Continuous Space & Nondeterministic Actions

Outline

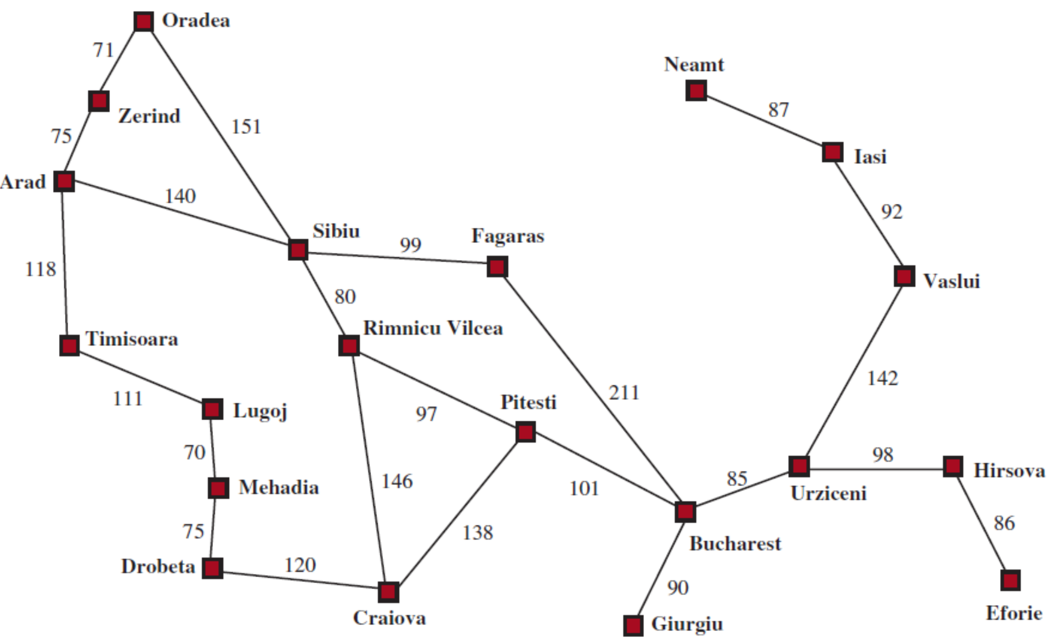
I. Local search in continuous spaces

II. Search with non-deterministic actions

* Figures are from the textbook site (or by the instructor) unless the source is specifically cited.

I. Local Search in Continuous Space

Problem Place three new airports anywhere in Romania to minimize the sum of square straight-line distances from every city to its nearest airport.



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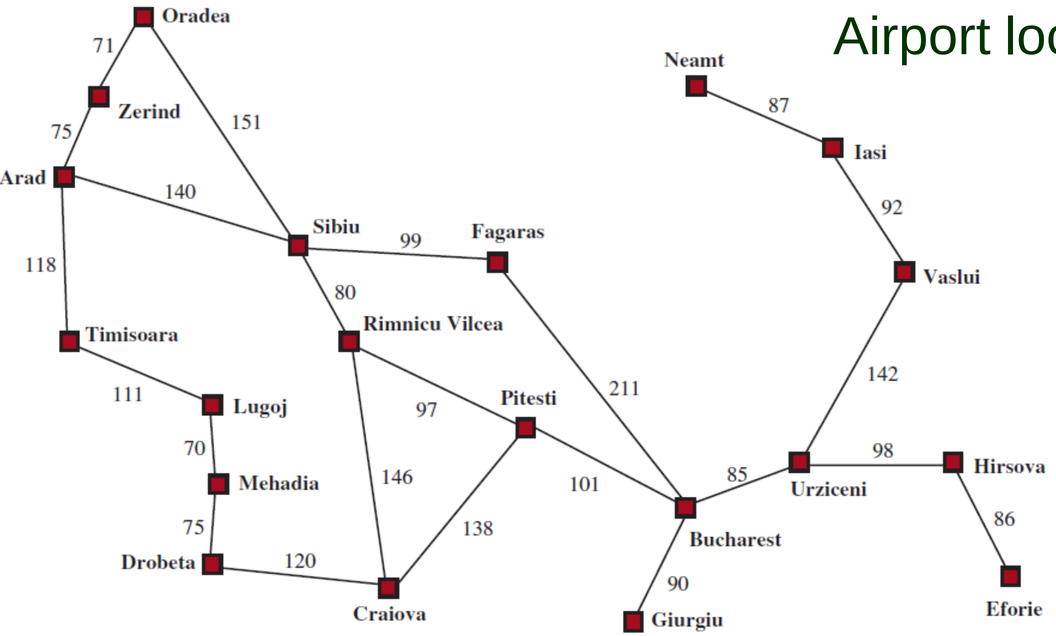

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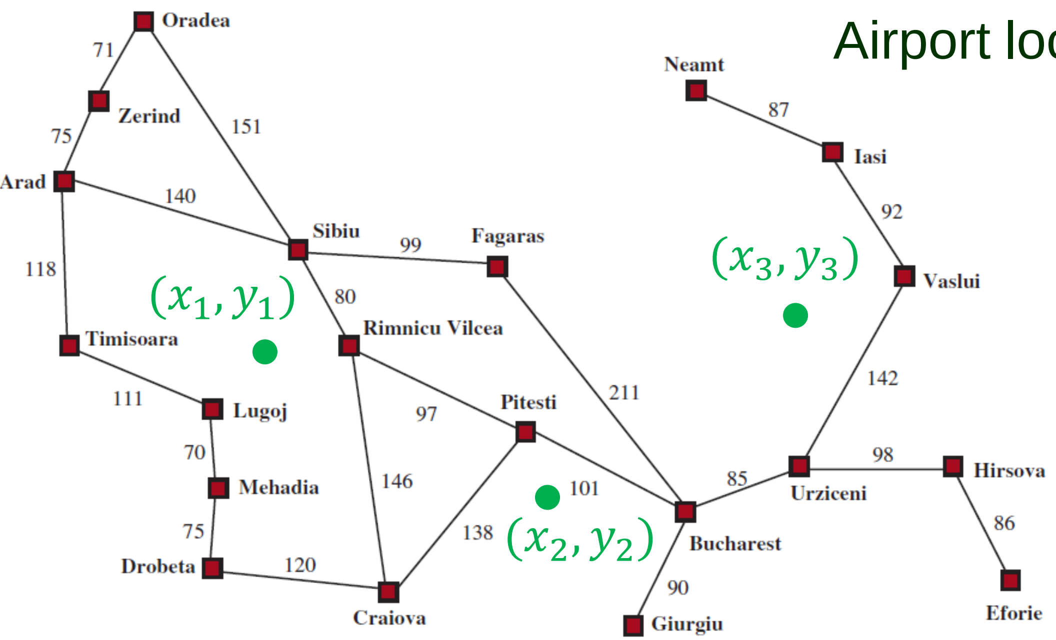

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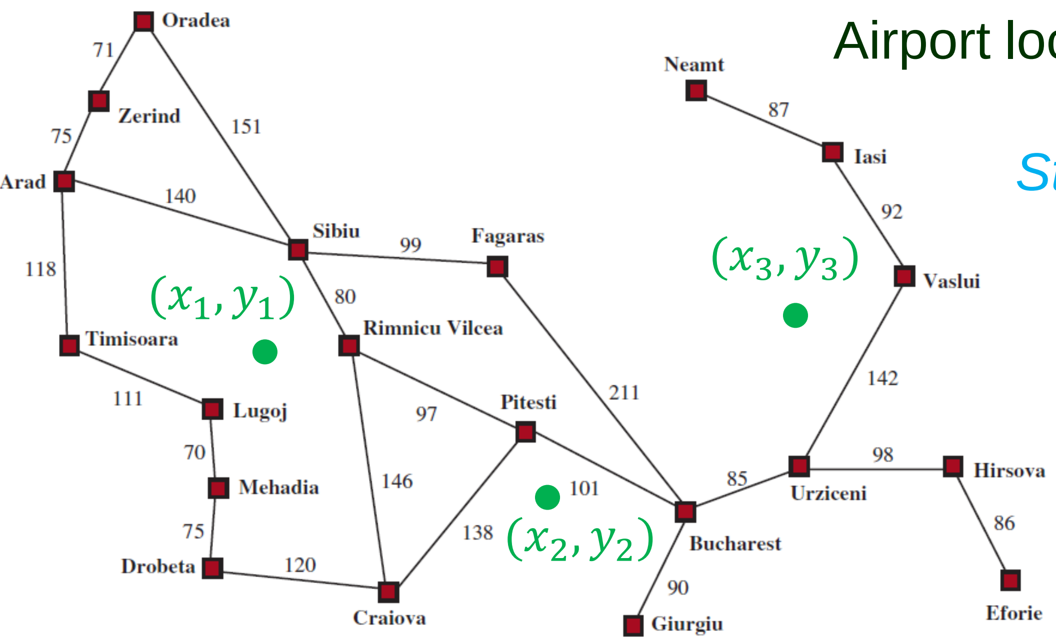

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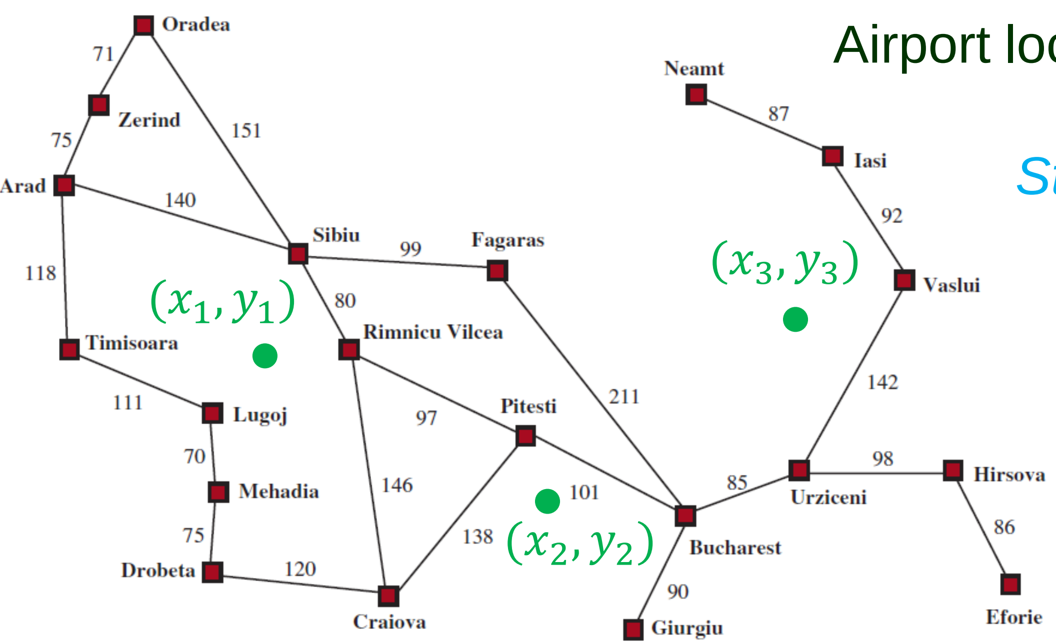

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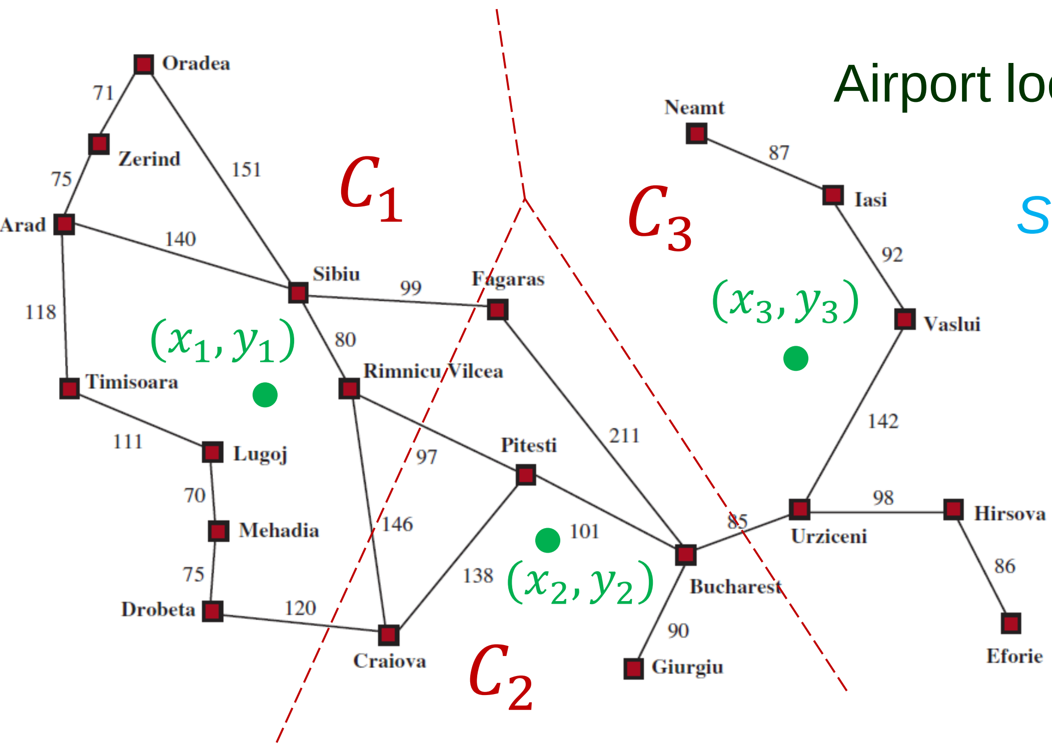

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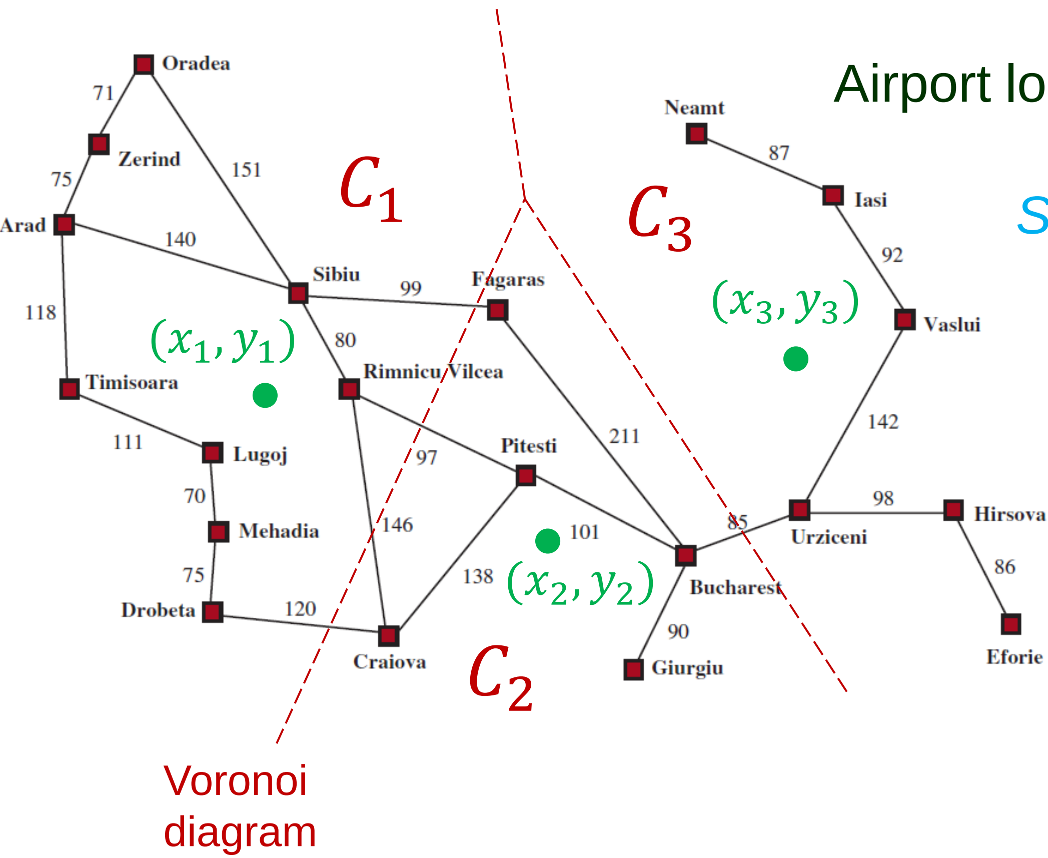

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Objective function:

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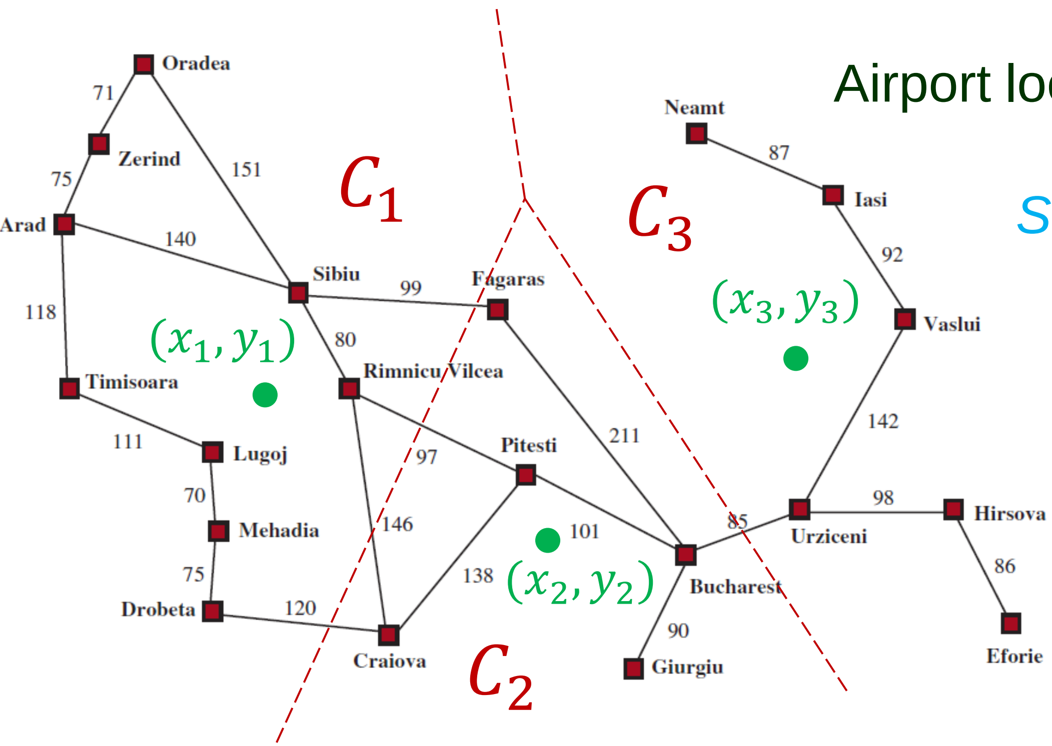

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$$\min_{x} f(\boldsymbol{x})$$

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Gradient-Based Method

Use the *gradient*: $\nabla f(x_1, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right)$

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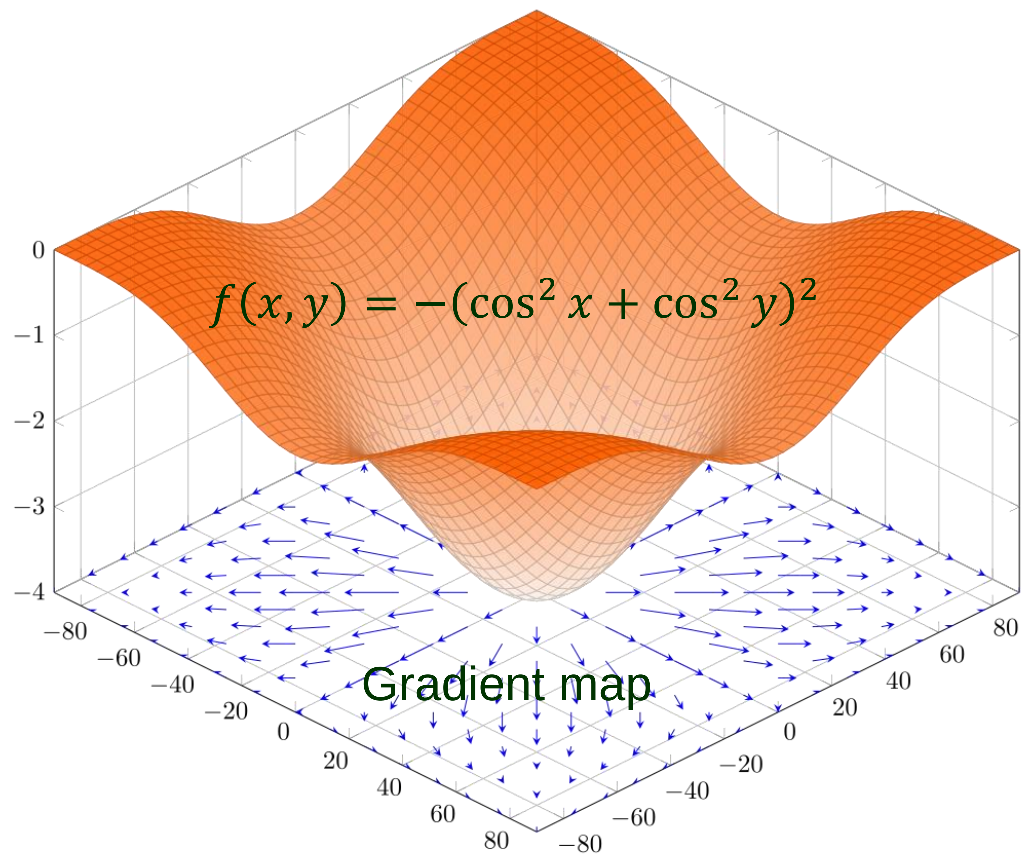

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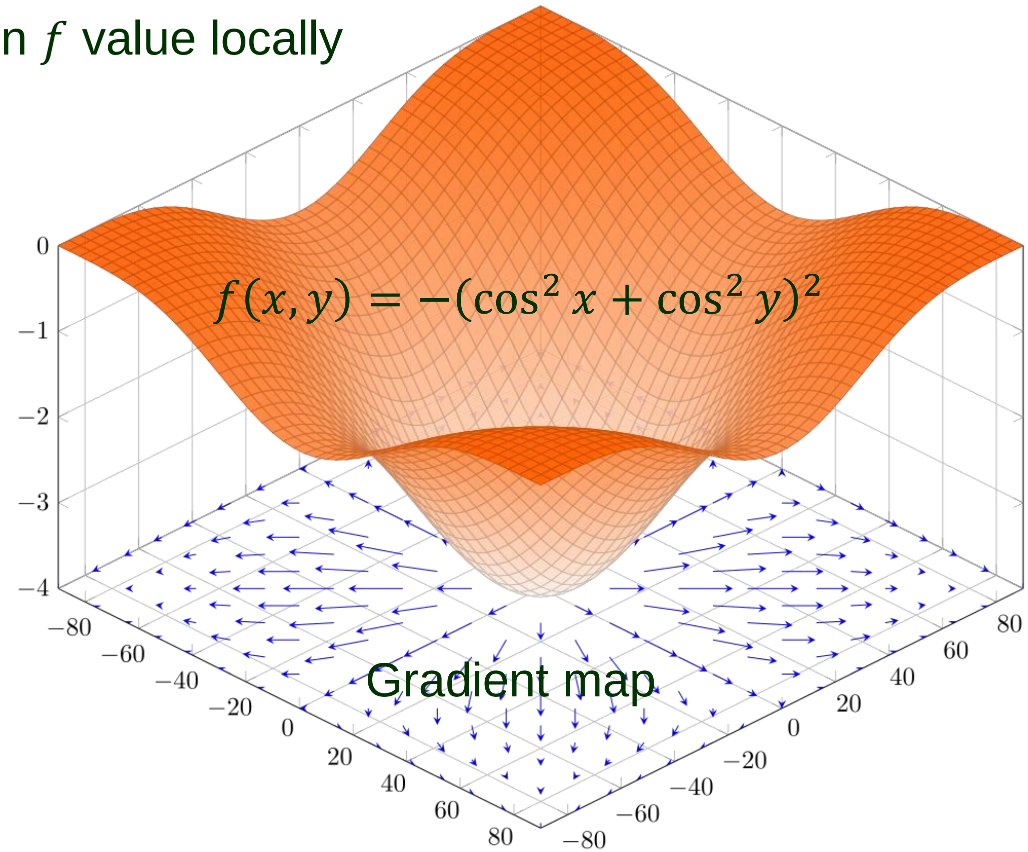
$-\nabla f$: direction of fastest decrease in f value locally



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Gradient Descent

Back to airport placements:

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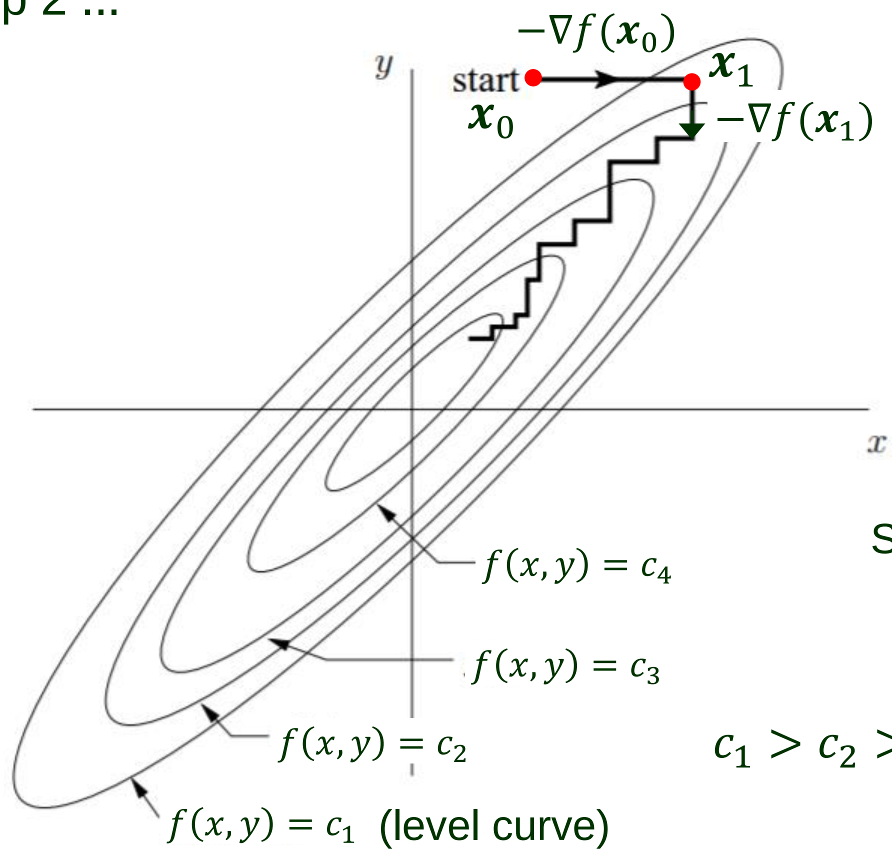
step size

Line Search

1. Start at an initial state $\boldsymbol{x} = \boldsymbol{x}_0$.
2. Move along $-\nabla f(\boldsymbol{x})$ until f no longer decreases.
3. $\boldsymbol{x} \leftarrow$ new stopping point.
4. Go back to step 2 ...



Steepest descent (along $-\nabla f$)

$c_1 > c_2 > c_3 > c_4$

Newton-Raphson Method

Solve

$$f(x) = 0 \qquad \text{// one variable}$$

with the iteration formula

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Hessian of f: matrix $\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)$

More on Continuous Optimization

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- Linear programming (i.e., linear optimization under linear constraints)

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  https://faculty.sites.iastate.edu/jia/files/inline-files/simplex.pdf

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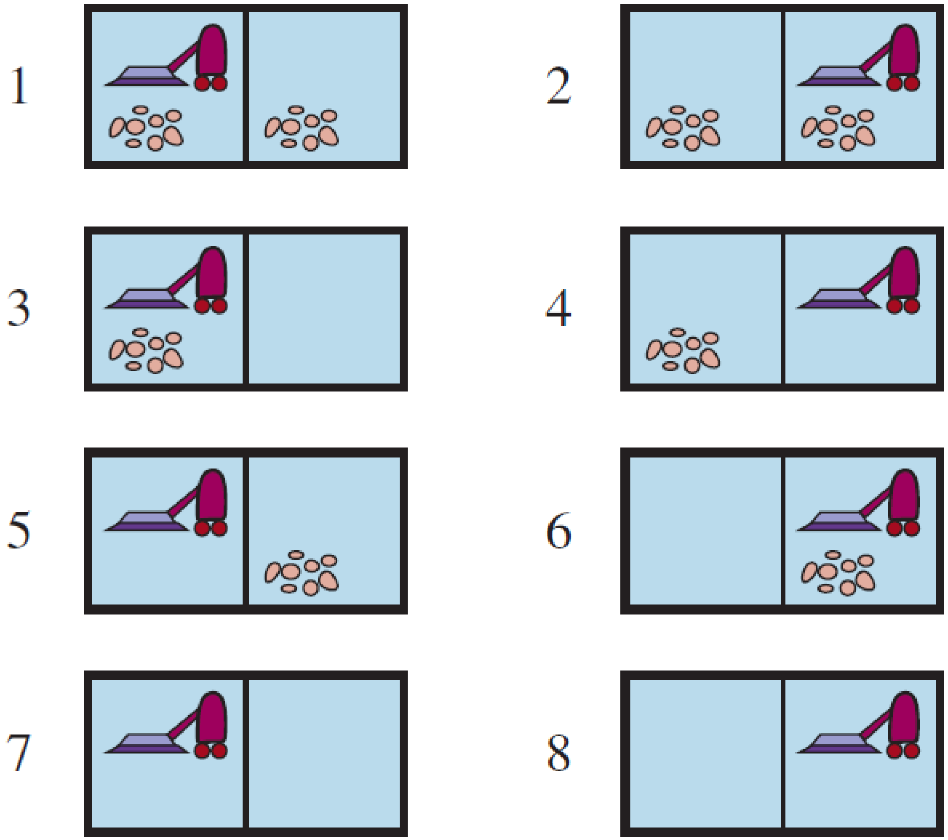
To specify what to do depending on what percepts the agent receives while executing the plan.

Perfect Vacuum World

Fully observable, deterministic, and completely known



8 possible states

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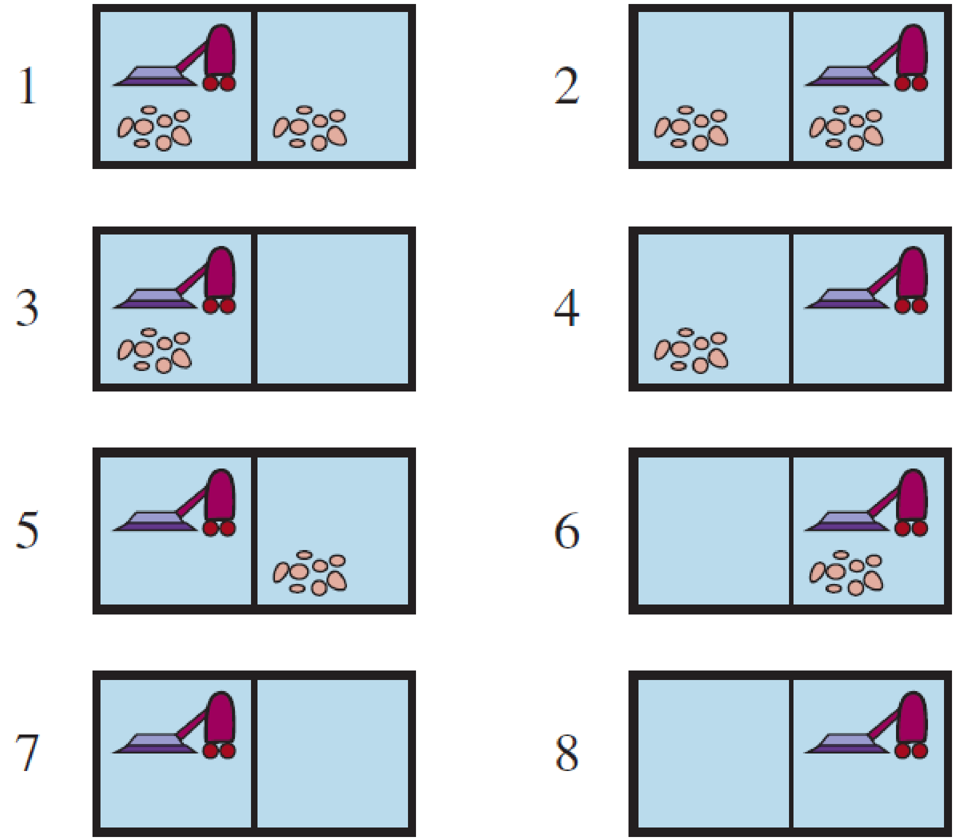
Solution: *Suck, Right, Suck* ⇦



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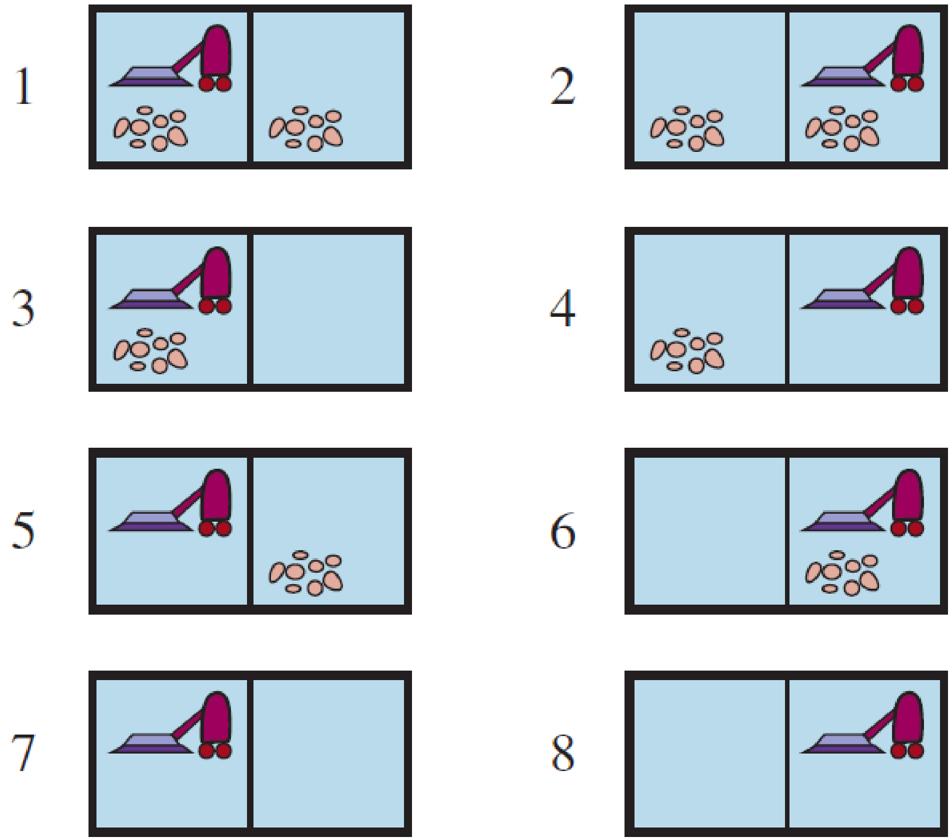

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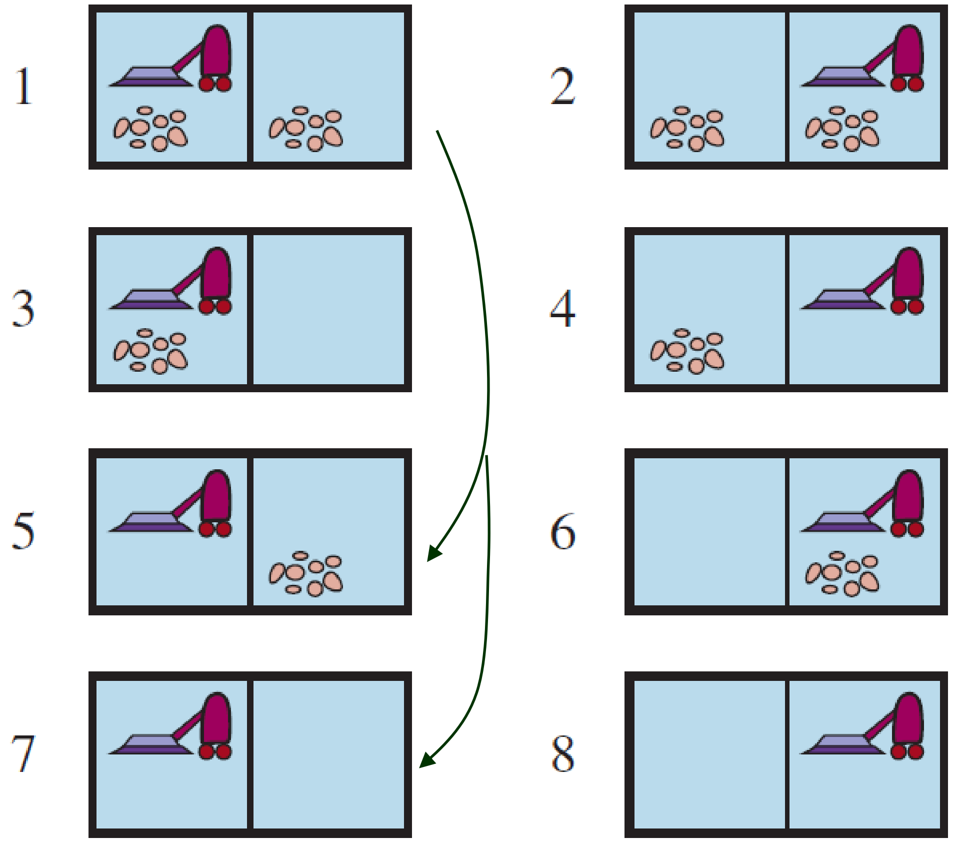

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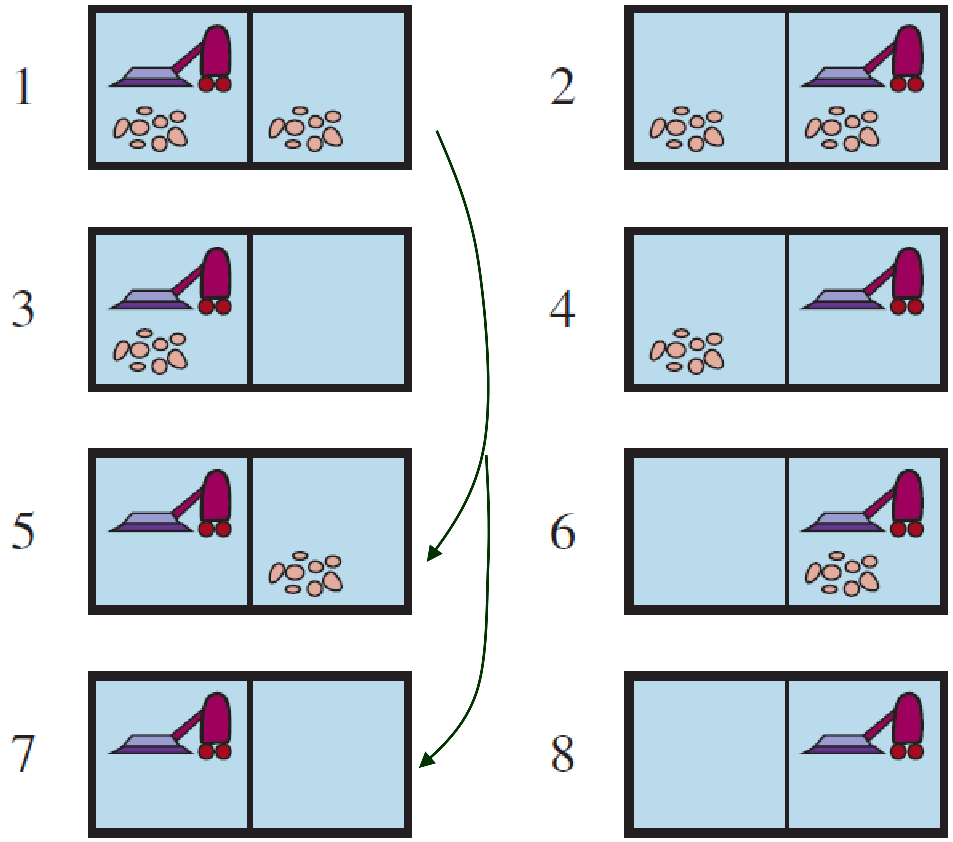

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RESULTS(1, *Suck*) = {5, 7}

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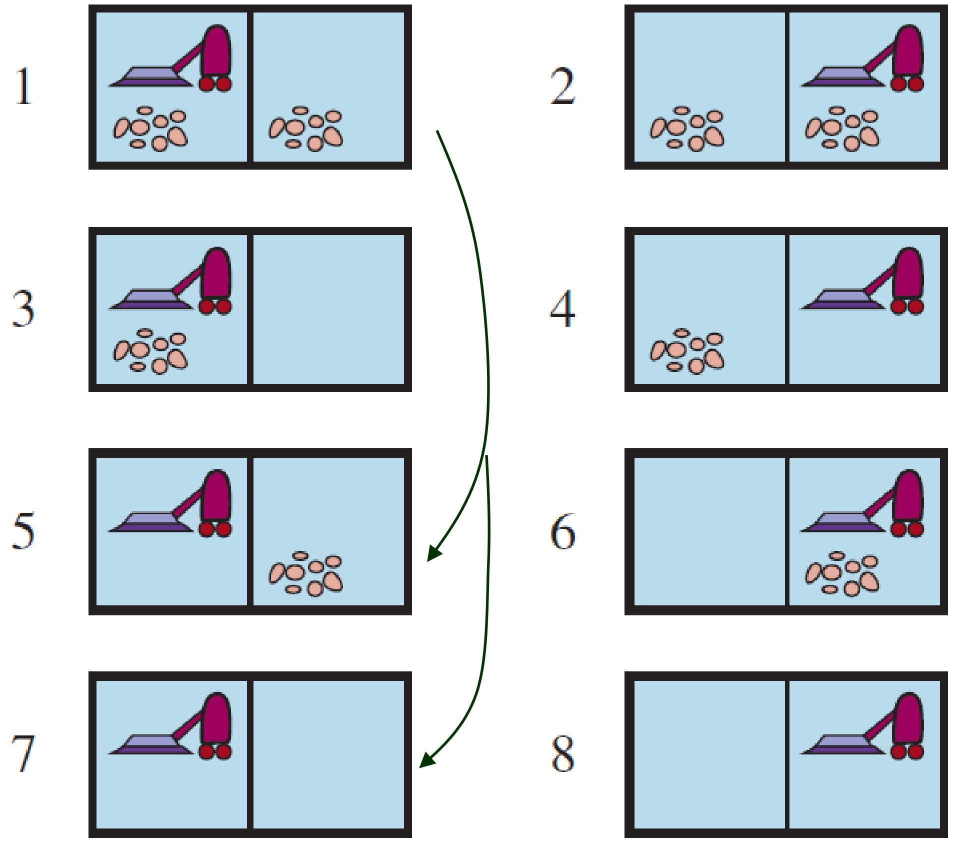

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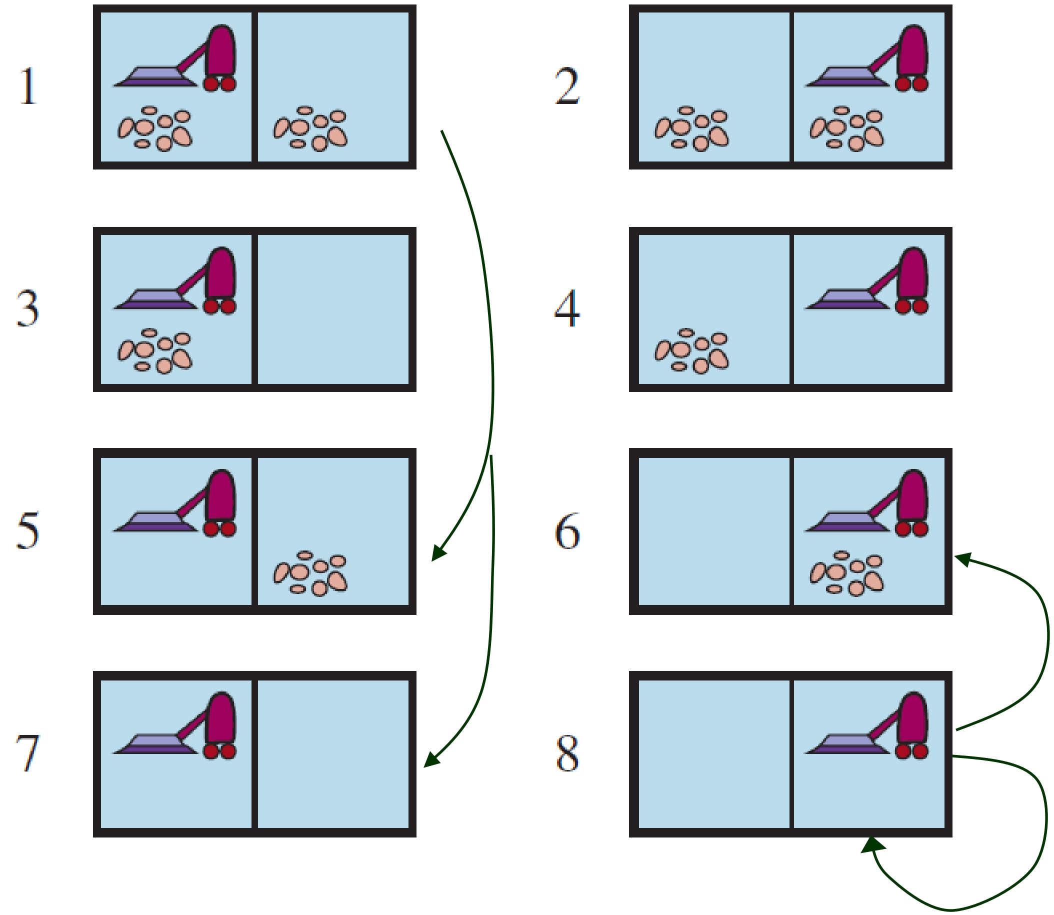

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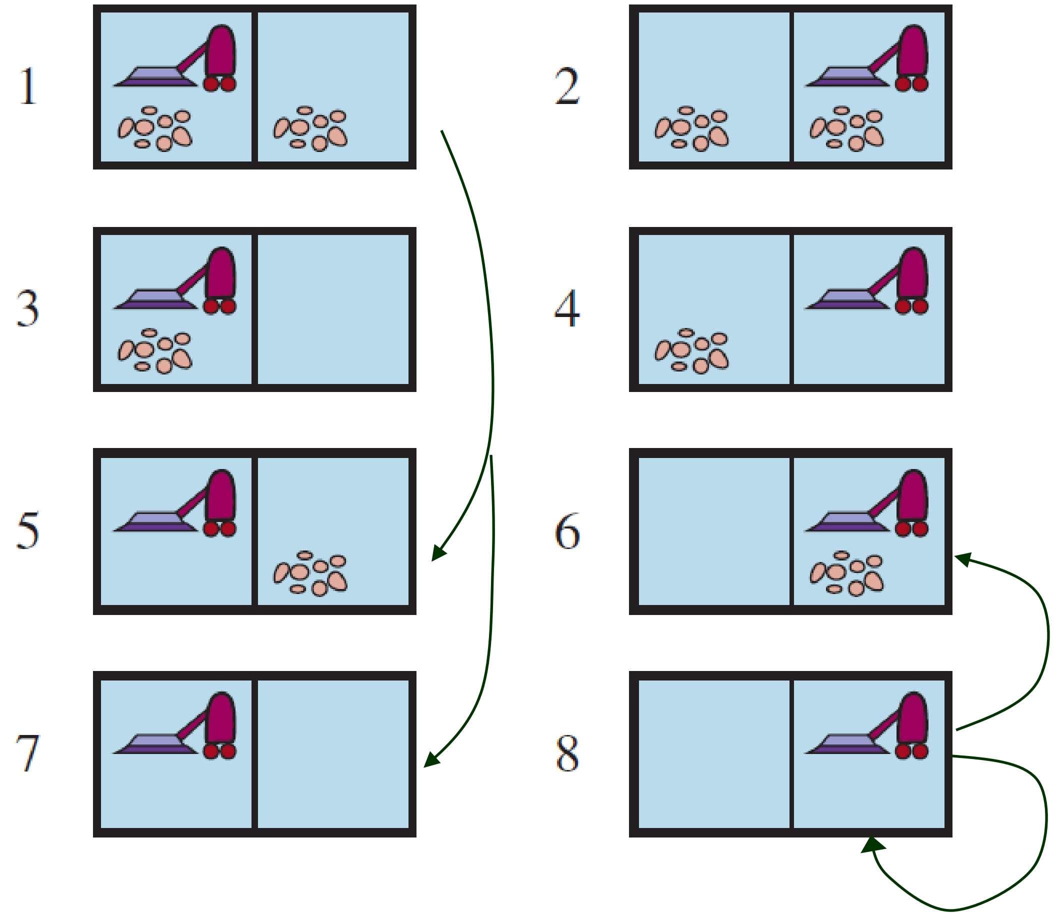

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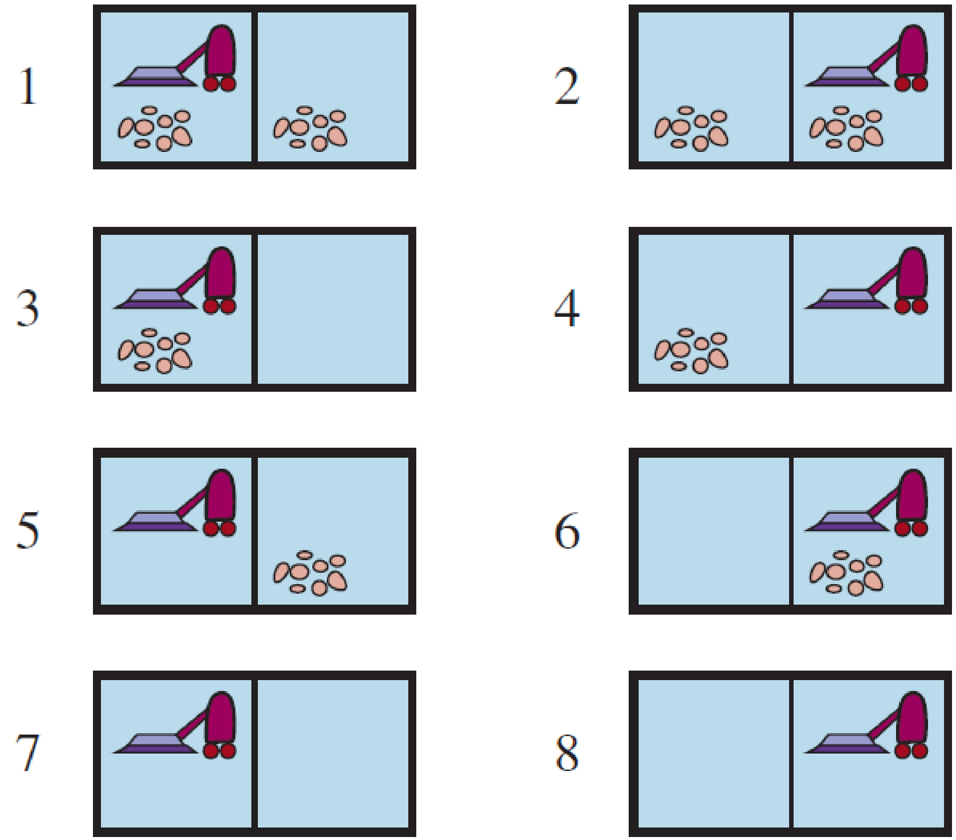

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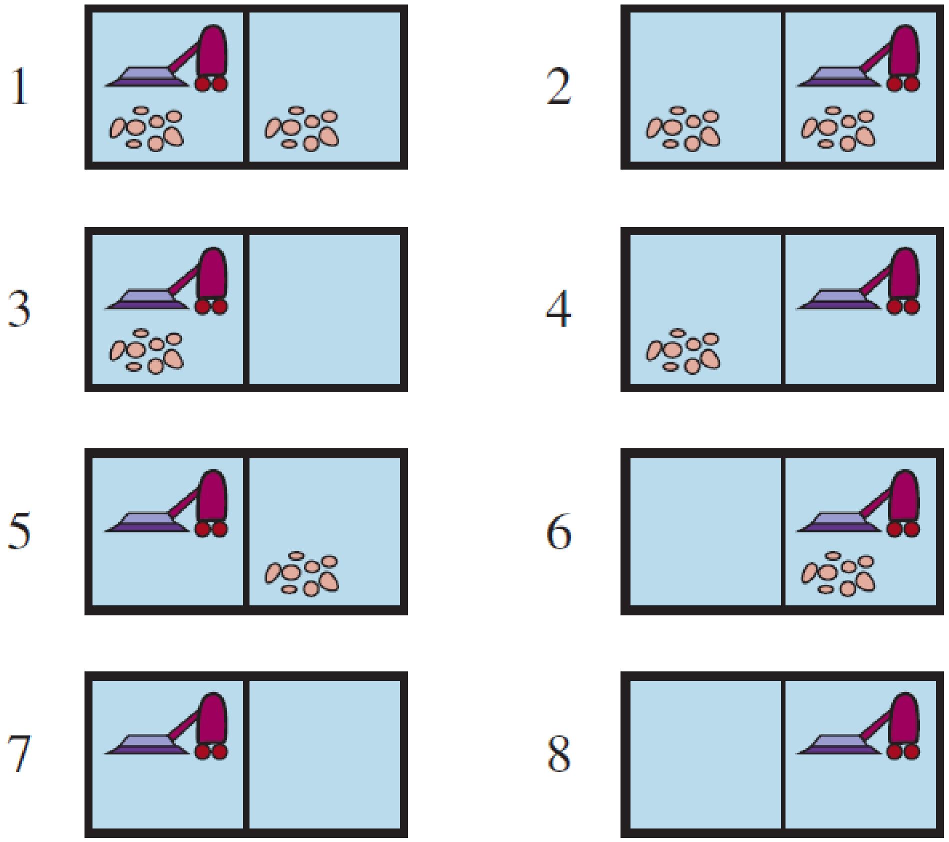

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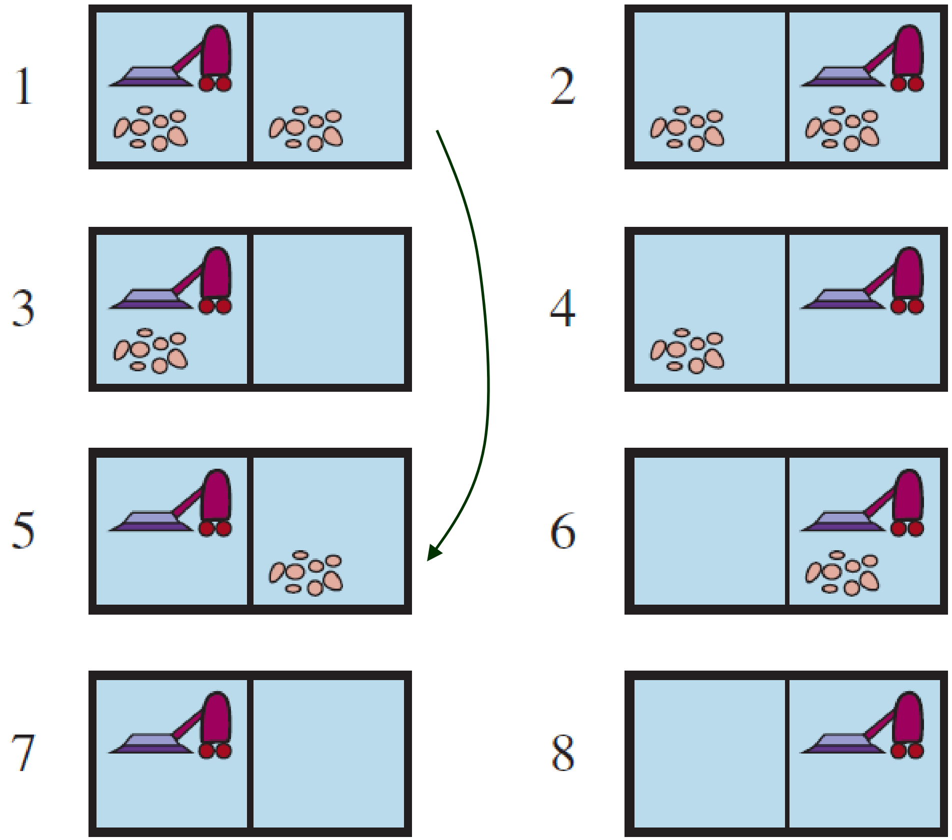

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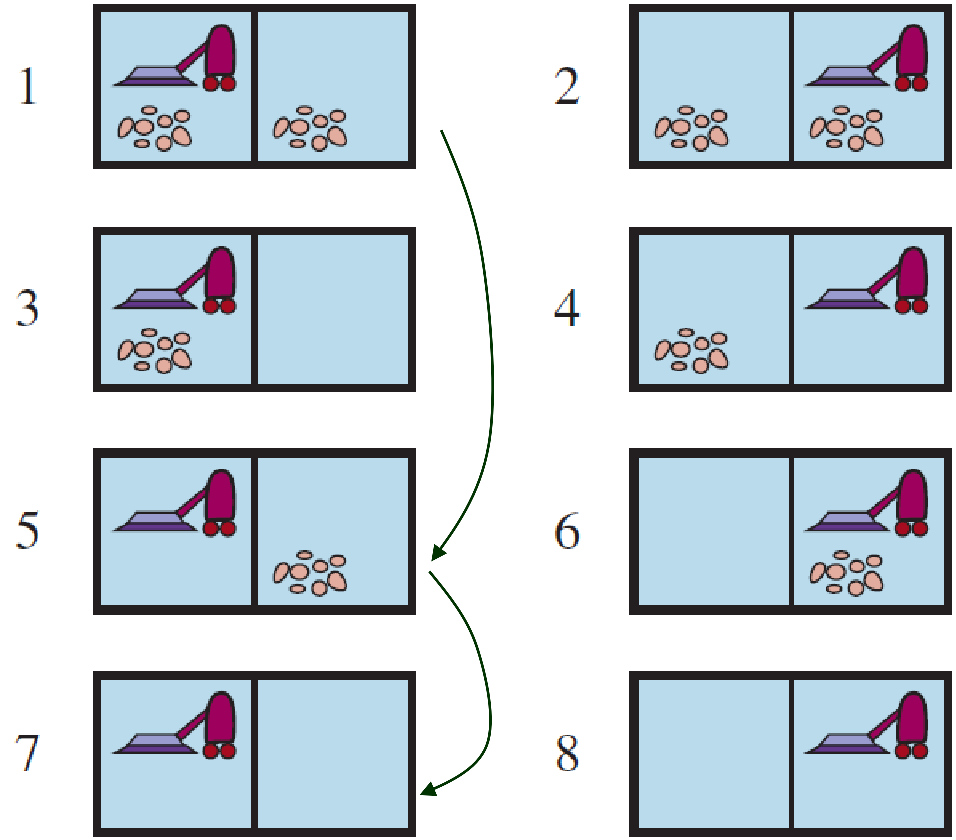

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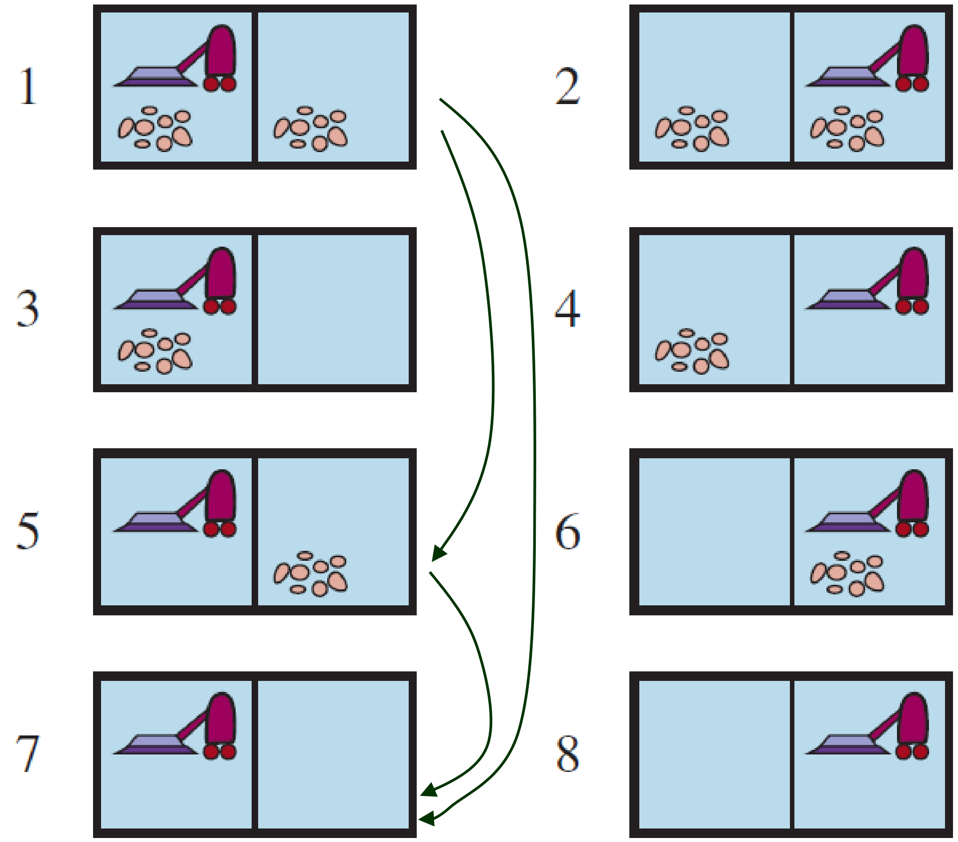

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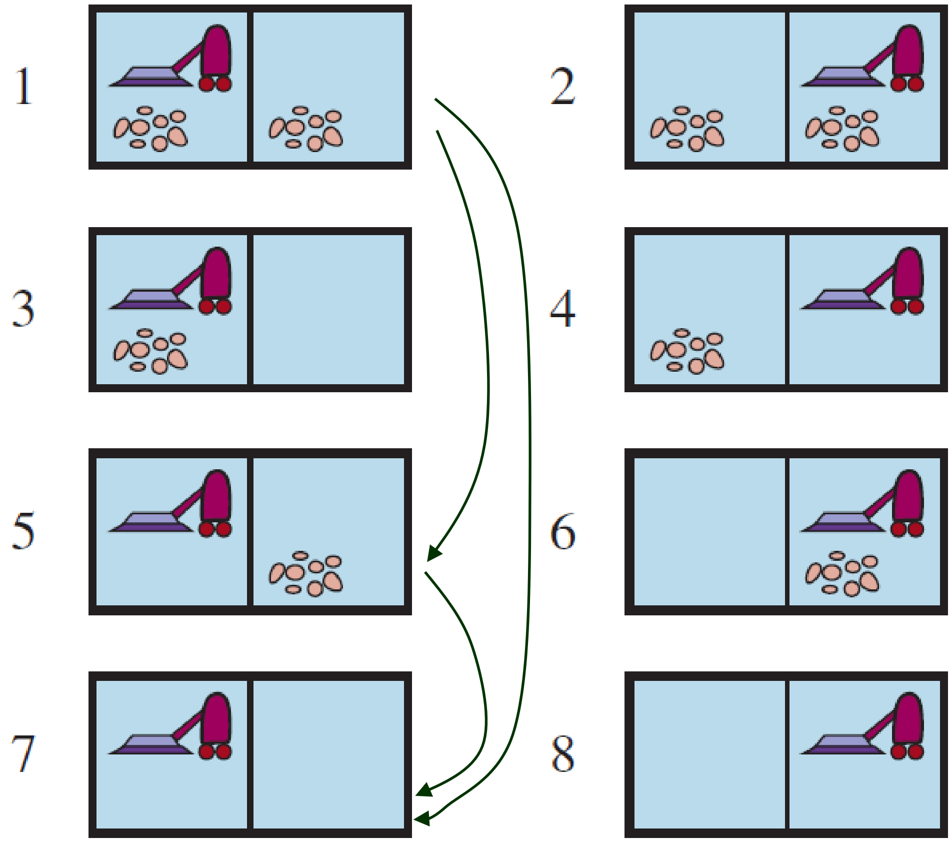


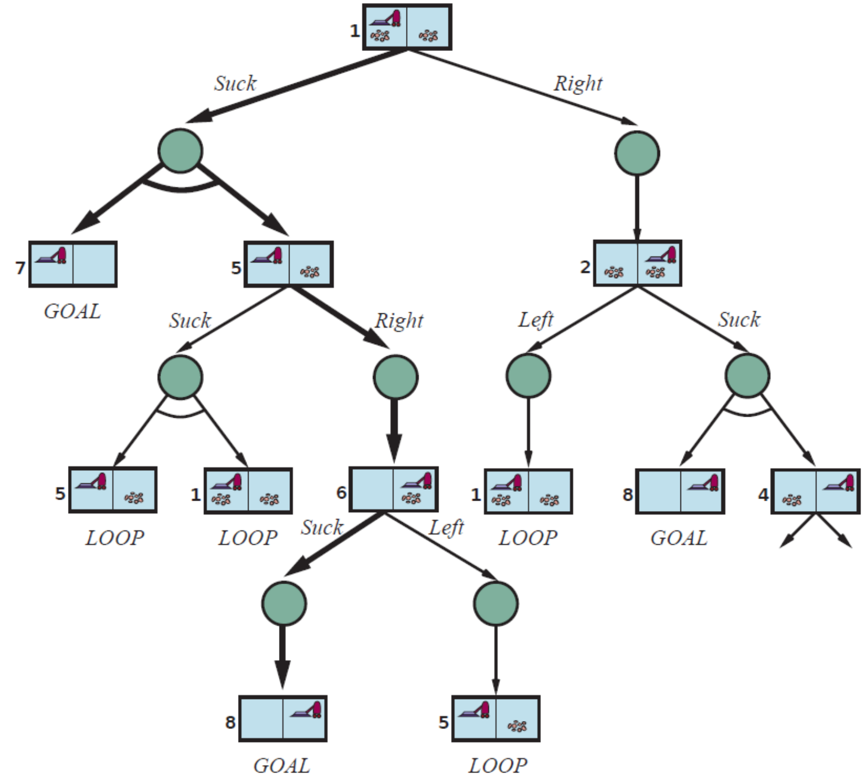
Solution is a tree – of a different character!

AND-OR Search Tree

OR-node (*deterministic*):

the agent chooses an action.

e.g., *Left*, *Right*, or *Suck*



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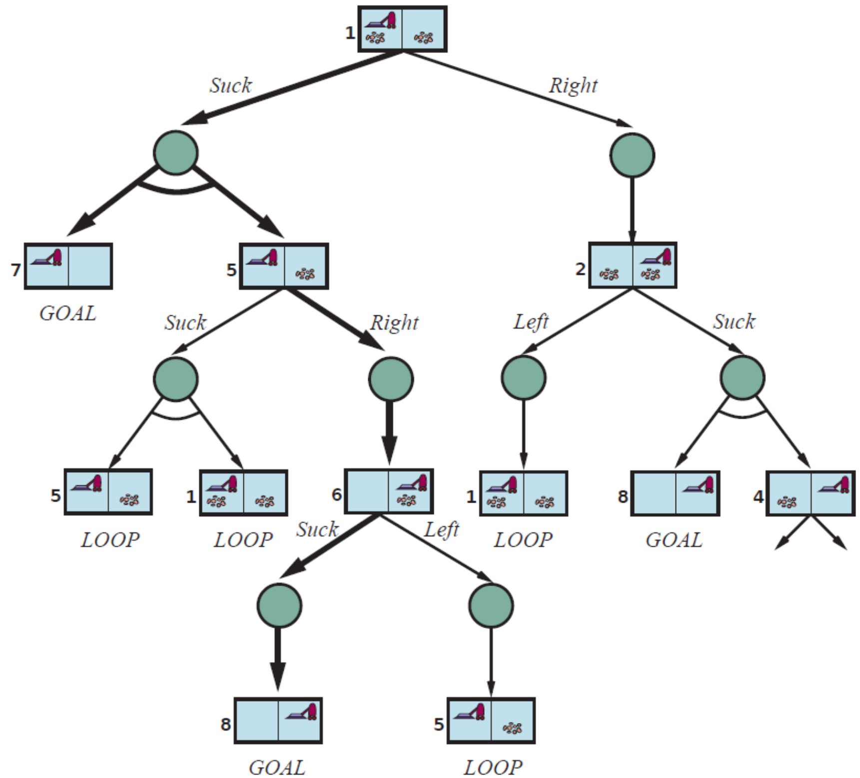

AND-node (*non-deterministic*): the environment "chooses" to have an outcome for each action.

e.g., *Suck* in state 1 results in the belief state {5, 7}.

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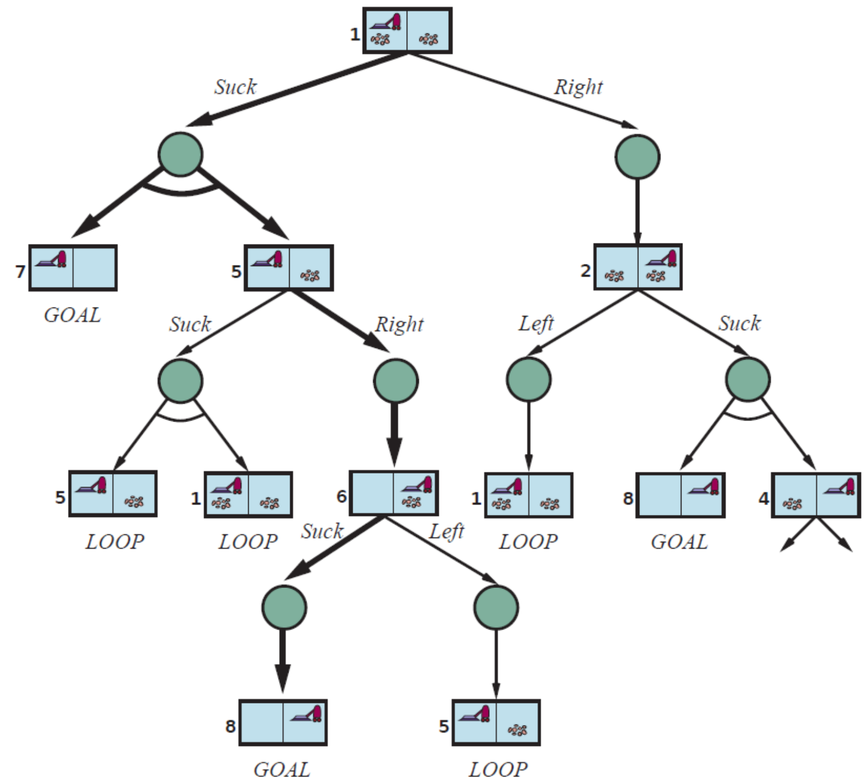

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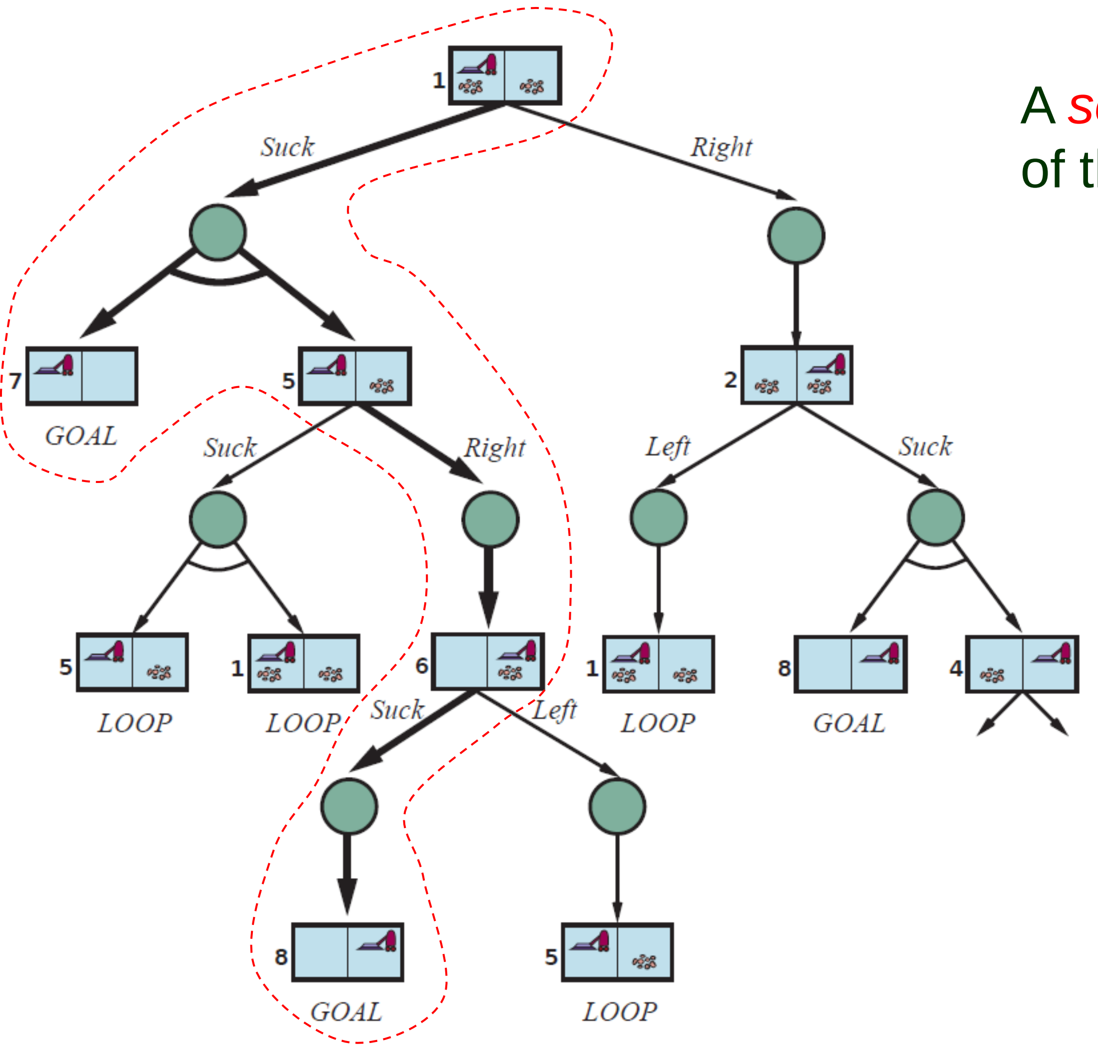
OR- and AND-nodes alternate in the tree.

Example of a (Partial) Tree



A *solution* is a connected portion of the AND-OR tree such that

- its root is the tree's root;
- every OR node has exactly one child (i.e., one of the actions);
- every AND node has all children (possible outcomes) from the corresponding action;
- all the leaves are goal nodes.

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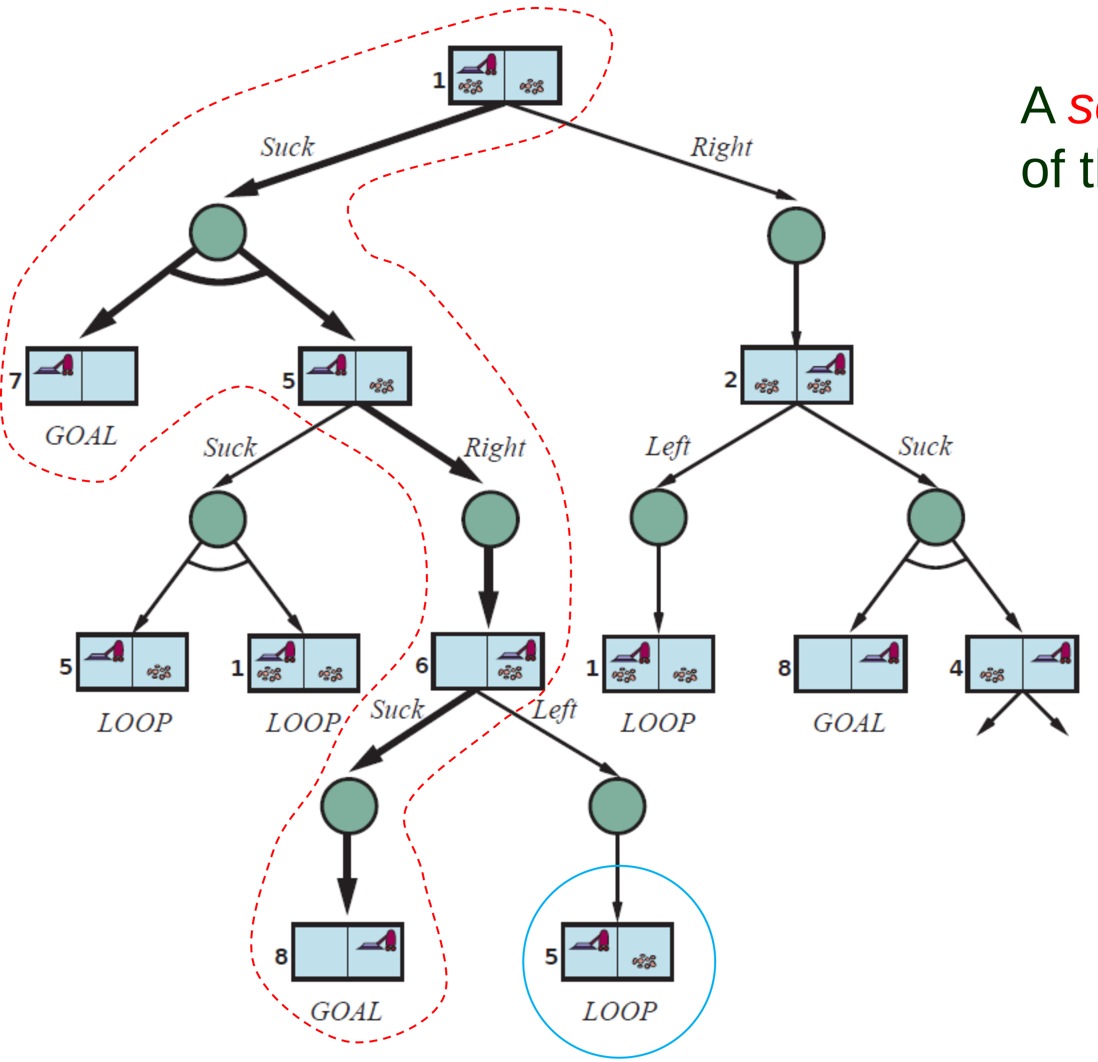

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DFS Implementation of AND-OR Tree Search

function AND-OR-SEARCH(*problem*) **returns** a conditional plan, or *failure*
  return OR-SEARCH(*problem*, *problem*.INITIAL, [ ])

function OR-SEARCH(*problem*, *state*, *path*) **returns** *a conditional plan, or failure*
  if *problem*.IS-GOAL(*state*) **then return** the empty plan
  if IS-CYCLE(*path*) **then return** *failure* // ignore a solution with a cycle. such a solution would
  for each *action* **in** *problem*.ACTIONS(*state*) **do** // imply the existence of a non-cyclic solution
    plan ← AND-SEARCH(*problem*, RESULTS(*state*, *action*), [*state*] + *path*]) // which can
    // be found.
    if $plan \neq failure$ **then return** [*action*] + *plan*]
  return *failure*

function AND-SEARCH(*problem*, *states*, *path*) **returns** *a conditional plan, or failure*
  for each s_i **in** *states* **do**
    $plan_i \leftarrow$ OR-SEARCH(*problem*, s_i, *path*)
    if $plan_i = failure$ **then return** *failure*
  return $\underbrace{[\textbf{if } s_1 \textbf{ then } plan_1 \textbf{ else if } s_2 \textbf{ then } plan_2 \textbf{ else } \ldots \textbf{if } s_{n-1} \textbf{ then } plan_{n-1} \textbf{ else } plan_n]}_{\text{Solution plan}}$

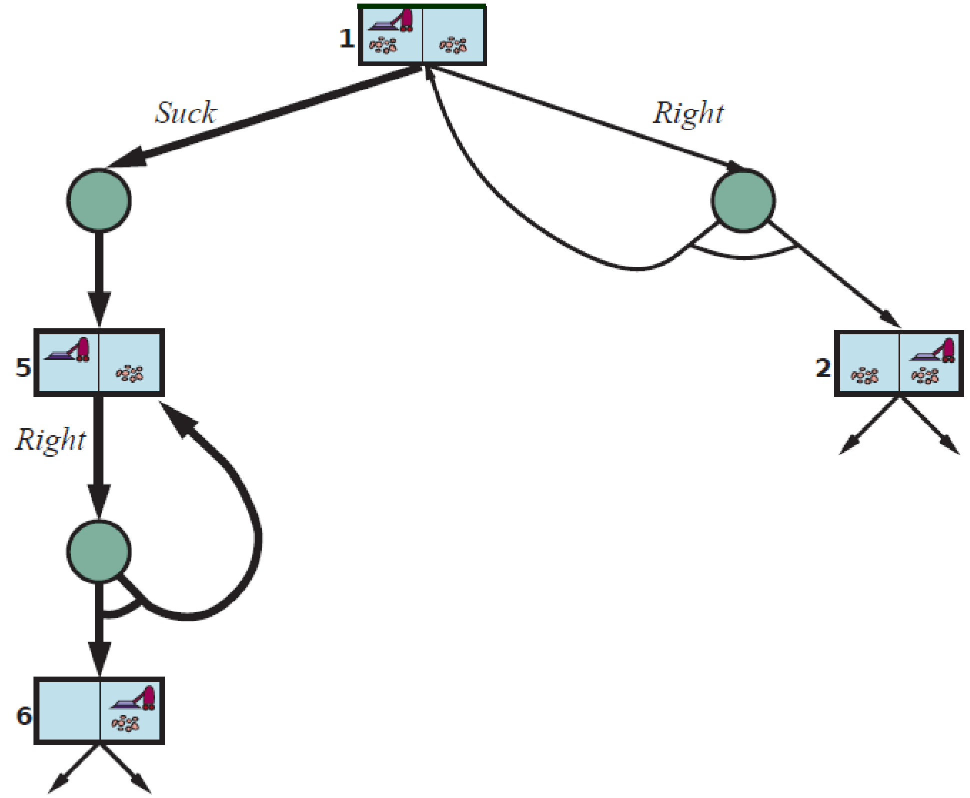
Cyclic Plan for a Solution

What if an action fails and the state is not changed?

Slippery vacuum world.

$$\text{State 1} \xrightarrow{\textit{Right}} \{1, 2\}$$

$$\text{State 5} \xrightarrow{\textit{Right}} \{5, 6\}$$



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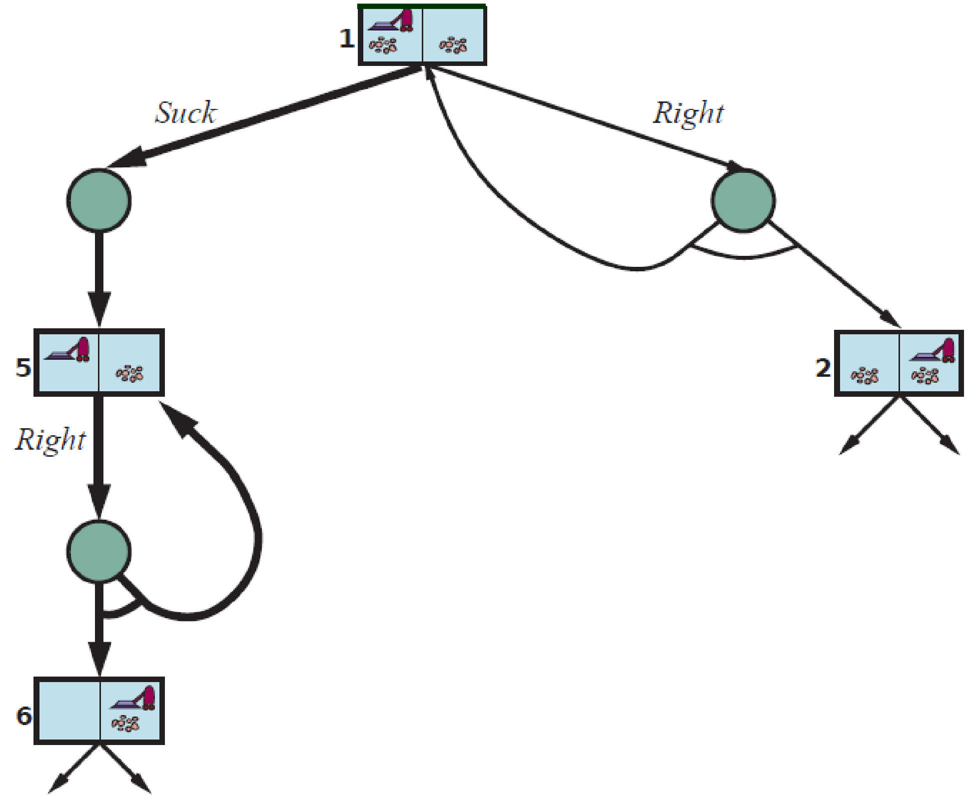

do
  Suck;
  if State = 5 then *Right*

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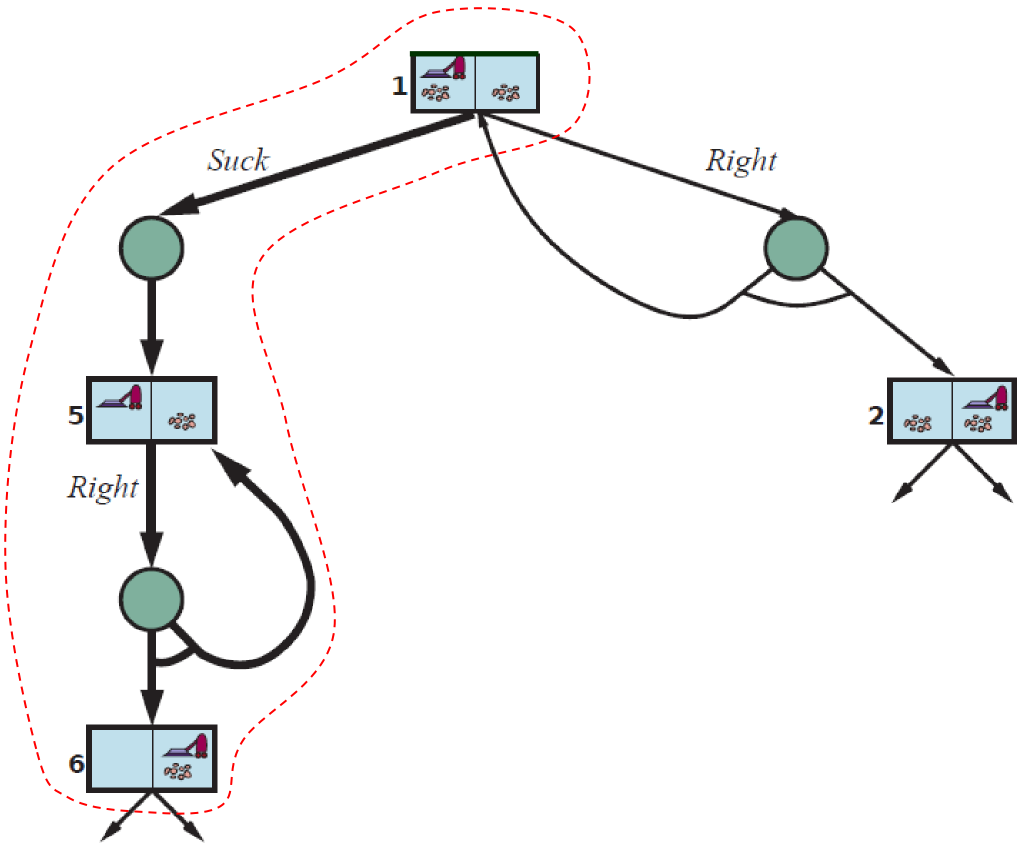

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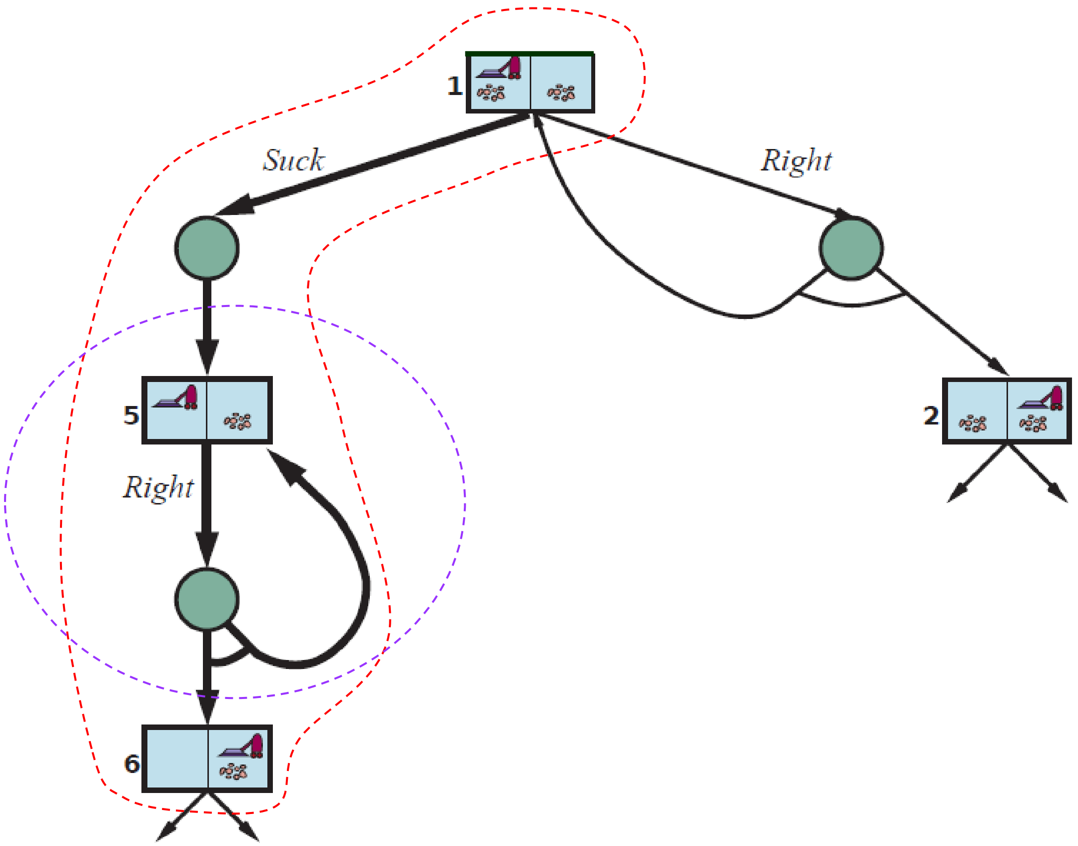

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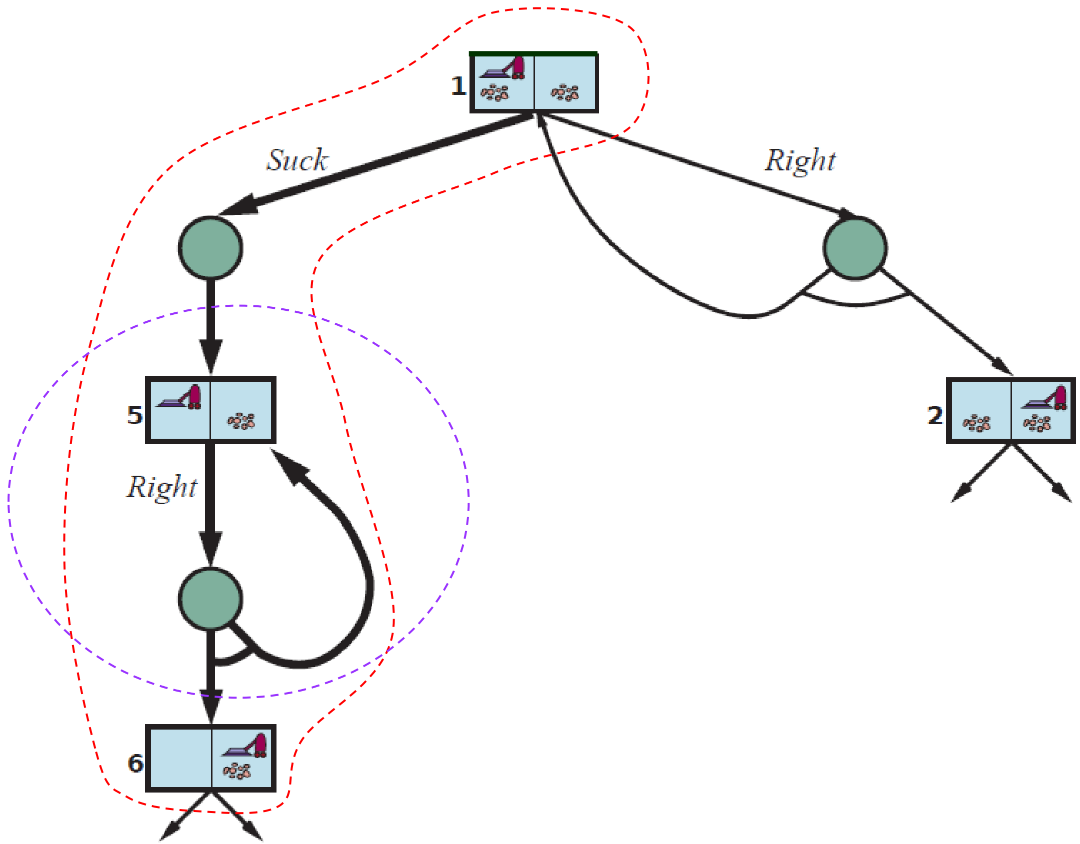

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The goal will be reached provided that each outcome of a nondeterministic action eventually occurs.