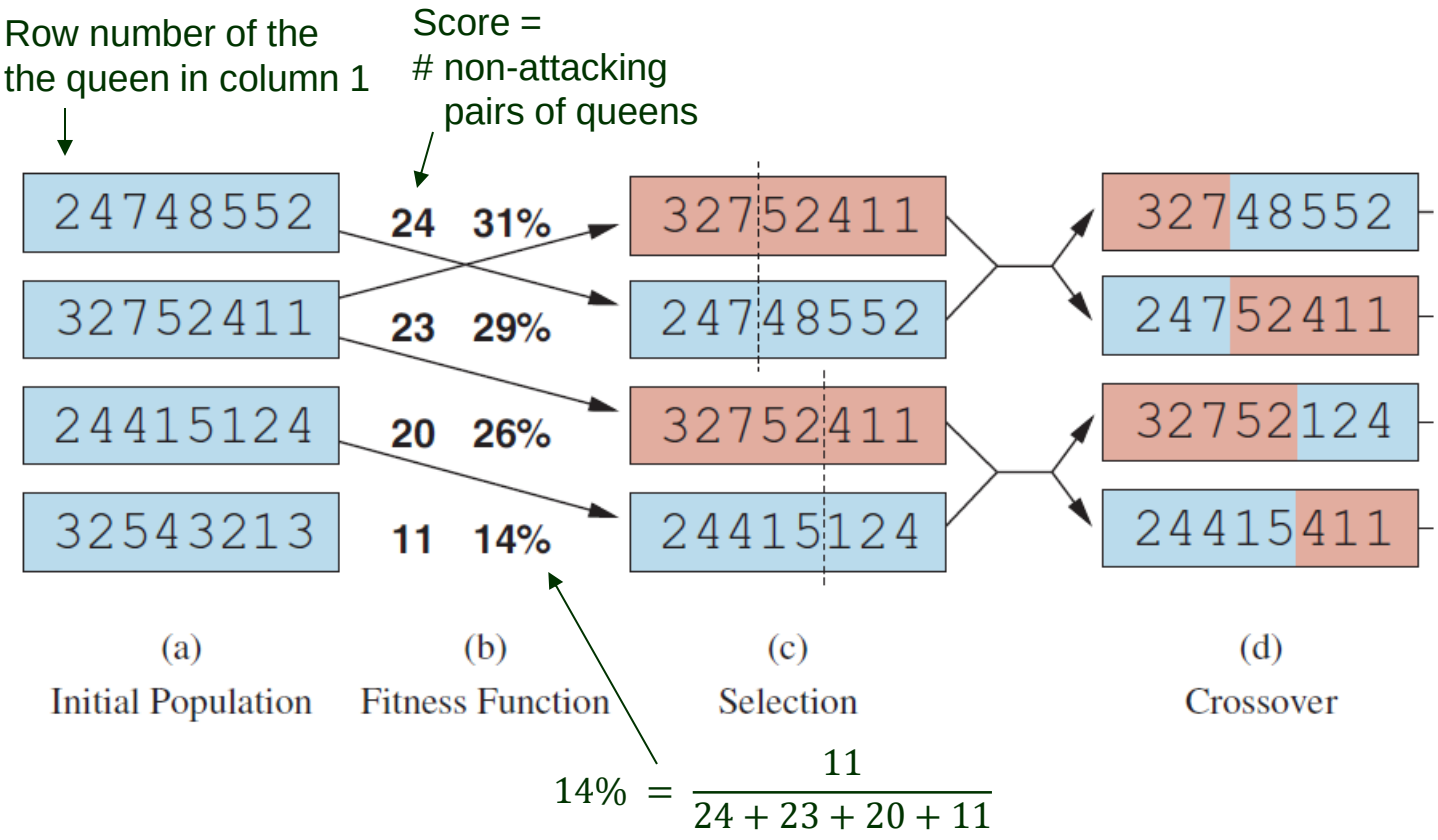
Genetic Algorithm on 8-Queen



Genetic Algorithm on 8-Queen

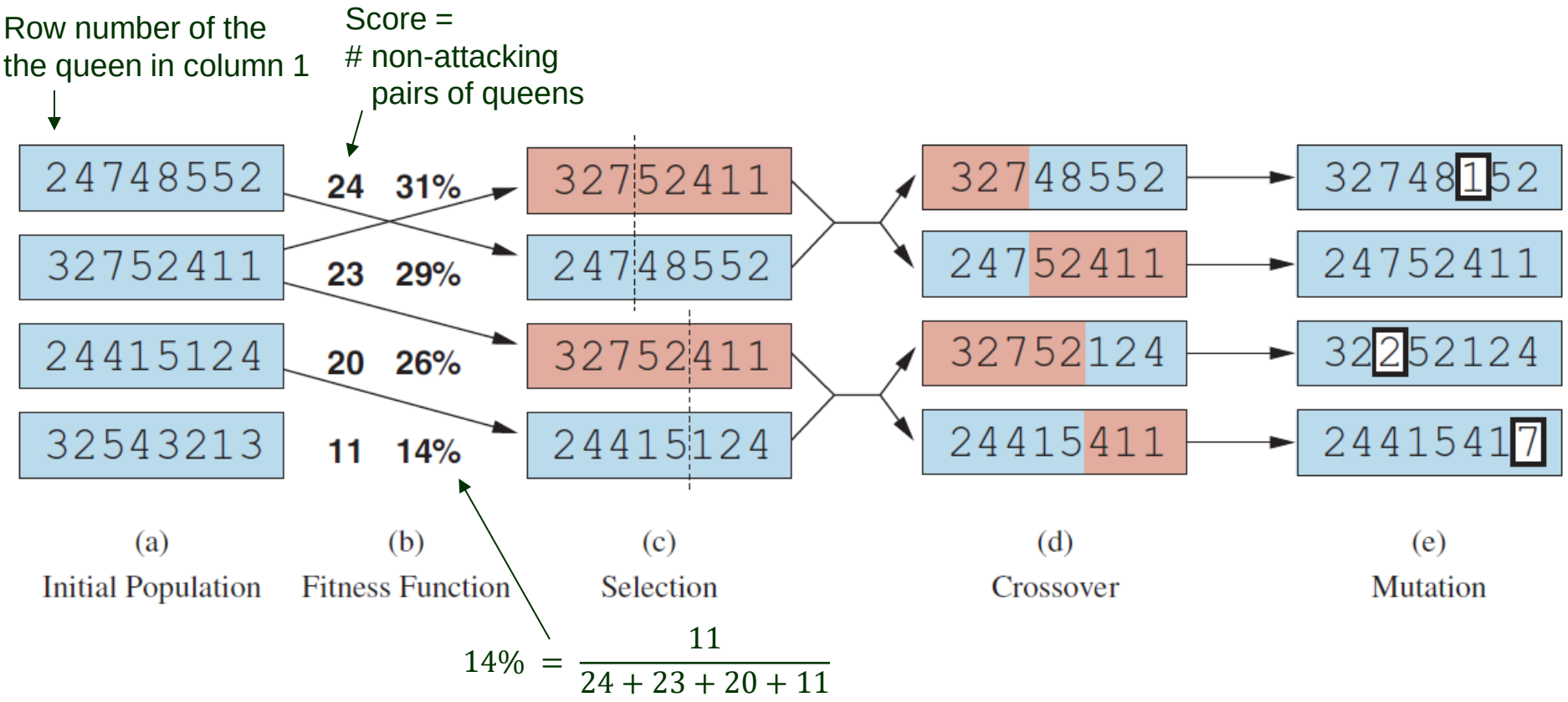


Genetic Algorithm on 8-Queen

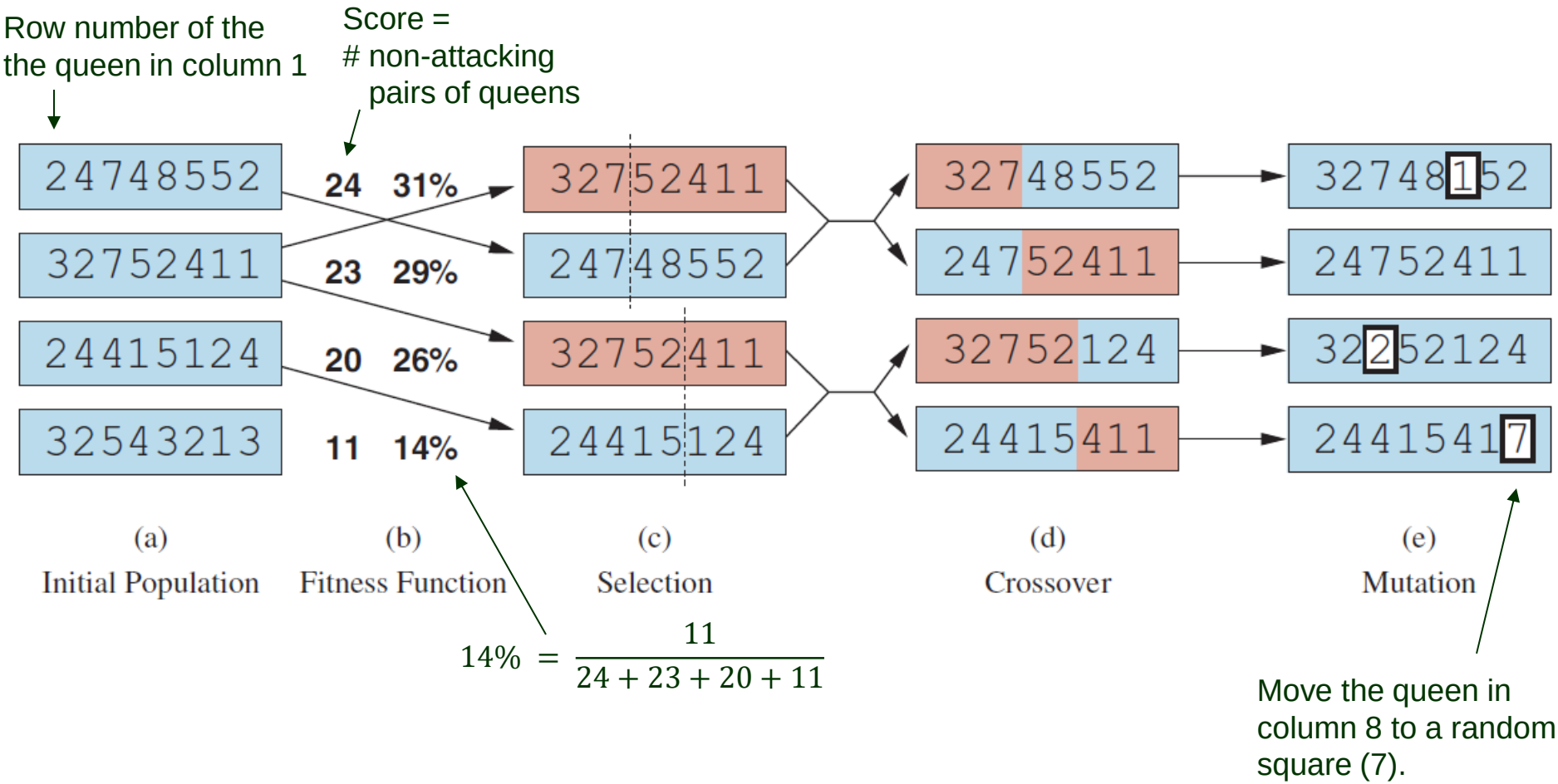


$$14\% = \frac{11}{24 + 23 + 20 + 11}$$

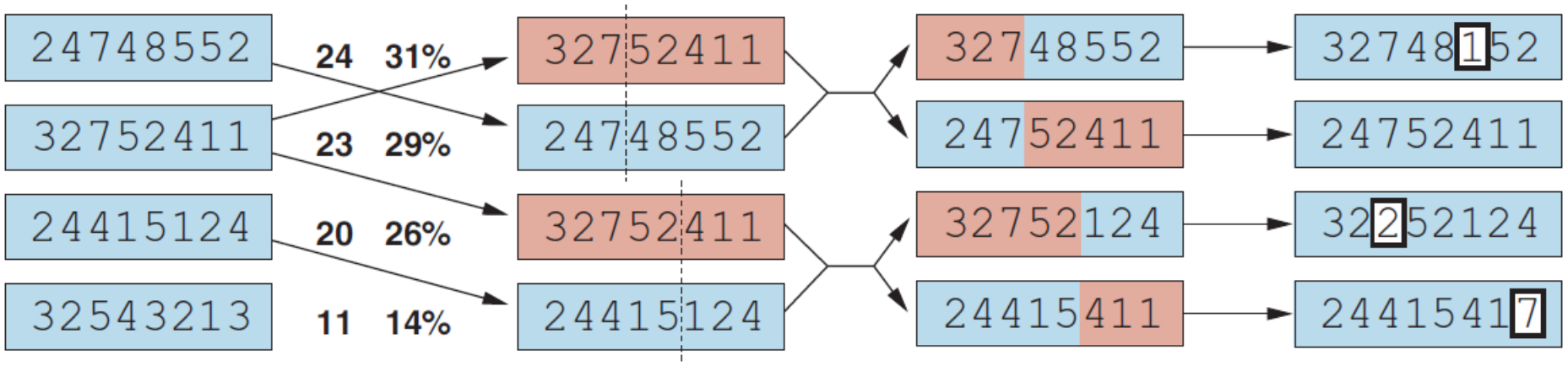
Genetic Algorithm on 8-Queen



Genetic Algorithm on 8-Queen

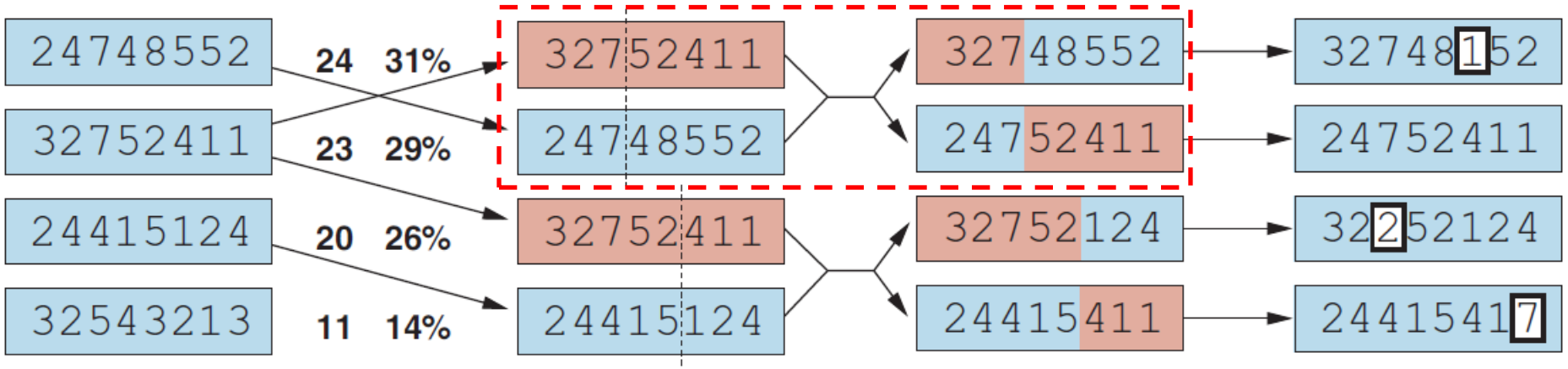


Crossover



(d)
Crossover

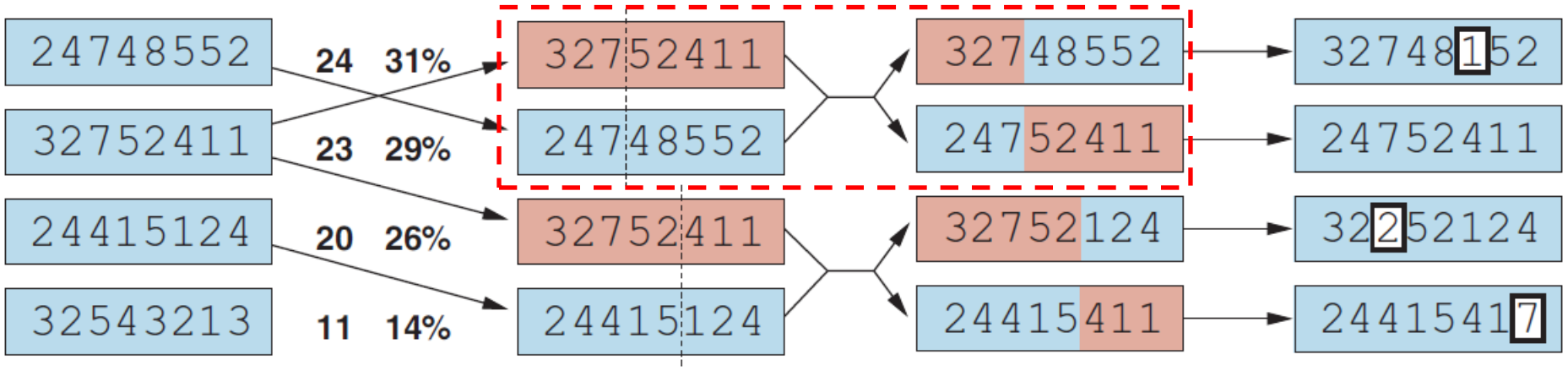
Crossover



(d)

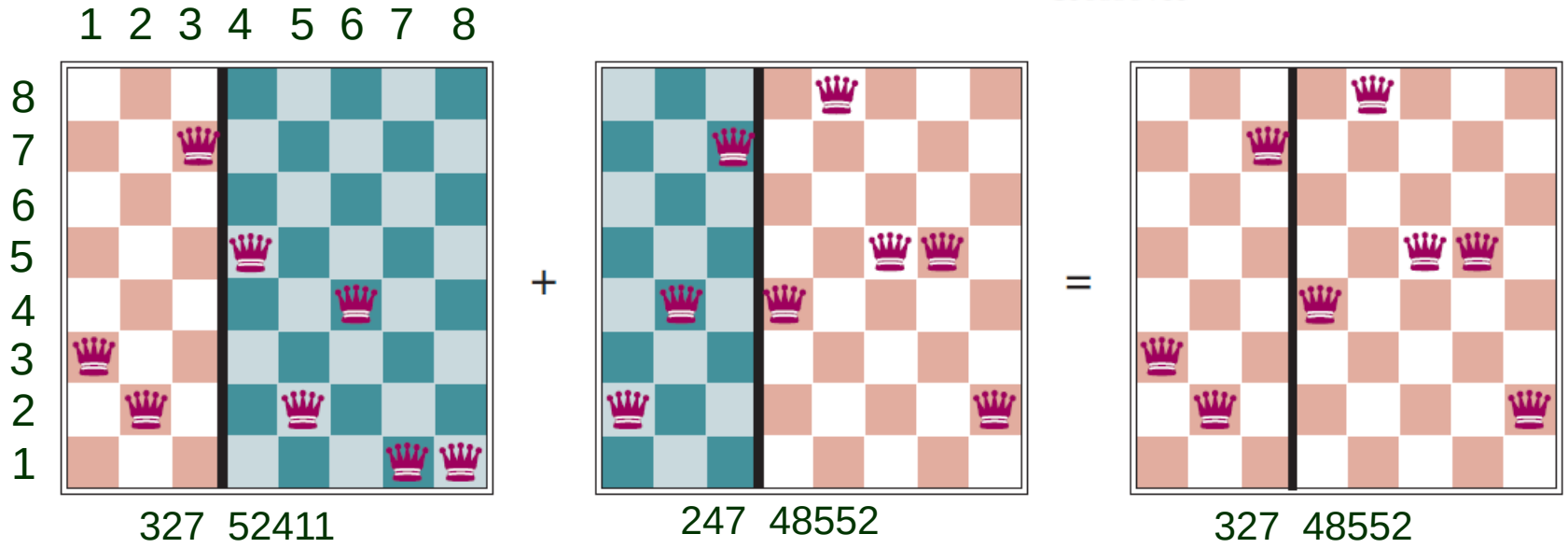
Crossover

Crossover



(d)

Crossover



Genetic Algorithm (Pseudocode)

```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
    weights ← WEIGHTED-BY(population, fitness)
    population2 ← empty list
    for i = 1 to SIZE(population) do
      parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
      child ← REPRODUCE(parent1, parent2)
      if (small random probability) then child ← MUTATE(child)
      add child to population2
    population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

Applications of GA

- Complex structured problems

  Circuit layout, job-shop scheduling

- Evolving the architecture of deep neural networks
- Finding bugs of hardware
- Molecular structure optimization
- Image processing.
- Learning robots, etc.