Heuristics in the A* Search

Outline

I. Properties of Heuristics

II. Comparison of Heuristics

* Figures are from the textbook site (or drawn by the instructor) unless the source is specifically cited.

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h_{SLD} is admissible because the actual distance to Bucharest cannot be less than the straight-line distance.

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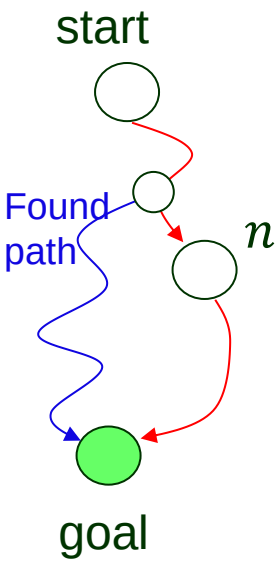

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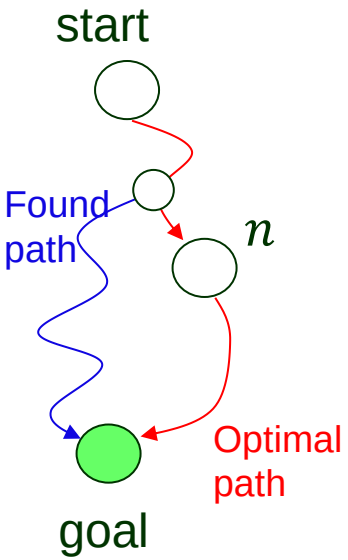

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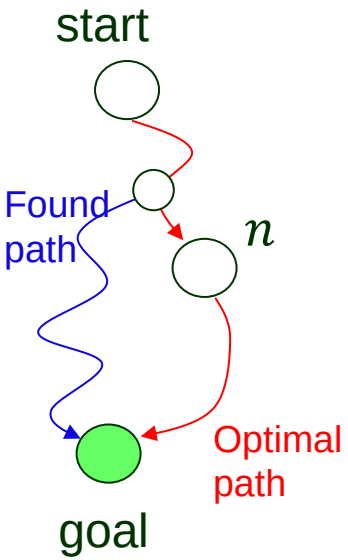

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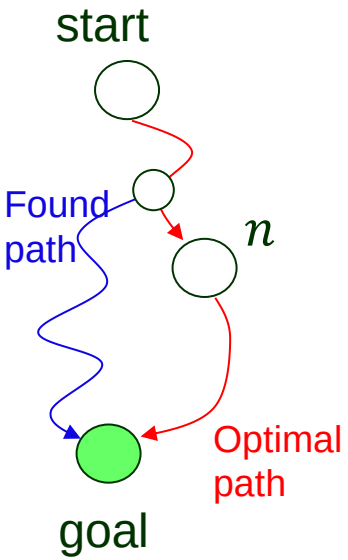

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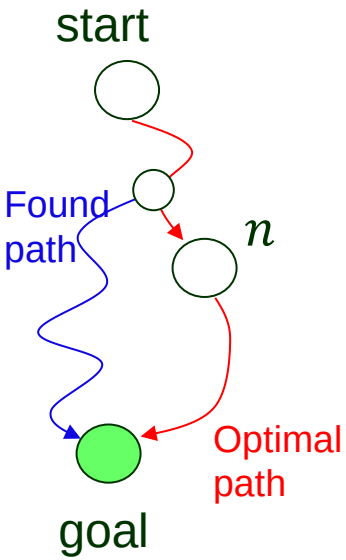

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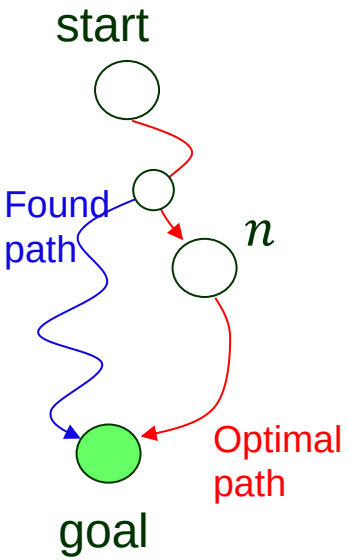

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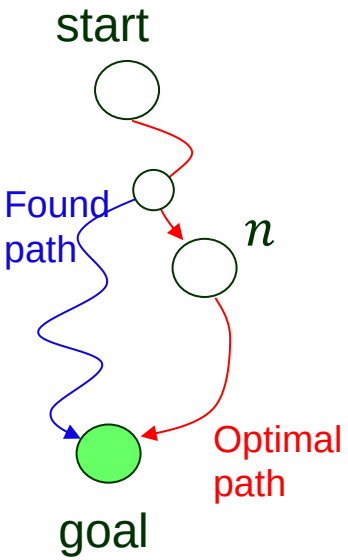

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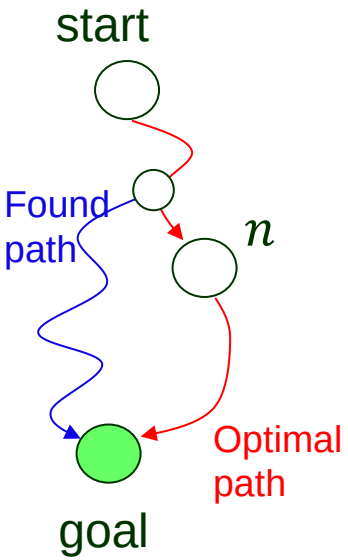

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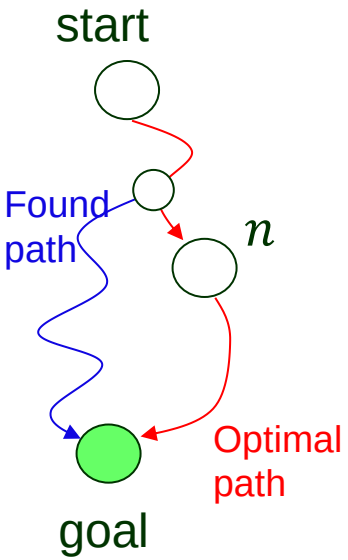

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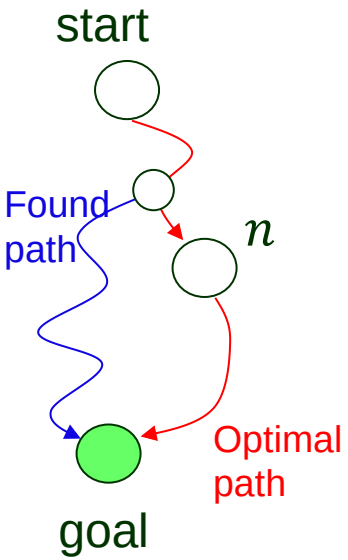

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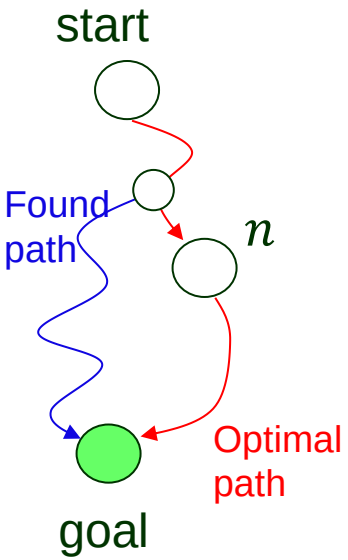

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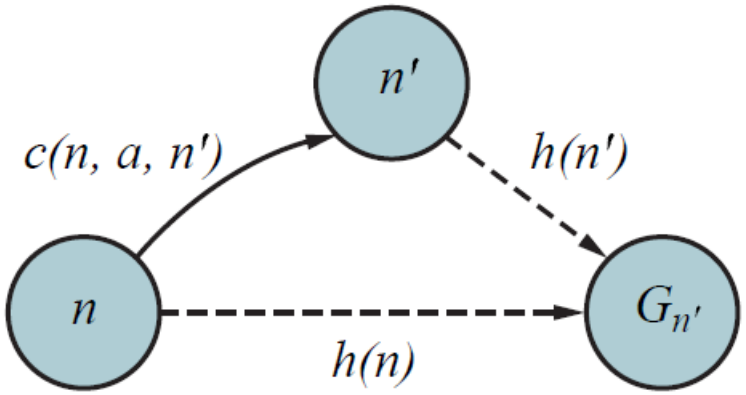
(*triangle inequality*)



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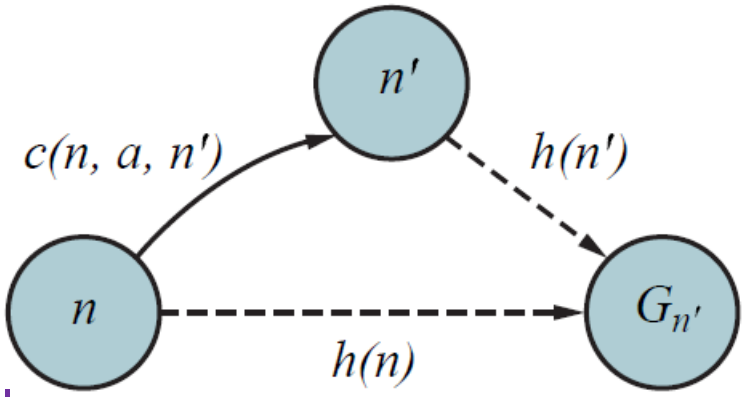

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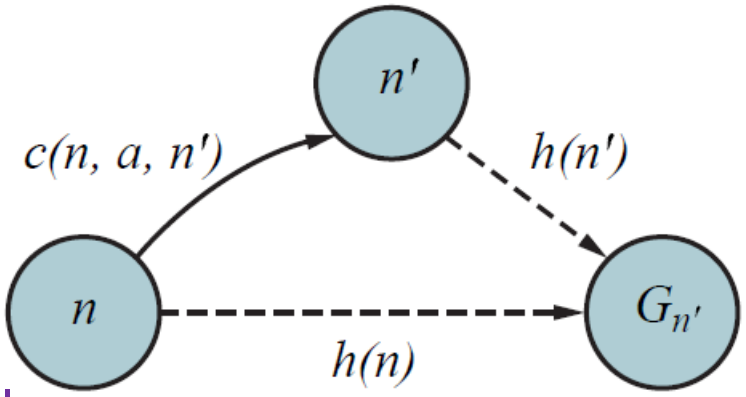

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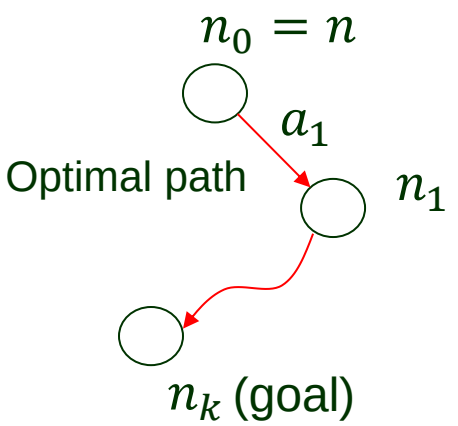

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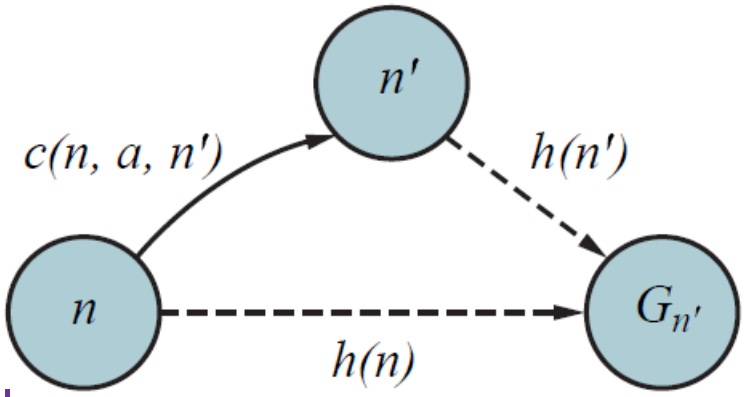

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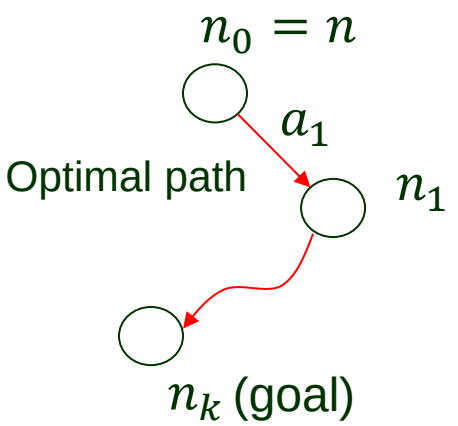

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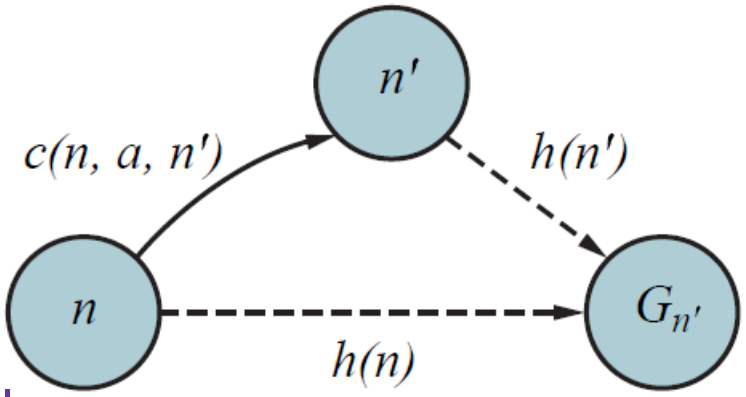

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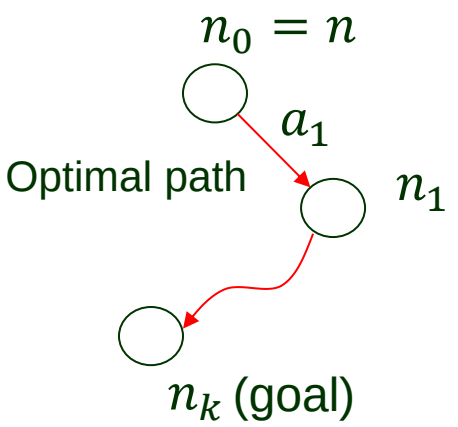

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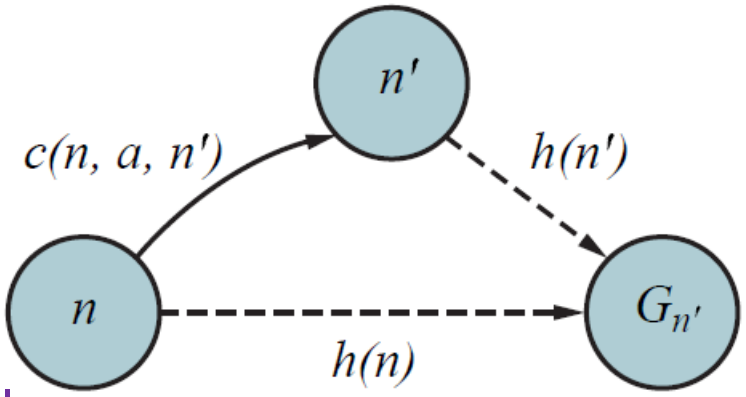

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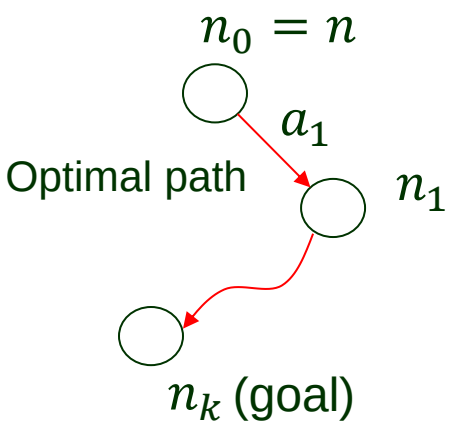

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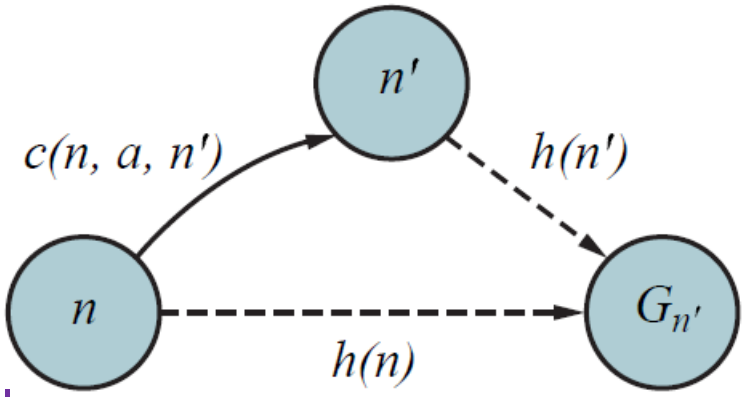

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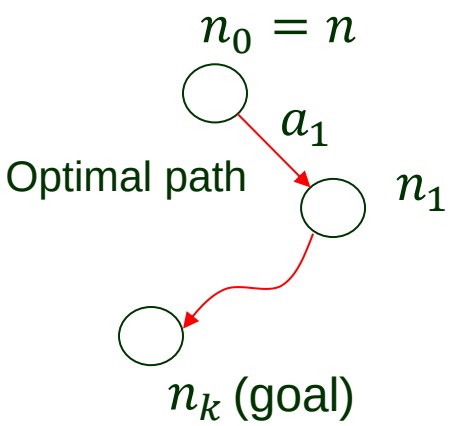

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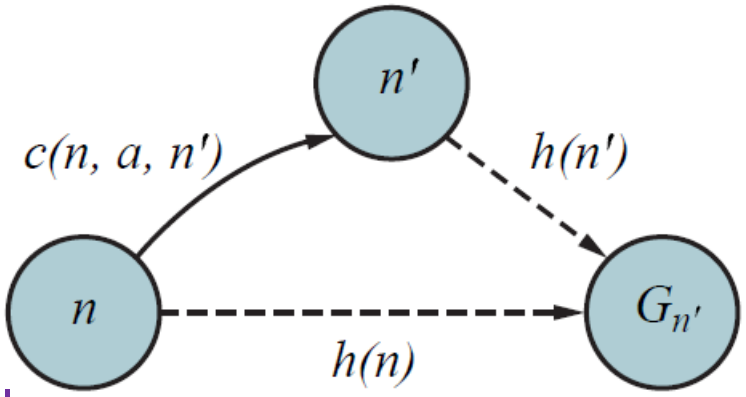

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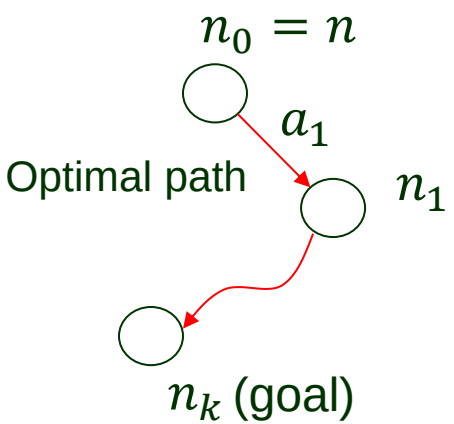

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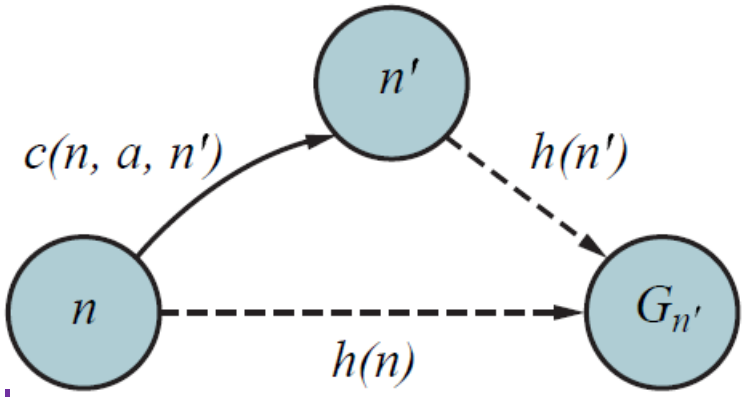

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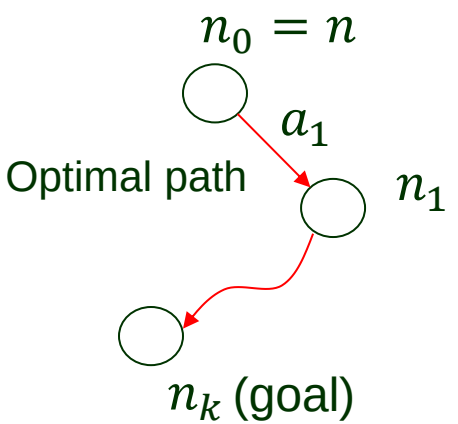
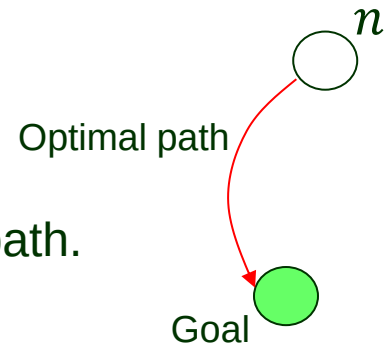


- A* with a consistent heuristic is cost-optimal.

What If the Heuristic Is Inadmissible?

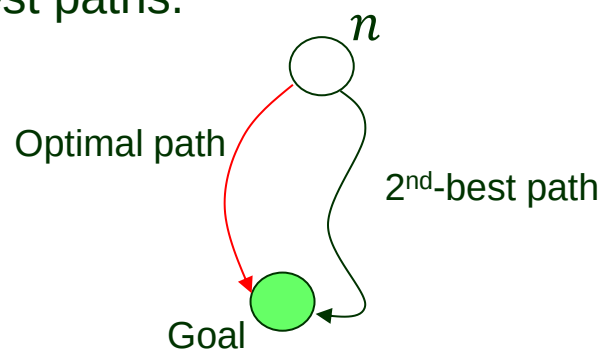
In such a situation, A* may or may not be cost-optimal.

Two of the cases where A* finds an optimal path:



- h is admissible for all the nodes on one optimal path.

- $h(n)$ does not overestimate the cost on each node n by more than the difference between the costs of the optimal and the second-best paths.



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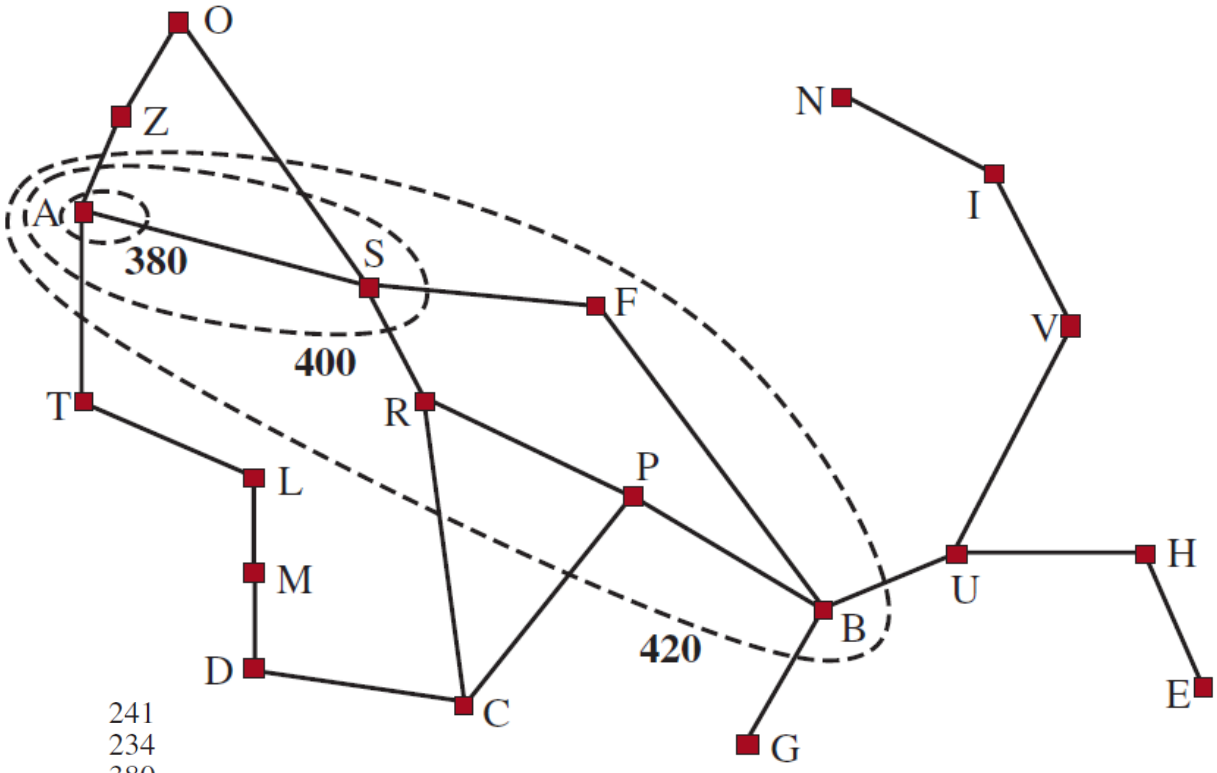

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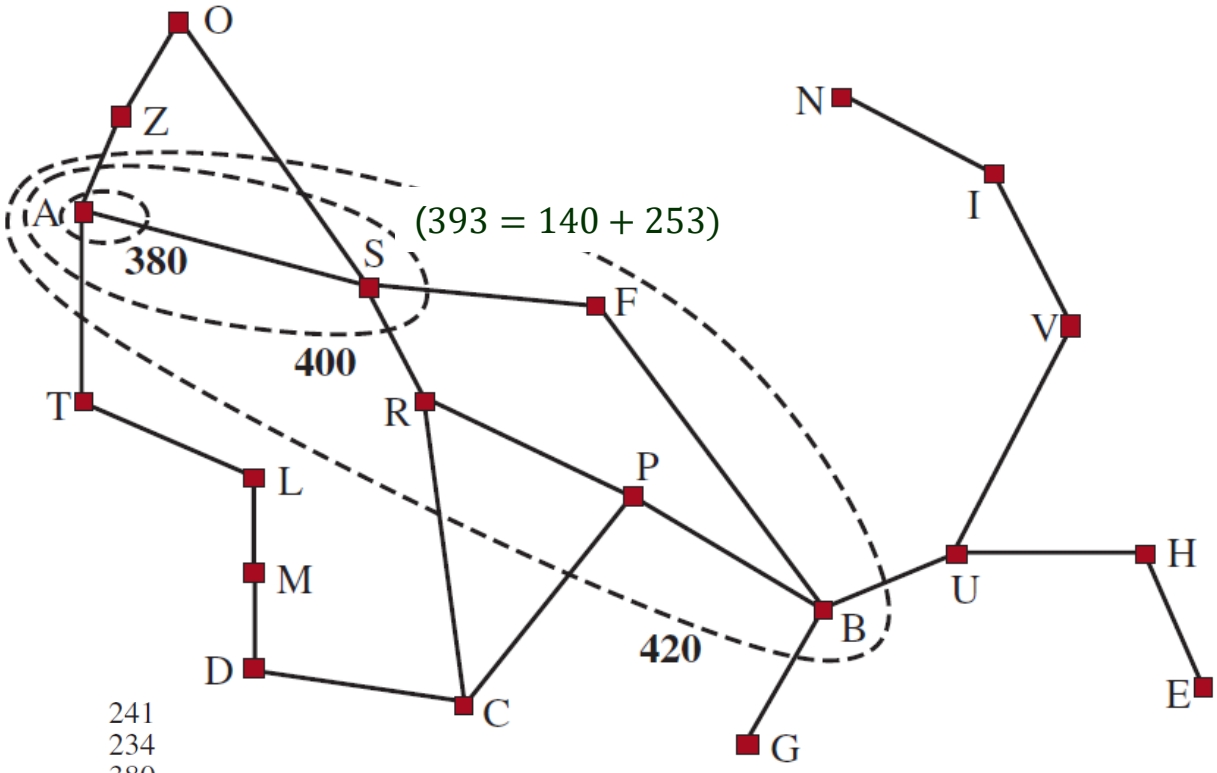

h_{SLD}: straight-line distance

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Some Search Contours

- Dijkstra's algorithm (i.e., uniform search) would have contours of g-cost to "circle" around the start state.

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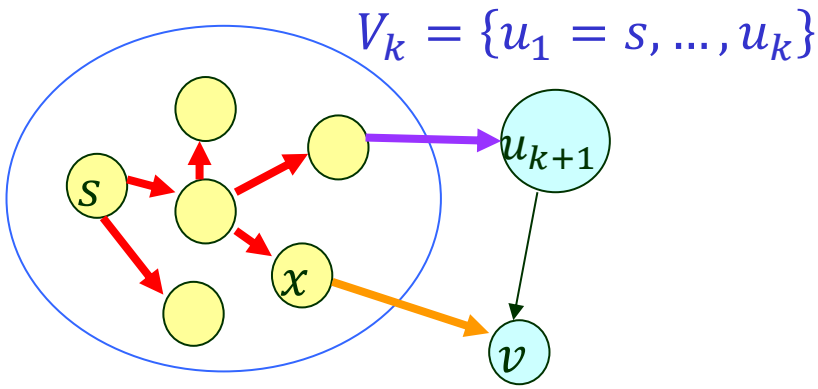

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$$d(v) = \min\{\, d(v), d(u_{k+1}) + w(u_{k+1}, v) \,\}$$

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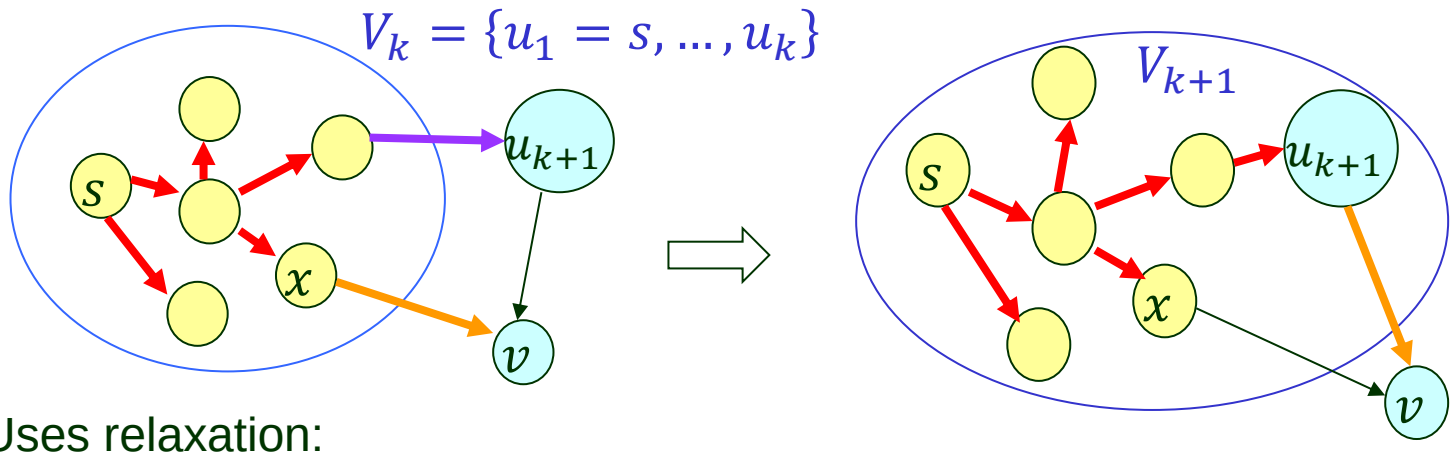

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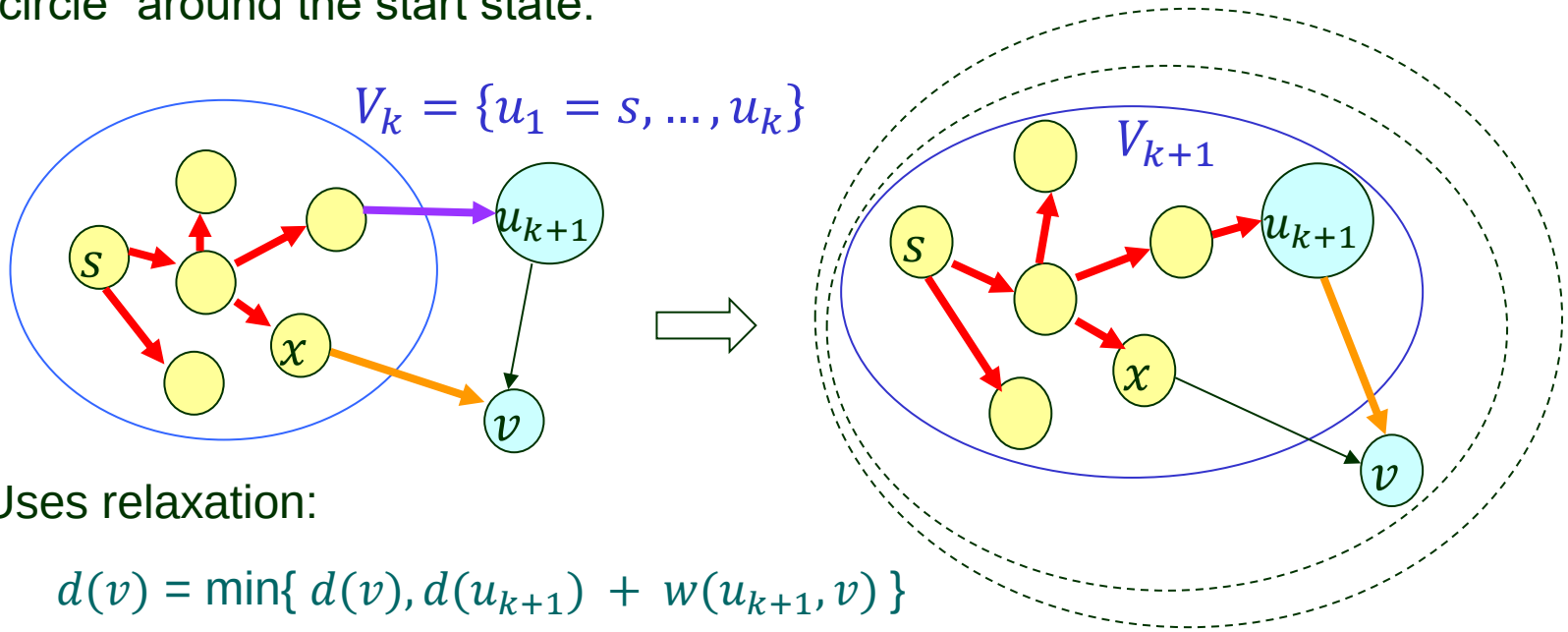

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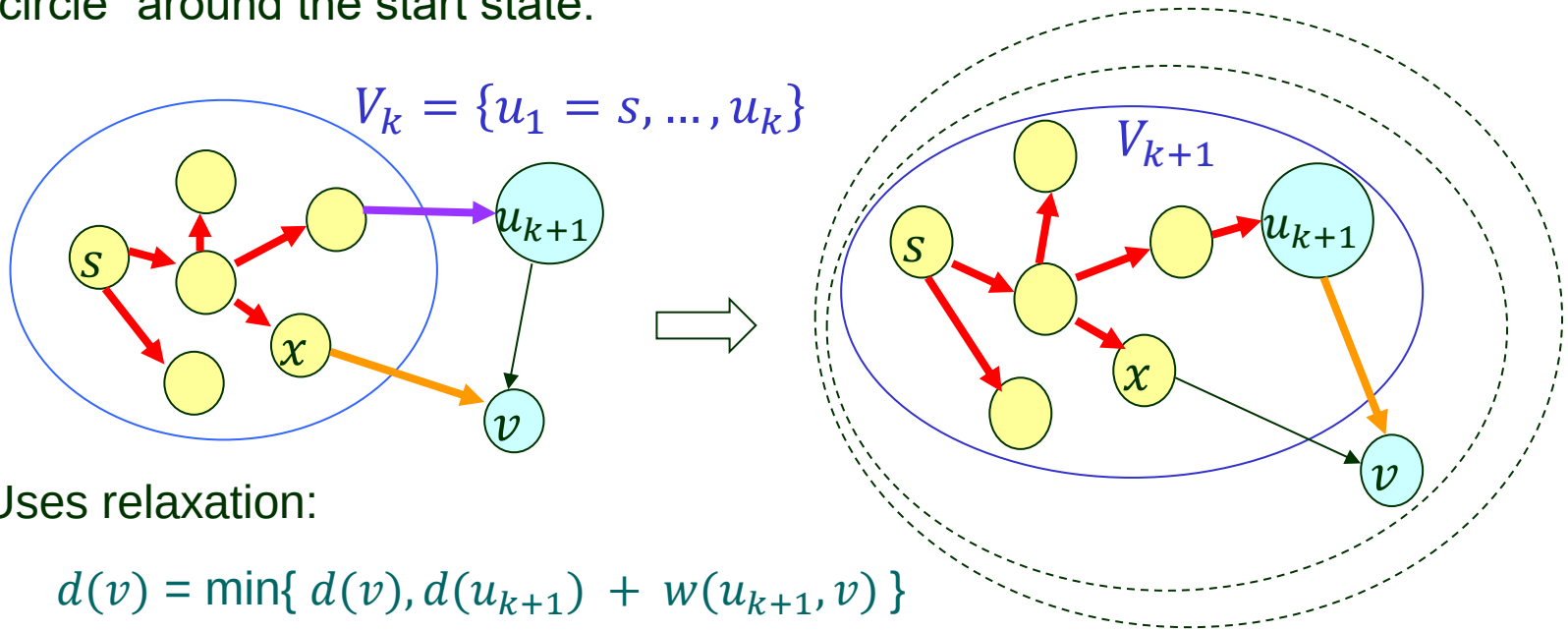

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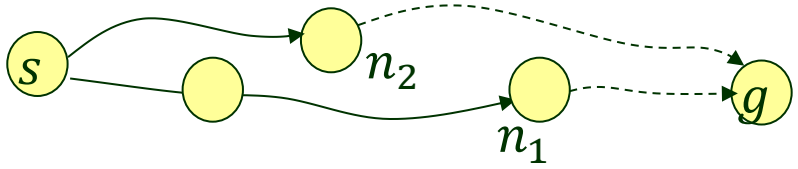
- With a good h, the $g + h$ bands will stretch toward a goal state.



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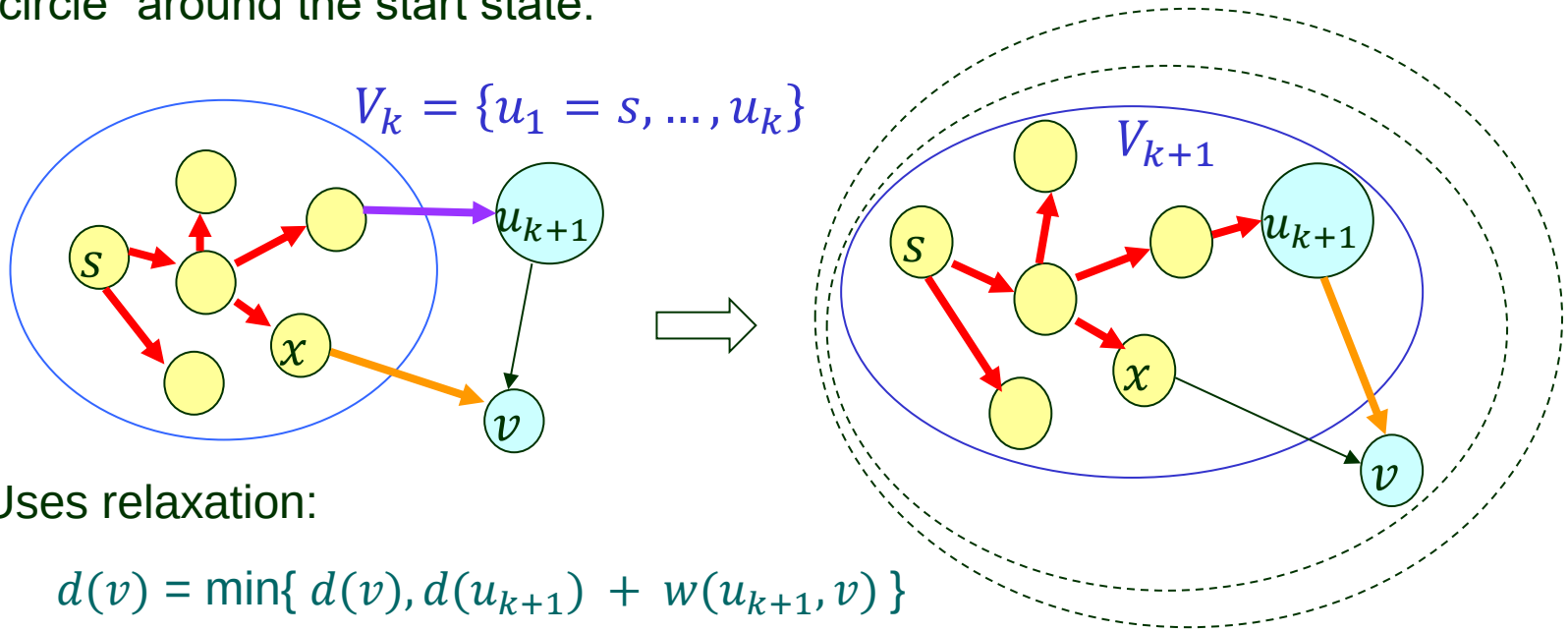

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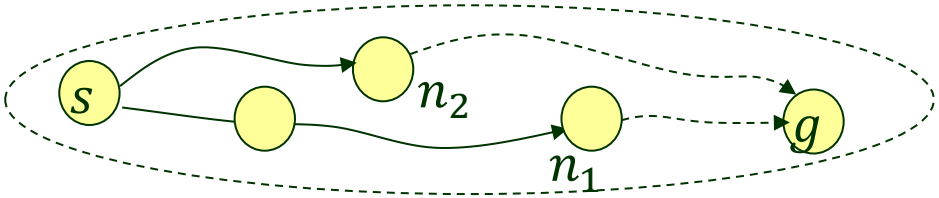
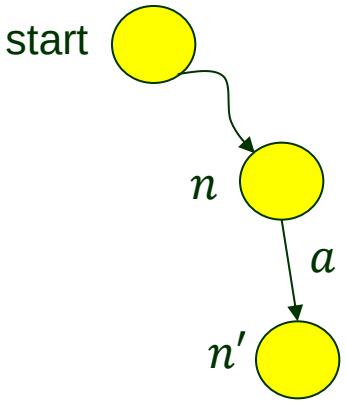


Monotonicity?

- The g cost increases along a path because action costs are positive.



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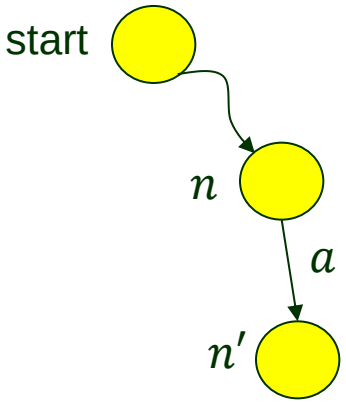

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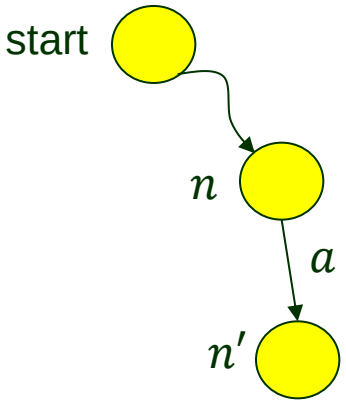

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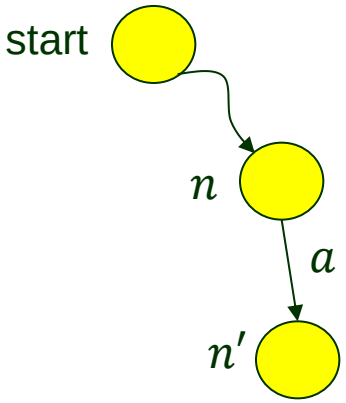

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Monotonicity is equivalent to consistency for a heuristic.

When Consecutive Nodes Are Scored the Same

$$g(n) + h(n) = \overbrace{g(n) + c(n, a, n')}^{= g(n')} + h(n')$$

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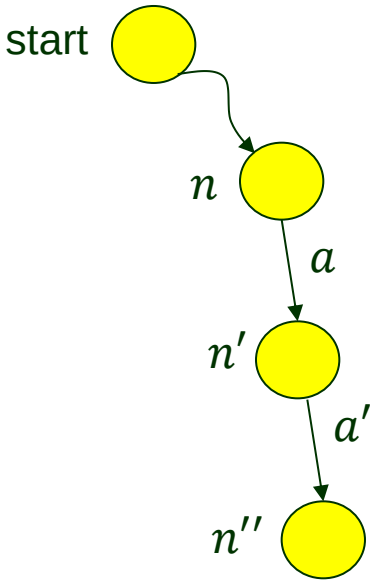

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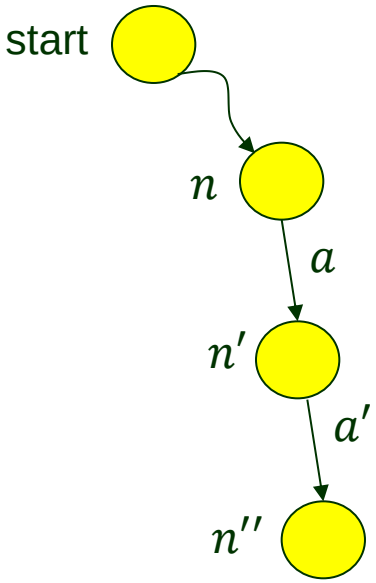

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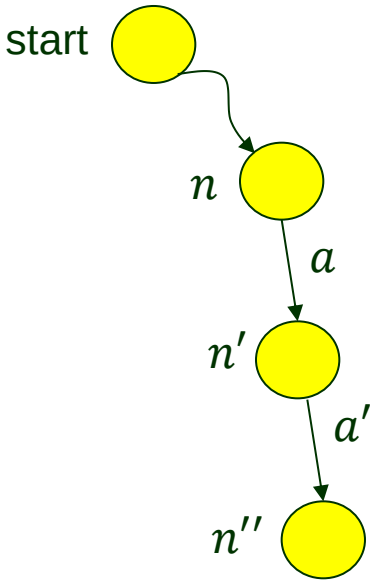

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h decreases as much as g increases after an action.

Efficiency of A*

h: admissible

C^*: cost of the optimal solution path

- A* will expand every node reachable via a sequence of nodes that have costs $< C^*$.
- A* will not expand any node n with $f(n) > C^*$.
- A* might expand a node n with cost $f(n) = C^*$ before selecting a goal node.

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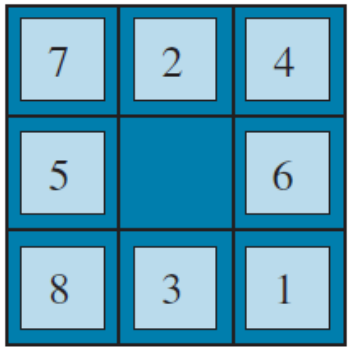
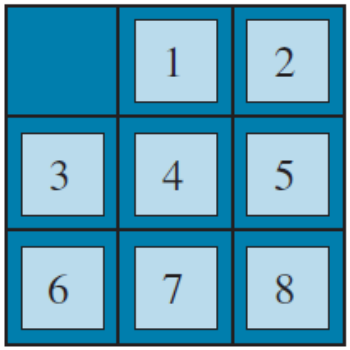
A* prunes away nodes unnecessary for finding an optimal solution.

- A* may take exponential time with a poor heuristic function.

II. 8-Puzzle: Large Search Space



Start State



Goal State

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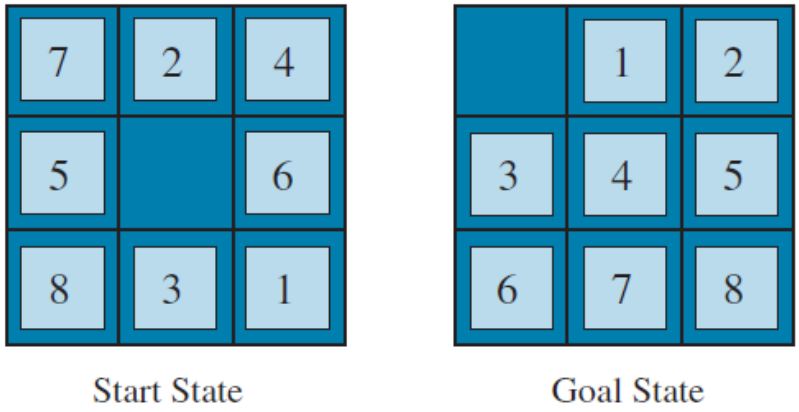

$\frac{9!}{2} = 181{,}400$ reachable states from start.

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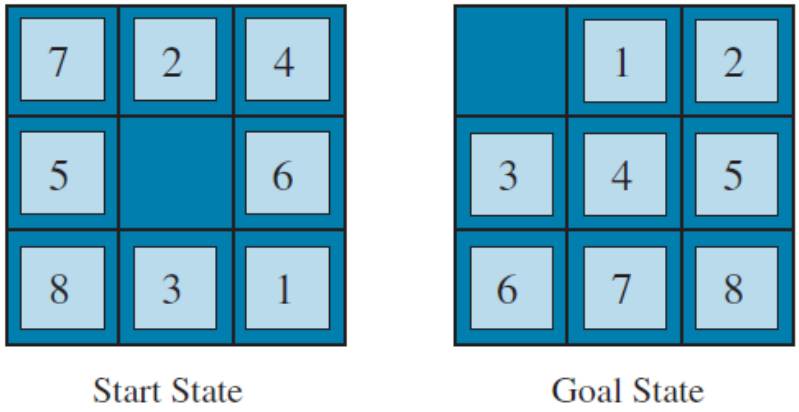
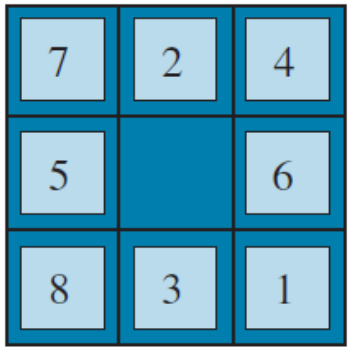


$\frac{9!}{2} = 181{,}400$ reachable states from start.

$\frac{16!}{2} > 10^{13}$ reachable states for the 15-puzzle!

Two Heuristics



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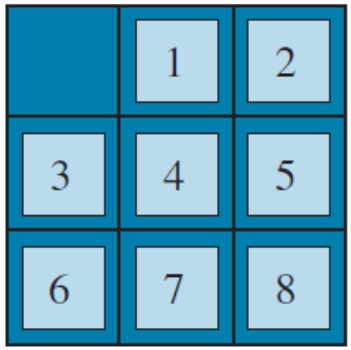
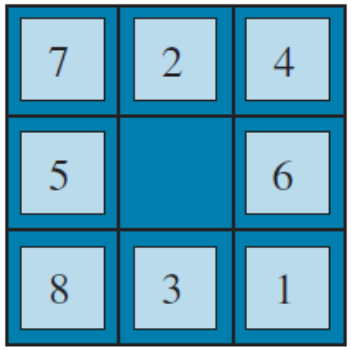


Goal State

Heuristics are needed for searching the vast state space.

Two Heuristics



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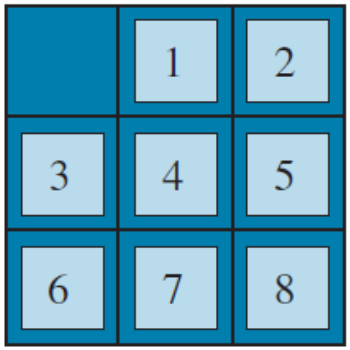

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Heuristics are needed for searching the vast state space.

- h_1 = # tiles misplaced

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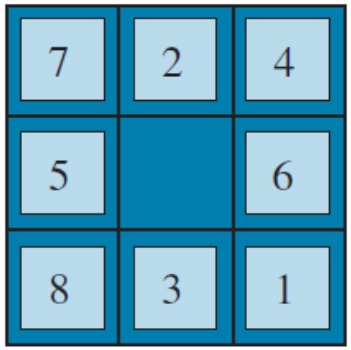

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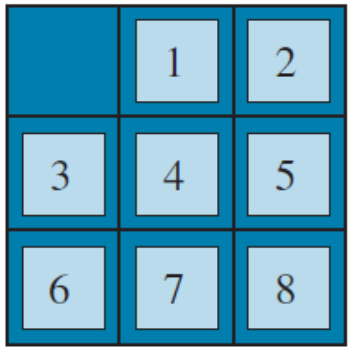

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Misplaced tiles:

1, 2, 3, 4, 5, 6, 7, 8

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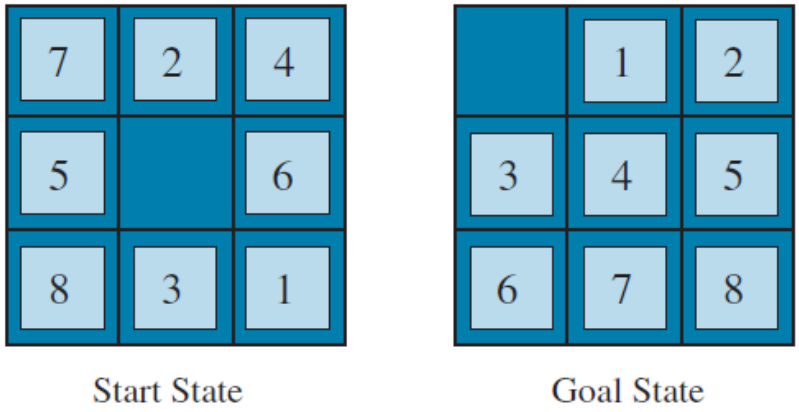

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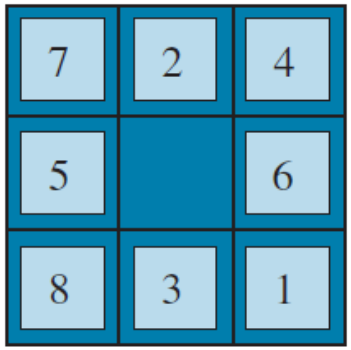

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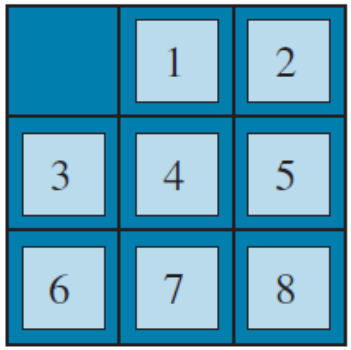

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Heuristics are needed for searching the vast state space.

♦ h_1 = # tiles misplaced

Admissible: any tile out of place will require ≥ 1 move to fix.

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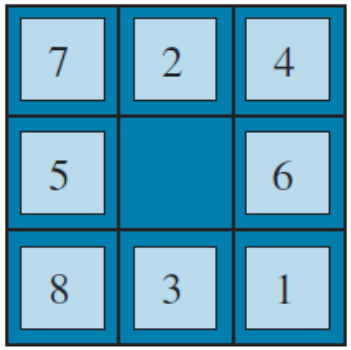

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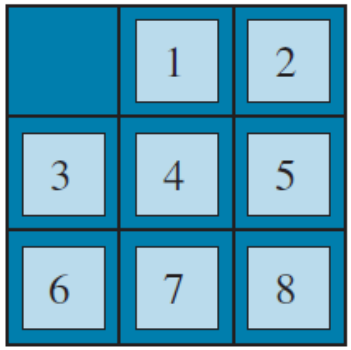

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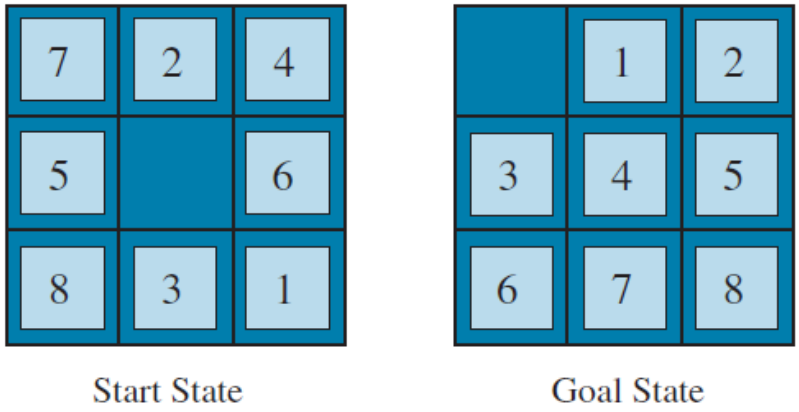

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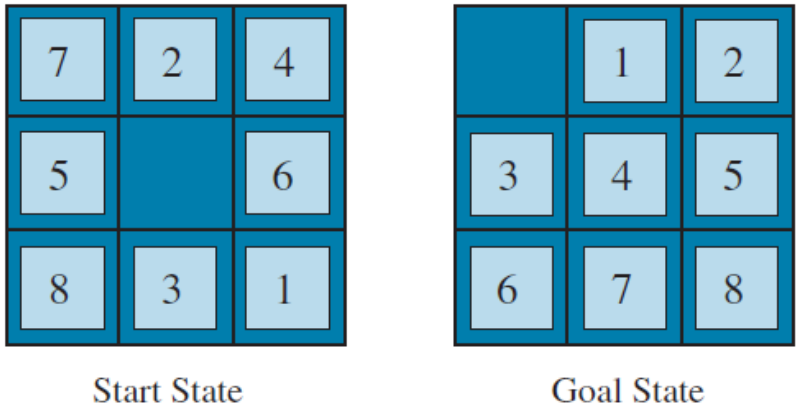

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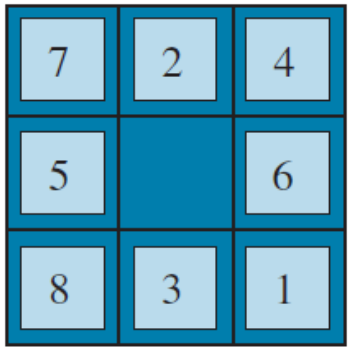

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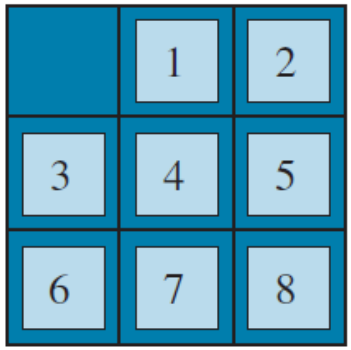

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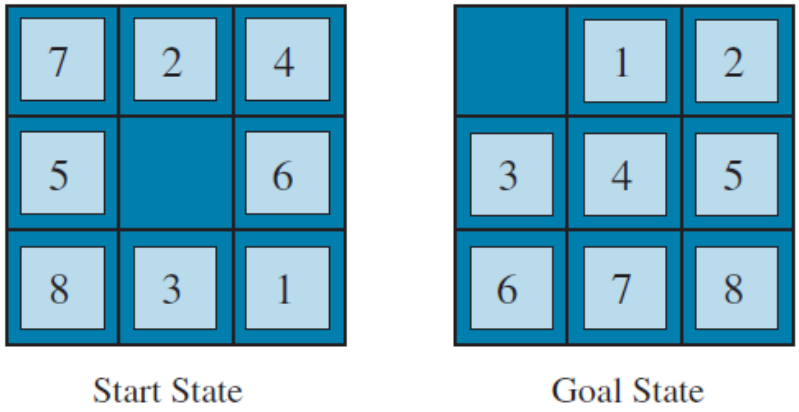

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♦ h_2 = sum of Manhattan distances of the tiles from their goal positions

Admissible: every move reduces the Manhattan distance of only one tile by ≤ 1.

Neither heuristic overestimates the shortest solution (26 actions for the problem instance).

Heuristic Accuracy on Performance

Quality of a heuristic is often measured by the effective branching factor.

If A* generates N nodes to find a solution at depth d, then its *effective branching factor* b^* is the root of the following equation:

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d$$
$$= \left(1 - (b^*)^{d+1}\right)/(1 - b^*)$$

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(polynomial root finding – Com S 4770/5770)

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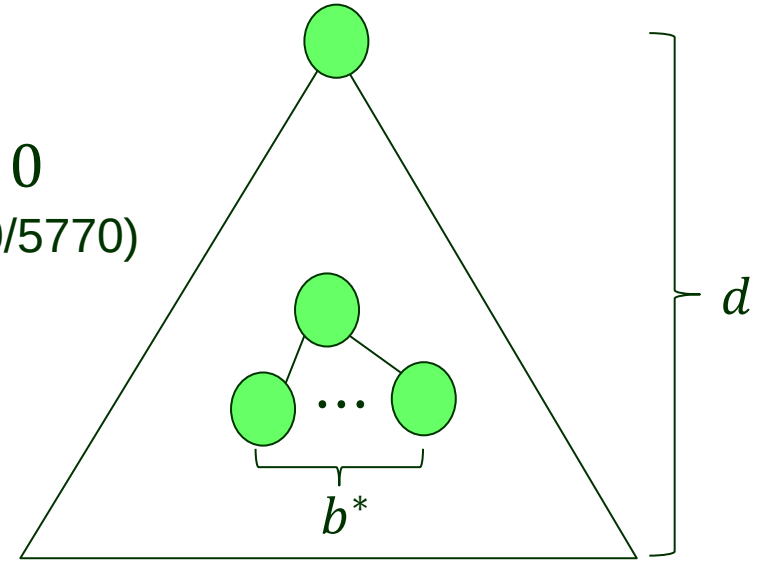
If A* generates N nodes to find a solution at depth d, then its *effective branching factor* b^* is the root of the following equation:

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d$$
$$= \left(1 - (b^*)^{d+1}\right)/(1 - b^*)$$

$$\Longrightarrow \quad (b^*)^{d+1} - (N+1)b^* + N = 0$$

(polynomial root finding – Com S 4770/5770)

Intuitively, the $N + 1$ nodes handled by A* would fill a tree of height d in which every node at depth $< d$ has exactly b^*children.



Heuristic Accuracy on Performance

Quality of a heuristic is often measured by the effective branching factor.

If A* generates N nodes to find a solution at depth d, then its *effective branching factor* b^* is the root of the following equation:

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d$$
$$= \left(1 - (b^*)^{d+1}\right)/(1 - b^*)$$

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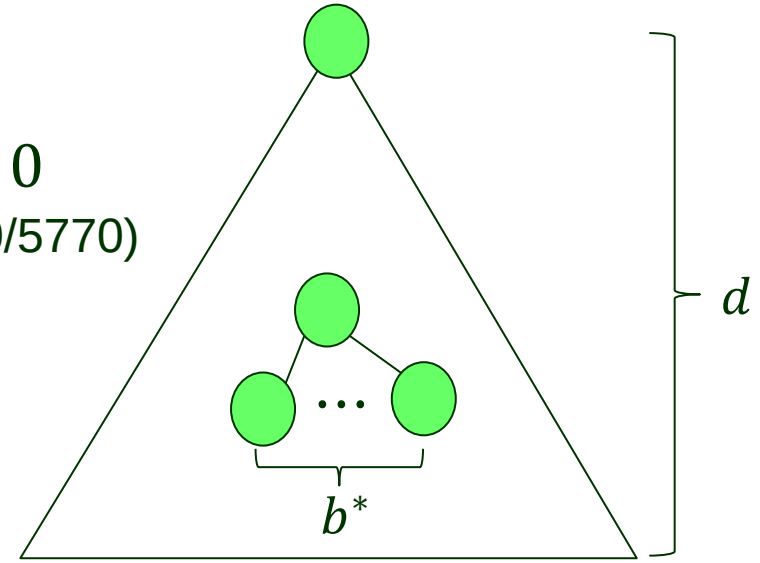
Intuitively, the $N + 1$ nodes handled by A* would fill a tree of height d in which every node at depth $< d$ has exactly b^*children.



e.g. A* finds a solution at depth 5 using 52 nodes has $b^* = 1.92$.

Performance Comparison on 8-Puzzle

	Search Cost (nodes generated)			Effective Branching Factor		
d	BFS	A*(h_1)	A*(h_2)	BFS	A*(h_1)	A*(h_2)
6	128	24	19	2.01	1.42	1.34
8	368	48	31	1.91	1.40	1.30
10	1033	116	48	1.85	1.43	1.27
12	2672	279	84	1.80	1.45	1.28
14	6783	678	174	1.77	1.47	1.31
16	17270	1683	364	1.74	1.48	1.32
18	41558	4102	751	1.72	1.49	1.34
20	91493	9905	1318	1.69	1.50	1.34
22	175921	22955	2548	1.66	1.50	1.34
24	290082	53039	5733	1.62	1.50	1.36
26	395355	110372	10080	1.58	1.50	1.35
28	463234	202565	22055	1.53	1.49	1.36

Figure 3.26 Comparison of the search costs and effective branching factors for 8-puzzle problems using breadth-first search, A* with h_1 (misplaced tiles), and A* with h_2 (Manhattan distance). Data are averaged over 100 puzzles for each solution length d from 6 to 28.