Lecture notes for *Principles of Artificial Intelligence* (COMS 4720/5720)
Yan-Bin Jia, Iowa State University

Problem Solving

Outline

I. Problem formulation

II. Example problems

* Figures are from the textbook site (or drawn by the instructor) unless the source is specifically cited.

I. Problem-Solving Agent

♣ A reflex agent cannot operate when the mapping from states to to actions becomes too large to store.

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if current percepts then action
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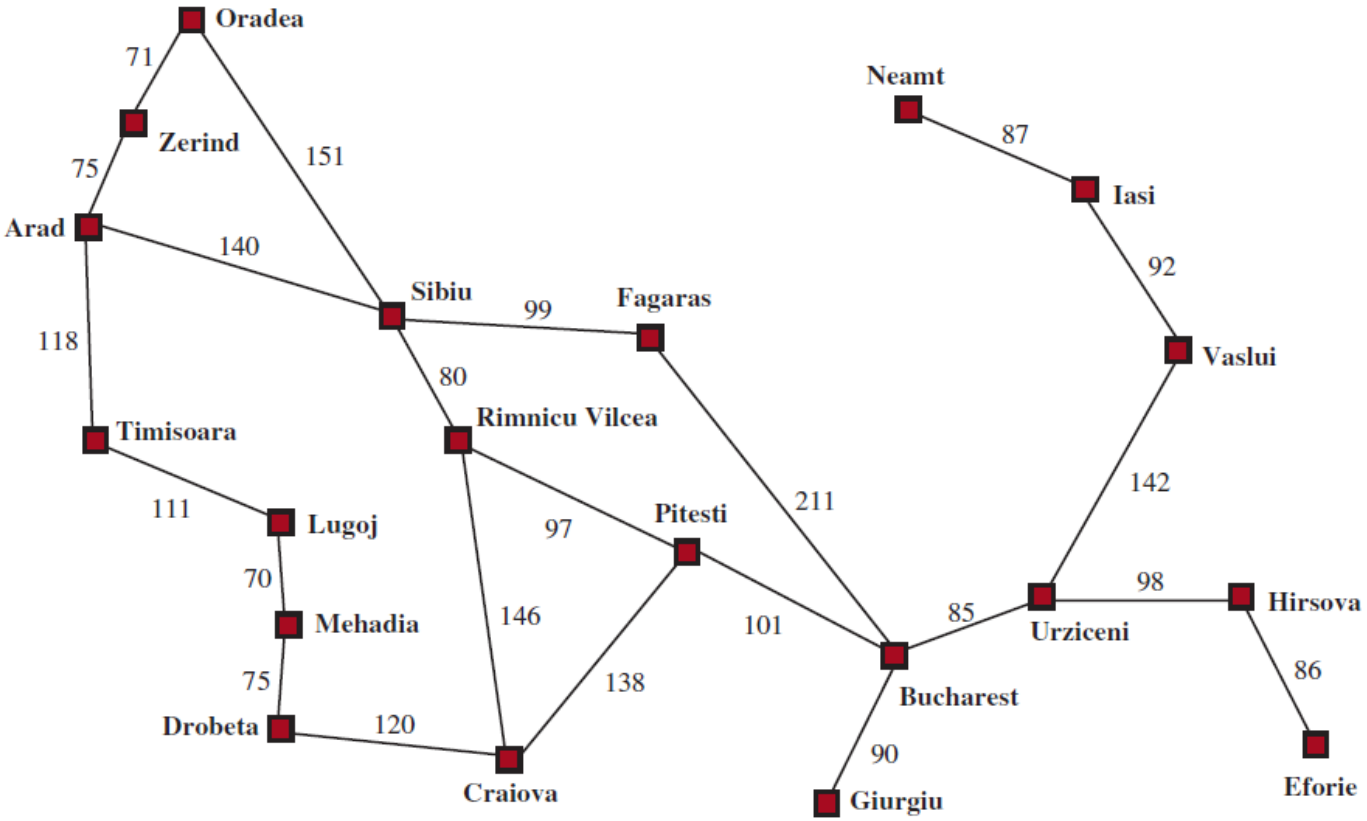
- A problem-solving agent is one kind of *goal-based* agent:
  - ♦ It considers states with no internal structure, i.e., in *atomic* representations.
- Problems are solved by general-purpose *search* algorithms.

Sightseeing Trip in Romania



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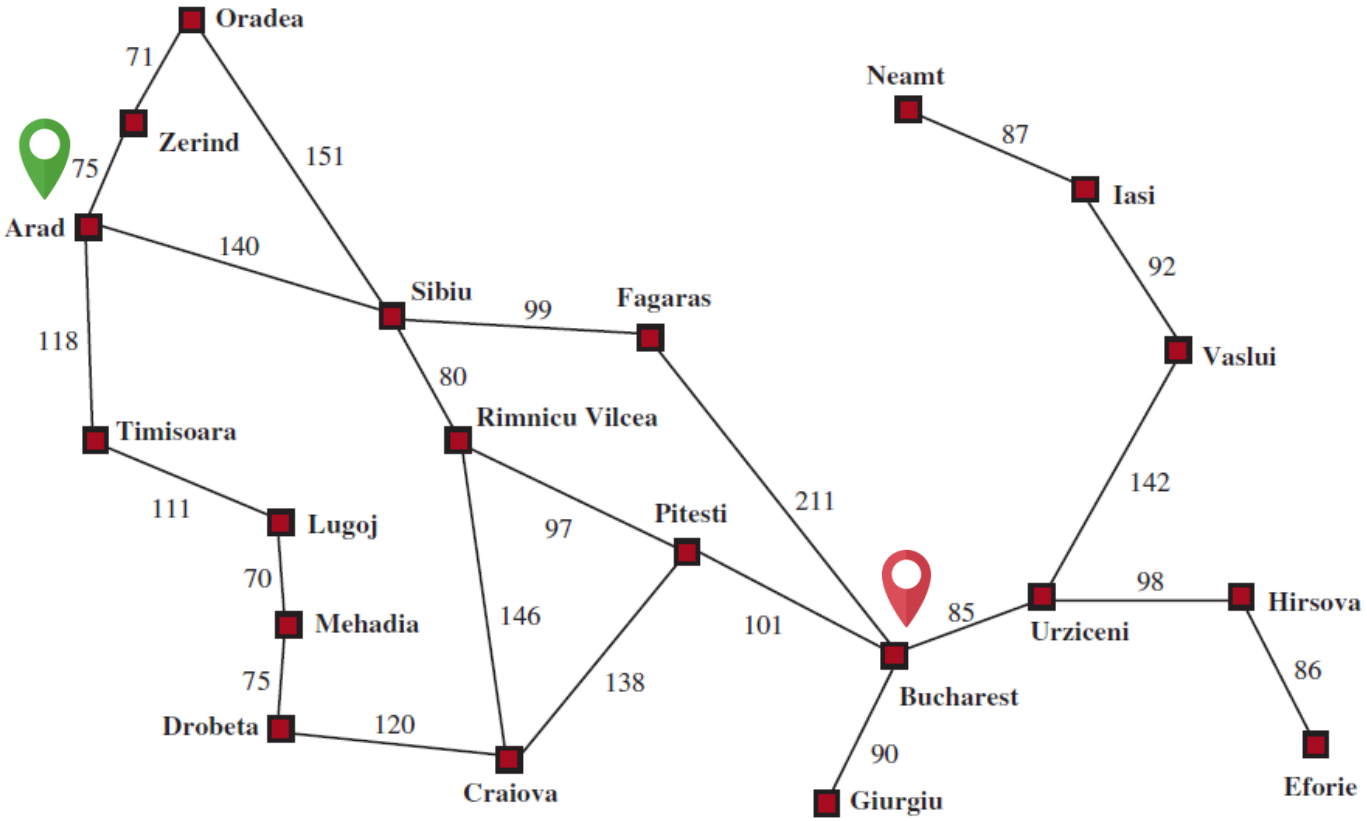

How to get to Bucharest from Arad?

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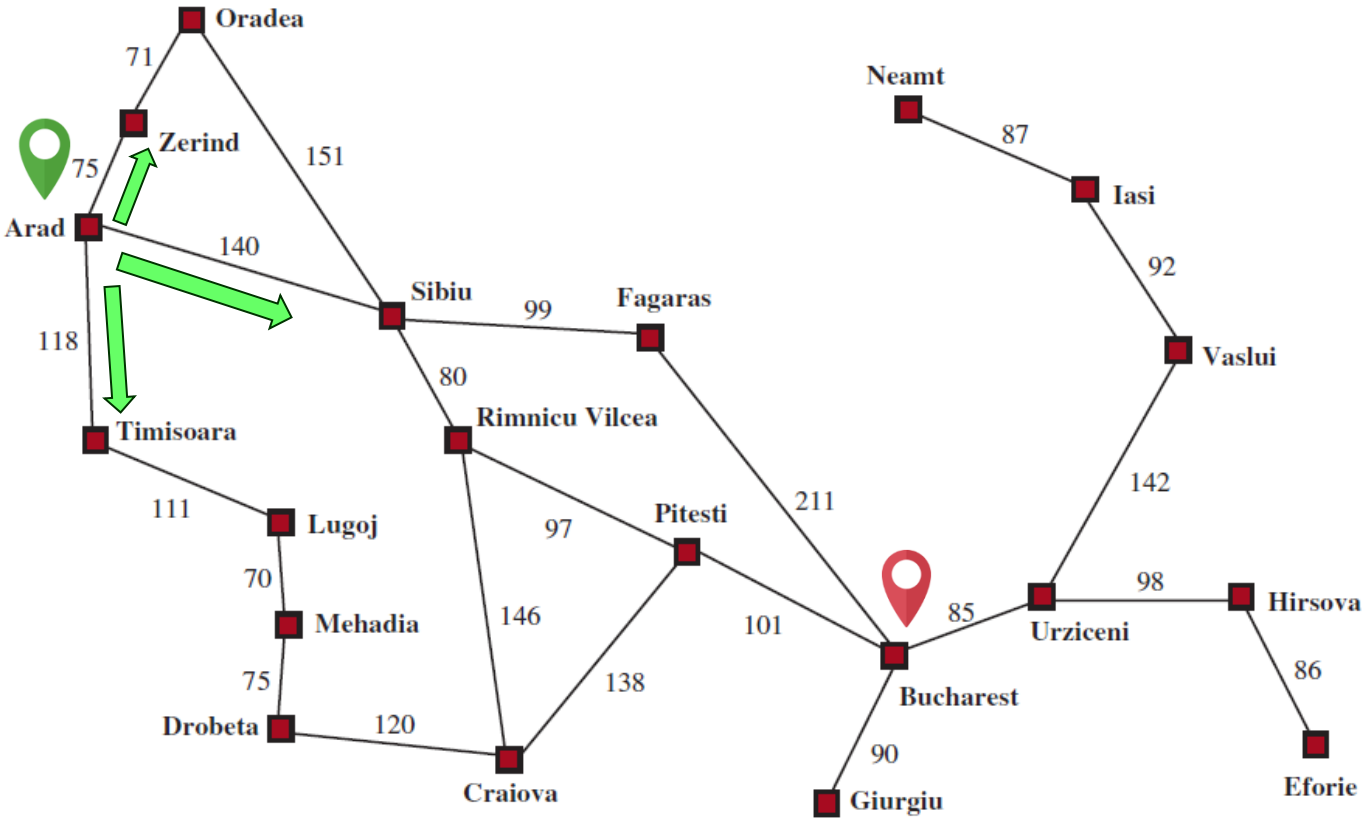

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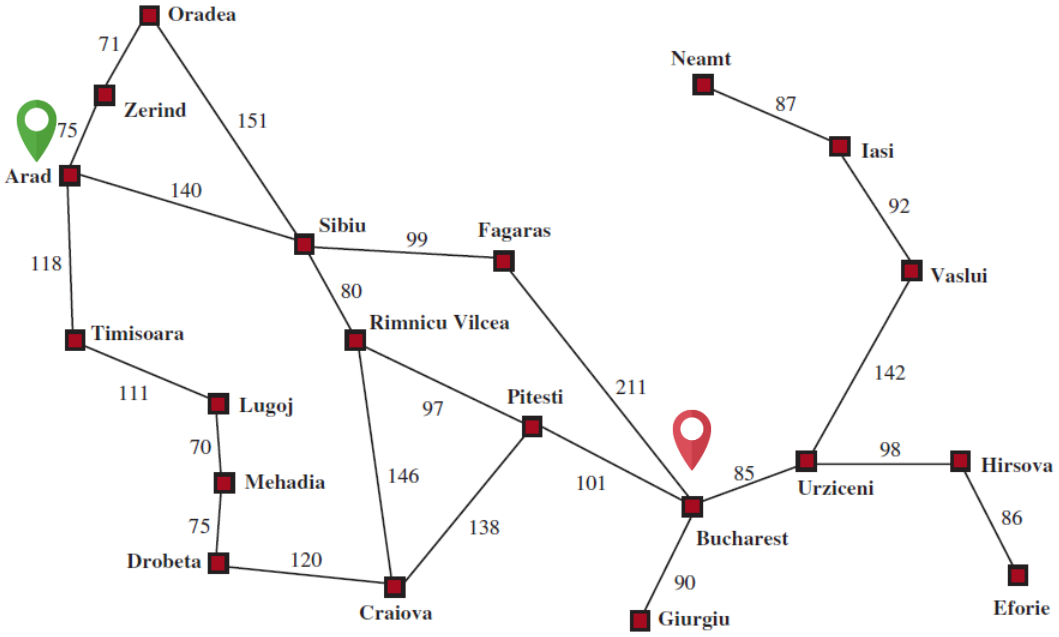
Four-Phase Problem Solving



- Goal formulation
- Problem formulation

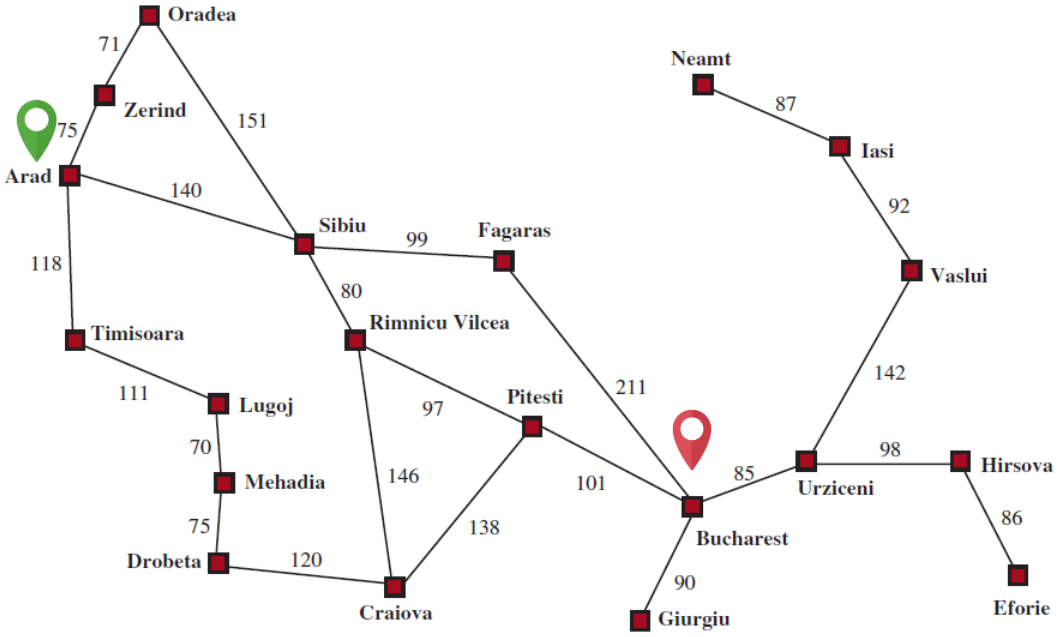
  states: cities

  action: travel from one city to an adjacent city
- Search

  Find a solution.

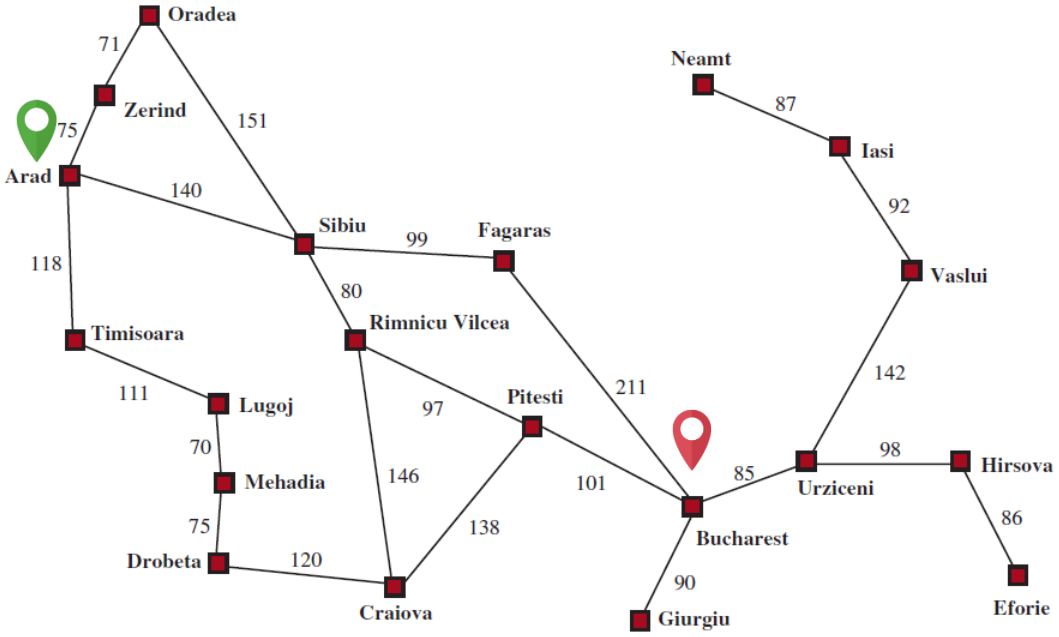
  A sequence of actions to reach the goal
- Execution

Search Problem



- State space (as a graph)

Search Problem



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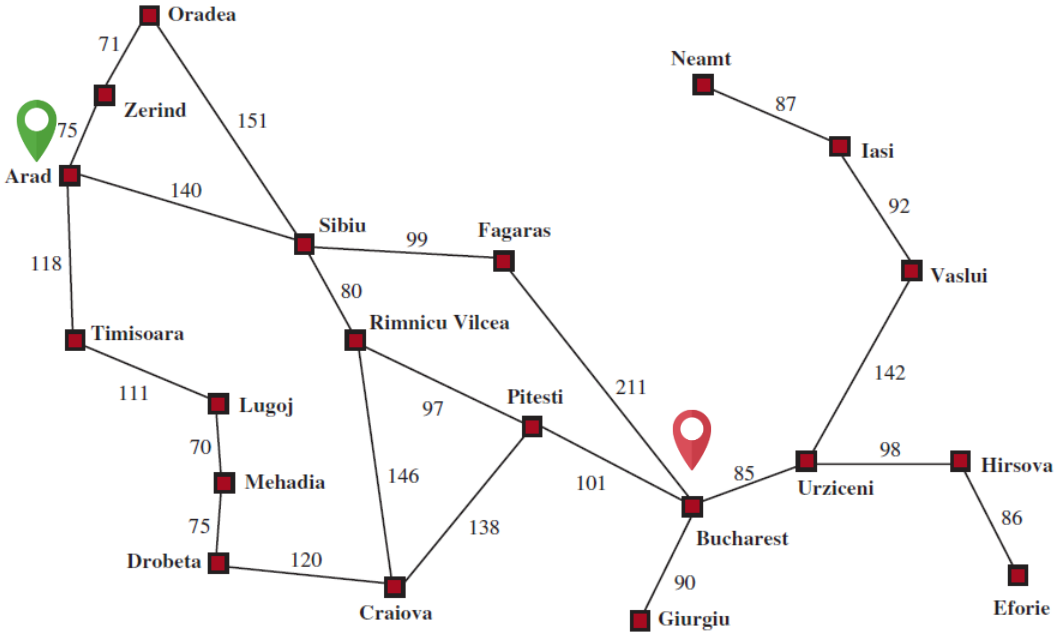

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- Goal state(s) (e.g., Bucharest)

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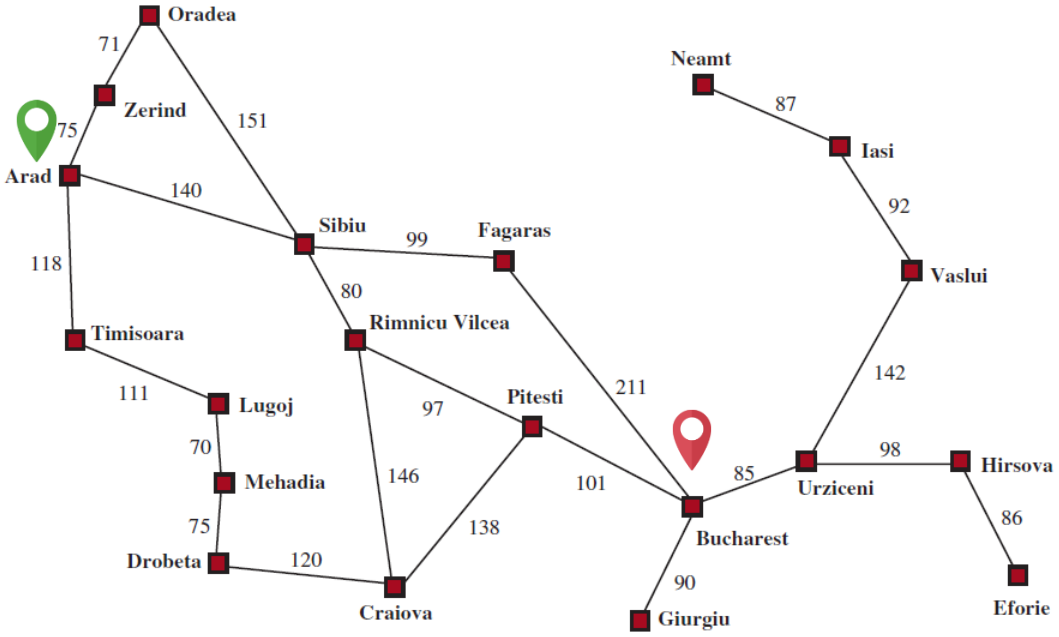
IS-GOAL(Fagaras)
```

Search Problem



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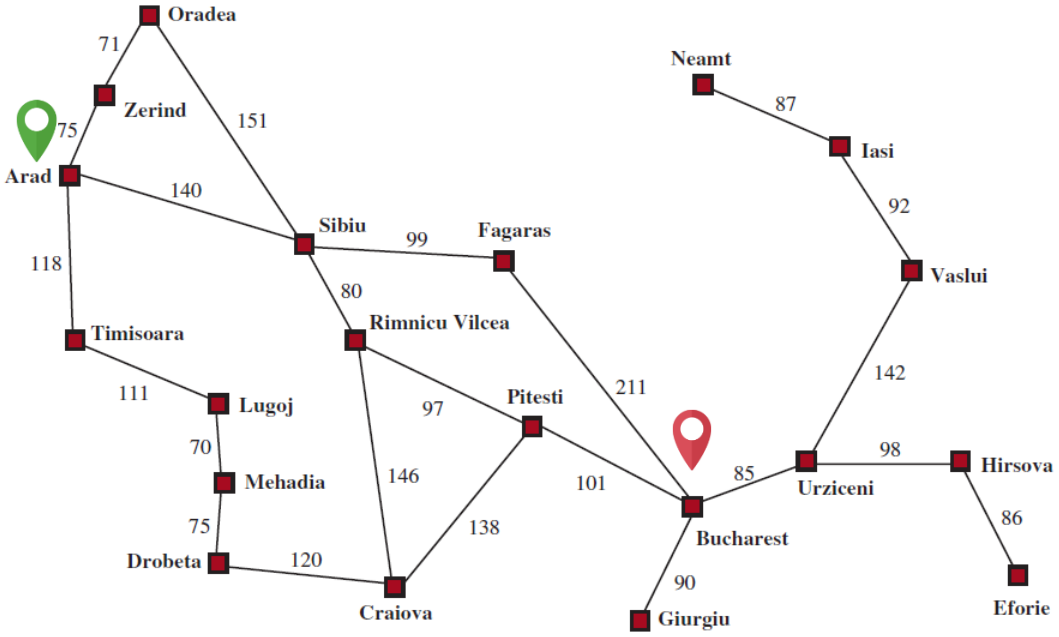
- Actions

Search Problem



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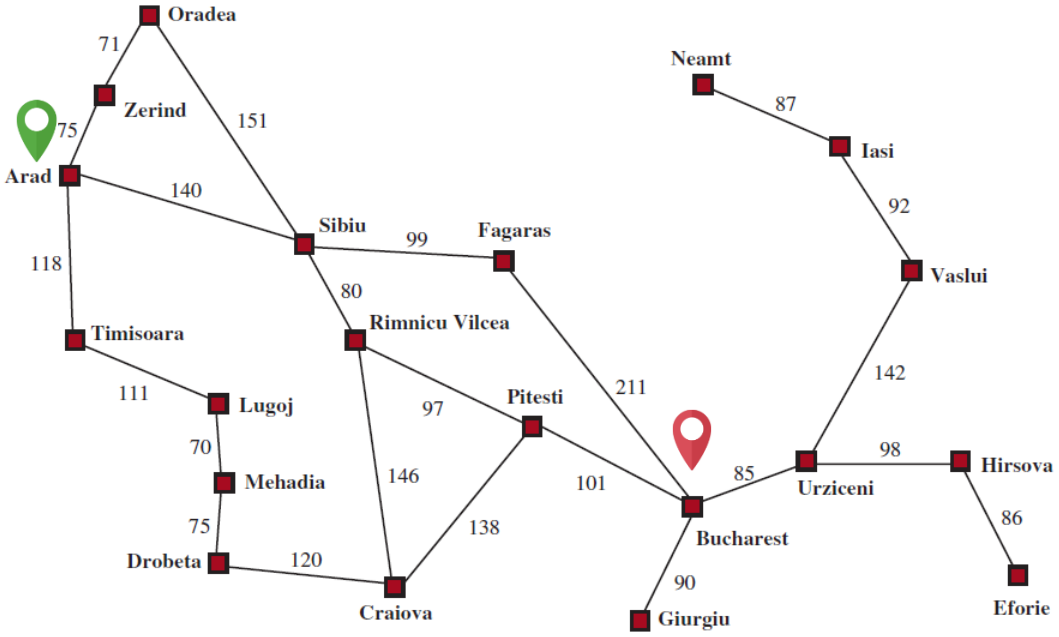
ACTIONS(s): a finite set of actions executable at state s.

ACTIONS($Arad$) = $\{ToSibiu, ToTimisoara, ToZerind\}$

Search Problem



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RESULT($Arad, ToZerind$) = $Zerind$

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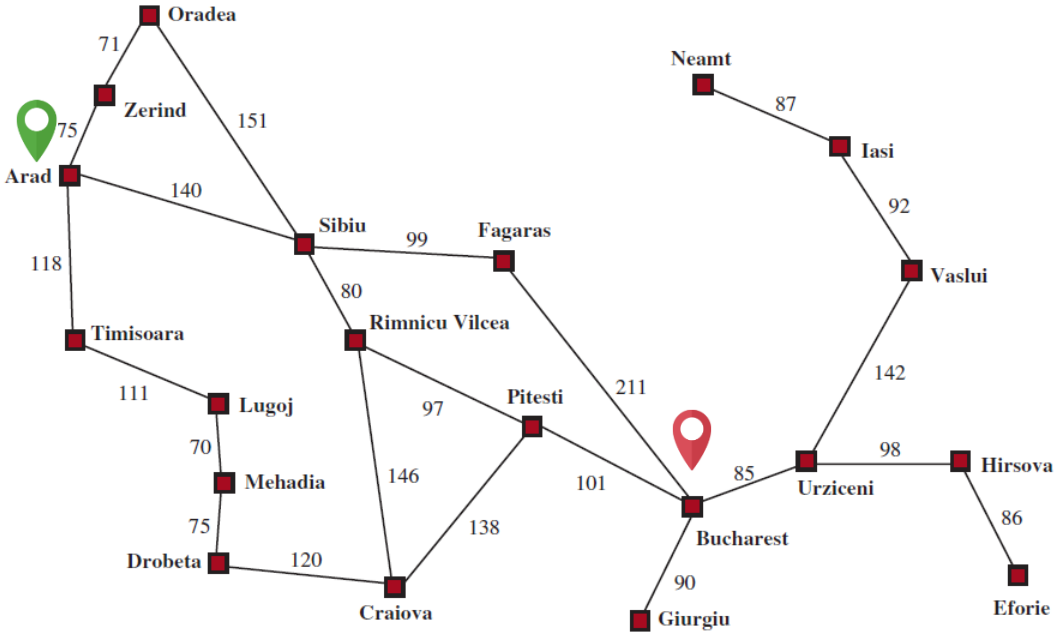

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$c(s, a, s')$: cost of applying action a in state s to reach state s'.

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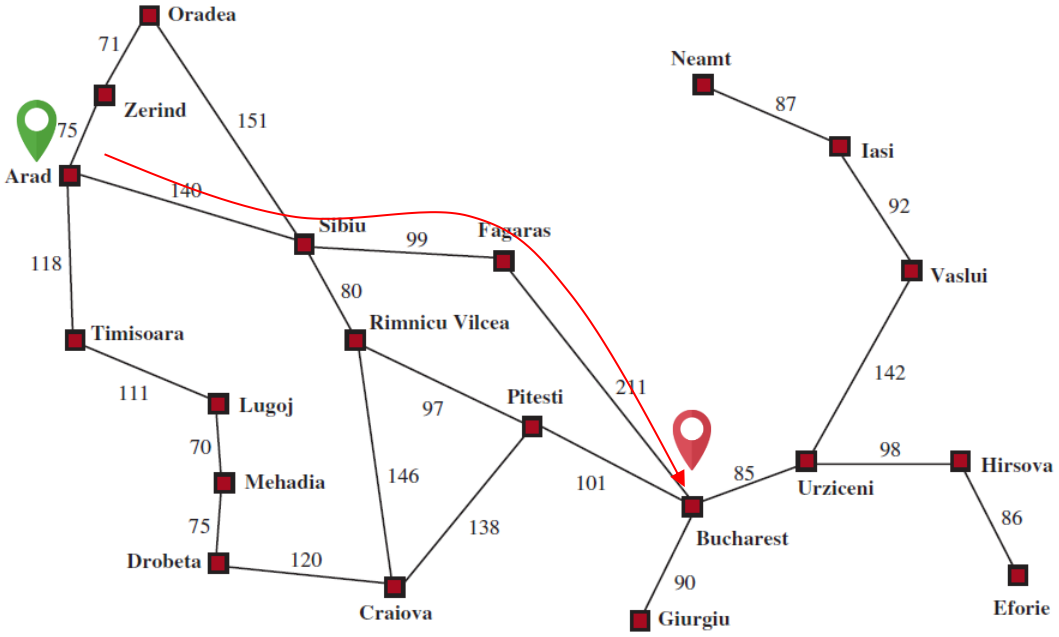

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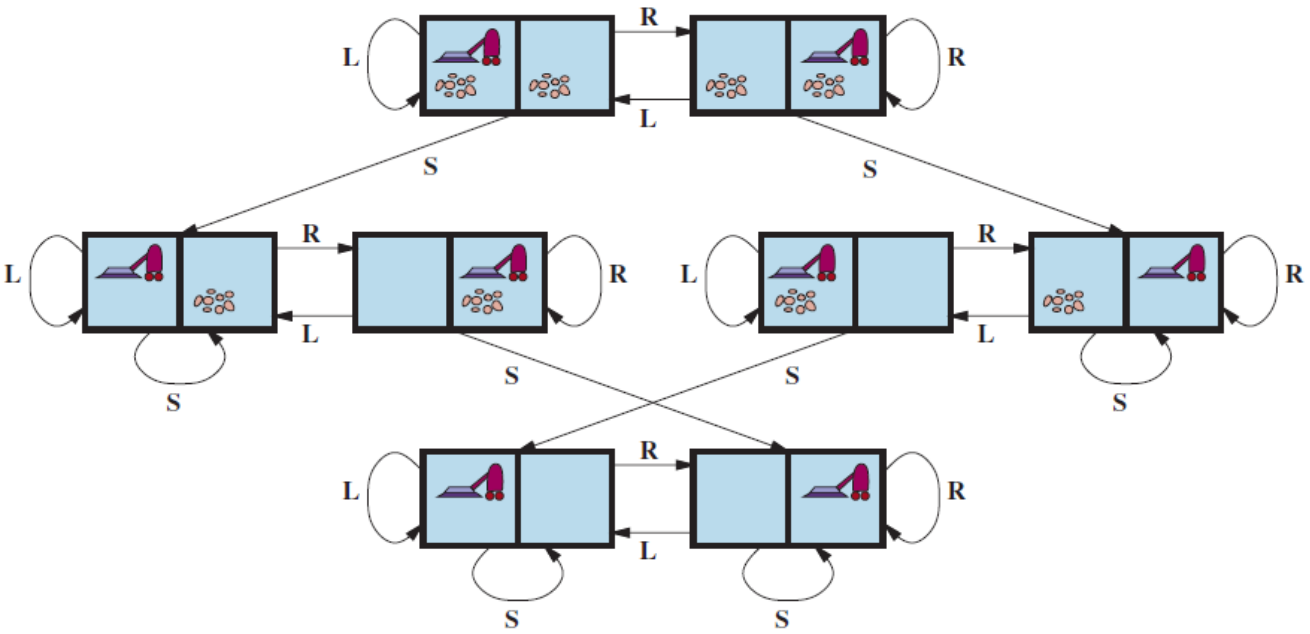
- Solution: initial state ⇝ goal state (e.g., Arad – Sibiu – Fagaras – Bucharest)

II. Example 1: Vacuum World

State-space graph:



- Actions: *Suck*, *Left*, *Right*
- Goal: every cell is clean.

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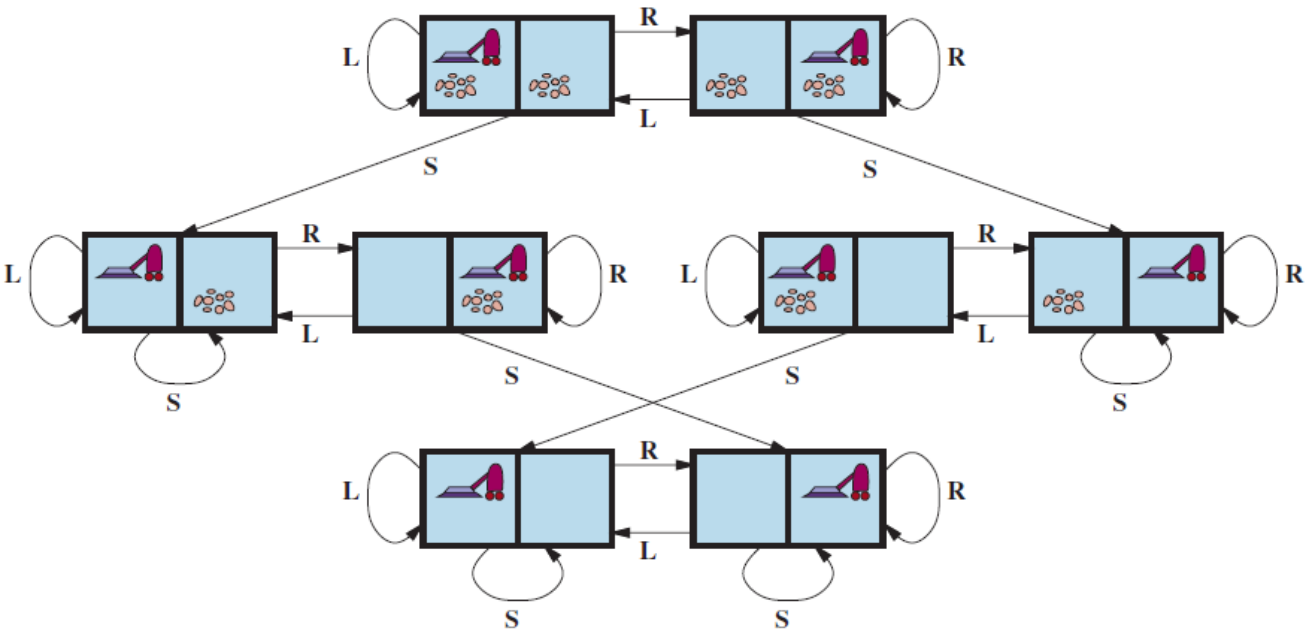

Agent in either cell

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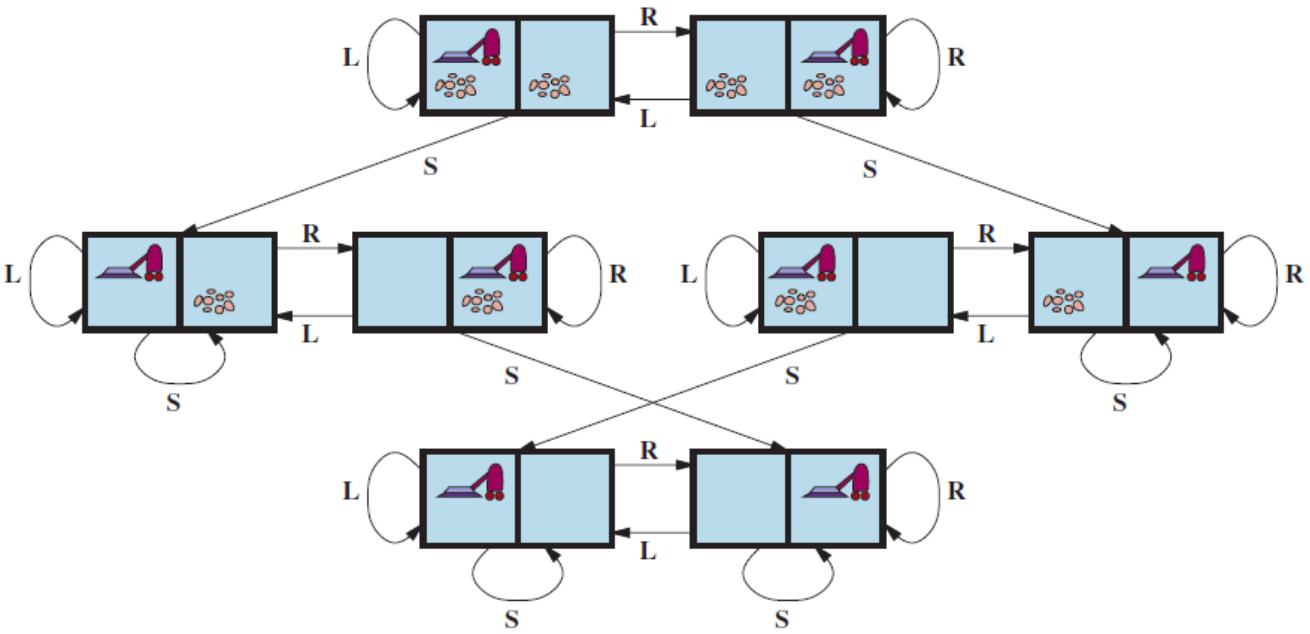

Agent in either cell

$2 \cdot 2$

Left cell has dirt or not

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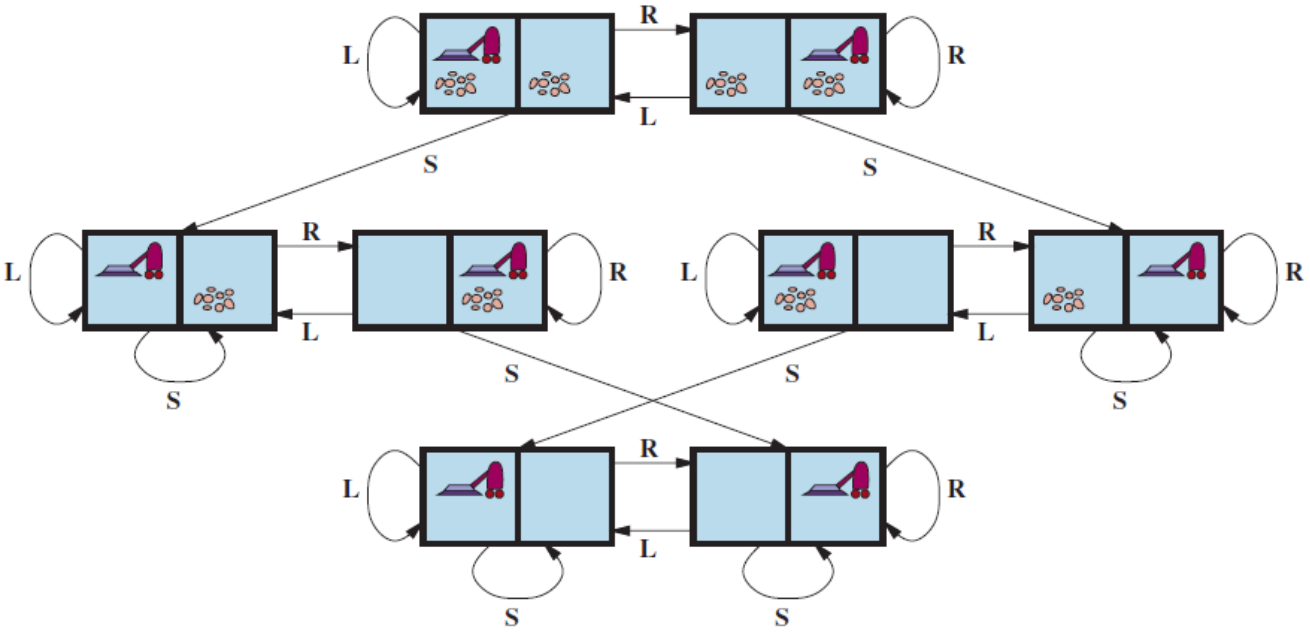

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$2 \cdot 2 \cdot 2 = 8$ states

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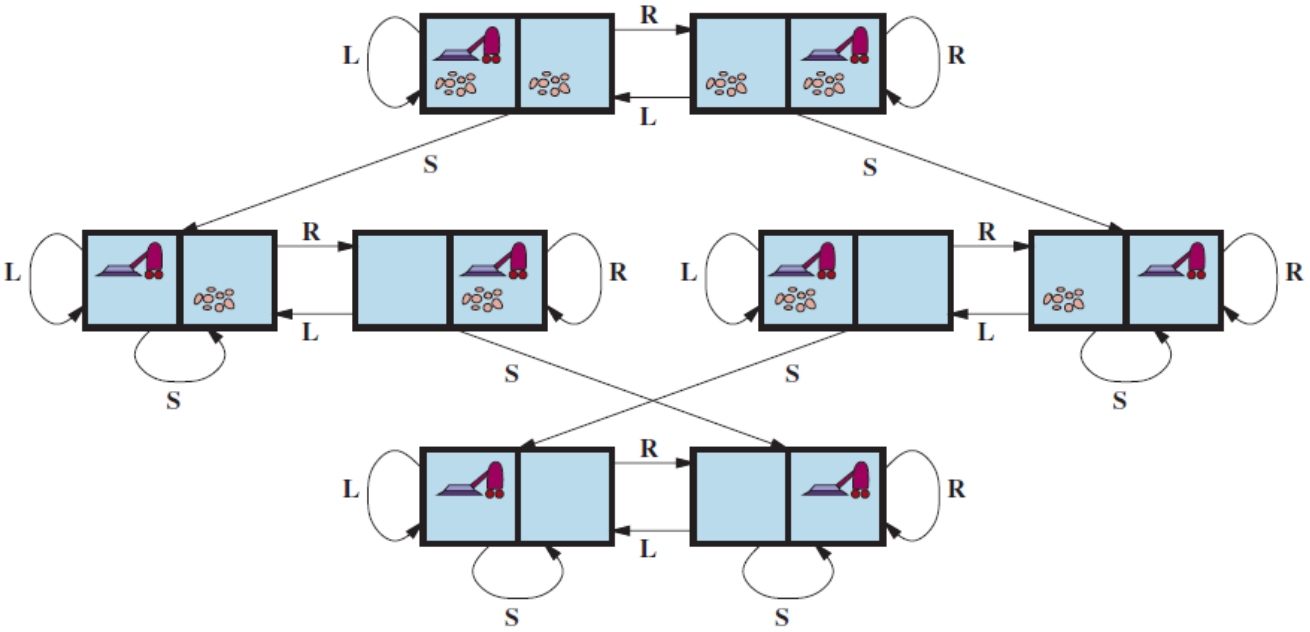

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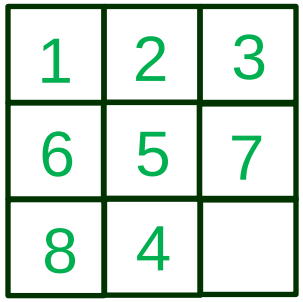
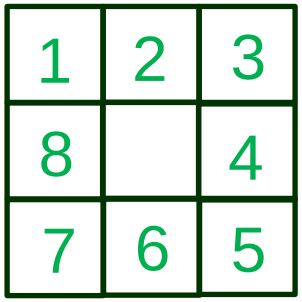
- Cost: 1 for each action

Example 2: 8-Puzzle



Initial state



Goal state

- Actions: ways of sliding a tile (adjacent to the blank space).

Left, Right, Up, Down

Applied to the
right neighbor
of the blank space

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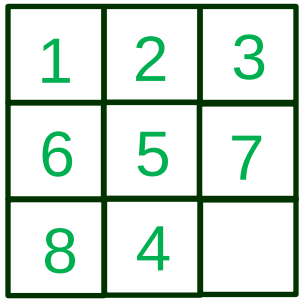

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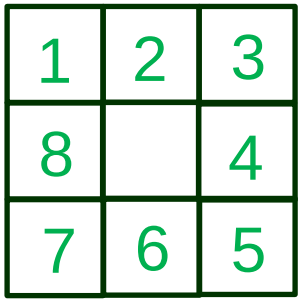

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Only two possible actions at the initial state: *Right* and *Down.*

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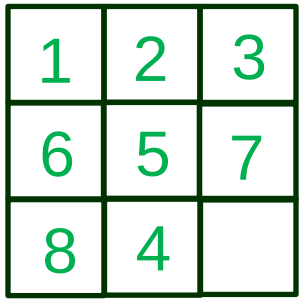

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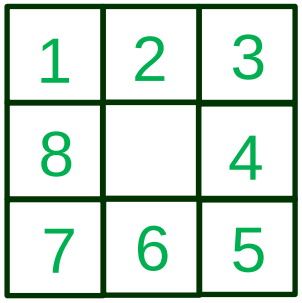

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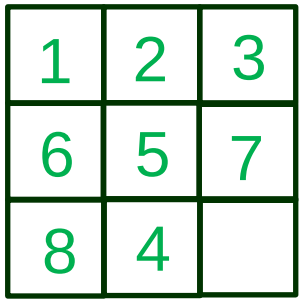
Solution to an 8-Puzzle



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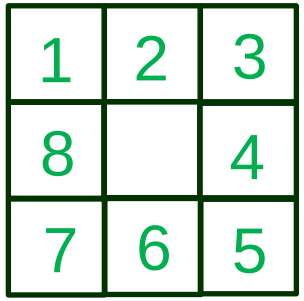

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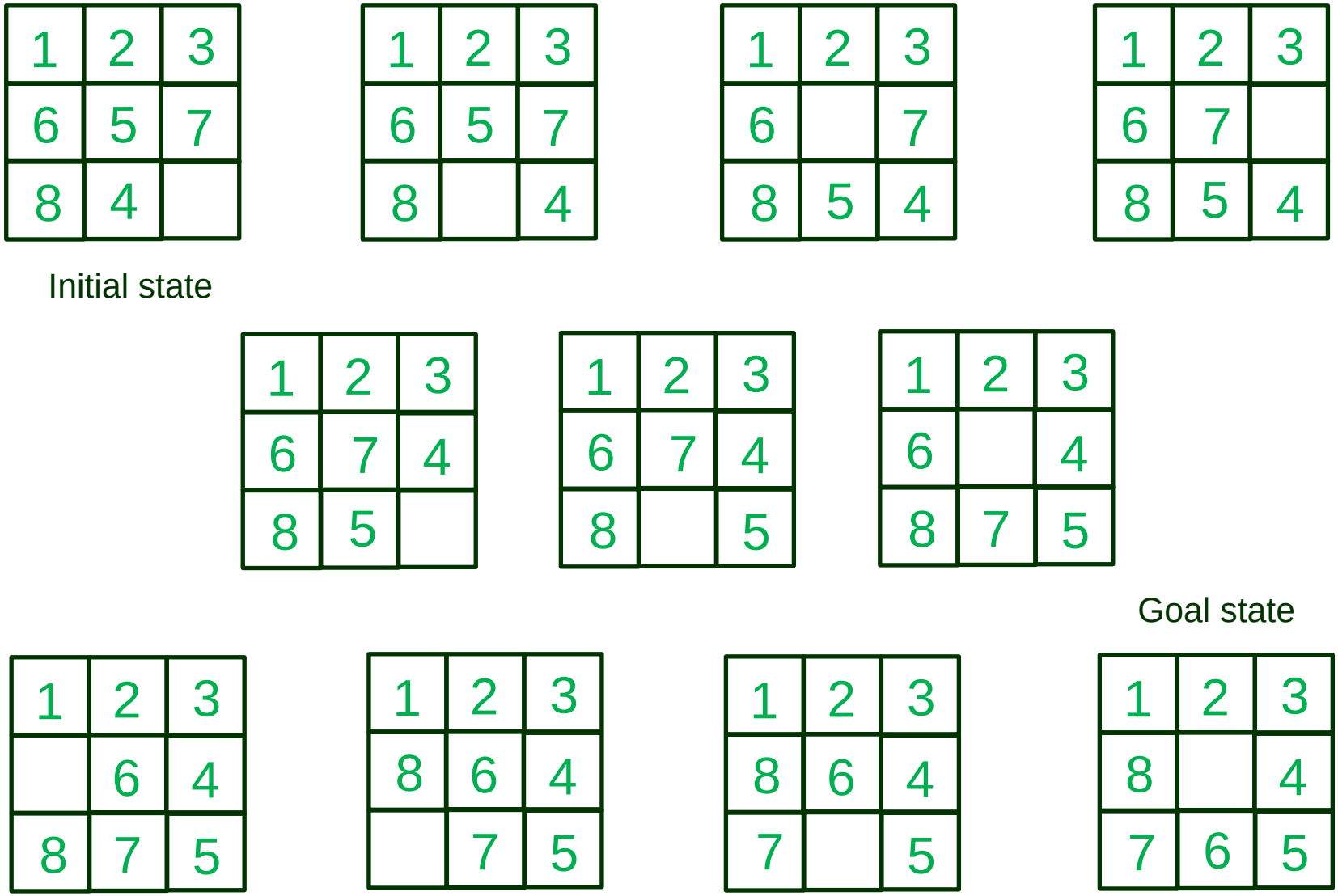
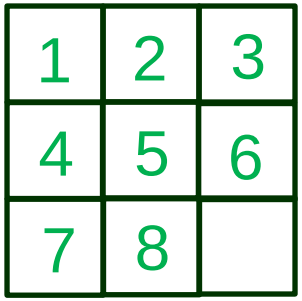


Inversions in an 8-Puzzle

State in which all the tiles appear in the correct order.



Every tile with a smaller number must be either above or, if in the same row, to the left of any tile with a larger number.

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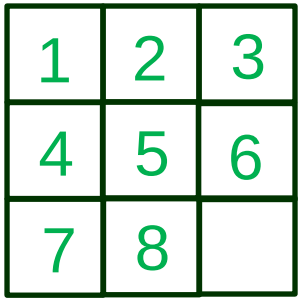

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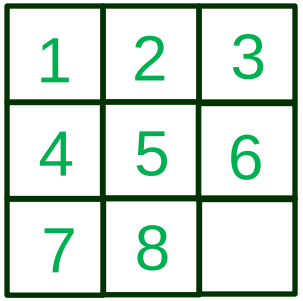

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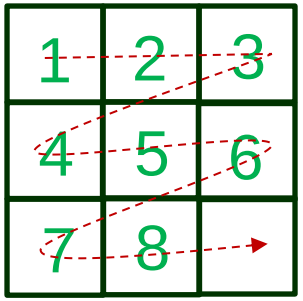

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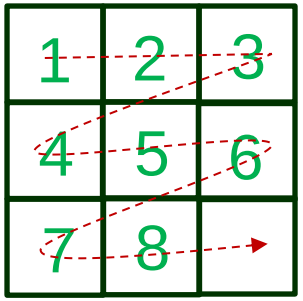

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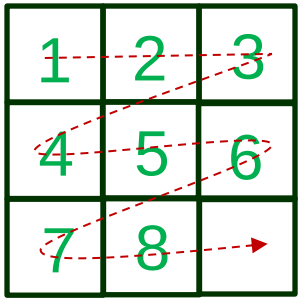

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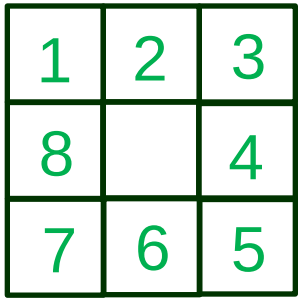
Starting from the top, we scan the tiles row by row, and within each row, from left to right. Ignore the blank tile.

- In the resulting sequence, every instance of a number appearing before a smaller one is counted as one inversion.

Counting Inversions

How many inversions are in the goal state?



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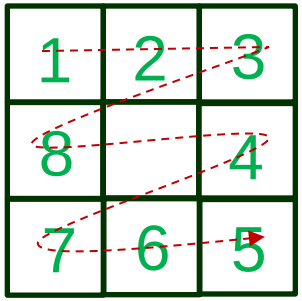

Sequence from the scan: 1, 2, 3, 8, 4, 7, 6, 5.

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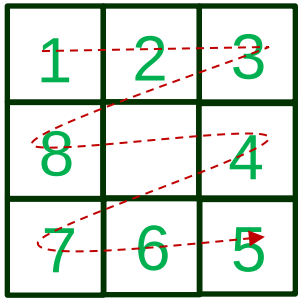

Sequence from the scan: 1, 2, 3, 8, 4, 7, 6, 5.

6 comes before 5
7 comes before 5
7 comes before 6
8 comes before 4
8 comes before 5
8 comes before 6
8 comes before 7

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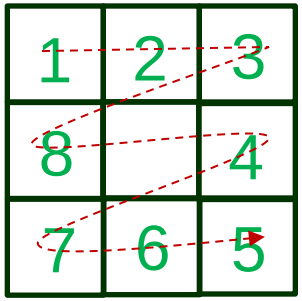

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7 inversions!

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If we fix the goal state at

1	2	3
8		4
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(7 inversions)

the puzzle is solvable if and only if the initial state has an *odd* number of inversions.

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4	1	2
3	5	
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5 inversions:

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Initial state 2:

4	1	2
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8	6	7

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6 inversions:

4 comes before 1, 2, 3

5 comes before 3

8 comes before 6, 7

Solvability of an 8-Puzzle

Theorem An 8-puzzle with an initial state and a goal state is solvable if and only if the two states differ by an ***even*** number of inversions

If we fix the goal state at

1	2	3
8		4
7	6	5

(7 inversions)

the puzzle is solvable if and only if the initial state has an *odd* number of inversions.

Initial state 1:

4	1	2
3	5	
8	6	7

5 inversions: ⇨ Solvable

4 comes before 1, 2, 3

8 comes before 6, 7

Initial state 2:

4	1	2
5	3	
8	6	7

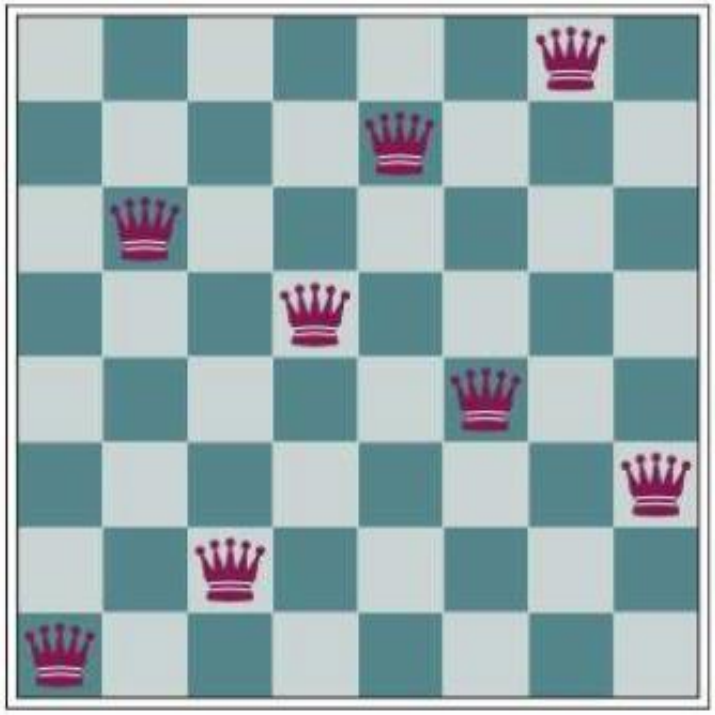
6 inversions: ⇨ Not solvable

4 comes before 1, 2, 3

5 comes before 3

8 comes before 6, 7

The 8-Queens Problem



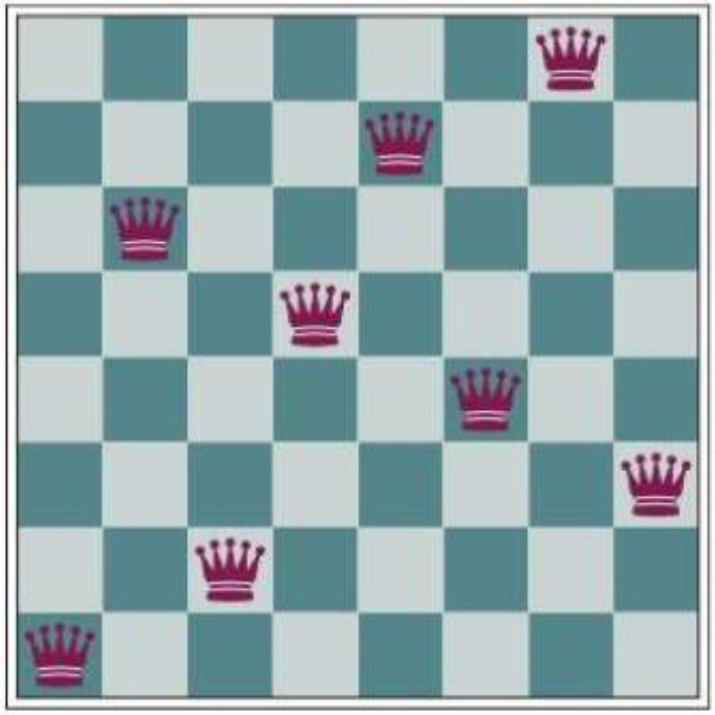
- Goal: A placement of 8 queens on the chess board in which no queen attacks another.

  ↓

  Same row, column, or diagonal

- Initial state: no queen on the board.

The 8-Queens Problem



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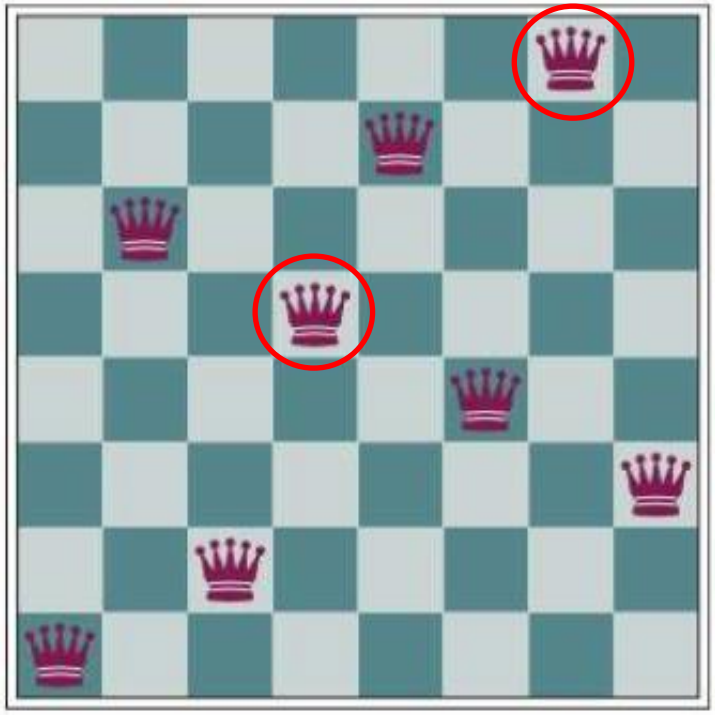

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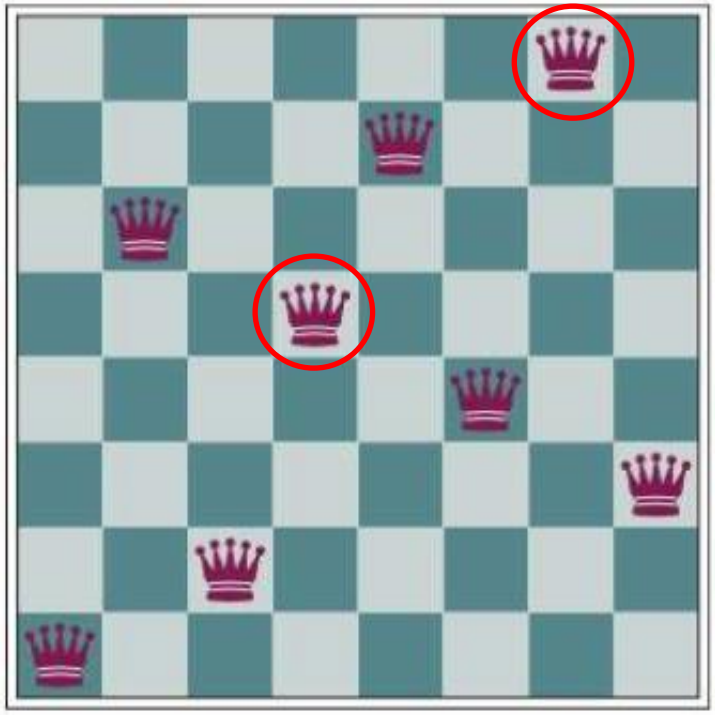

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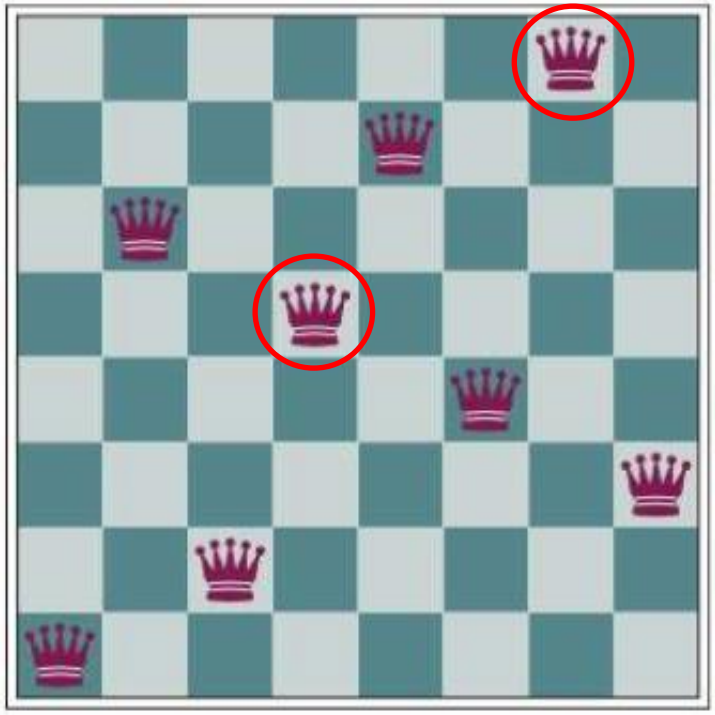
Constraint satisfaction!

The 8-Queens Problem



Close to a solution

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  ↓

  Same row, column, or diagonal

- Initial state: no queen on the board.

Constraint satisfaction!

The n-queens problem

Place n queens on an $n \times n$ chess board so that no queen attacks another.

Knuth's Conjecture (1964)

Donald Knuth (Stanford)

"father of the analysis of algorithms"
ACM Turing Award (1974)
National Medal of Science (1979)

Any integer > 4 can be reached from 4 via a sequence of square root, floor, and factorial operations.

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}} \right\rfloor = 5$$

* Photo from https://amturing.acm.org/byyear.cfm.

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- States: positive real numbers.
- Initial state: 4.
- Goal state: the desired integer > 4.
- Actions: square root, floor, or factorial operation.
- Action cost: 1.

* Photo from https://amturing.acm.org/byyear.cfm.

Real-World Problems

- Route finding (e.g., from Ames, IA to Mountain View, CA)
- Traveling salesman problem

Round trip visiting 13,509 US cities
with population size > 500 exactly once*

* Figure from http://www.crpc.rice.edu/CRPC/newsletters/sum98/news_tsp.html.

Real-World Problems

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- VLSI layout

Positioning of millions of components and connections on a chip.

- Robot navigation
- Autonomous assembly sequencing (e.g., protein design)

* Figure from http://www.crpc.rice.edu/CRPC/newsletters/sum98/news_tsp.html.