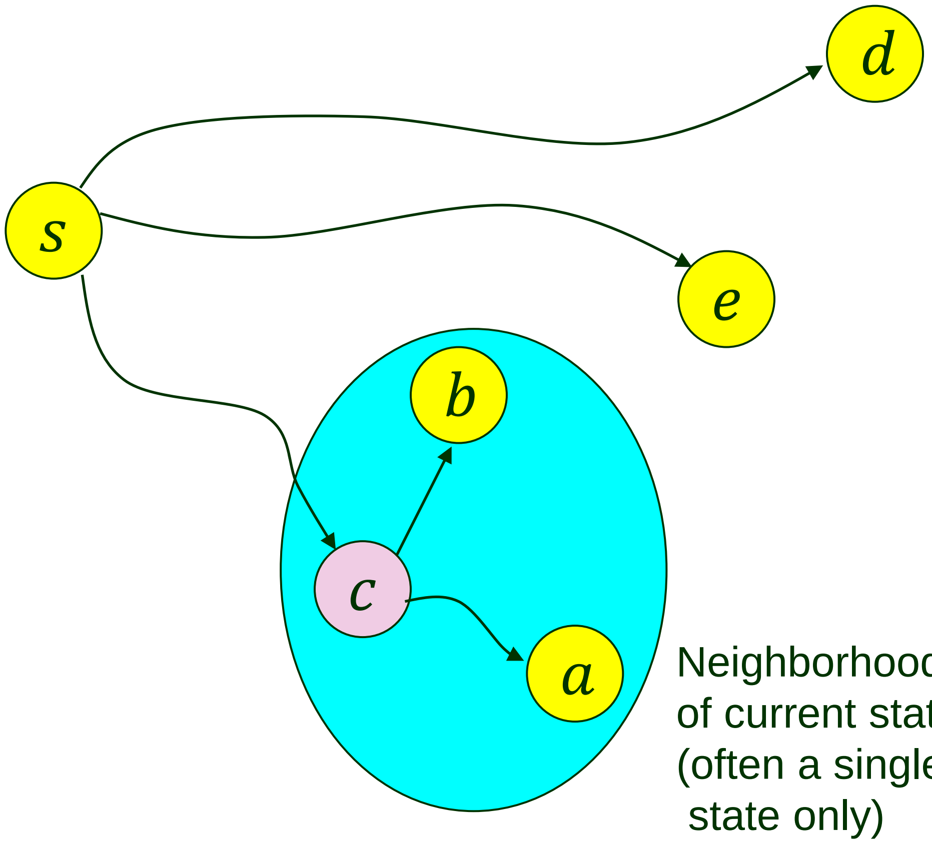
# Local Search

Evaluate and modify one or more *current states* rather than systematically exploring paths from an initial state.




Neighborhood
of current states
(often a single
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* Figures are from the textbook site (or drawn by the instructor).

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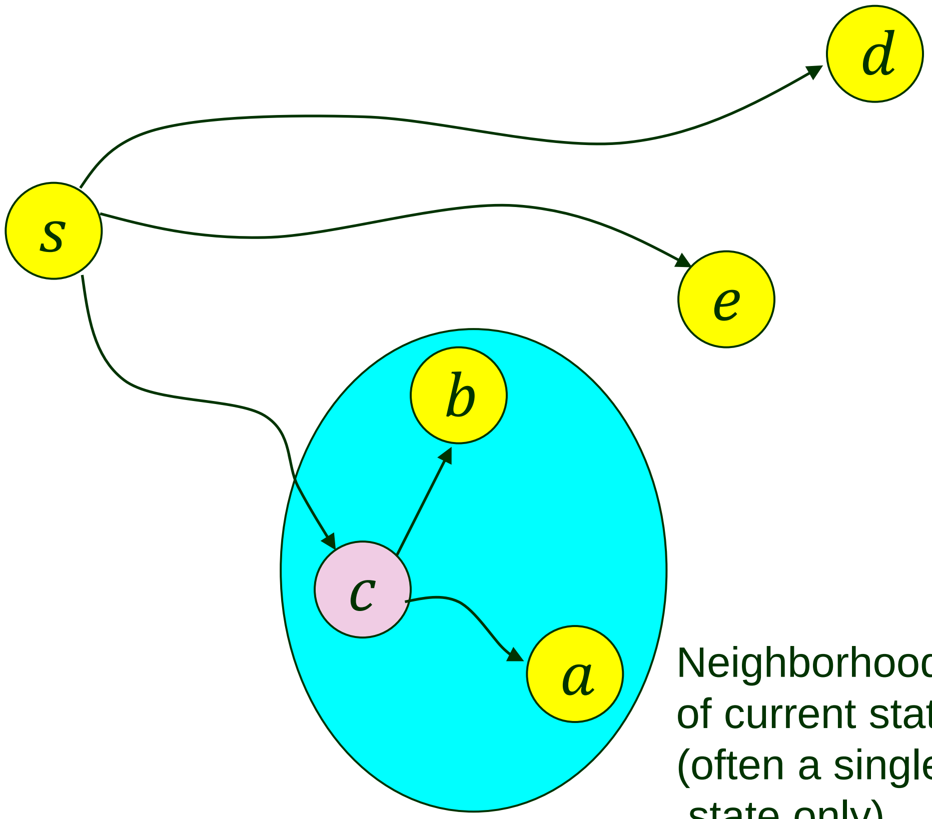
## Outline

I. Hill climbing

II. Simulated annealing

III. Genetic algorithms

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# Advantages of Local Search

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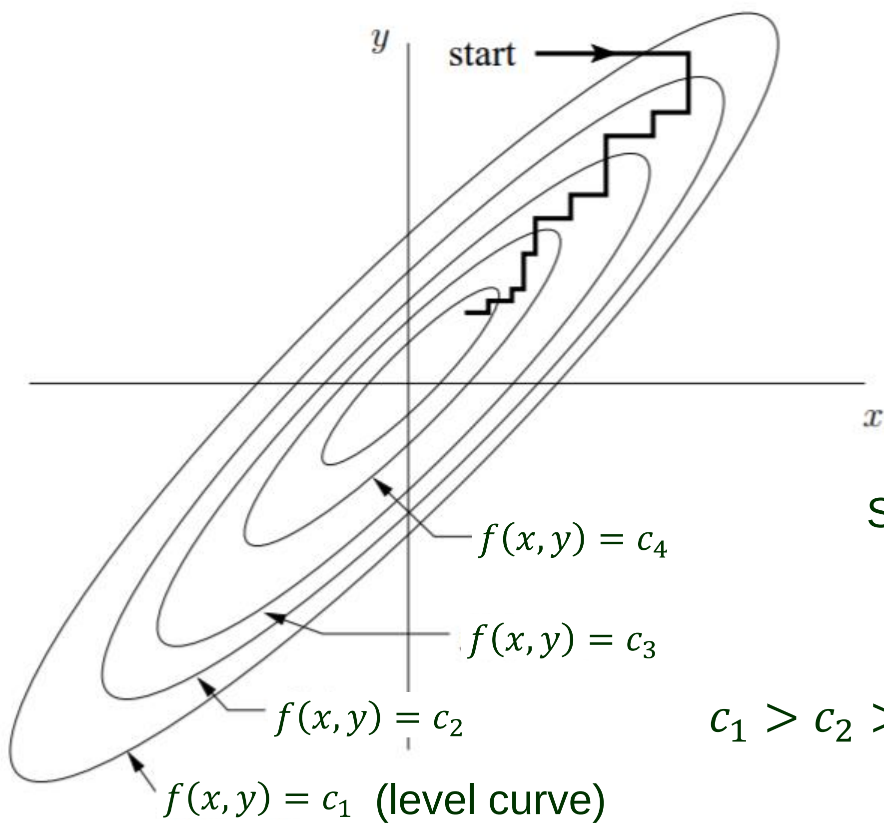
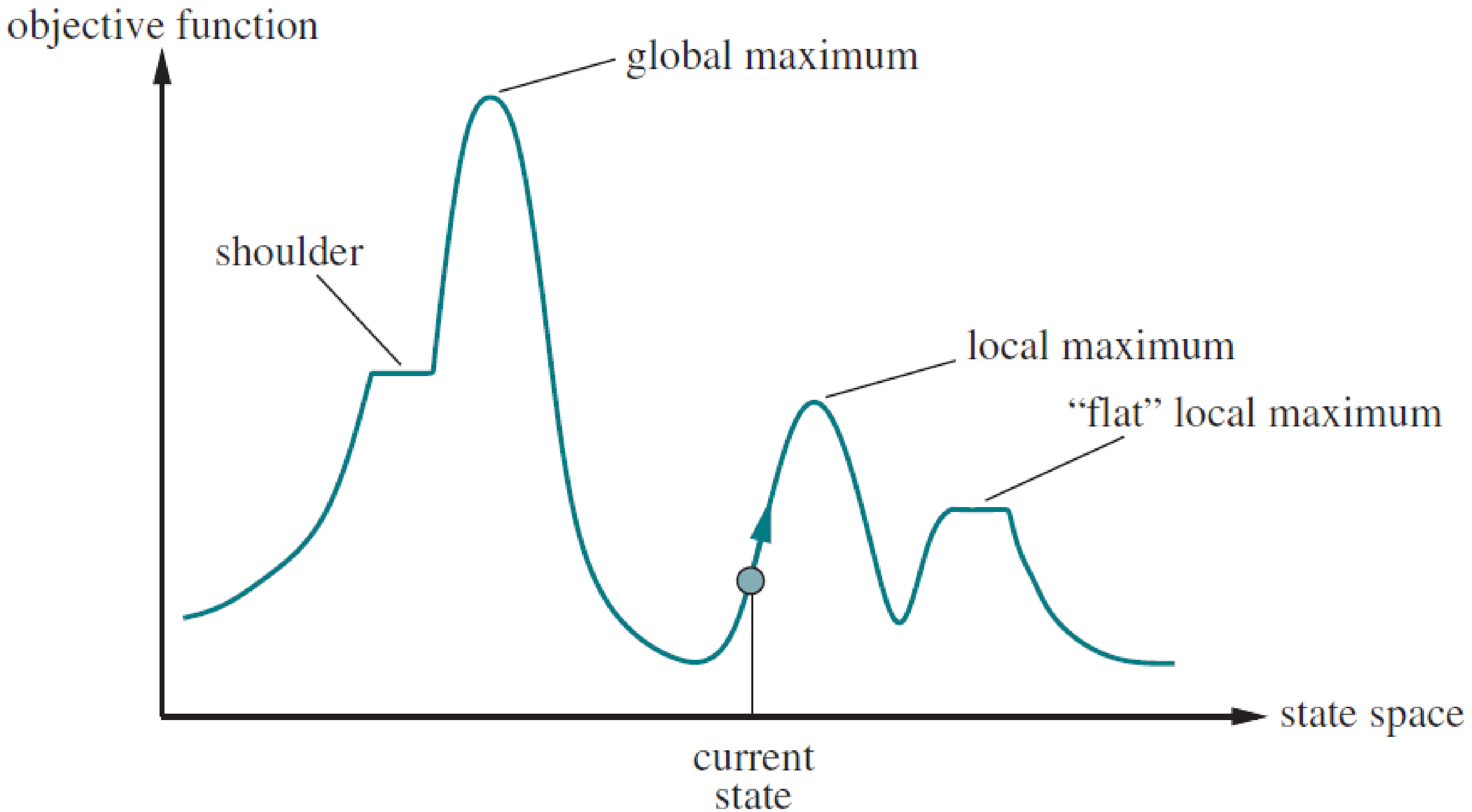



Maximize $f(x, y)$

Steepest ascent
(along $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$)

$c_1 > c_2 > c_3 > c_4$

# State Space Landscape



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Hill climbing randomly picks one.

# Efficiency?


18 12 14 13 13 12 14 14
14 16 13 15 12 14 12 16
14 12 18 13 15 12 14 14
15 14 14 13 16 13 16
14 17 15 14 16 16
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18 14 15 15 14 16
14 14 13 17 12 14 12 18

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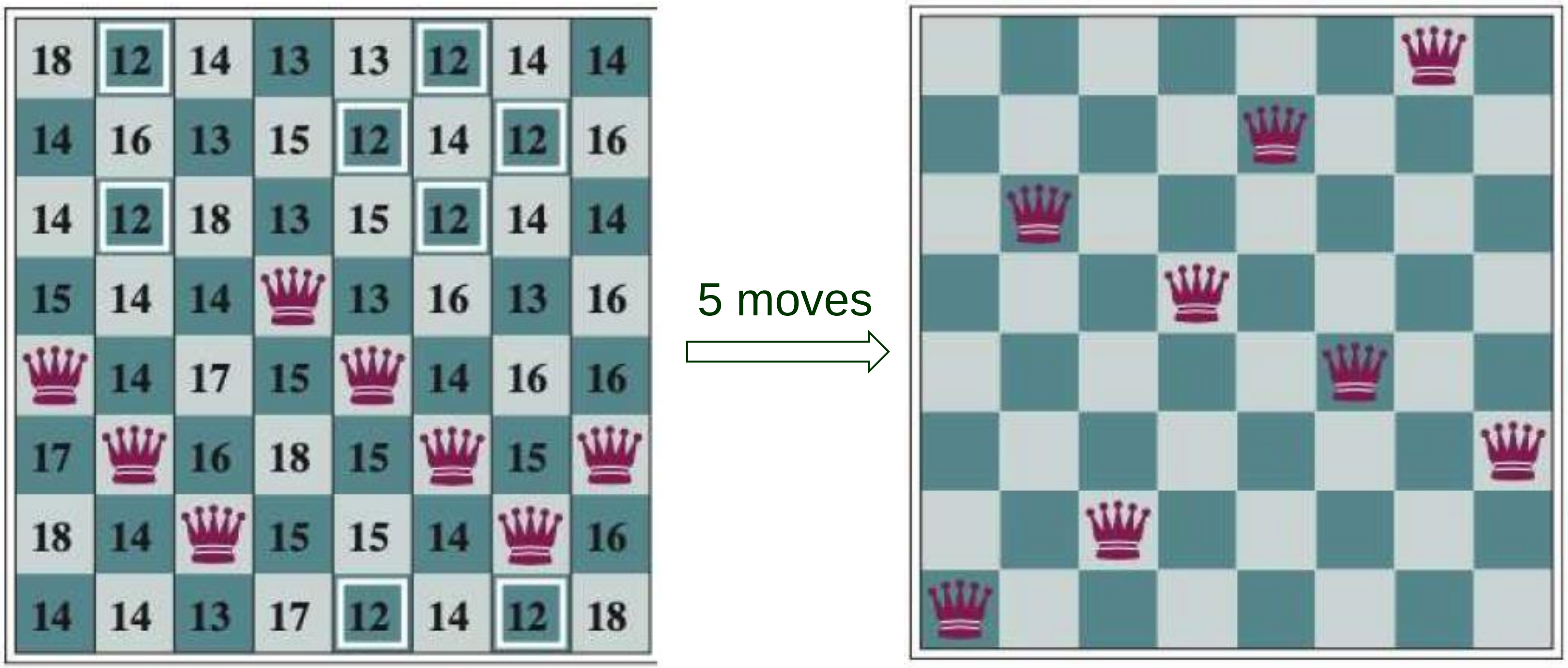
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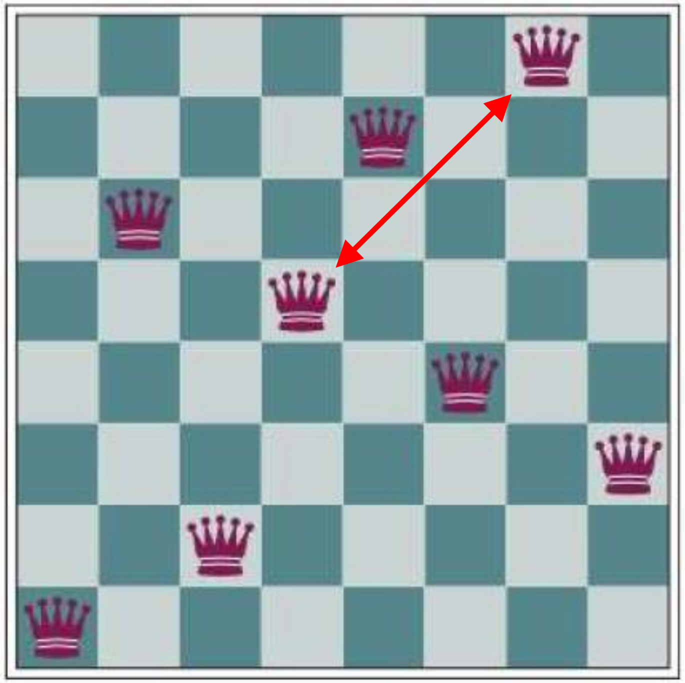

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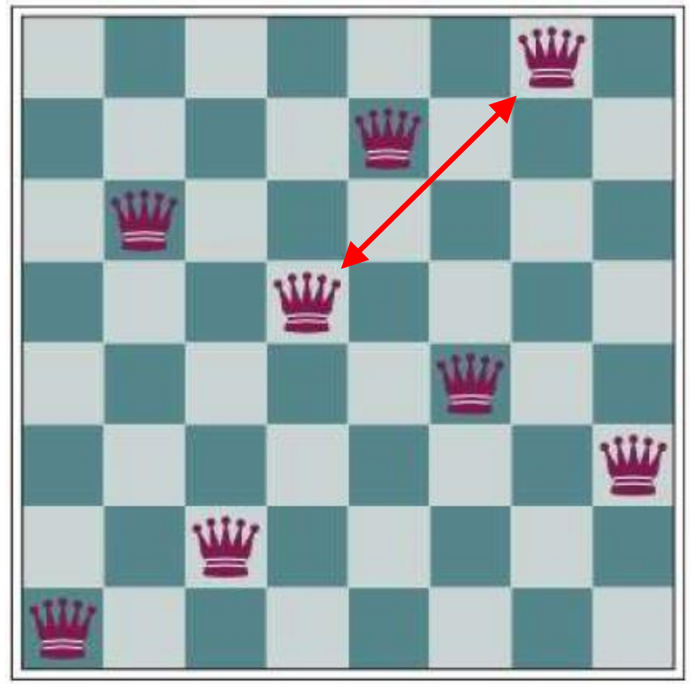

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- ♦ Hill climbing can make rapid progress toward a solution.

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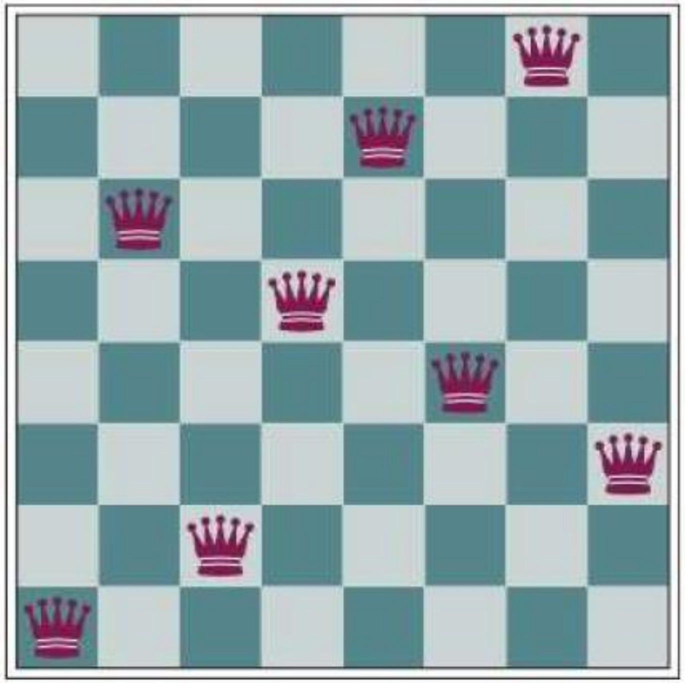

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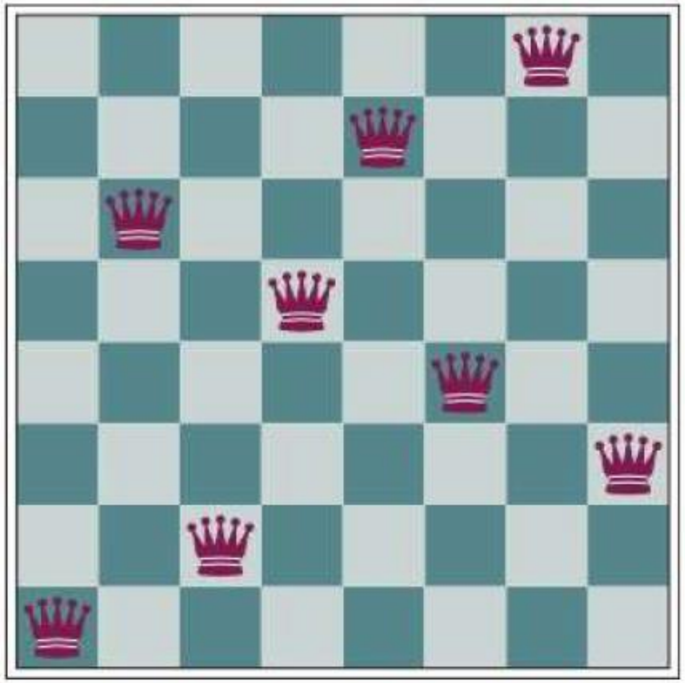

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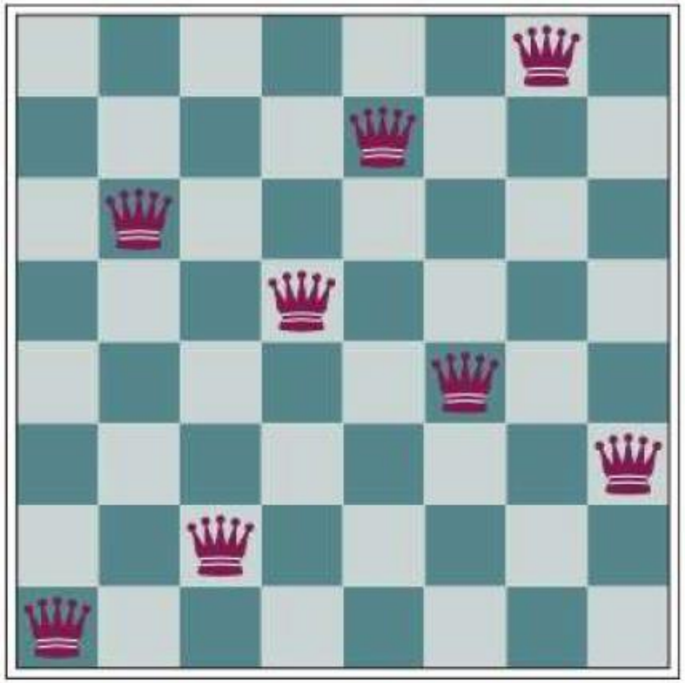
  Hill climbing in the vicinity of a local maximum will be drawn toward it and then get stuck there.



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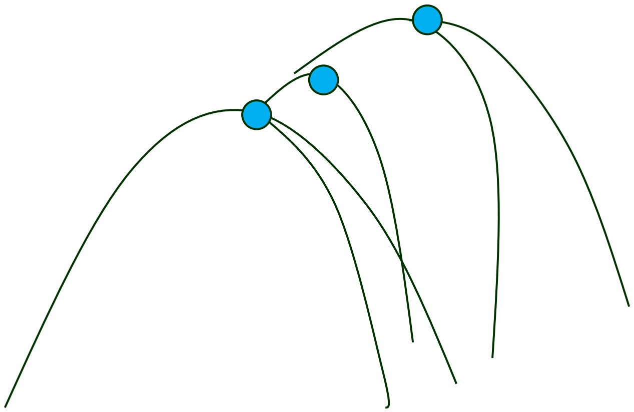
# Drawback of Hill Climbing (2)

- Ridge: A sequence of local maxima difficult to navigate.



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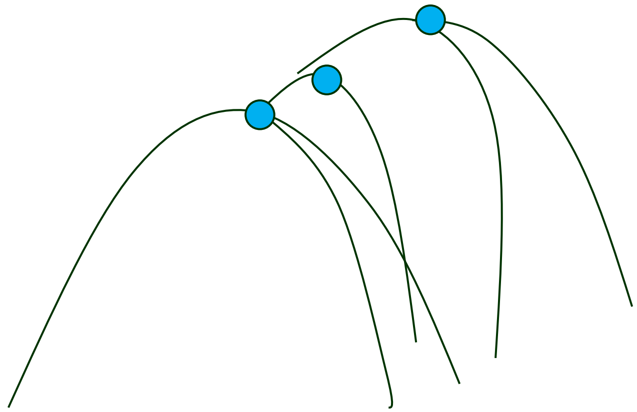

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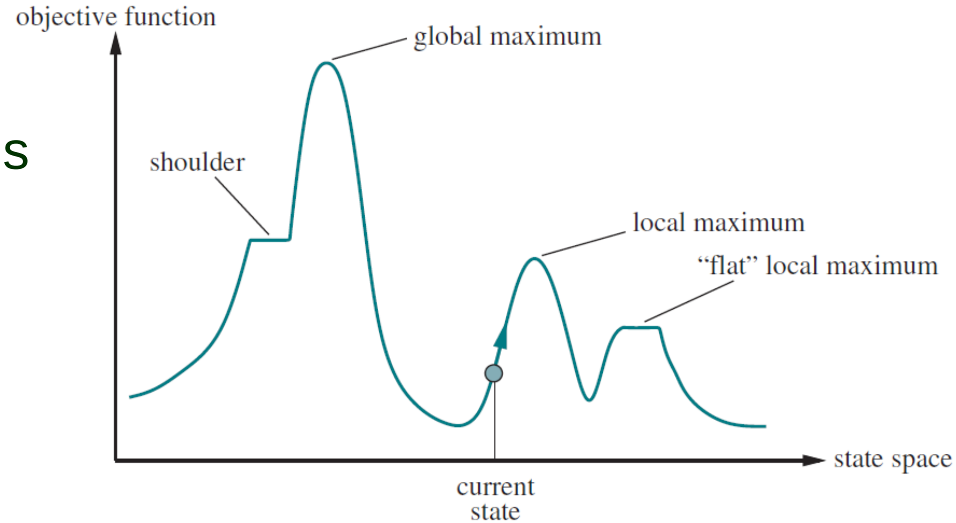

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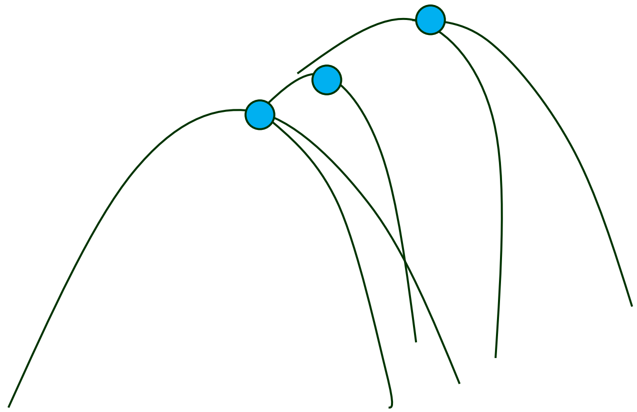

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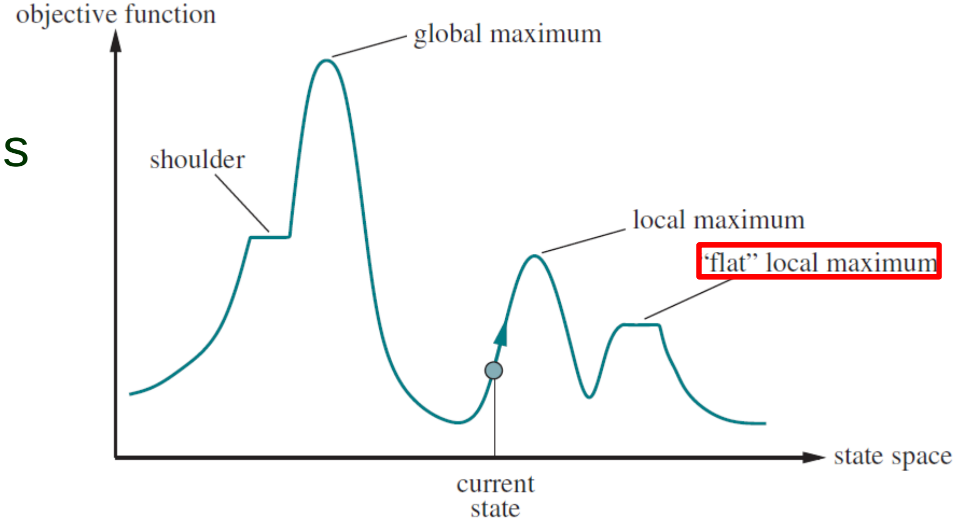

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- Random restart hill climbing
  - Restart search from random initial state.

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Annealing: Heat a metal to a high temperature and then gradually cool it, allowing the material to reach a low-energy crystalline state so it is hardened.

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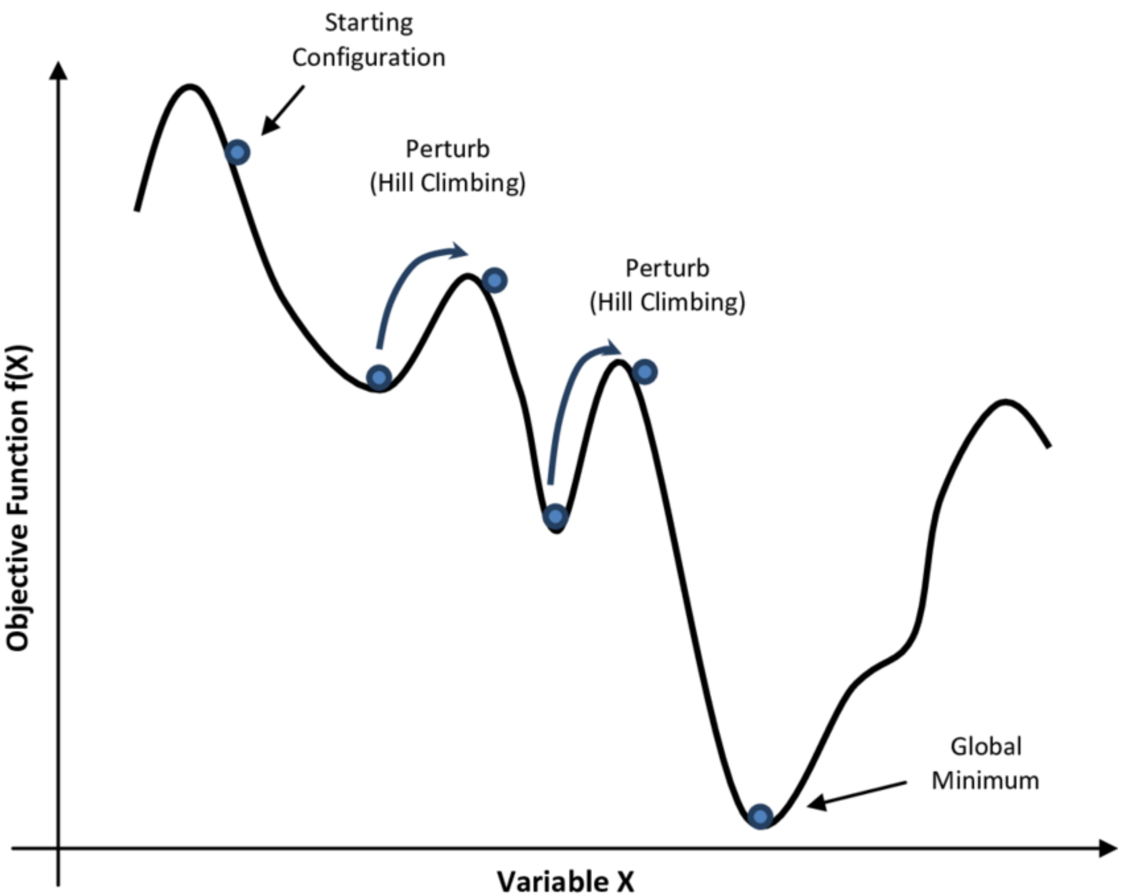


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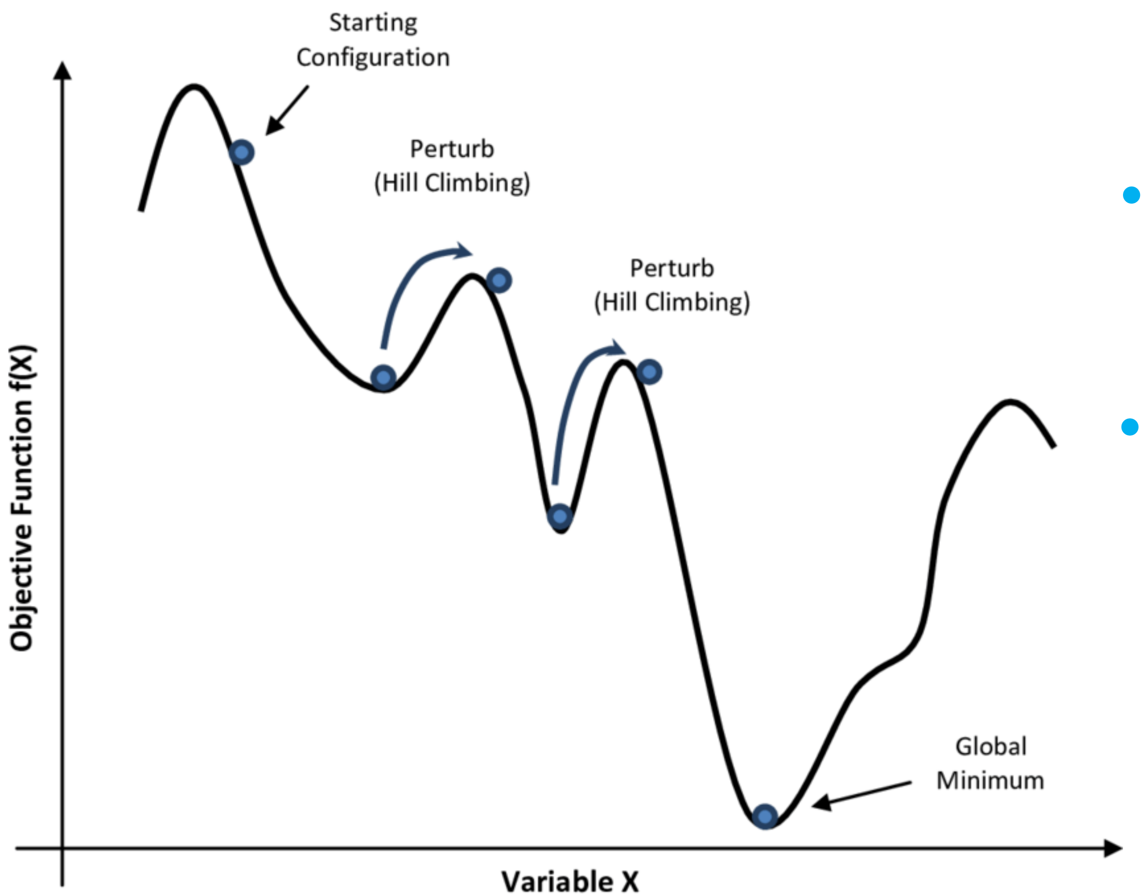


- Start by shaking hard (i.e., at high temperature).
- Gradually reduce the intensity of shaking (i.e., lower the temperature).

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# Simulated Annealing Algorithm

Minimization

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- Escape local minima by allowing bad moves.

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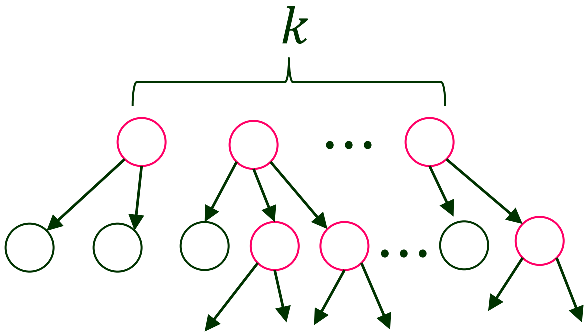
- Applied to many problems:
  - VLSL layout
  - factory scheduling
  - aircraft trajectory planning
  - NP-hard optimization (i.e., the traveling salesman problem)
  - large-scale stochastic optimization tasks

# Local Beam Search

Keep track of $k$ states rather than one.

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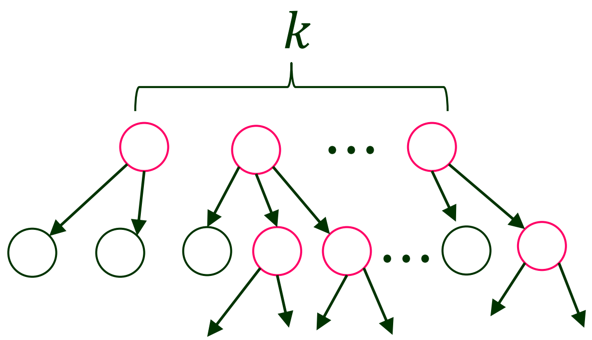


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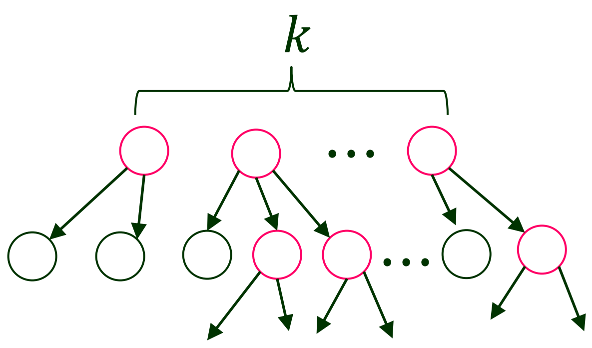


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**Solution**: stochastic beam search which chooses successors with probabilities proportional to their values.

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Mutation: Randomly change the bits of an offspring.

Culling: All individuals below a threshold are discarded from the population.

# III. Evolutionary Algorithms

- Also called *genetic algorithms*.
- Inspired by natural selection in biology.

1. Start with a population of $k$ randomly generated states (individuals).
2. Select the *most fit* individuals to become parents of the next generation
3. Combine every $\rho$ parents to form an offspring (typically $\rho = 2$).

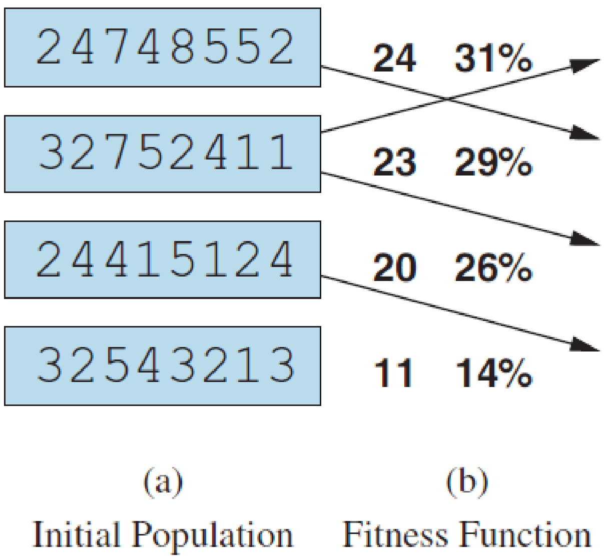
   Crossover: Split each of the parent strings and recombine the parts to form two children.

   Mutation: Randomly change the bits of an offspring.

   Culling: All individuals below a threshold are discarded from the population.

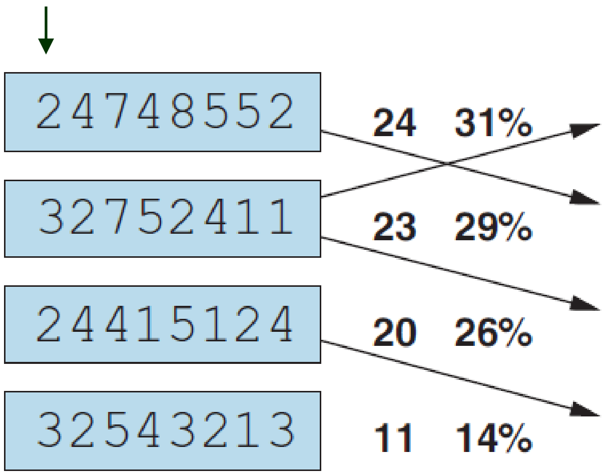
4. Go back to step 2 and repeat until *sufficiently fit* states are discovered (in which case the best one is chosen as a solution).

# Genetic Algorithm on 8-Queen



# Genetic Algorithm on 8-Queen

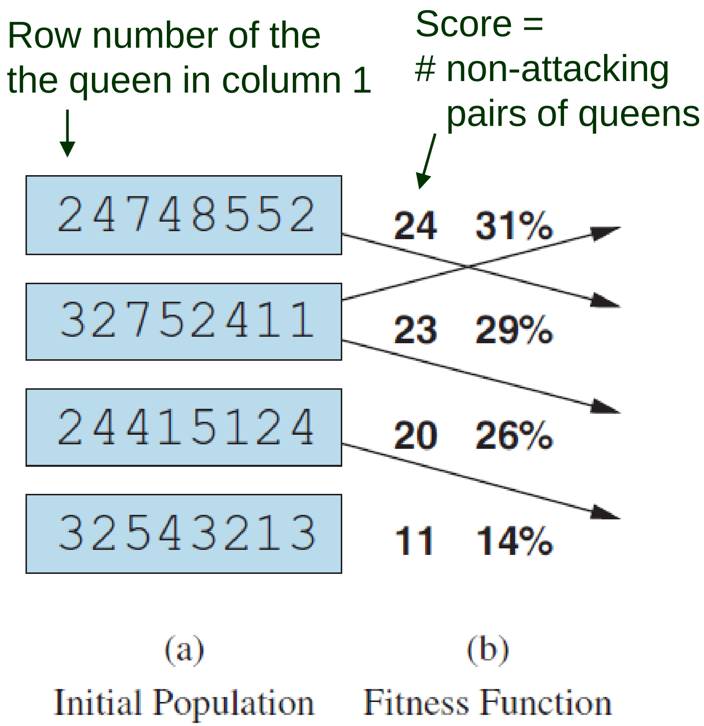
Row number of the
the queen in column 1




(a) Initial Population

(b) Fitness Function

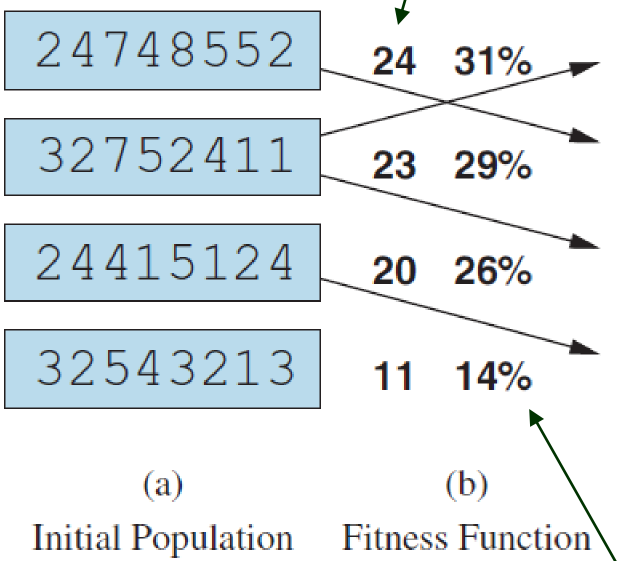
# Genetic Algorithm on 8-Queen



# Genetic Algorithm on 8-Queen

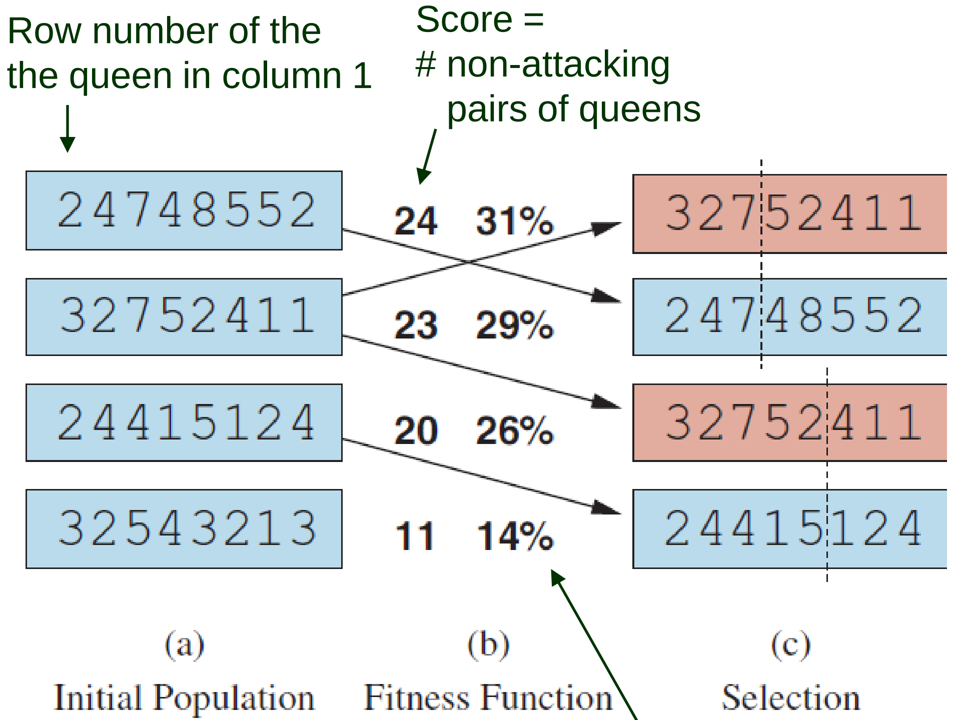
Row number of the
the queen in column 1

Score =
# non-attacking
pairs of queens




$$14\% = \frac{11}{24 + 23 + 20 + 11}$$

# Genetic Algorithm on 8-Queen

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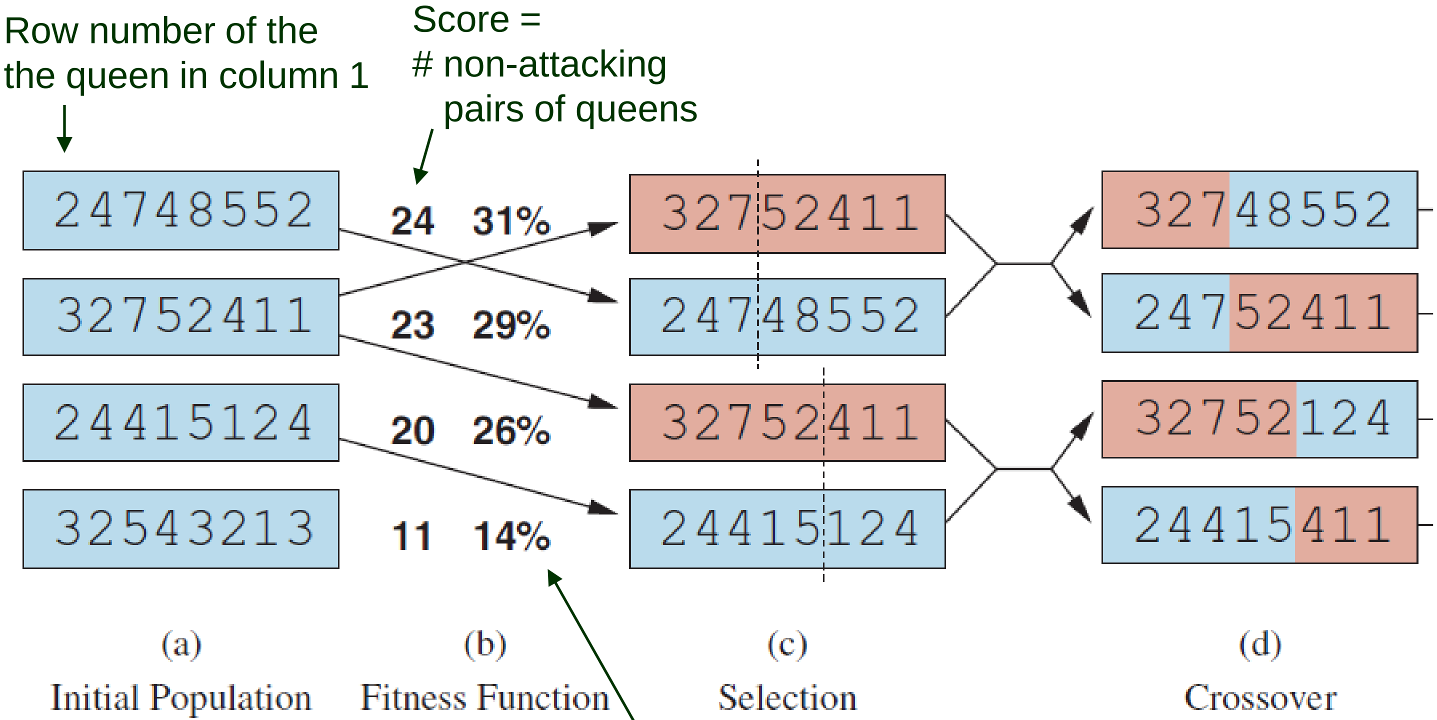
# Genetic Algorithm on 8-Queen

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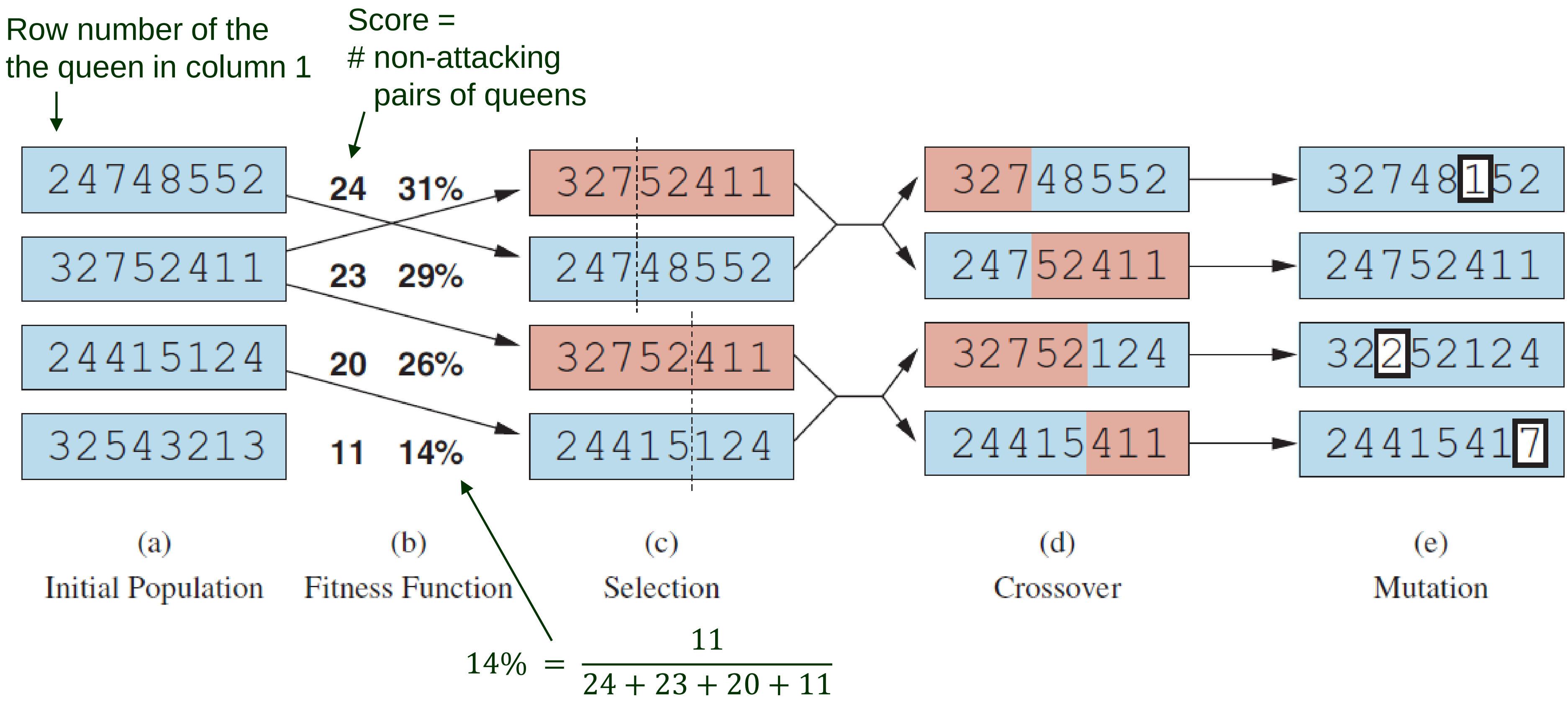
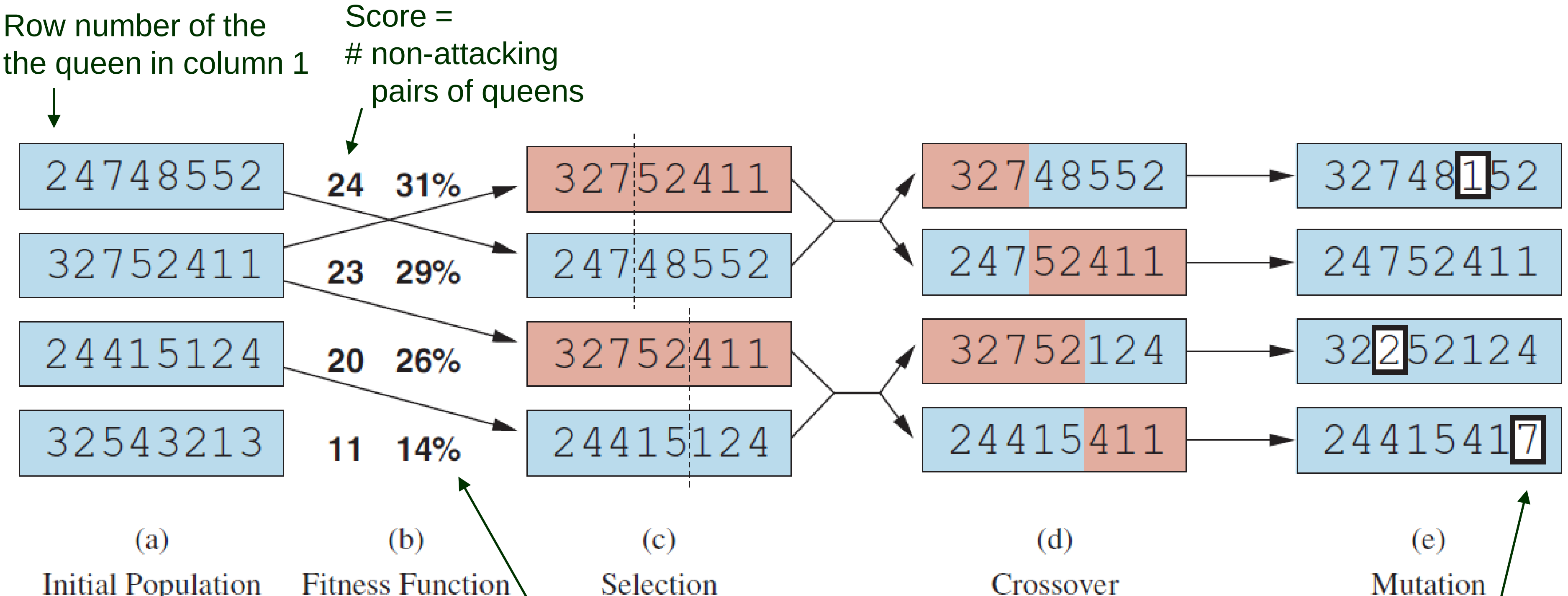
# Genetic Algorithm on 8-Queen



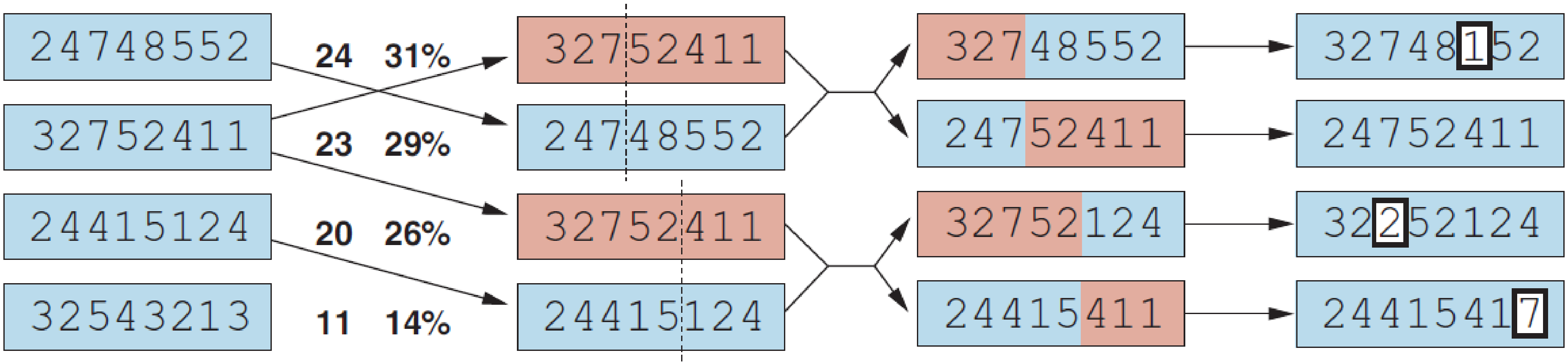
# Genetic Algorithm on 8-Queen




$$14\% = \frac{11}{24 + 23 + 20 + 11}$$

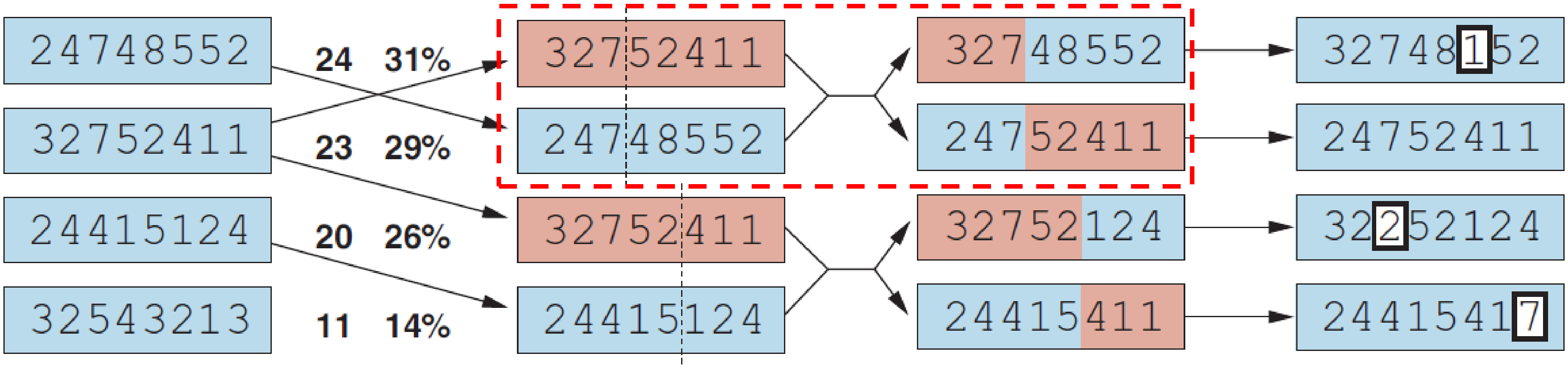
Move the queen in column 8 to a random square (7).

# Crossover



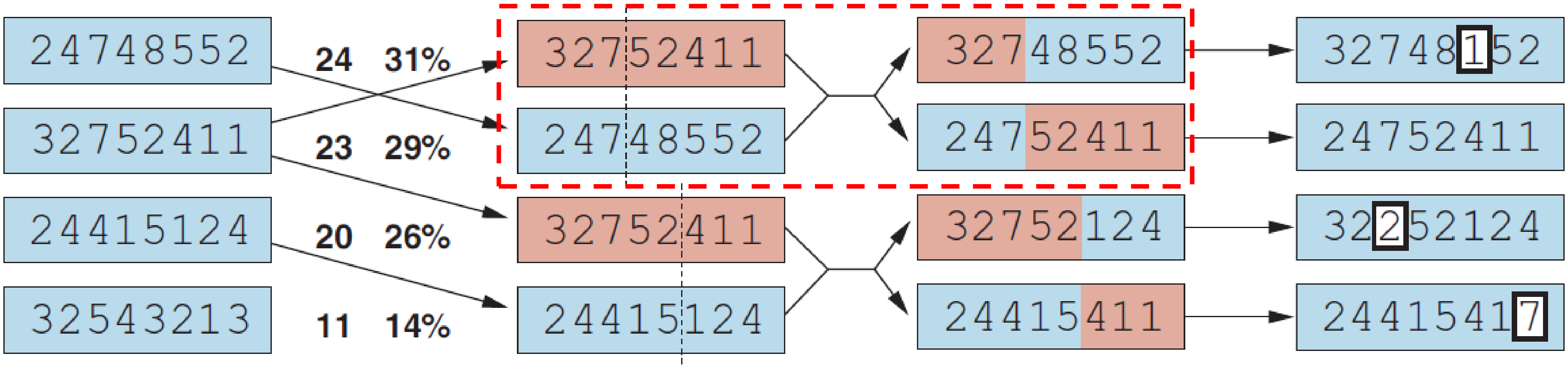

(d)
Crossover

# Crossover



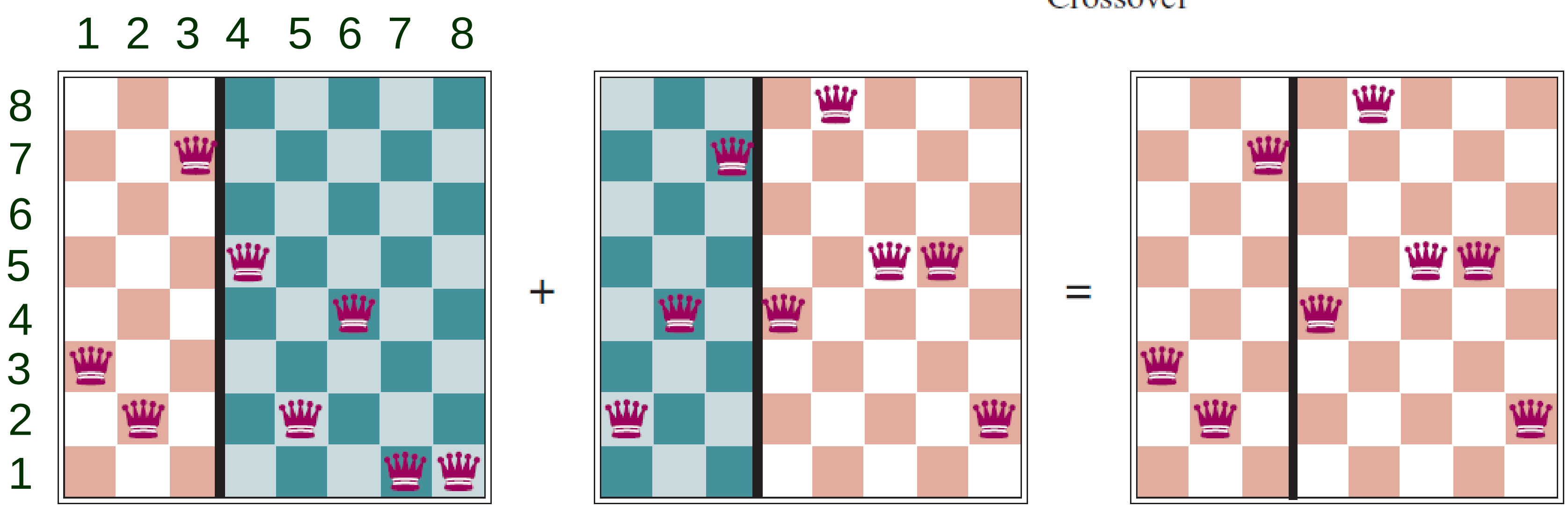

(d)
Crossover

# Crossover




(d)

Crossover



# Genetic Algorithm (Pseudocode)

**function** GENETIC-ALGORITHM(*population*, *fitness*) **returns** an individual
  **repeat**
    *weights* ← WEIGHTED-BY(*population*, *fitness*)
    *population2* ← empty list
    **for** $i$ = 1 **to** SIZE(*population*) **do**
      *parent1*, *parent2* ← WEIGHTED-RANDOM-CHOICES(*population*, *weights*, 2)
      *child* ← REPRODUCE(*parent1*, *parent2*)
      **if** (small random probability) **then** *child* ← MUTATE(*child*)
      add *child* to *population2*
    *population* ← *population2*
  **until** some individual is fit enough, or enough time has elapsed
  **return** the best individual in *population*, according to *fitness*

**function** REPRODUCE(*parent1*, *parent2*) **returns** an individual
  $n$ ← LENGTH(*parent1*)
  $c$ ← random number from 1 to $n$
  **return** APPEND(SUBSTRING(*parent1*, 1, $c$), SUBSTRING(*parent2*, $c + 1$, $n$))

# Applications of GA

- Complex structured problems

  Circuit layout, job-shop scheduling

- Evolving the architecture of deep neural networks
- Finding bugs of hardware
- Molecular structure optimization
- Image processing.
- Learning robots, etc.

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