Lecture notes for *Principles of Artificial Intelligence* (COMS 4720/5720)
Yan-Bin Jia, Iowa State University

Generation of Heuristics & A* Variations

Outline

I. Generating heuristics

II. Variations of A* search

* Figures are from the textbook site (or drawn by the instructor) unless the source is specifically cited.

I. Better Heuristics

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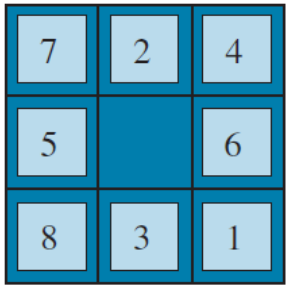
♣ h_1 might cause other nodes to be expanded as well.

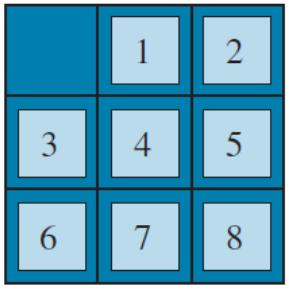
Generating Heuristics by Relaxation

- An admissible heuristic can be derived from exact solution cost of a *relaxed problem*.

with fewer restrictions on the actions



Start State



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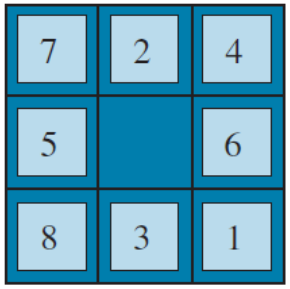

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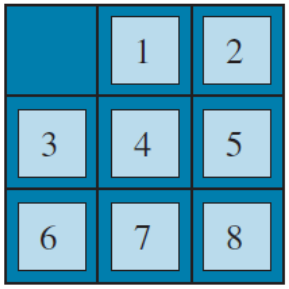

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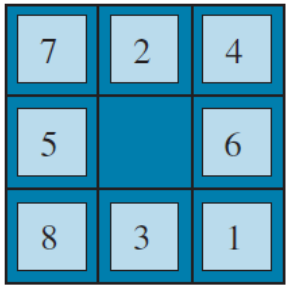

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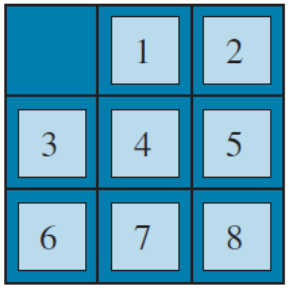

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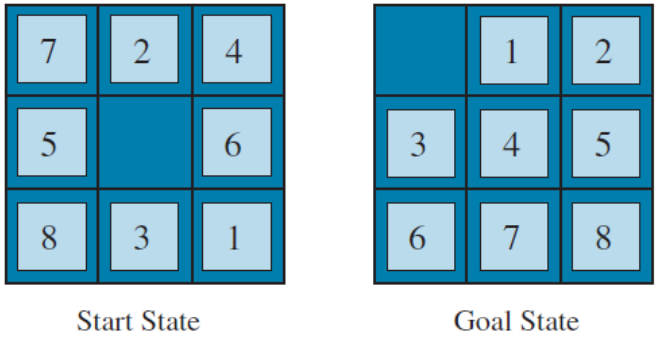

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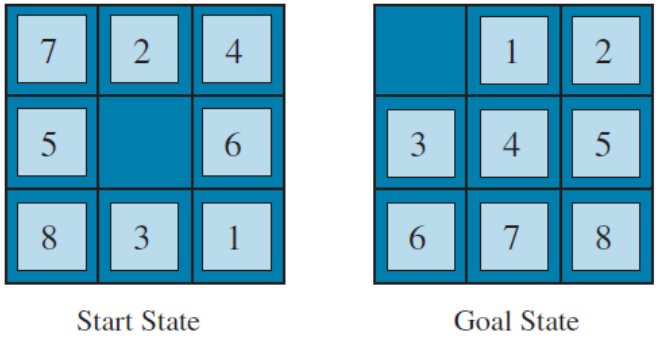

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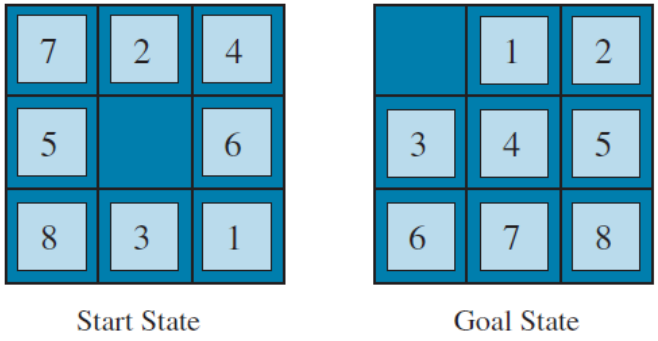

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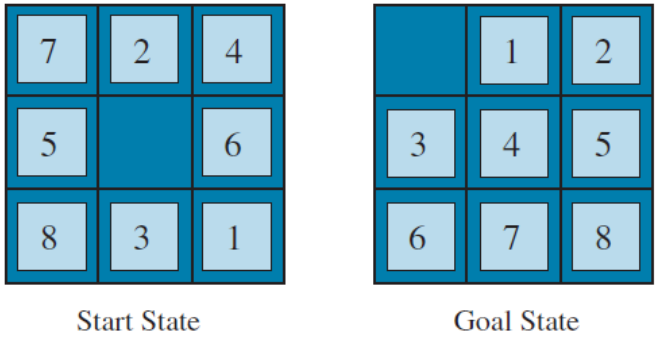

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The heuristic is consistent.

Heuristics from Formal Specification

Formal specification of a problem (8-puzzle):

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  Otherwise, evaluation of the corresponding heuristic will be expensive.
- Program ABSOLVER generates heuristics automatically from problem definitions, including the best one for the 8-puzzle and the first one for the Rubik's Cube puzzle.

Multiple Heuristics Available

Admissible heuristics $h_1, h_2, \cdots, h_k$ are available but none is clearly better than the others.

Use a composite heuristic:

$$h(n) = \max\{h_1(n), h_2(n), \cdots, h_k(n)\}$$

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- ♦ h is *admissible* because $h_1, h_2, \cdots, h_k$ are.
- ♦ h *dominates* $h_1, h_2, \cdots, h_k$.
- ♣ h takes longer to compute – at least $O(k)$.
  May randomly select one heuristic at each evaluation.

II. Sacrificing Search

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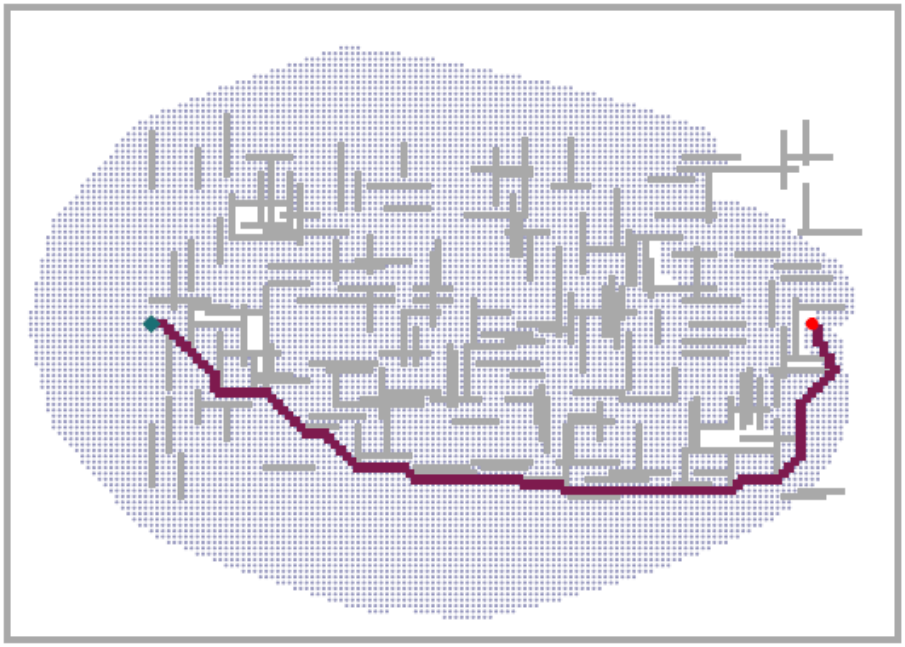
Detour index: multiplier applied to the straight-line distance.

e.g., a detour index of 1.3 implies a good estimate of 13 miles between two cities that are 10 miles apart.

Weighted A*

Evaluation function: $f(n) = g(n) + W \times h(n)$ for some $W > 1$.



A* search

Gray bars: obstacles
Dots: reached states

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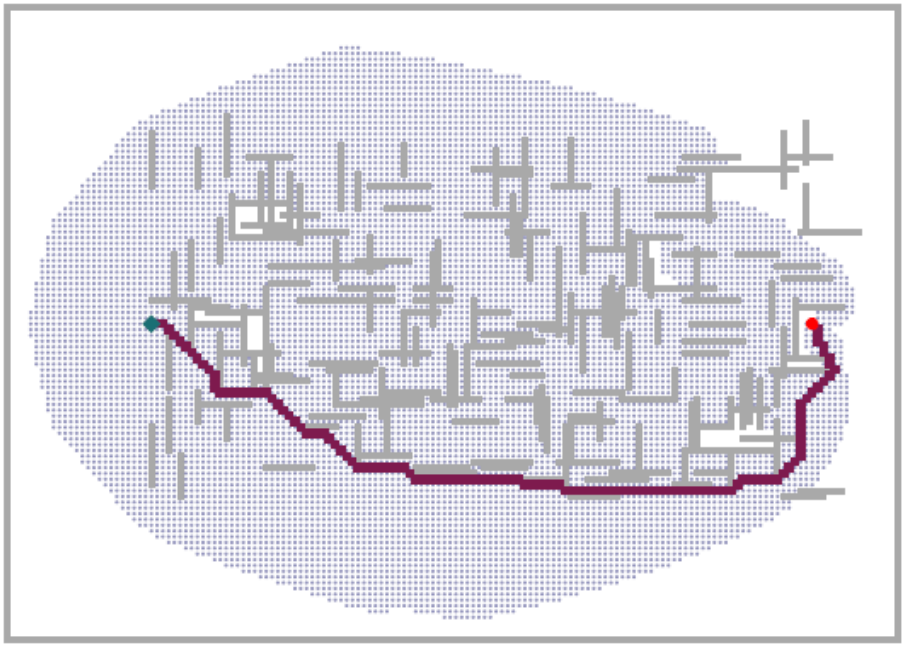

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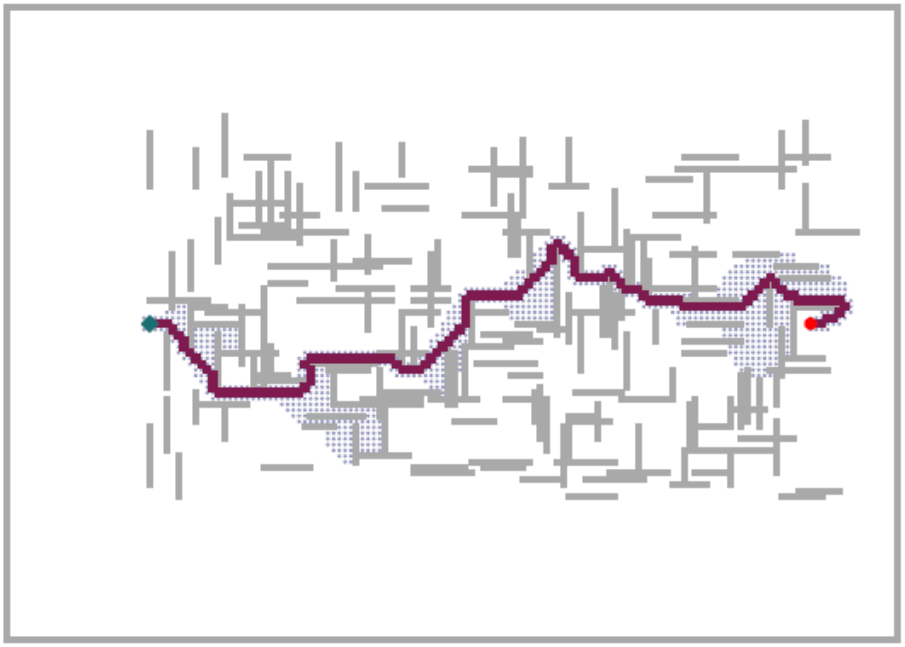

Weighted A* search
($W = 2$ on the same grid)

Weighted A* As a Generalization

- Weighted A* finds a solution with cost between C^* and $W \times C^*$.
- Cost is usually much closer to C^* in practice.

Weighted A* search $g(n) + W \times h(n)$ $(1 \leq W < \infty)$

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A* search	$g(n) + h(n)$	$(W = 1)$
Uniform-cost search	$g(n)$	$(W = 0)$
Best-cost search	$h(n)$	$(W = \infty)$
Weighted A* search	$g(n) + W \times h(n)$	$(1 \leq W < \infty)$

Memory-Bounded Search

Beam search keeps the k nodes with the best f scores.

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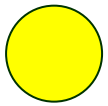
Example

$k = 2$

$f = 5$



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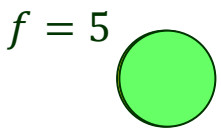

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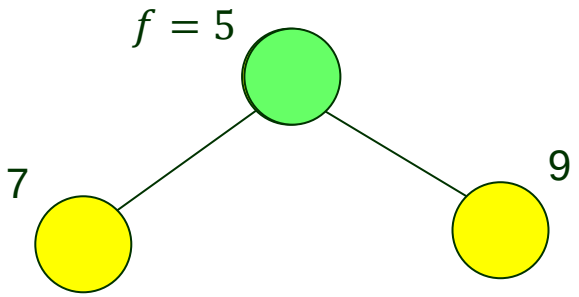

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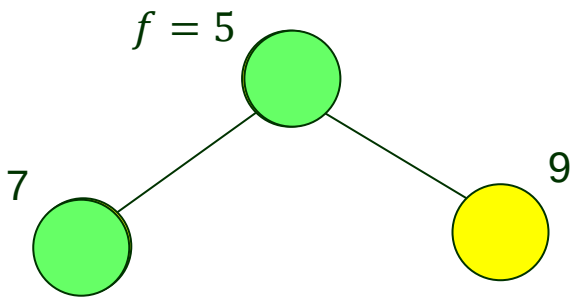

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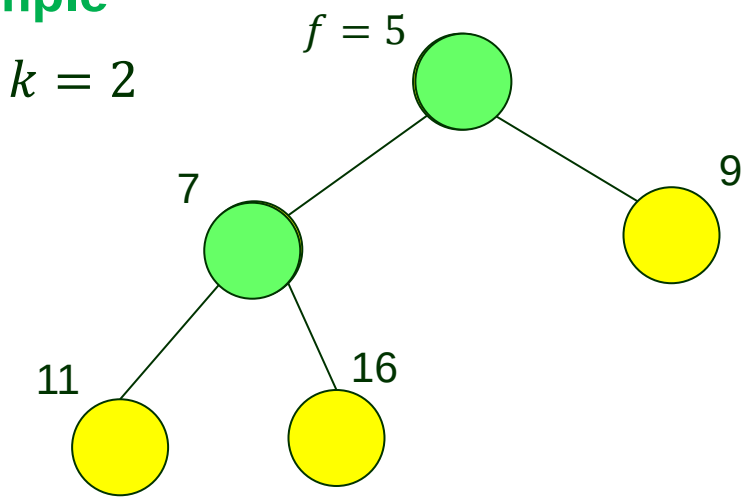

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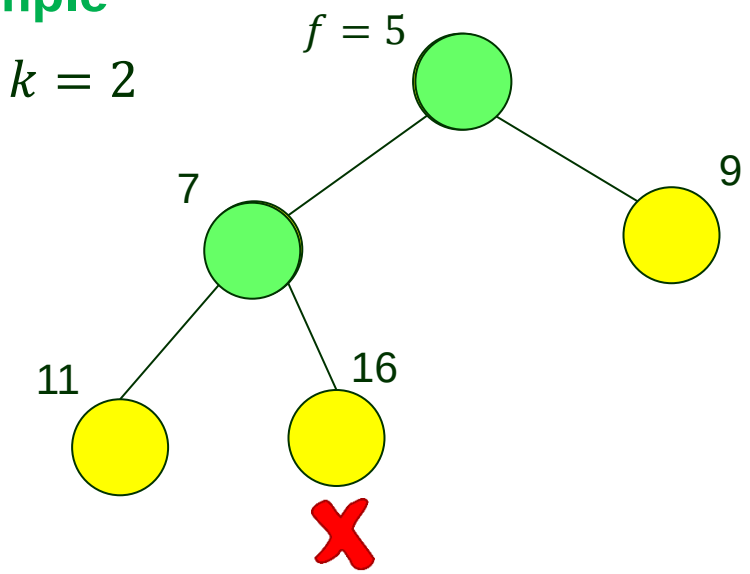

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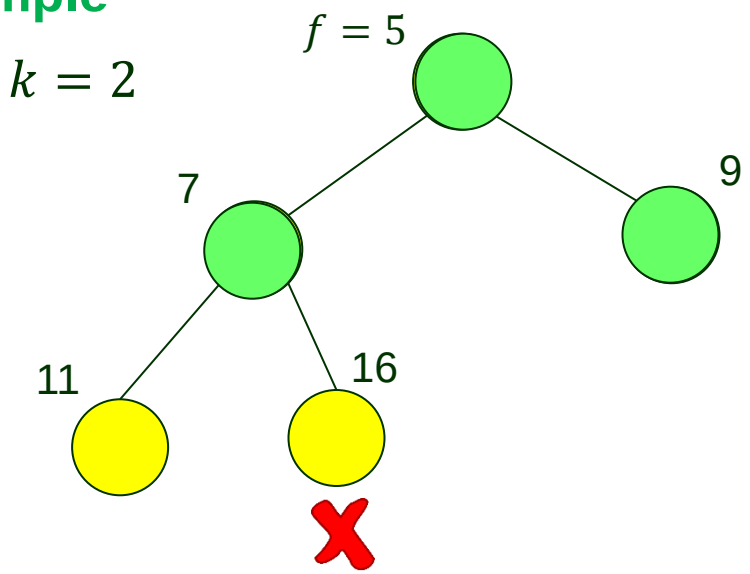

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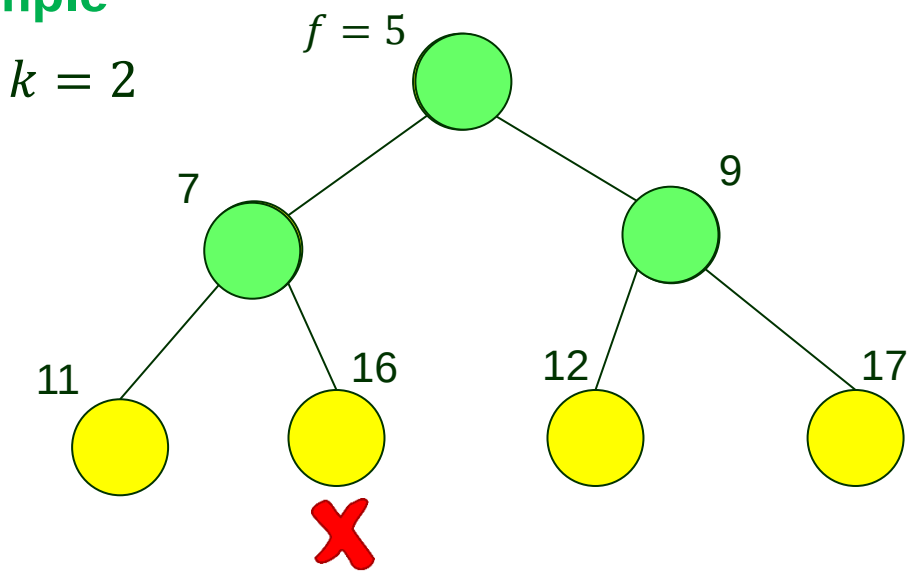

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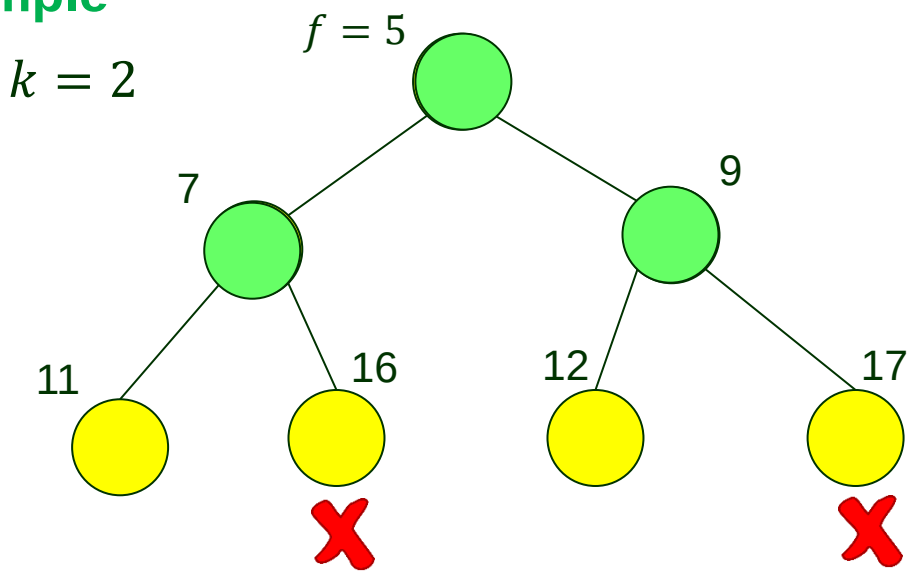

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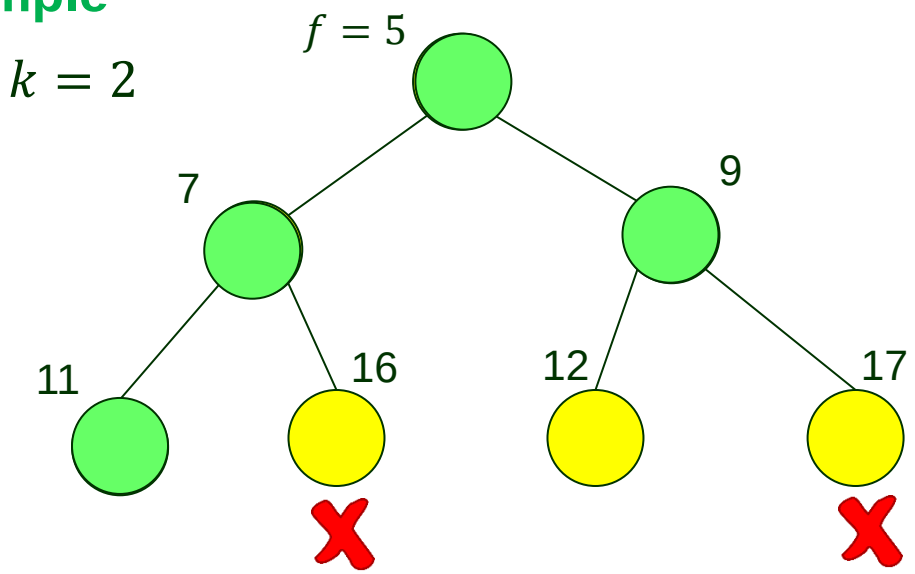
Example

$k = 2$



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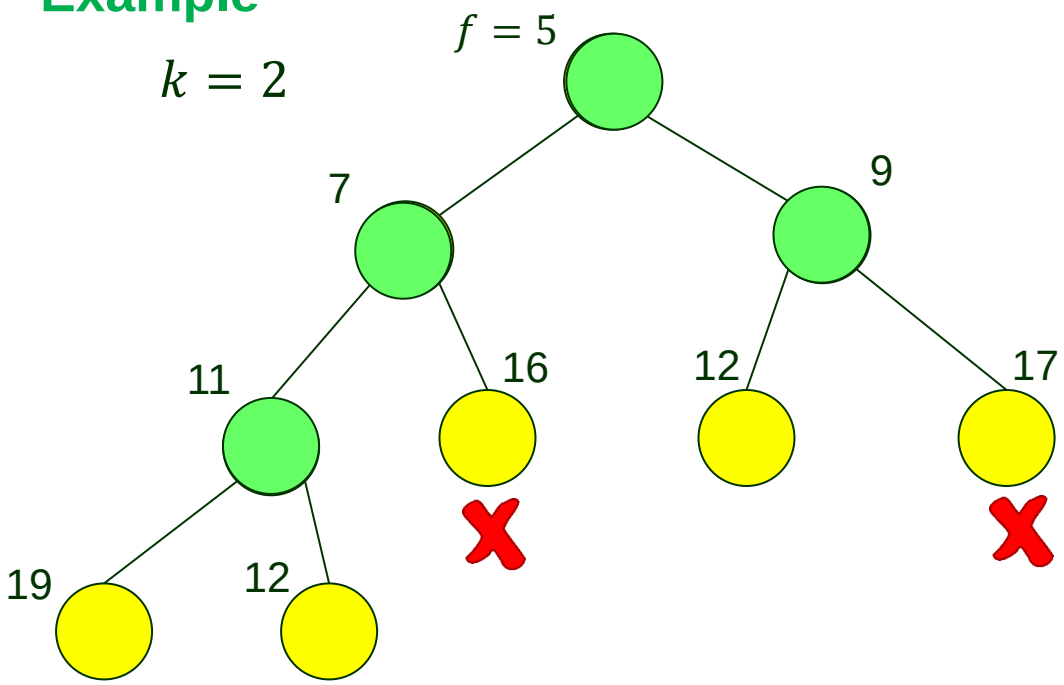

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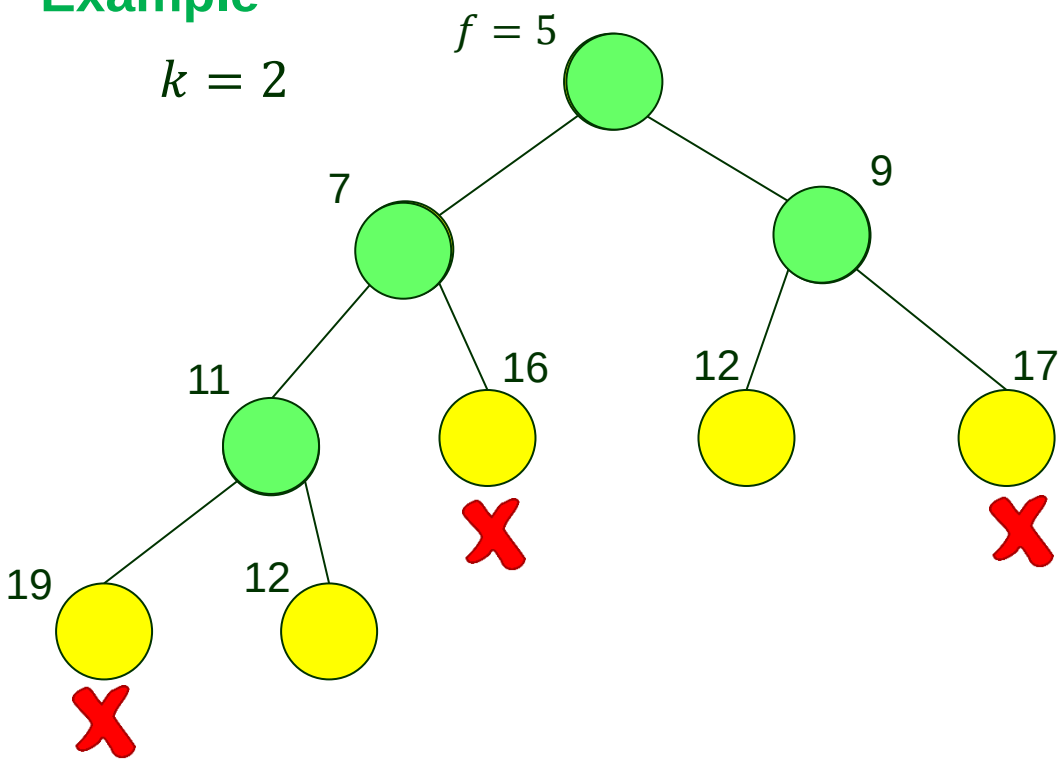
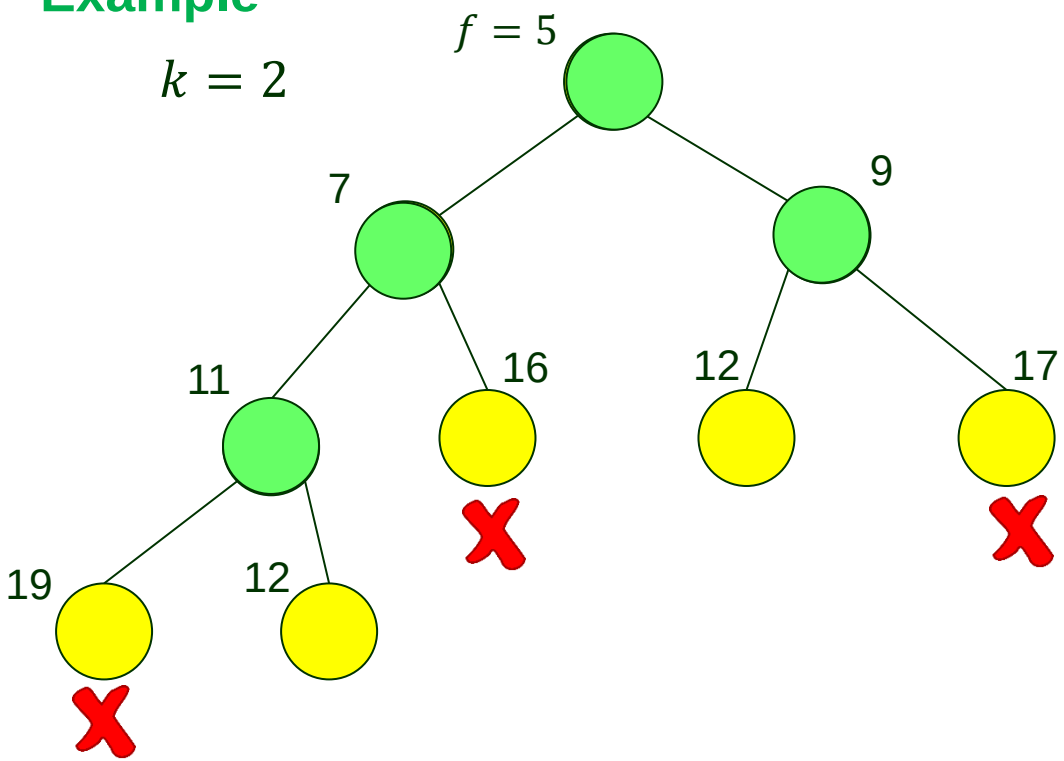


Memory-Bounded Search

Beam search keeps the k nodes with the best f scores.

- ♦ Less memory and faster execution.
- ♣ Incomplete and suboptimal.

Example



Iterative-Deepening A* Search (IDA*)

- The cutoff at each iteration is the f-cost rather than depth.
- Determine the (new) cutoff value for the current iteration as

$S = \{n \mid$ node n generated in the previous iteration and $f(n) >$ old $f_{\text{cutoff}}\}$

new $f_{cutoff} \leftarrow \min_{n \in S} f(n)$

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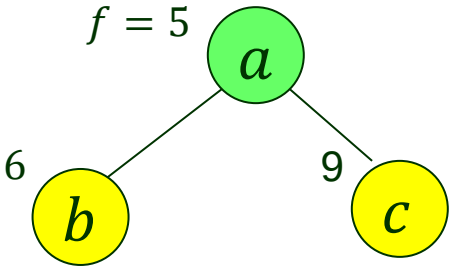

Iteration 1 ($f_{cutoff} = 5$)

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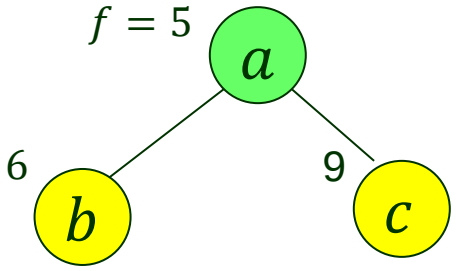

Iteration 1 ($f_{cutoff} = 5$) Iter. 2 ($f_{cutoff} = 6$)

Iterative-Deepening A* Search (IDA*)

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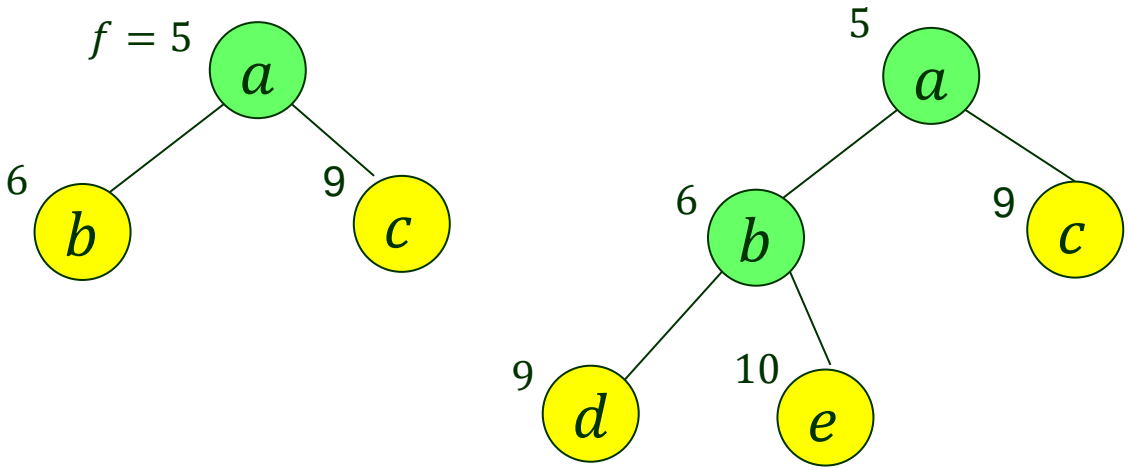
Example



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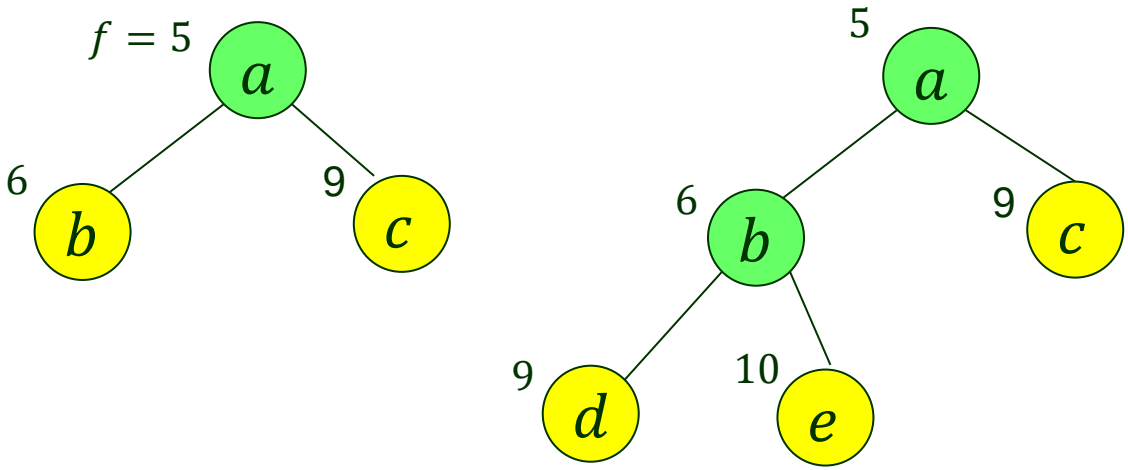

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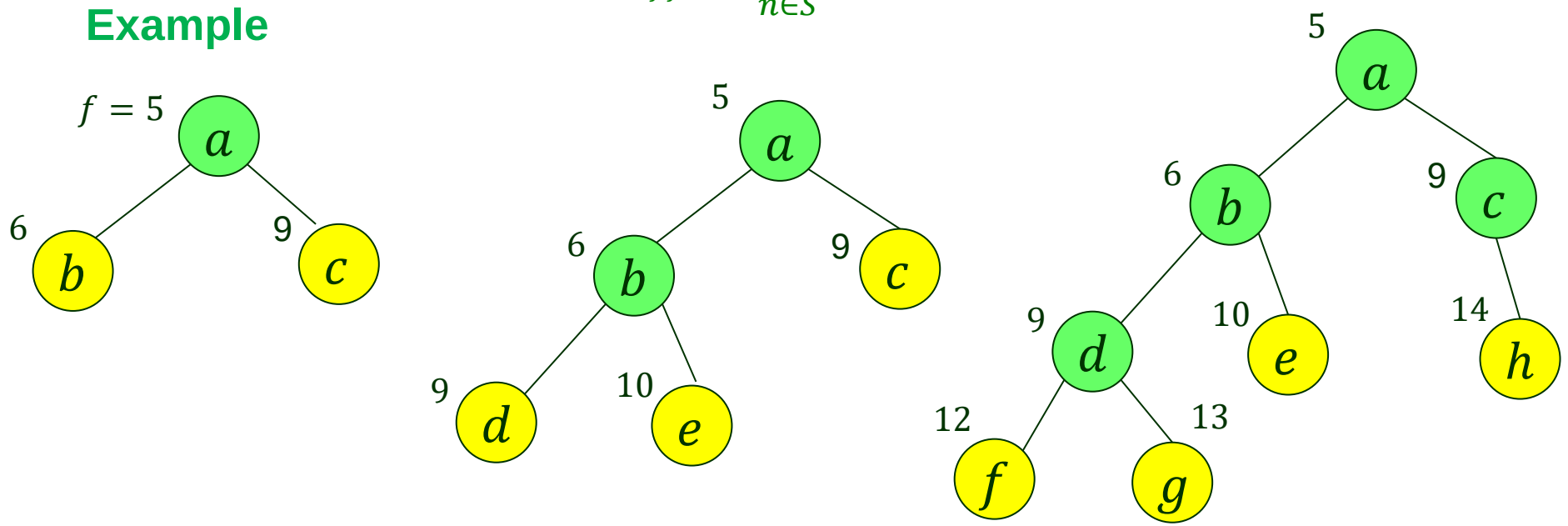
Example



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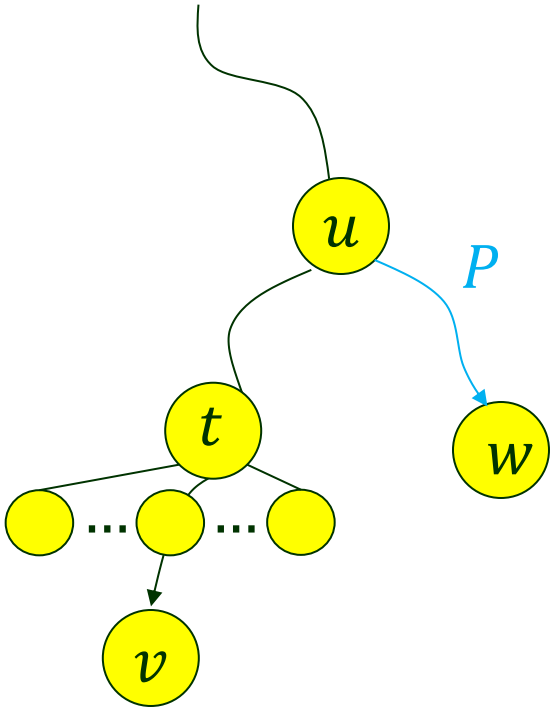
IDA* (cont'd)

- Steady progress towards the goal if f-cost of every path is an integer.

  $\leq C^*$ iterations

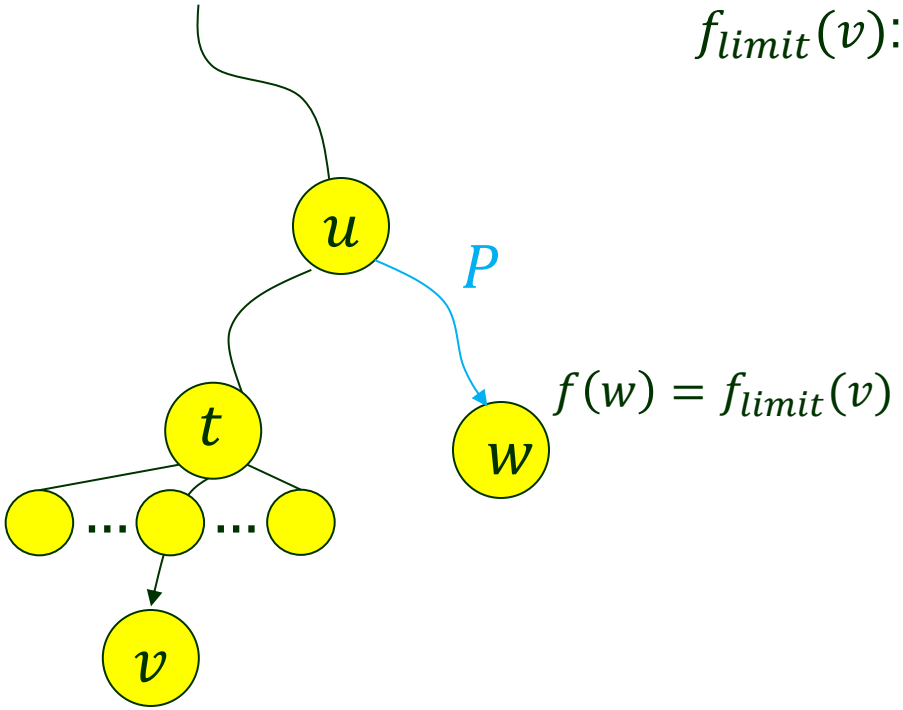
- In the worst case, #iterations = #states

Recursive Best-First Search (RBFS)



Idea: Remembers the f-value of the best leaf in the forgotten subtree in case it's worth a re-expansion at some later time.

Recursive Best-First Search (RBFS)



$f_{limit}(v)$: f-value of the *best alternative path* from any *ancestor* of the node v.

$f(w) = f_{limit}(v)$

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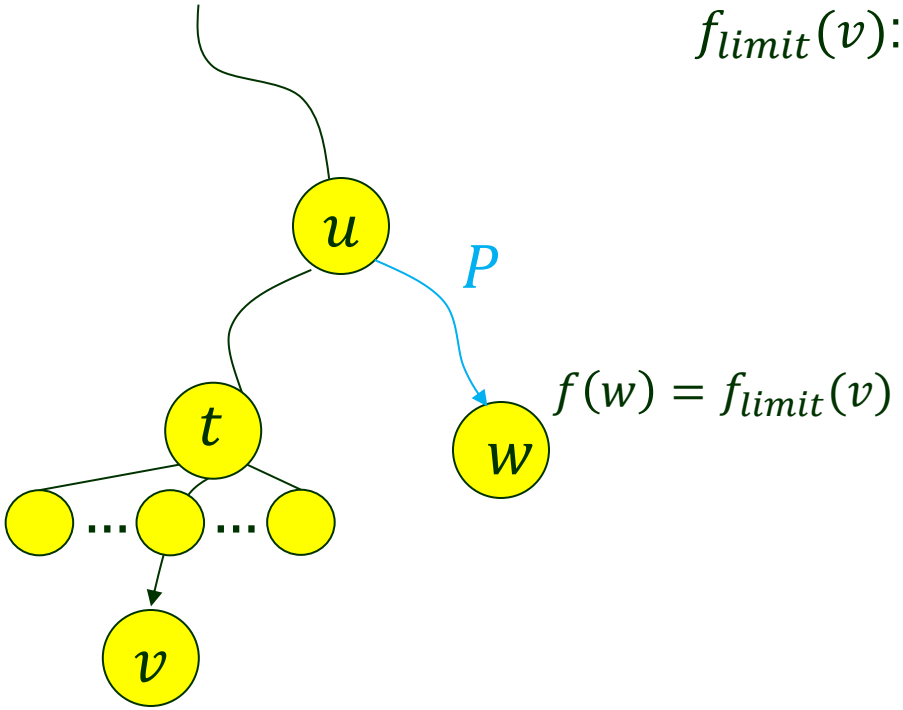

$f_{limit}(v)$: f-value of the *best alternative path* from any *ancestor* of the node v.

- Best-first search if, at the currently visited node v, it holds that $f(v) \leq f_{limit}(v)$.

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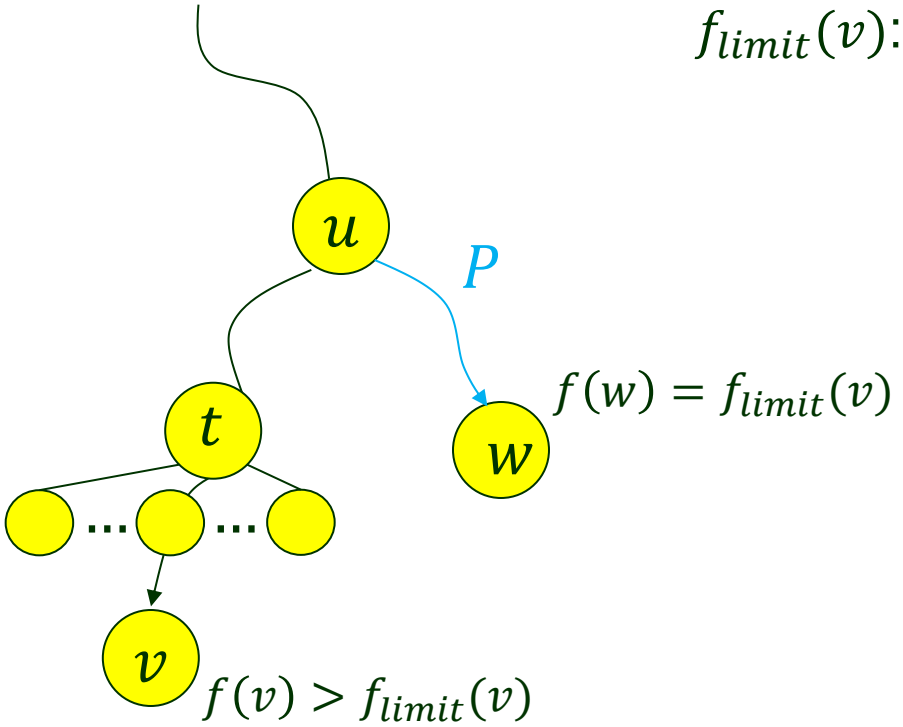

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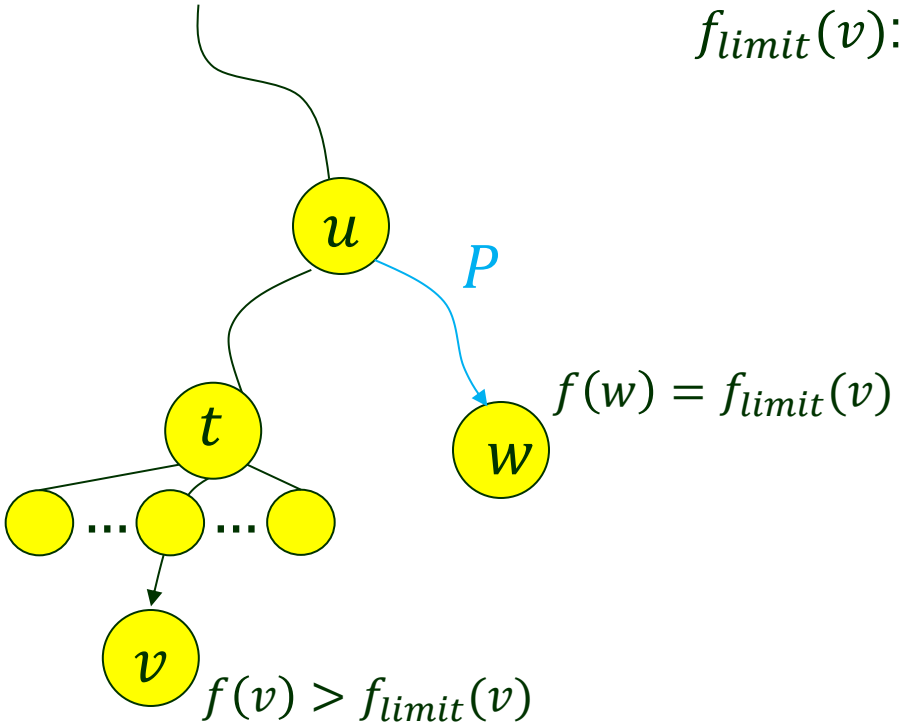

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  - unwinds back to the alternative path P;

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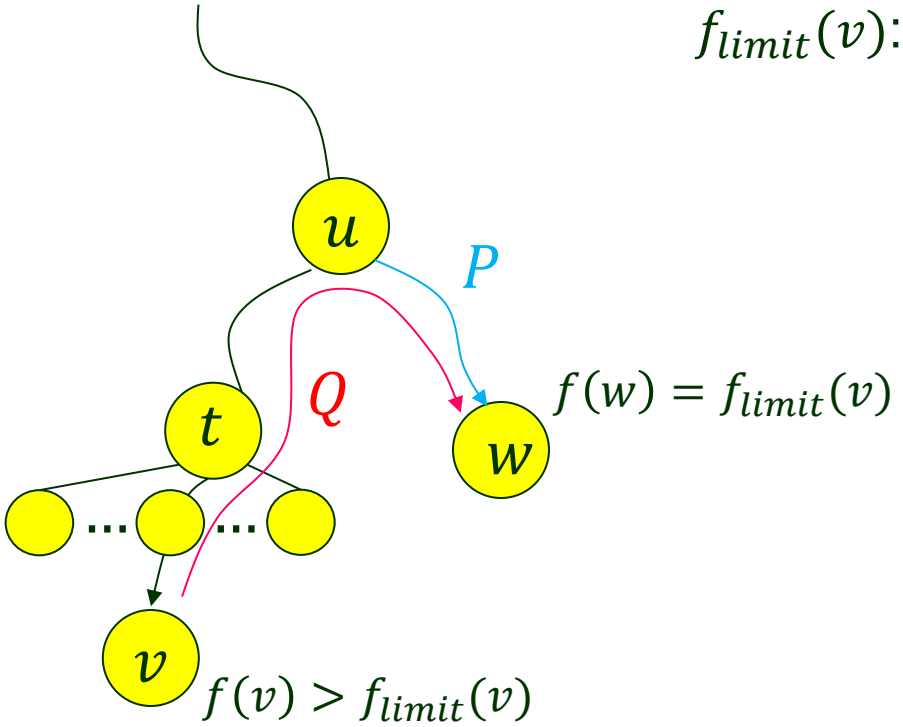

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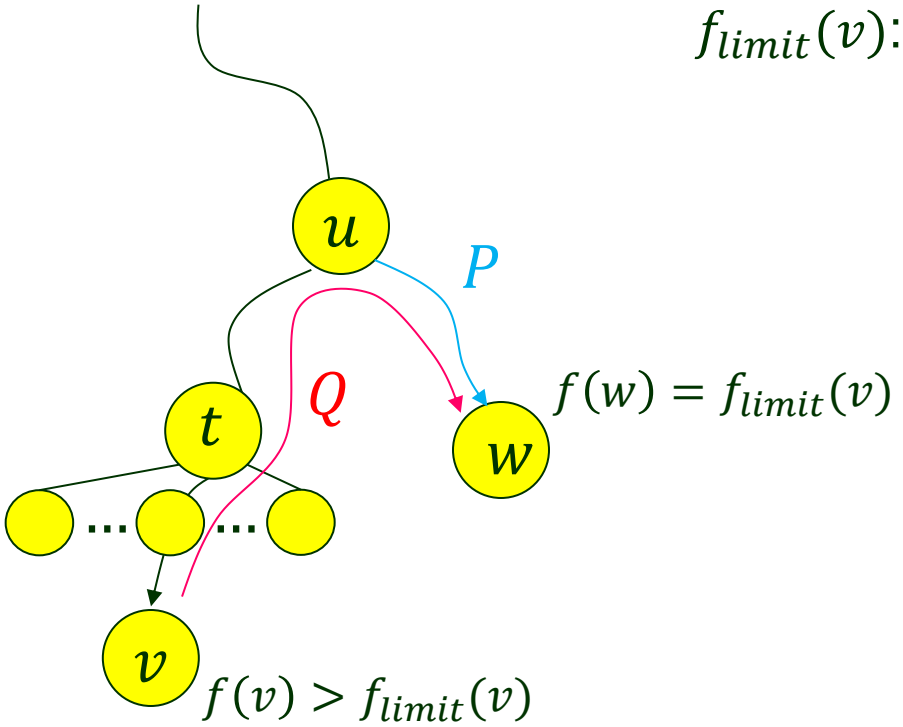

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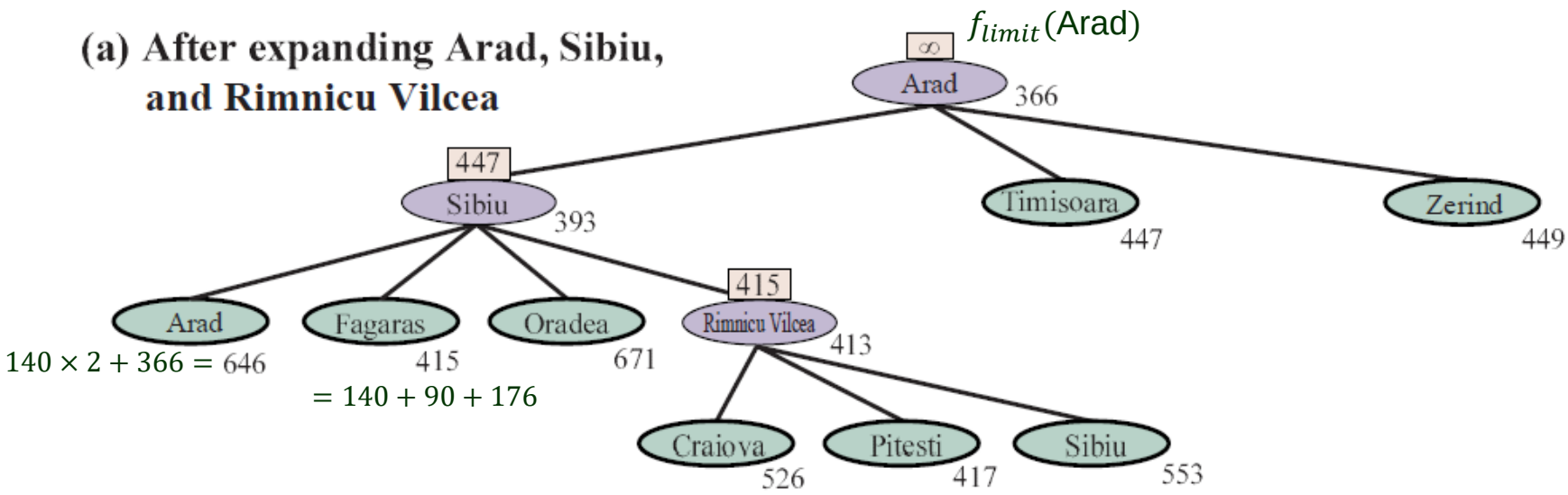
- Otherwise (i.e., $f(v) > f_{limit}(v)$),
  - unwinds back to the alternative path P;
  - the visited nodes during the unwinding form a path Q.
  - set the f-value of every node t on Q to be the best f- value of its children.

Idea: Remembers the f-value of the best leaf in the forgotten subtree in case it's worth a re-expansion at some later time.

RBFS Example



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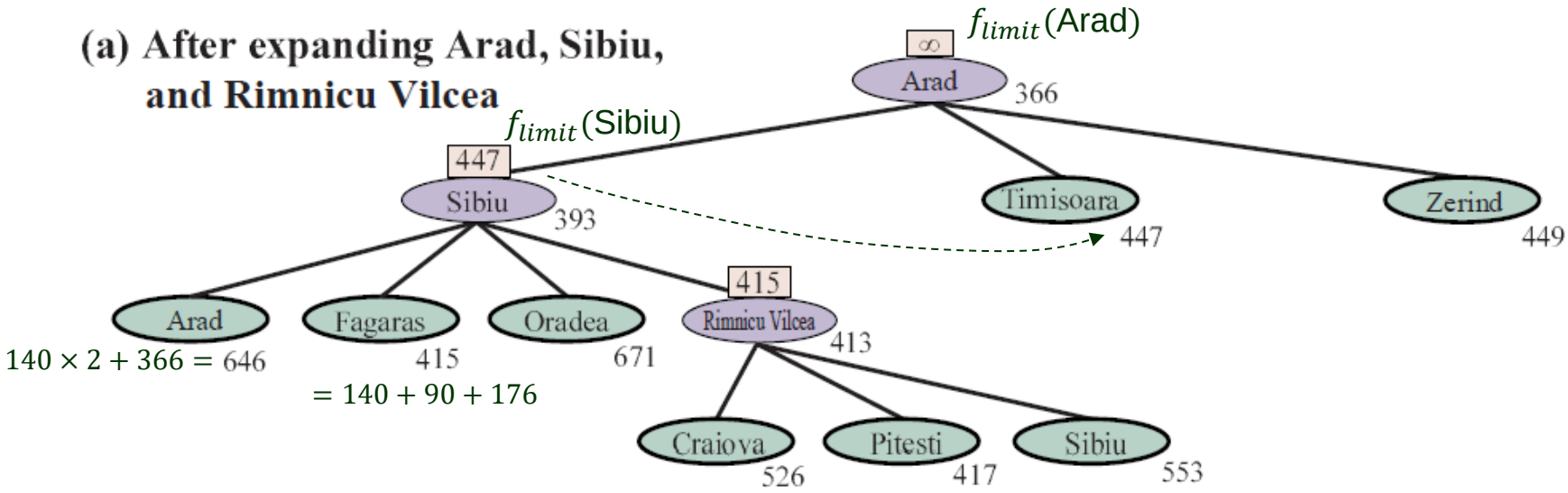

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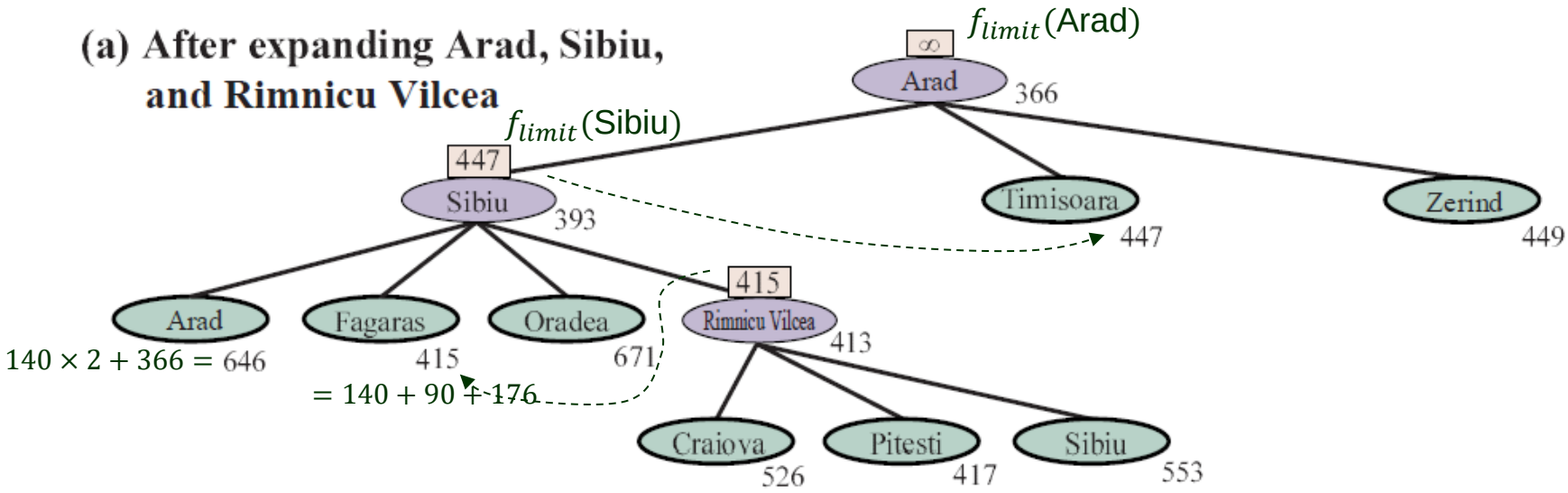

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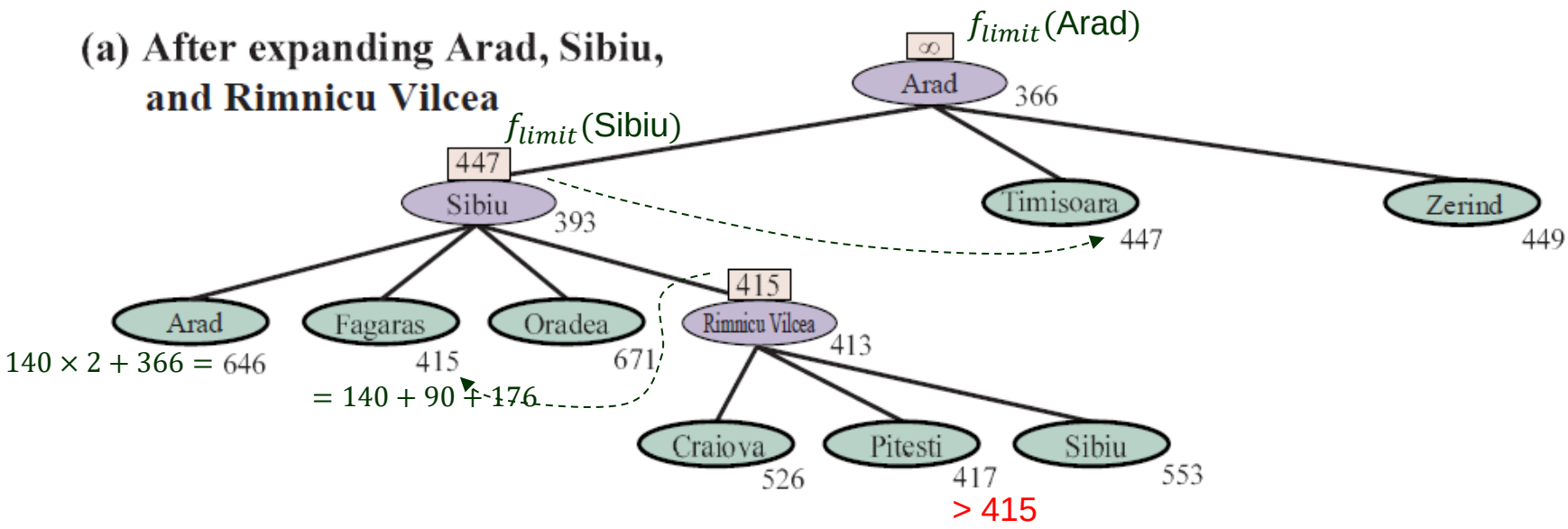

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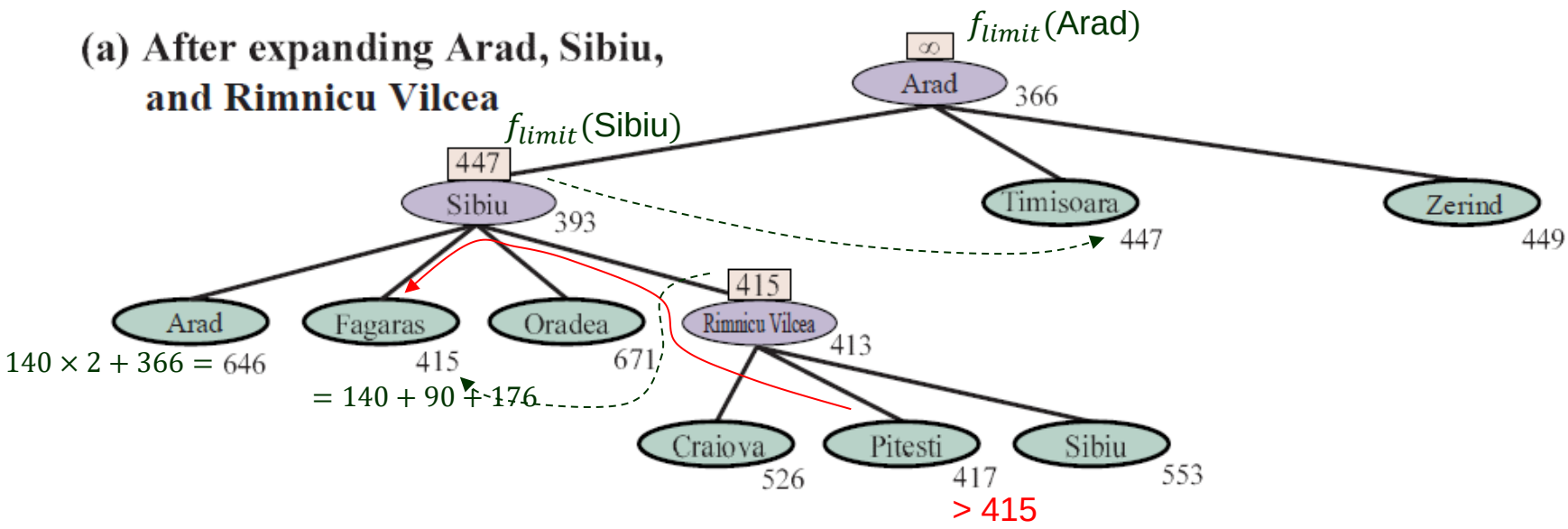

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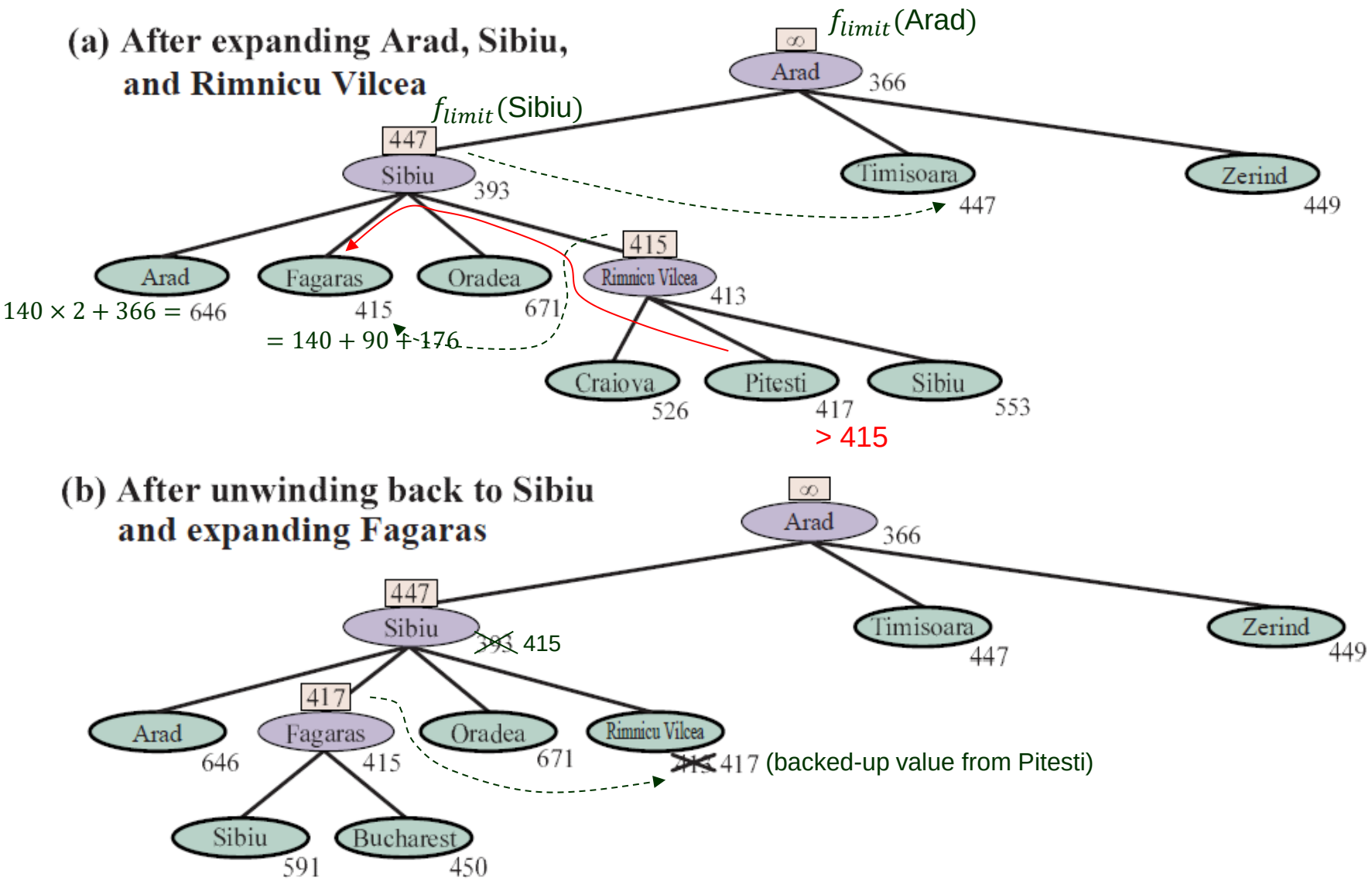

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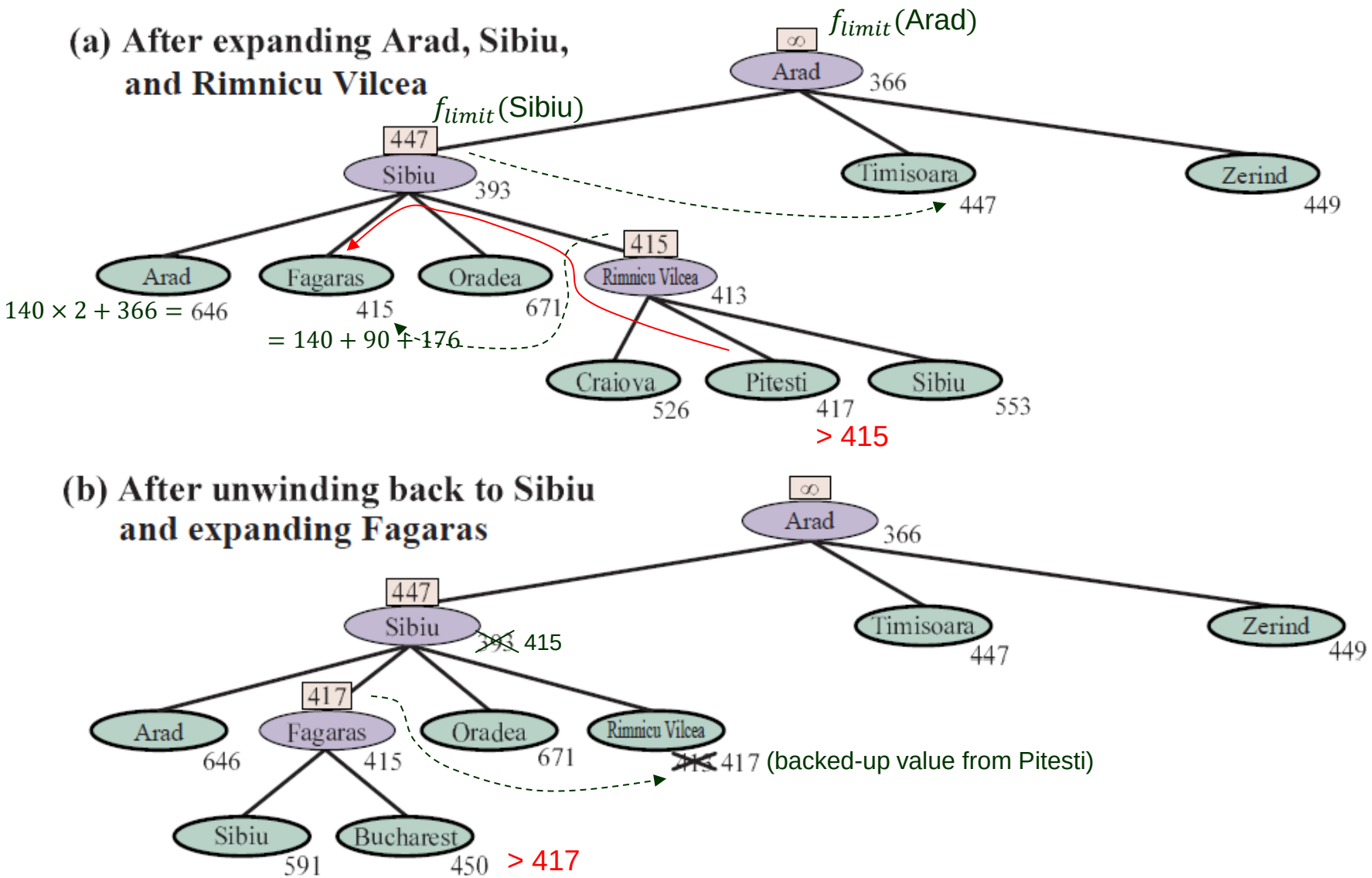

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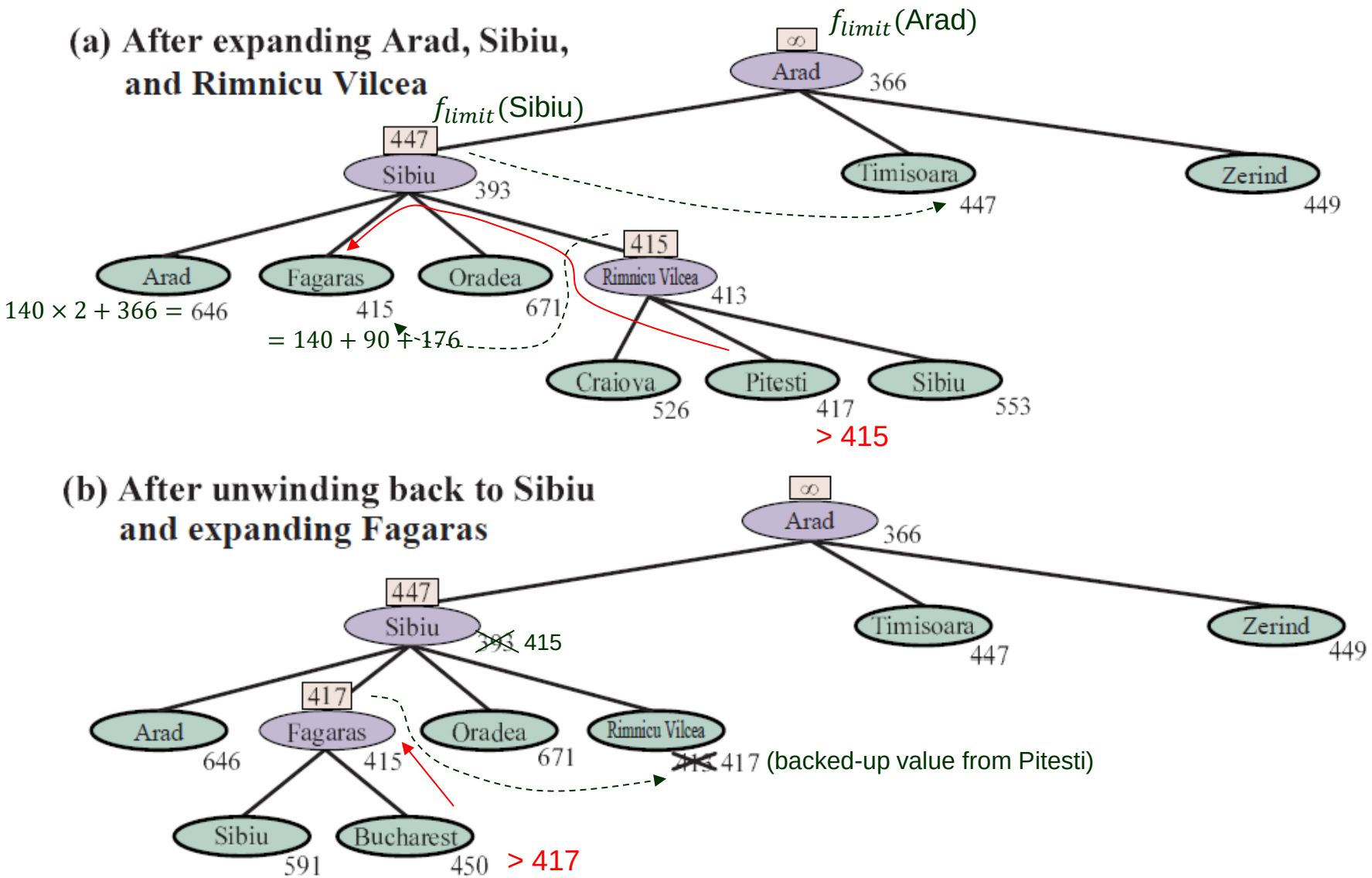
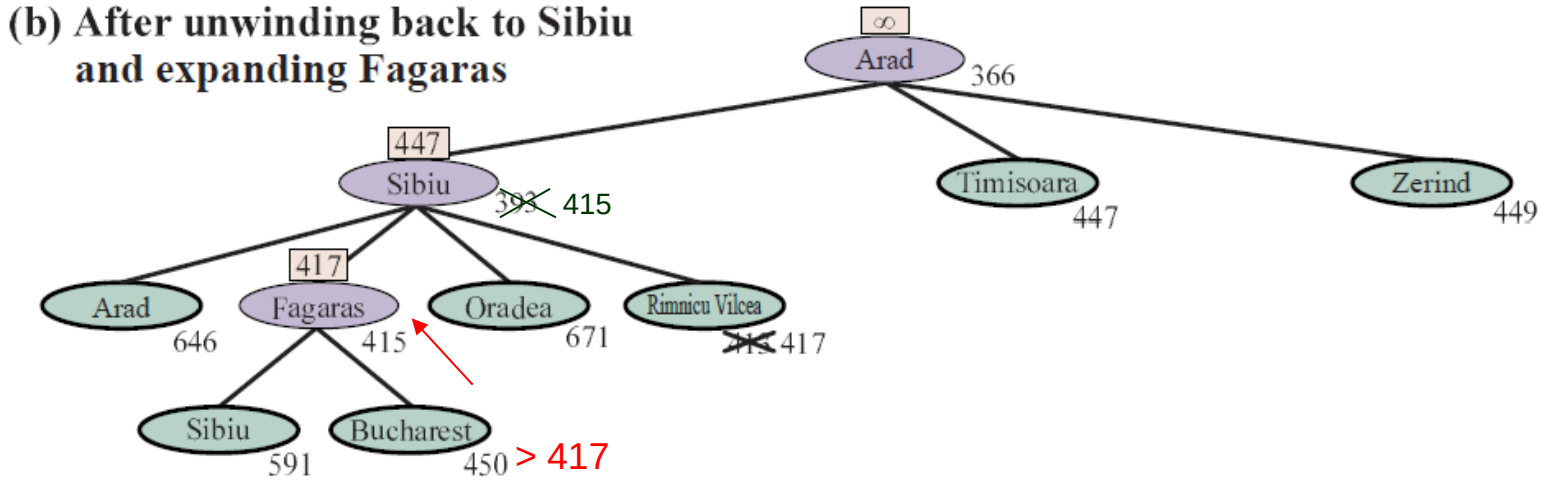
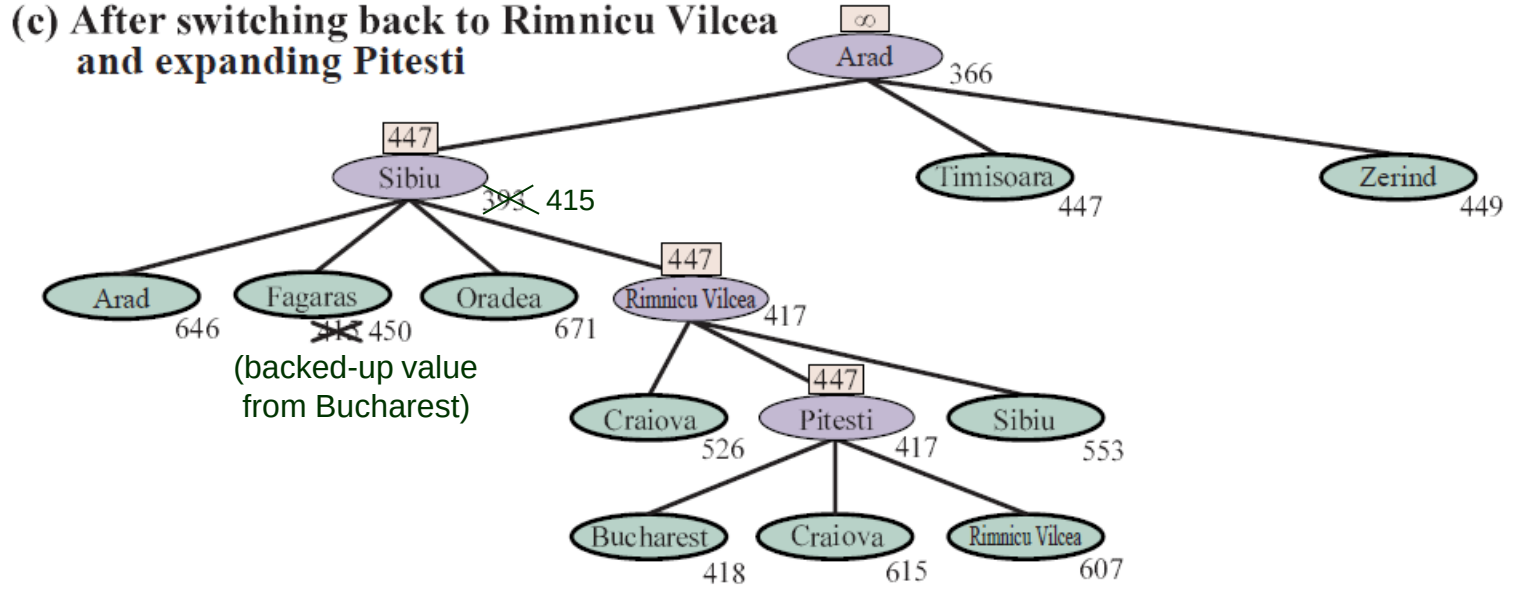


RBFS Example (cont'd)





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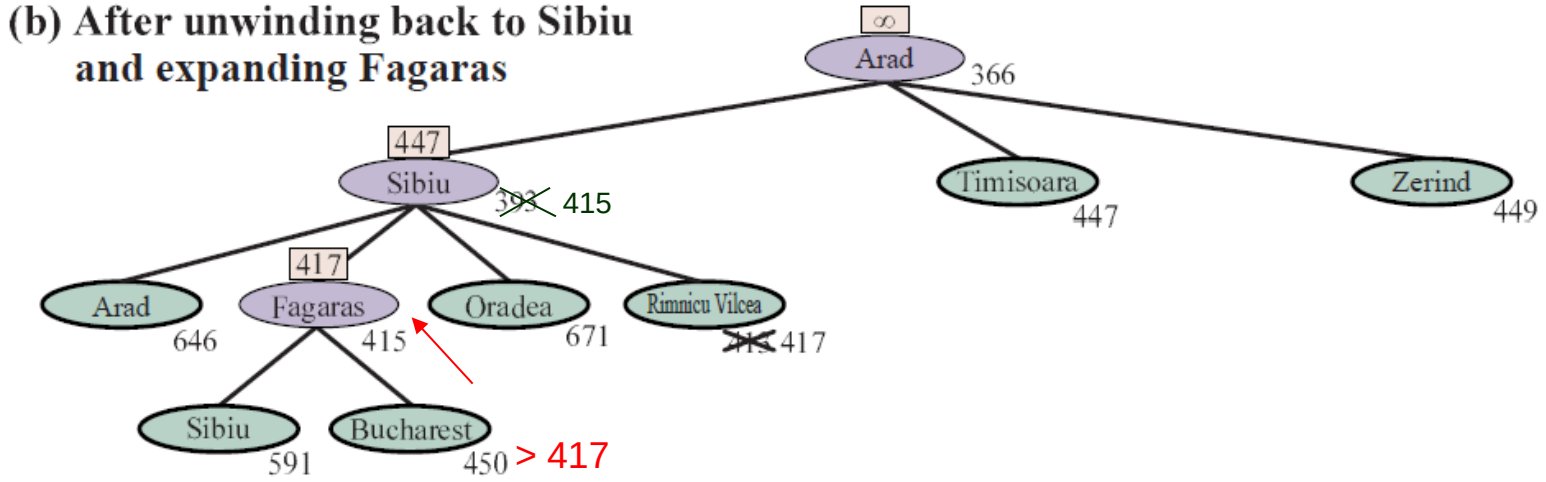

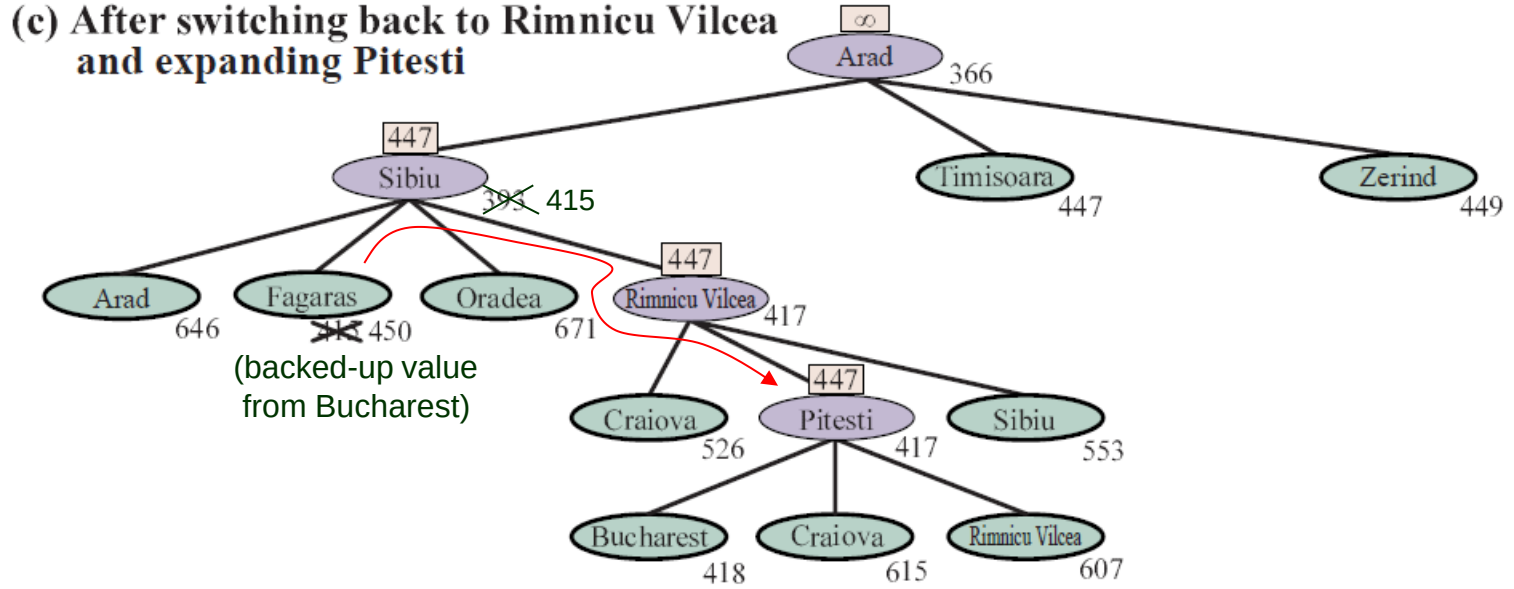

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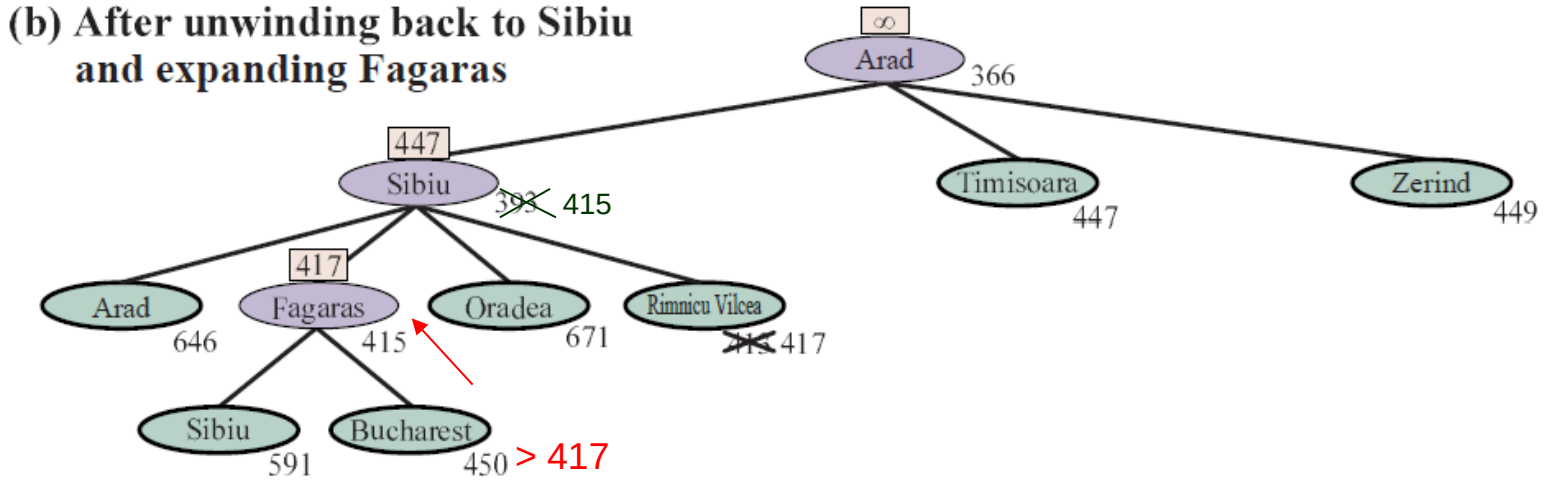

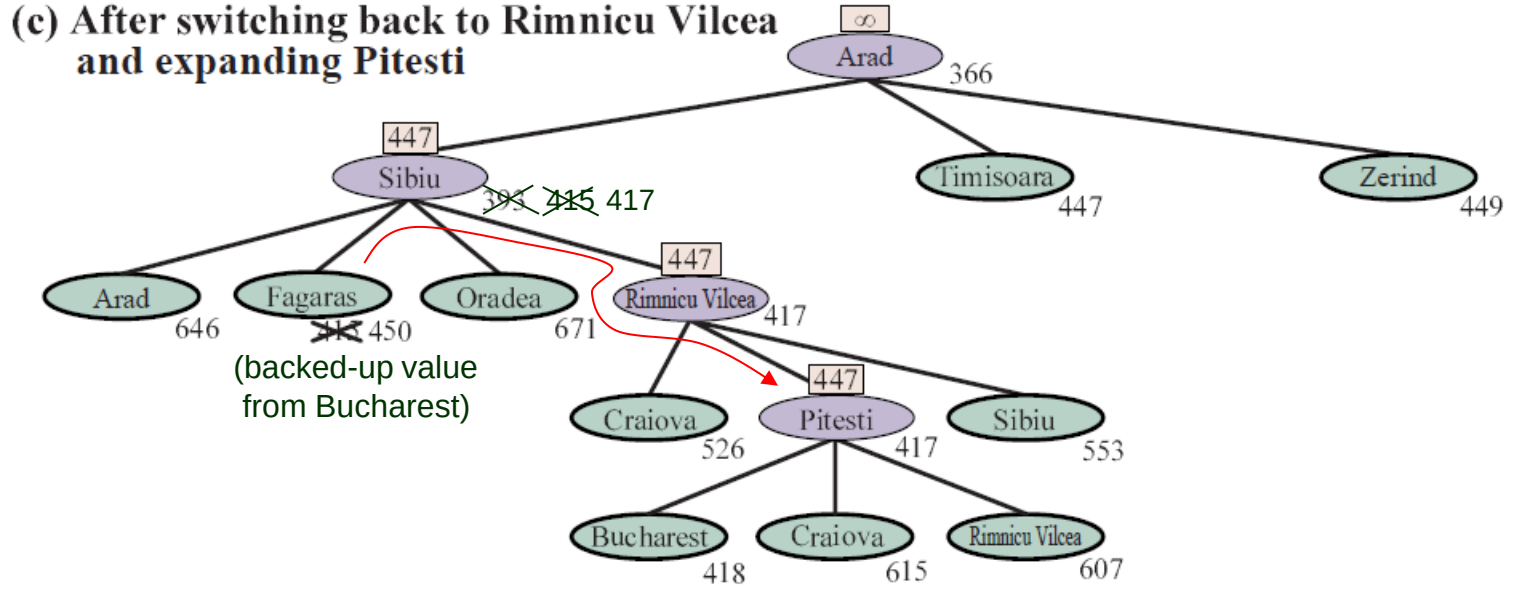

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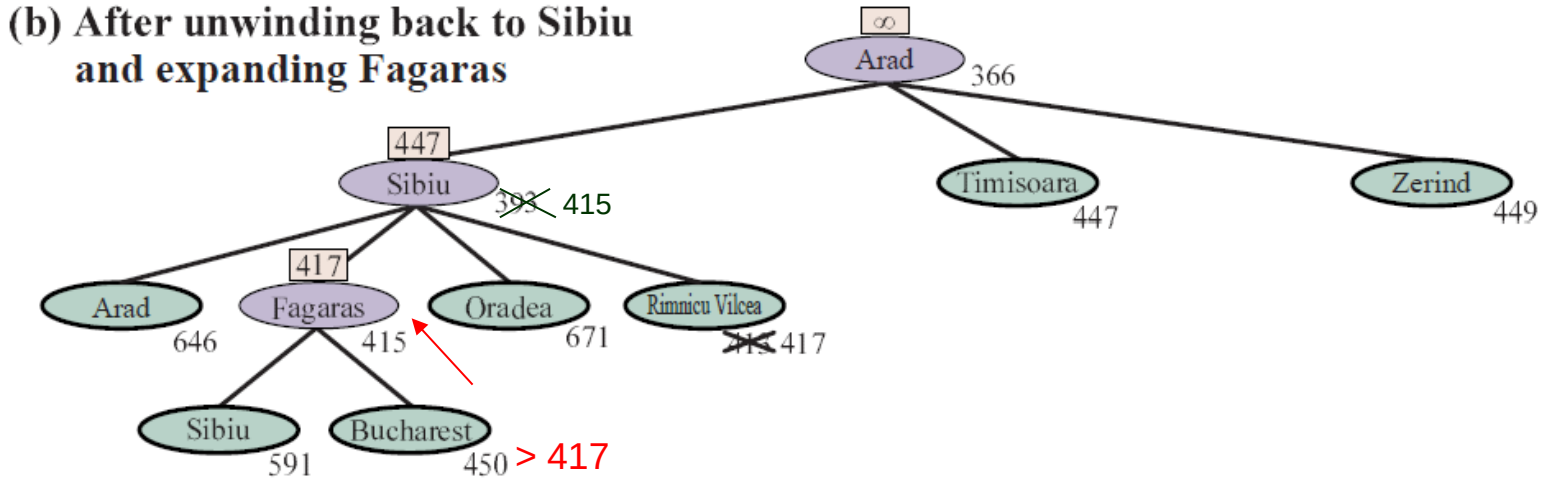
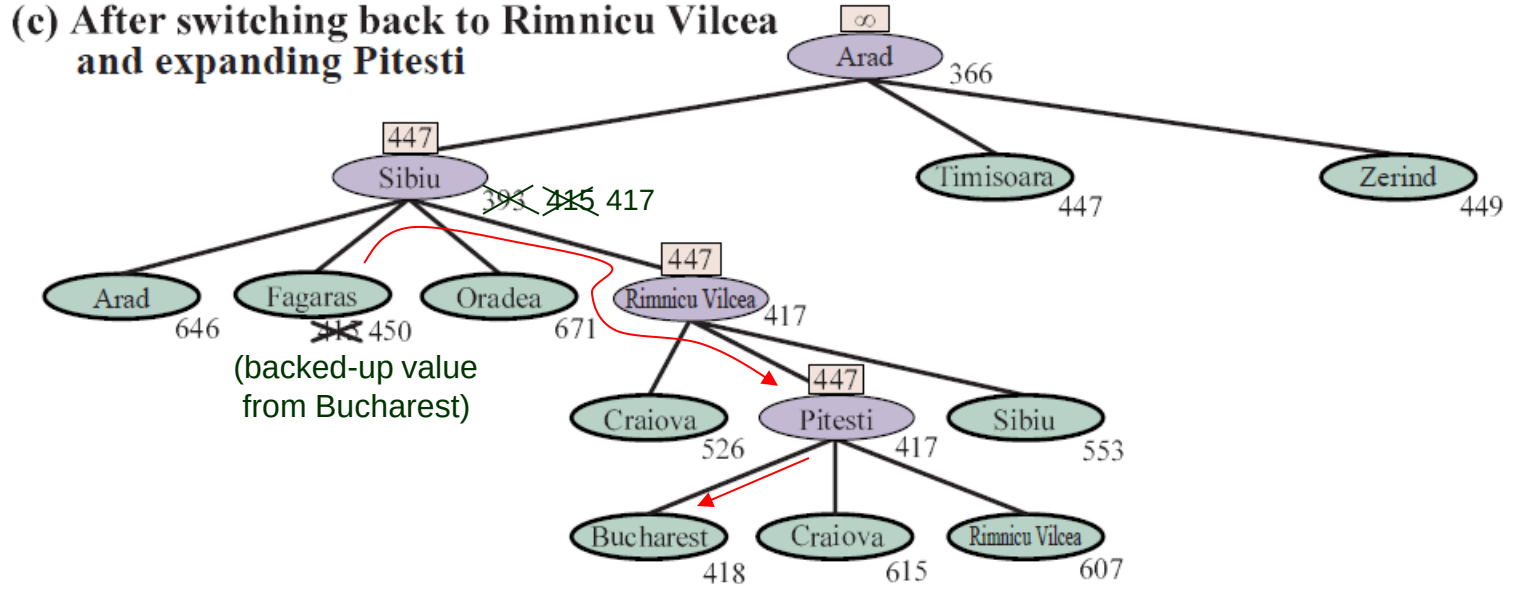
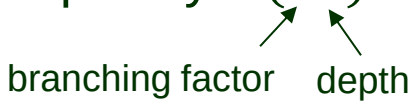




RBFS Summary

- Optimal with admissible heuristic function $h(n)$.
- Space complexity $O(bd)$.



- Time complexity difficult to analyze, depending on
  - accuracy of $h(n)$
  - how often the best path changes
- Slightly more efficient than IDA*.
- Both IDA* and RBFS suffering from using too little memory and may explore the same state multiple times.