Conditional Independence & Distributions

Outline

I. Conditional independence relations

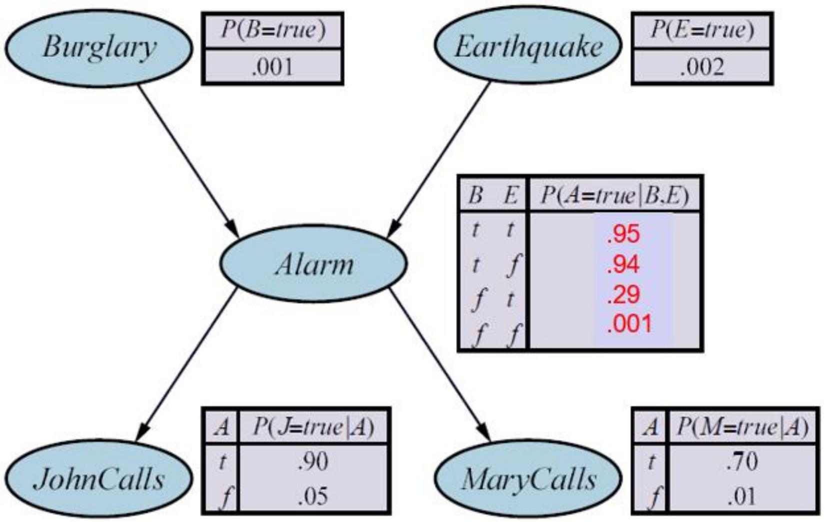
II. Independence within Context

III. Case Studies

* Figures are either from the textbook site or by the instructor.

I. Non-Descendants Property of BNs

"Non-descendants" property: Every variable is conditionally independent of its non-descendants, given its parents.



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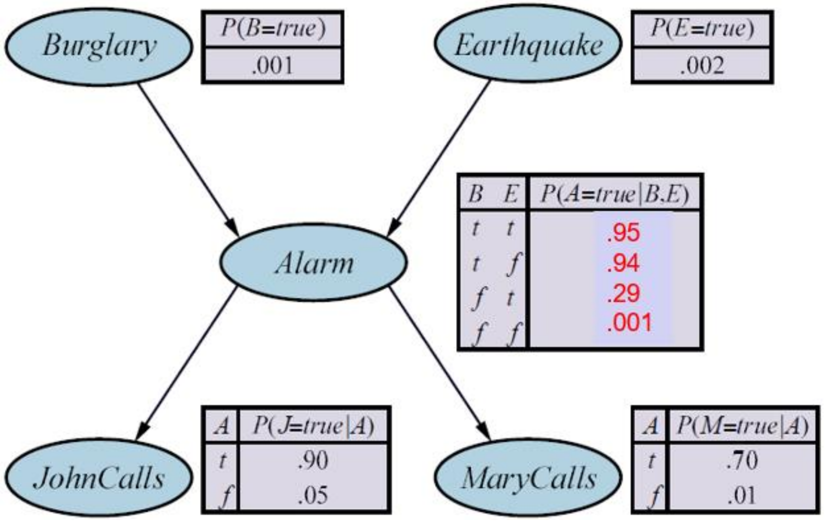

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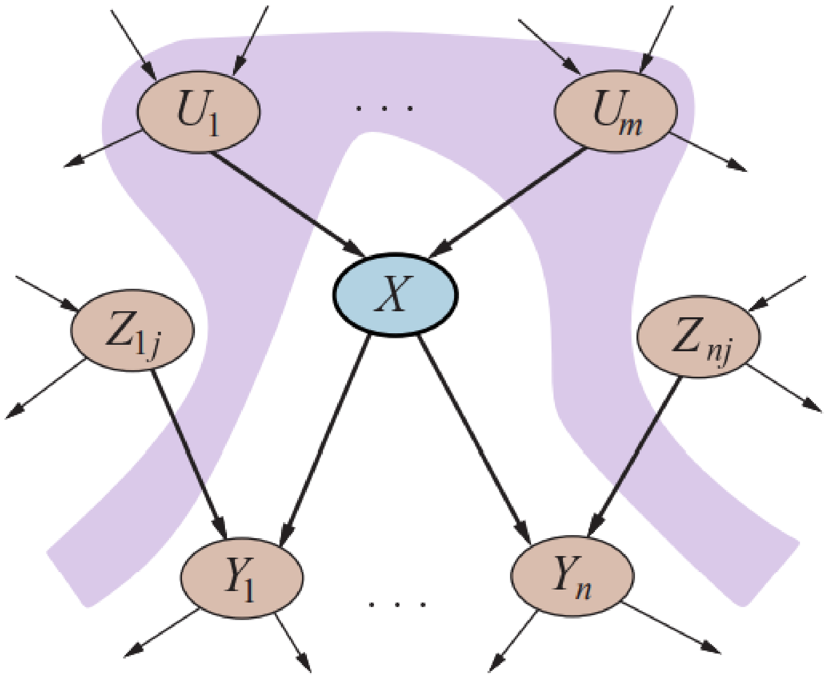
// network parameter interpretation

Illustration



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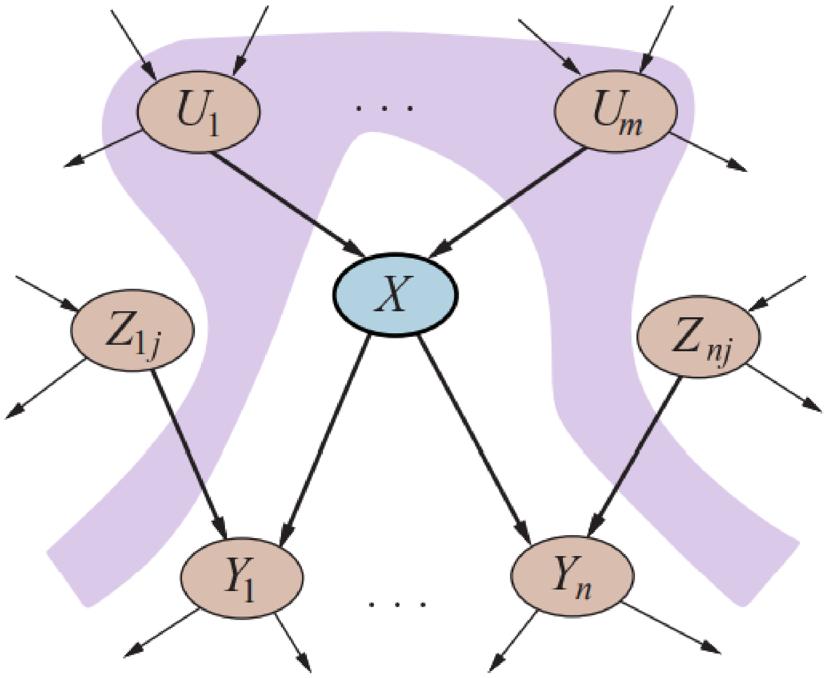

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$$\Downarrow$$

The full joint distribution $P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid parents(X_i))$

Markov Blanket

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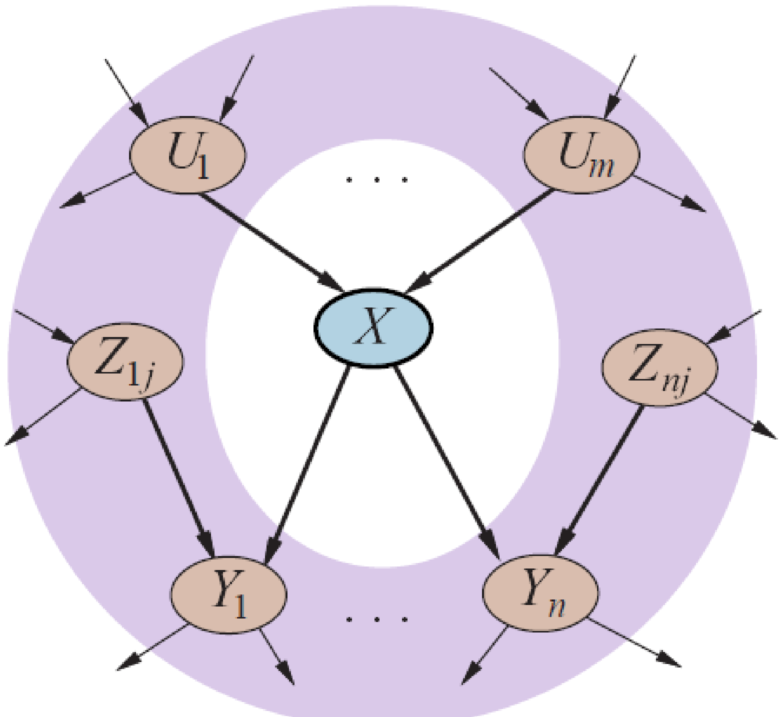

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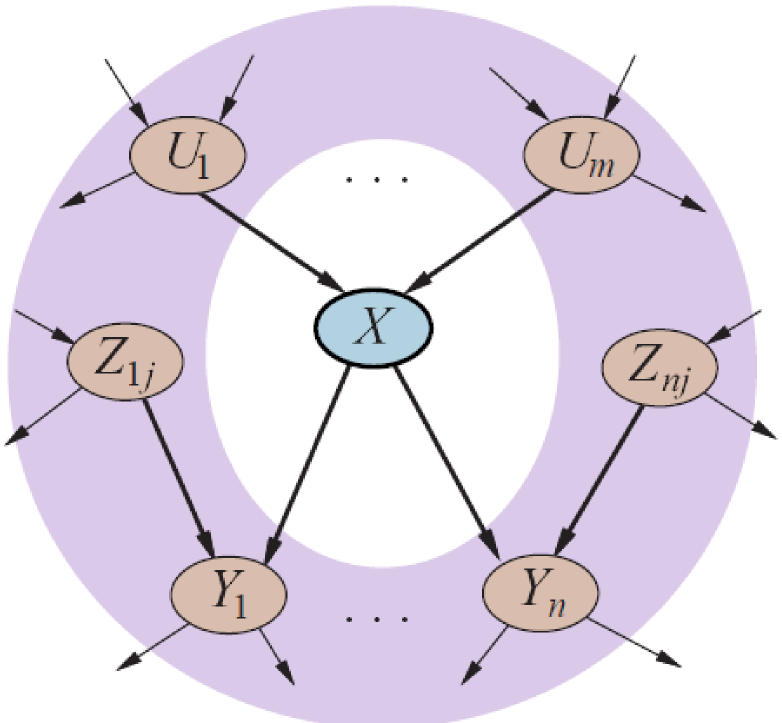

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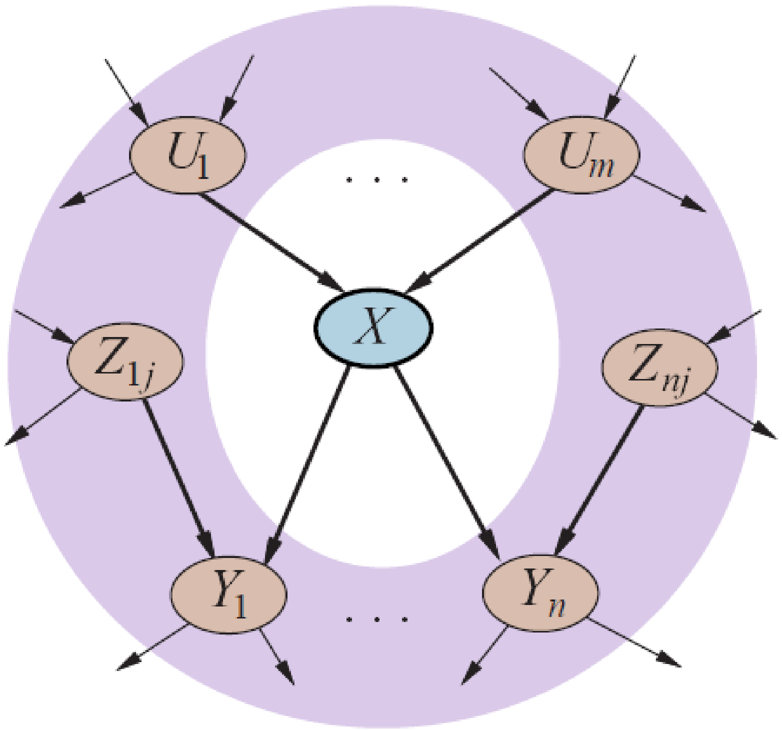

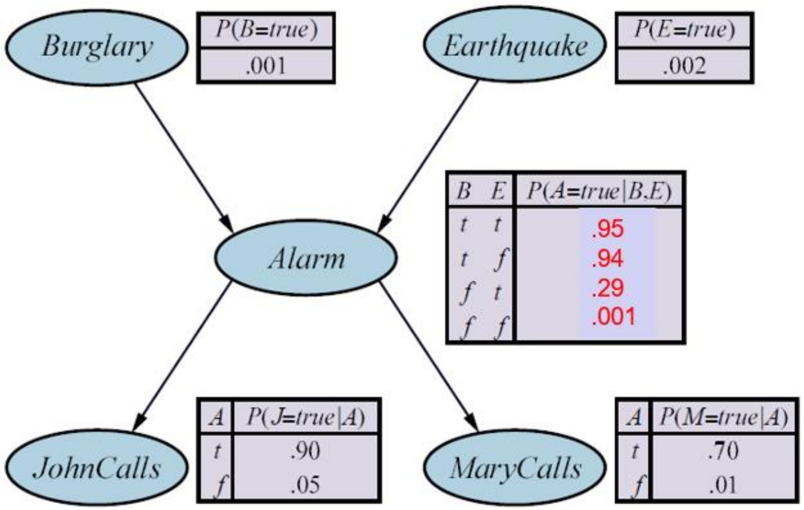

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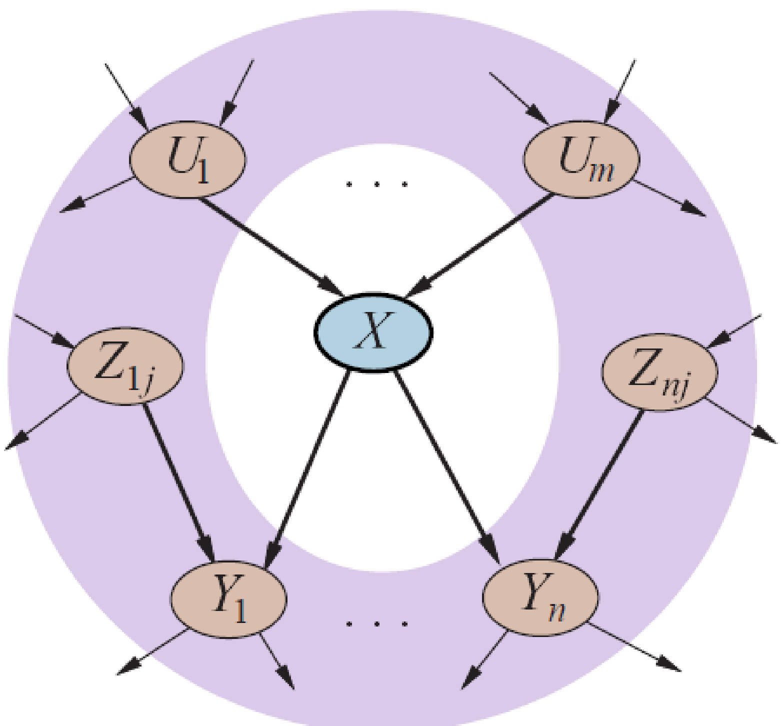

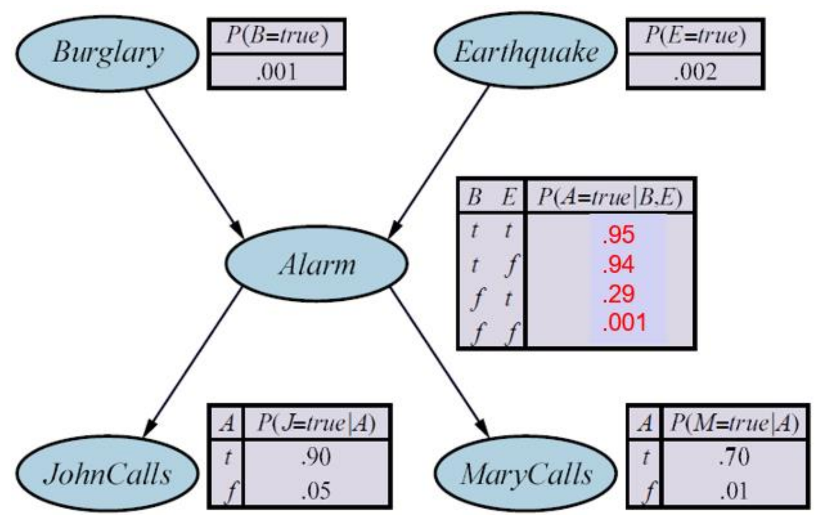

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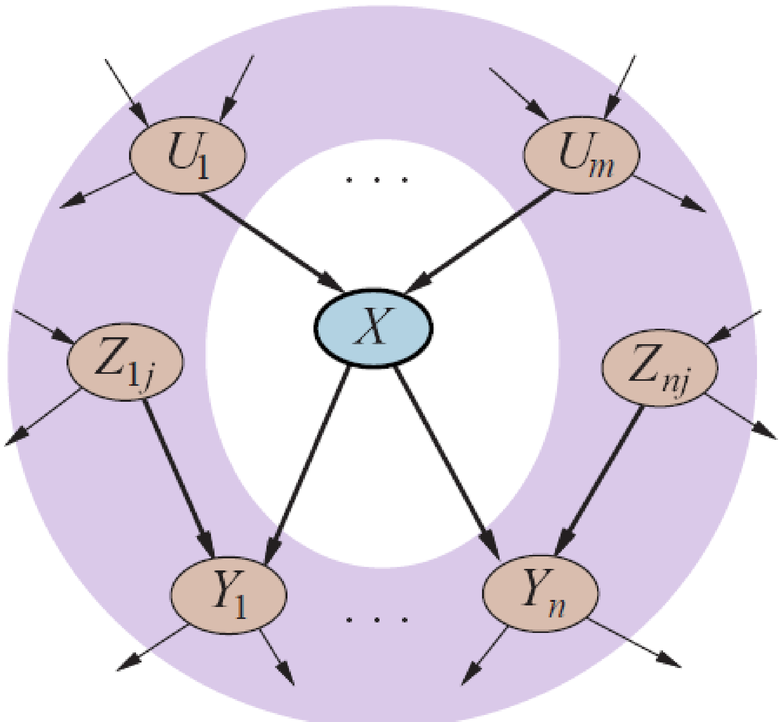

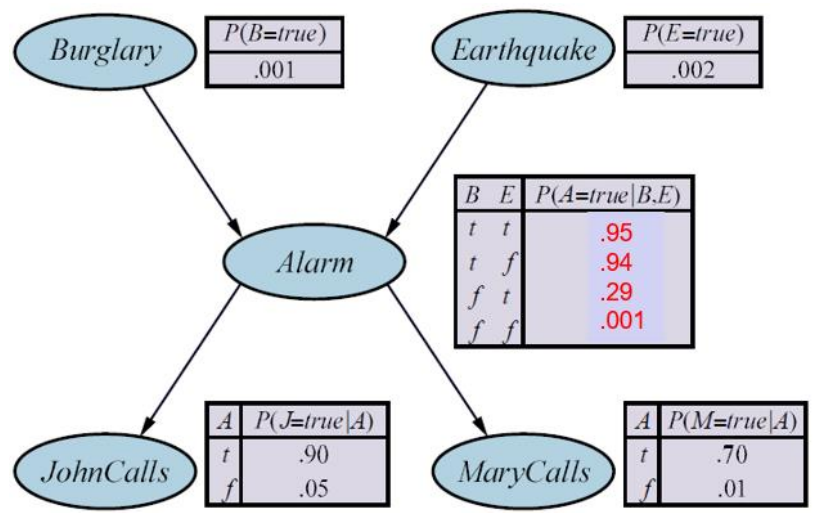

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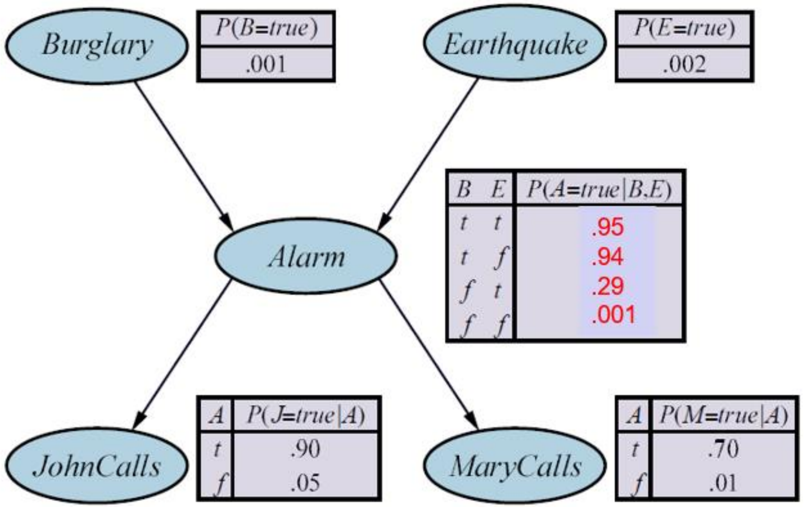

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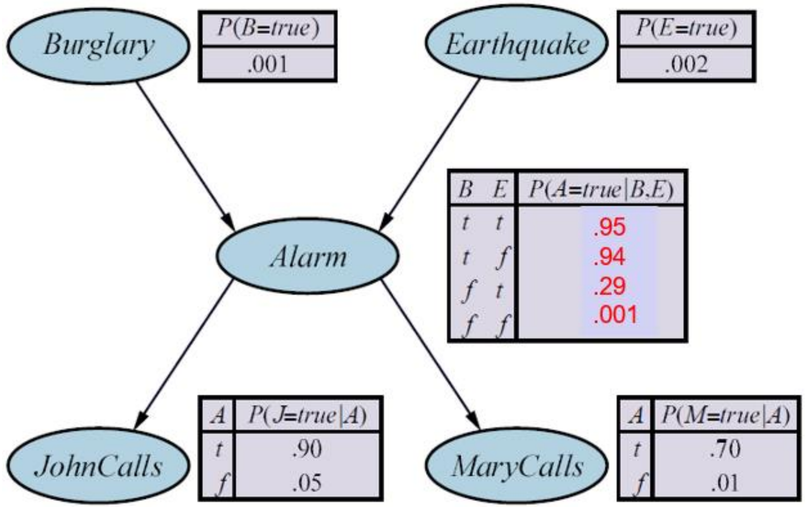

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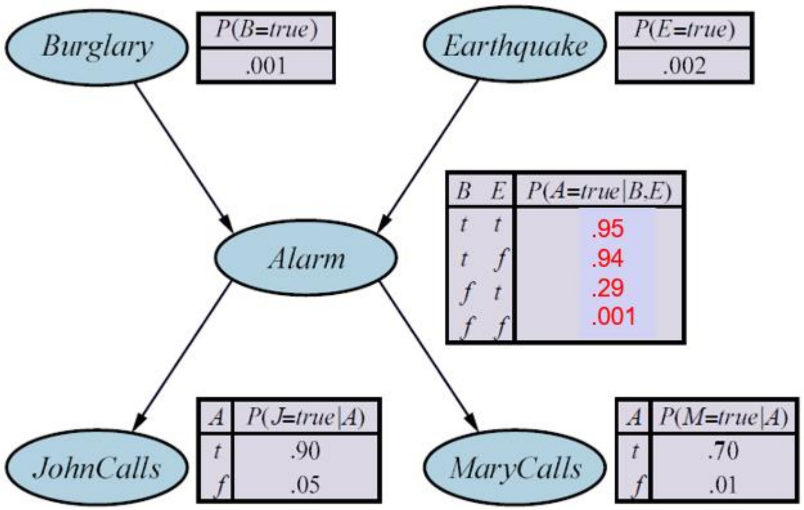

Markov blanket will be used for inferences over stochastic sampling processes.

D-Separation

Q: Is a set of nodes $\boldsymbol{X}$ conditionally independent of another set $\boldsymbol{Y}$, given a third set $\boldsymbol{Z}$? In short, does $\boldsymbol{P}(\boldsymbol{X} \mid \boldsymbol{Y}, \boldsymbol{Z}) = \boldsymbol{P}(\boldsymbol{X} \mid \boldsymbol{Z})$ hold?

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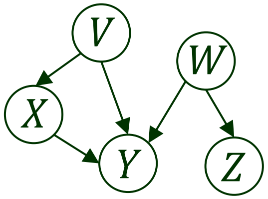

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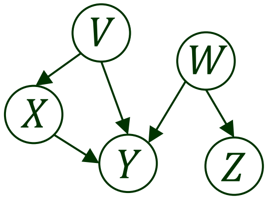

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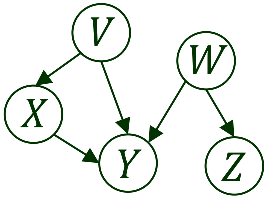

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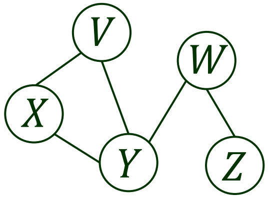

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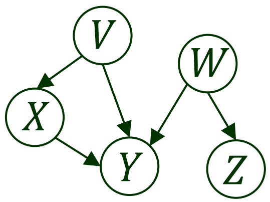

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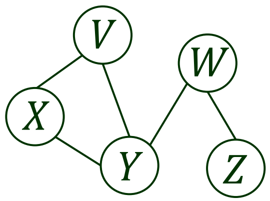

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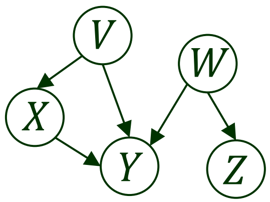

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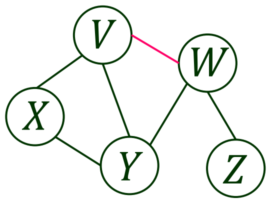

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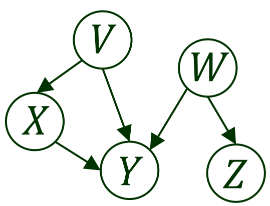

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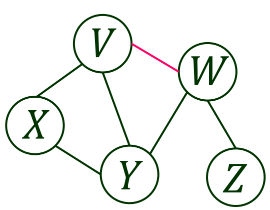

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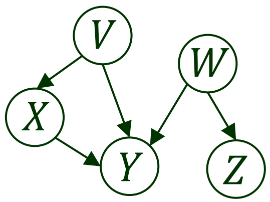

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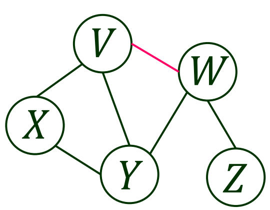

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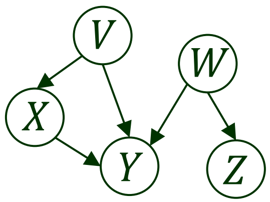

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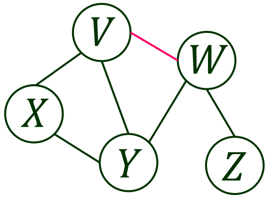

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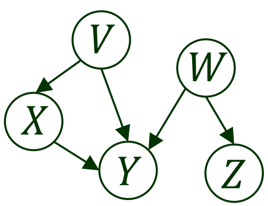

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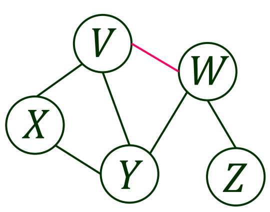

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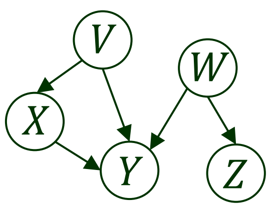

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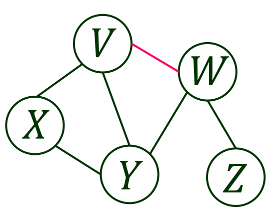
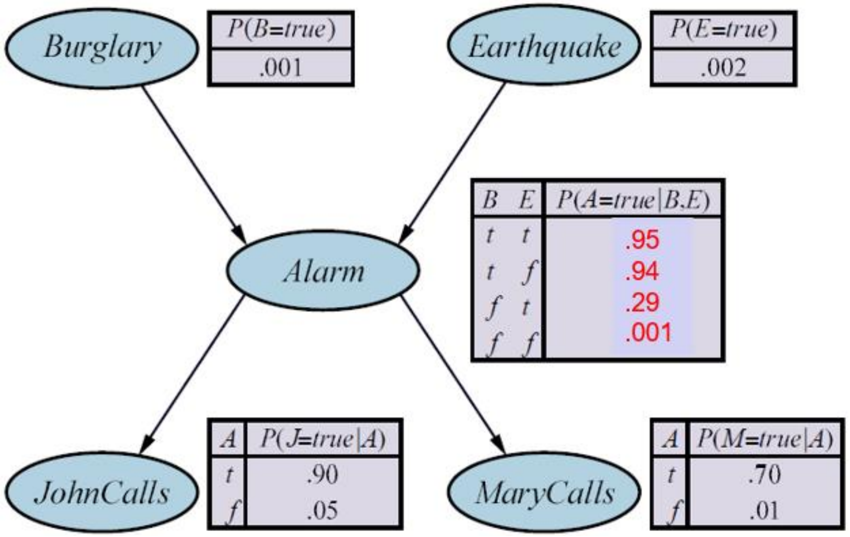


Moral graph

Example 1



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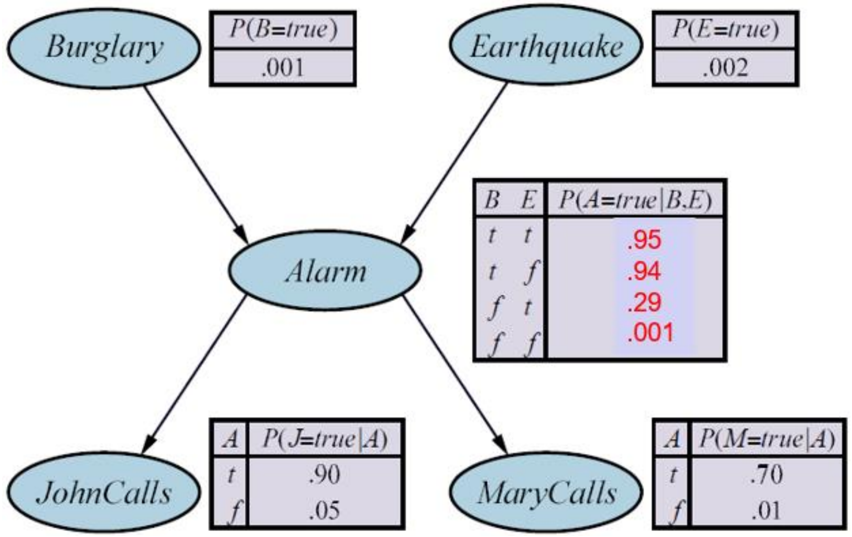

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Ancestral graph:

$\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ and their ancestors (and edges)

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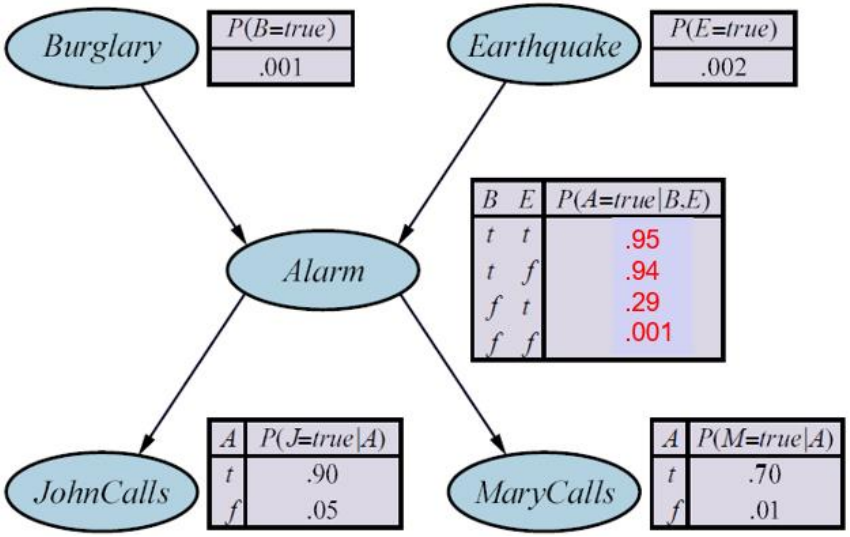

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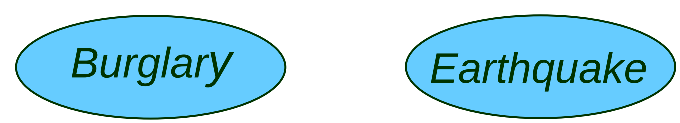
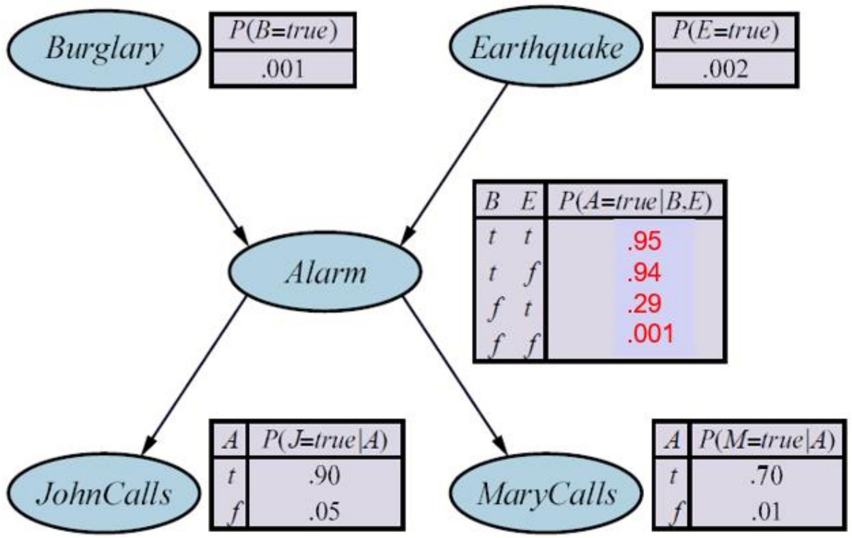
$\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ and their ancestors (and edges)

Moral graph:
also edges between two parents of the same child



Example 1



1. $\boldsymbol{X}$ = { *Burglary* }
   $\boldsymbol{Y}$ = { *Earthquake* }
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$\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ and their ancestors (and edges)

Moral graph:
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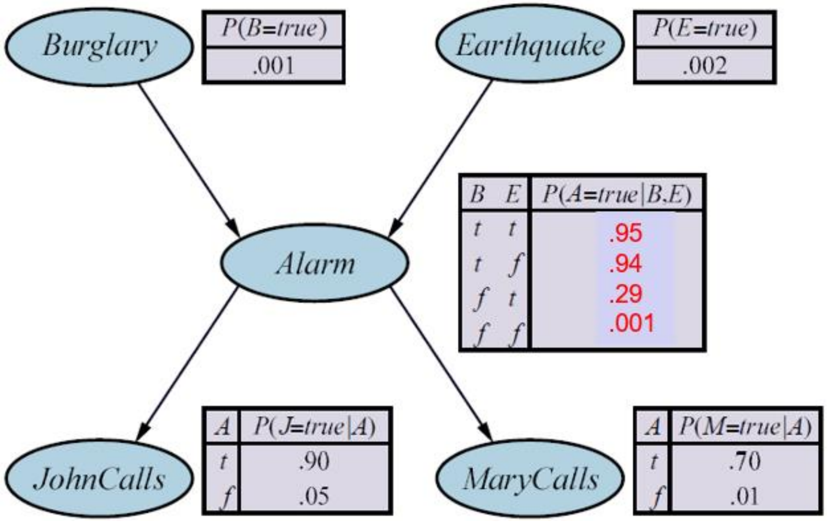
$\boldsymbol{X}$ and $\boldsymbol{Y}$ are separated, thus d-separated by $\boldsymbol{Z}$.
Burglary and *Earthquake* are independent given the empty set.

Example 2



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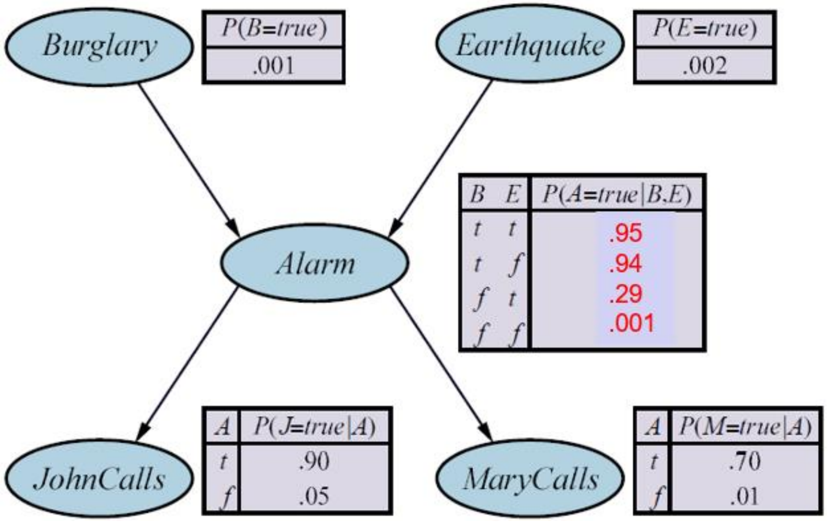

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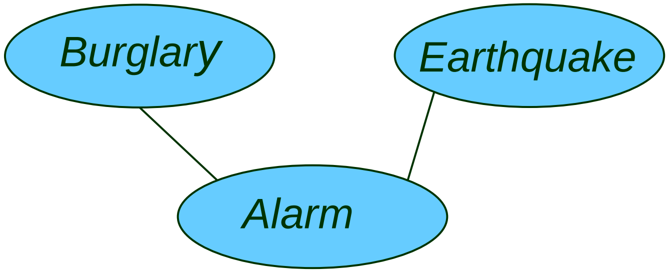

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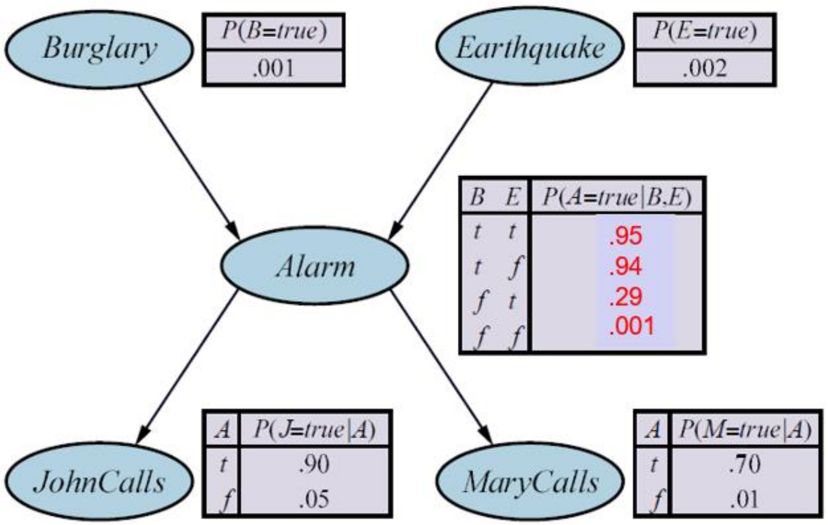

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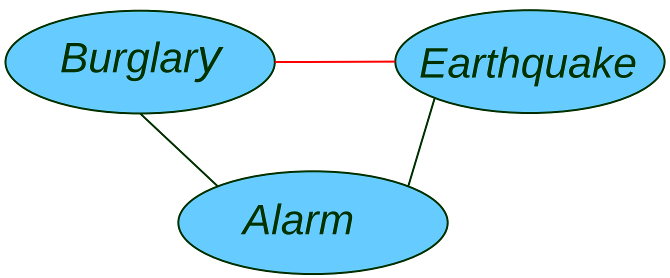

Moral graph

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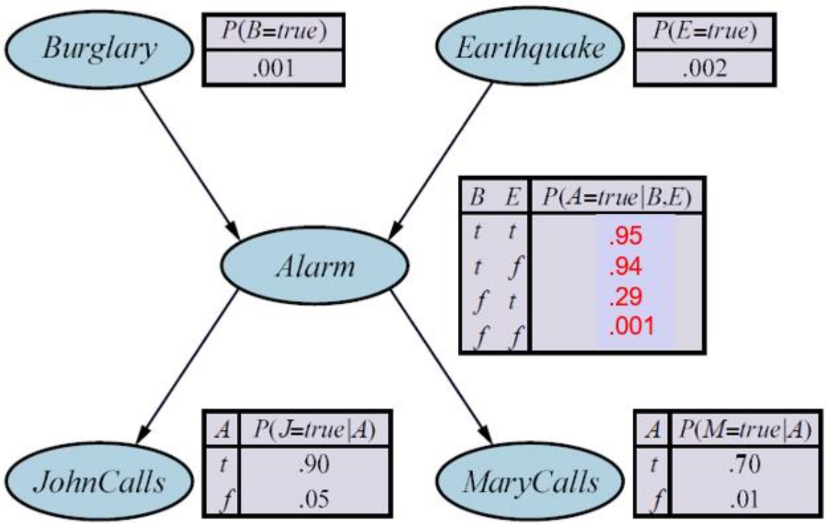

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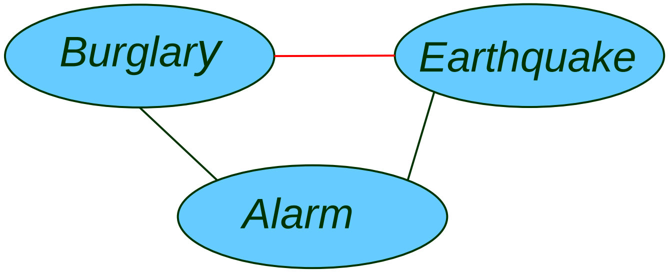
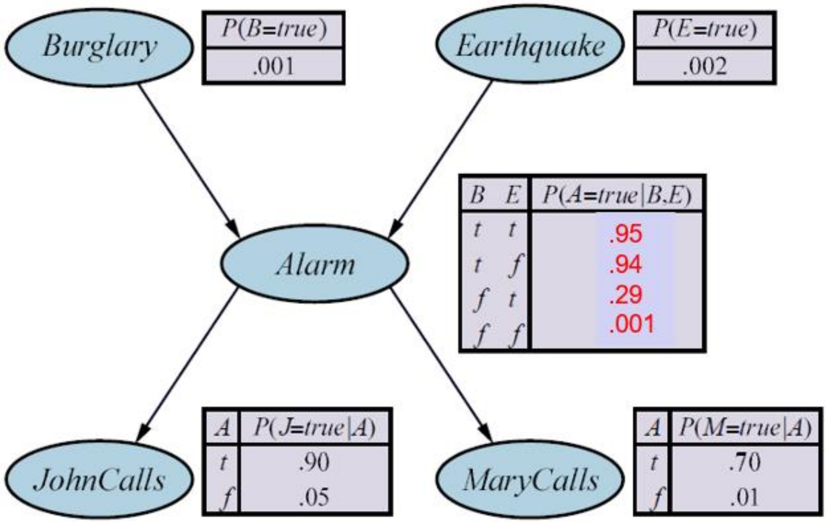


Moral graph

Burglary and *Earthquake* are not necessarily independent given *Alarm*.

Example 3



3. $\boldsymbol{X}$ = { *JohnCalls* }

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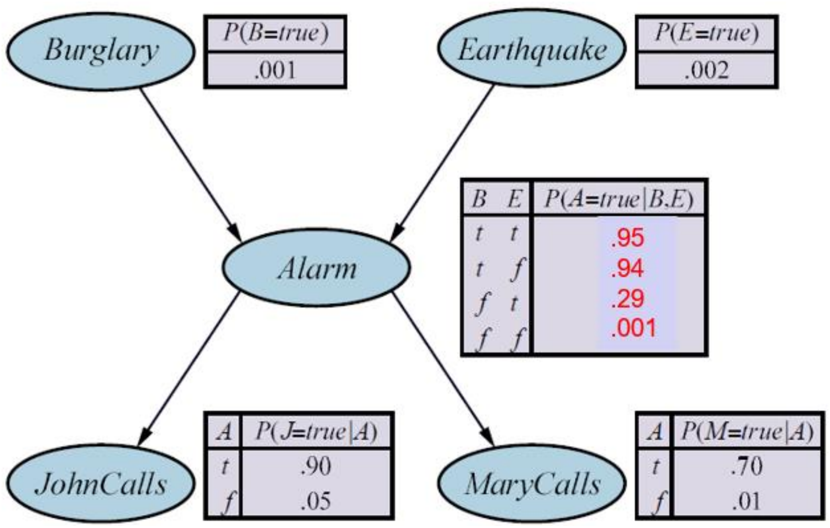

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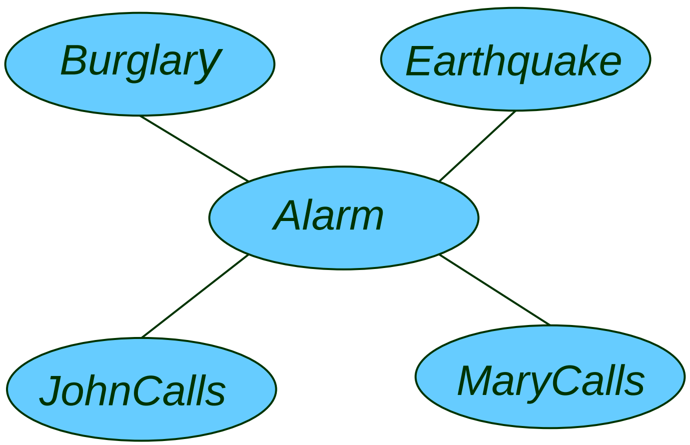

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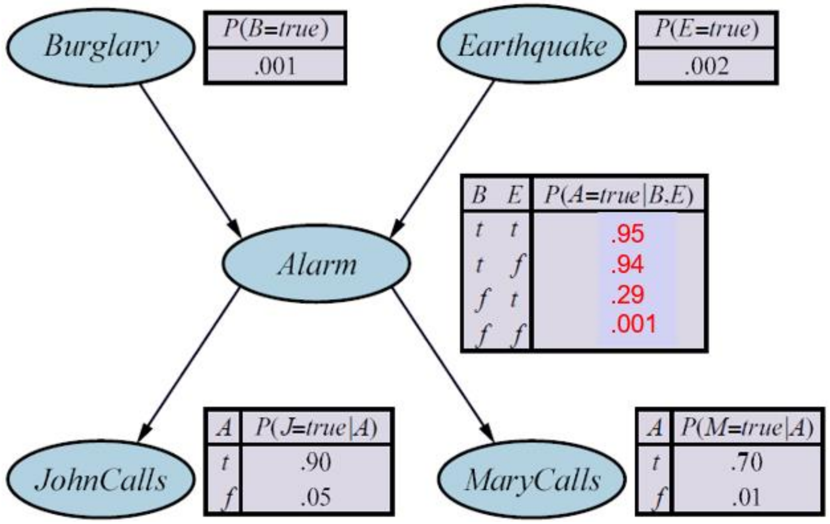

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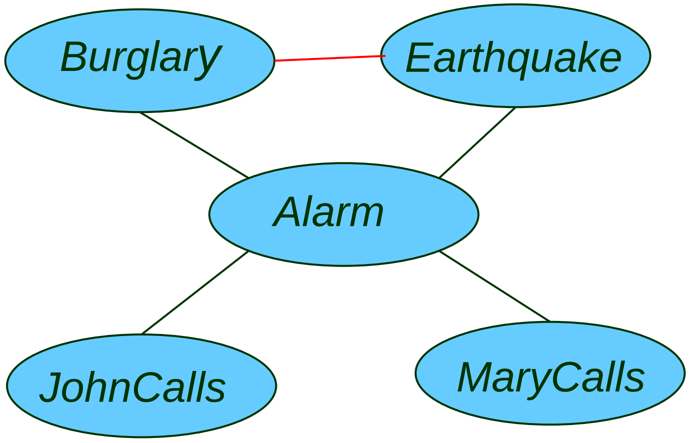

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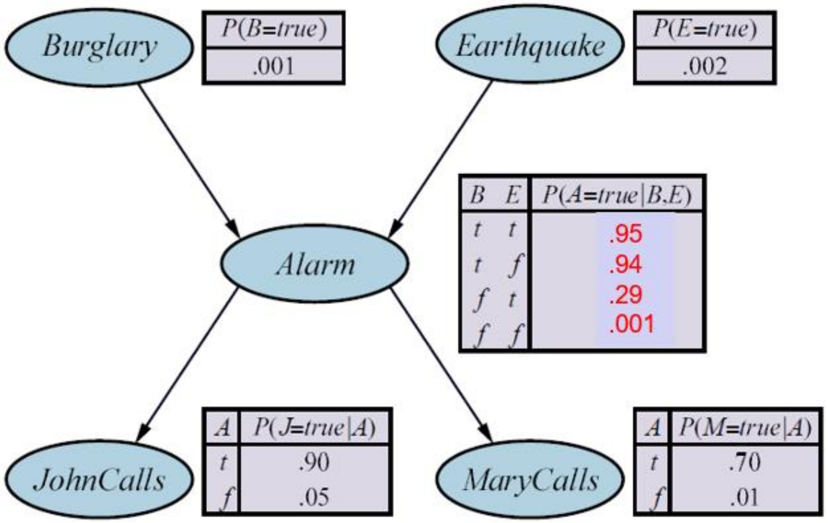

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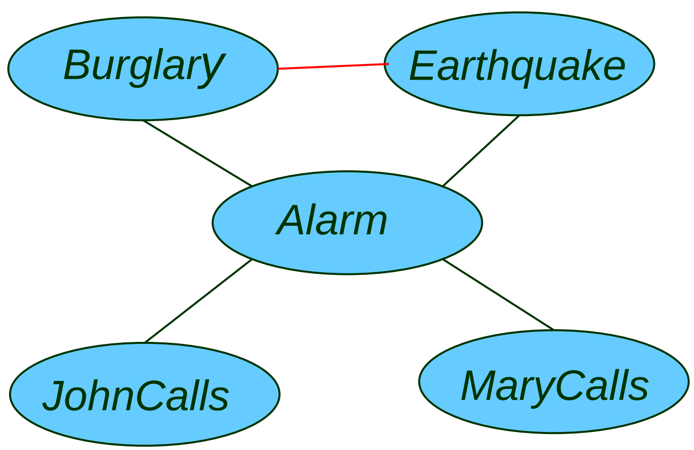
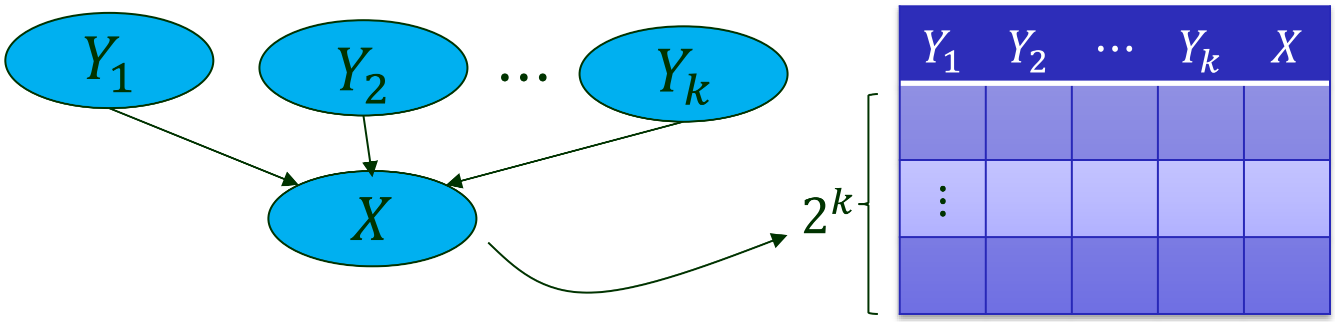
JohnCalls and *MayCalls* are conditionally independent given *Alarm*.

Moral graph



II. Efficient Representations

- The size 2^k of the *conditional probability table* (CPT) for a node with k parents is the worst-case scenario in which *the relationships with the parents are arbitrary*.



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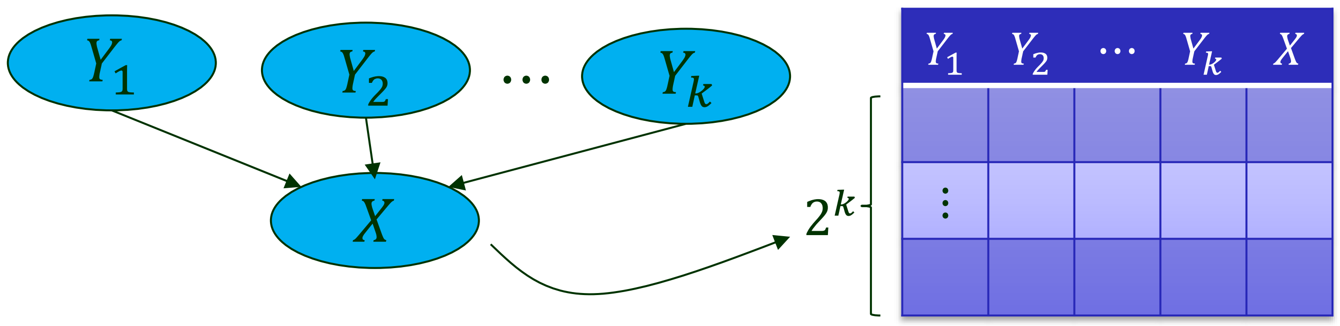

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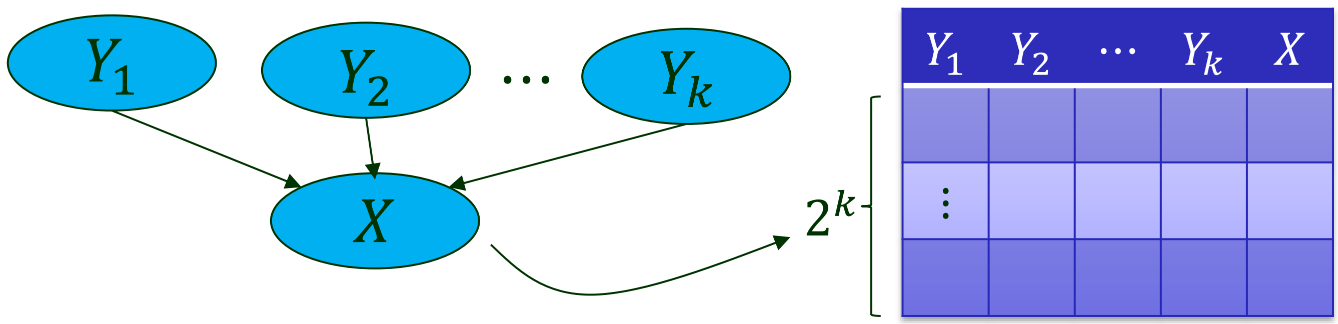

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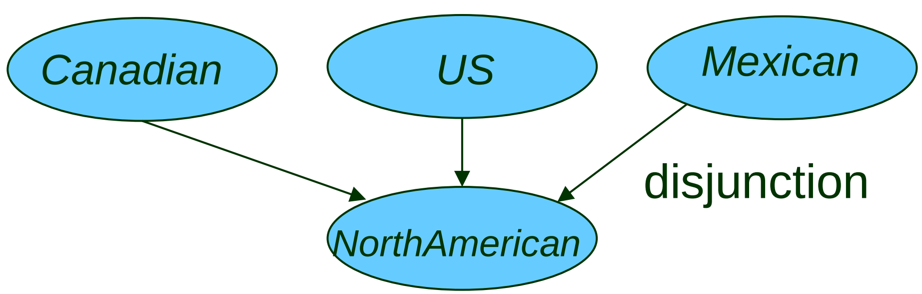

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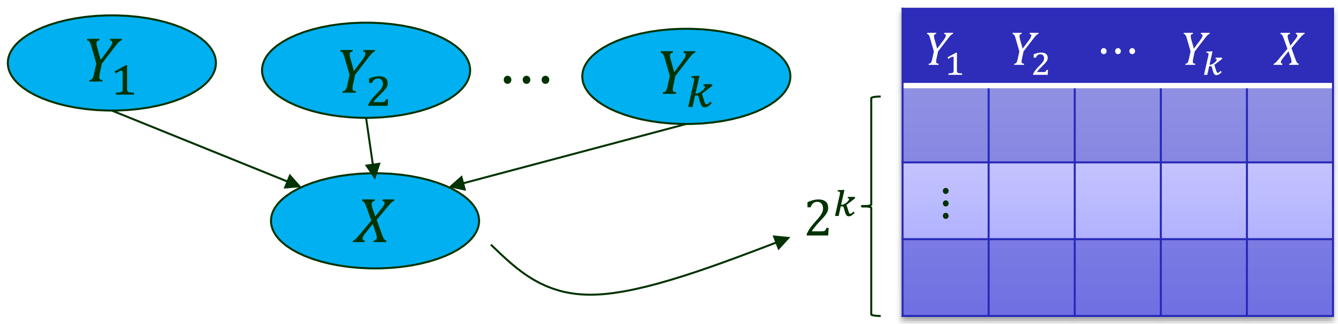

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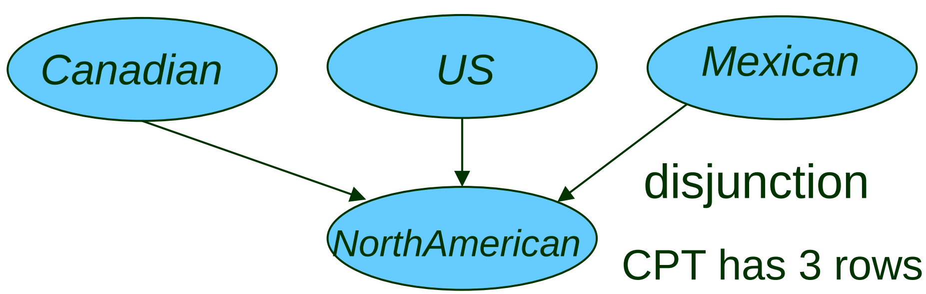

CPT has 3 rows instead of 8.

Independence Within Context

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// if no accident, damage to
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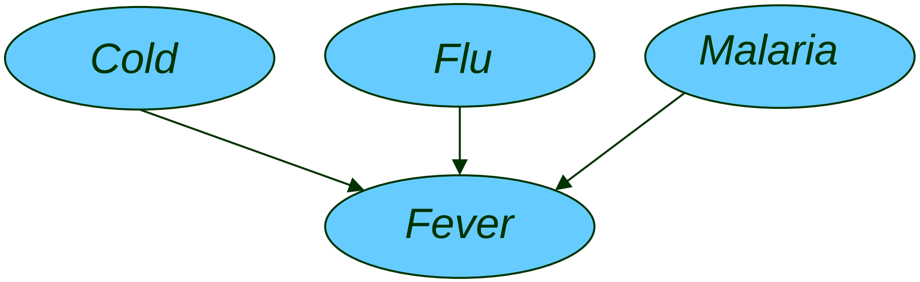
$Fever \Leftrightarrow Cold \vee Flu \vee Malaria$ (Propositional logic)



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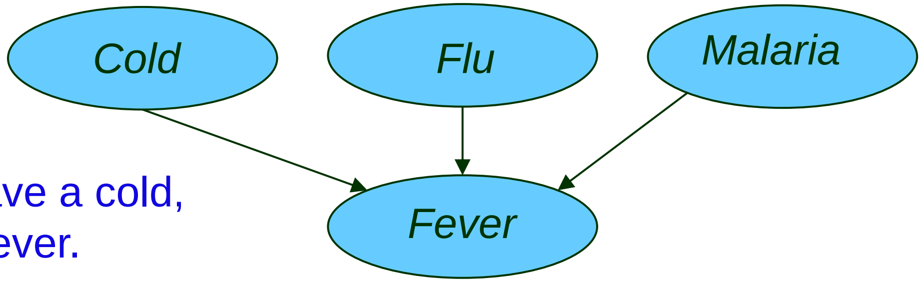

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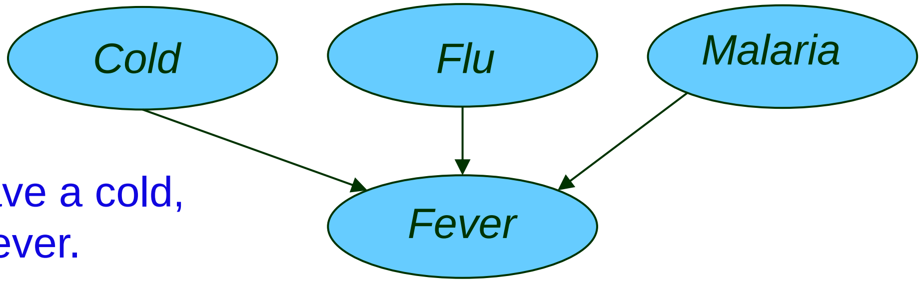

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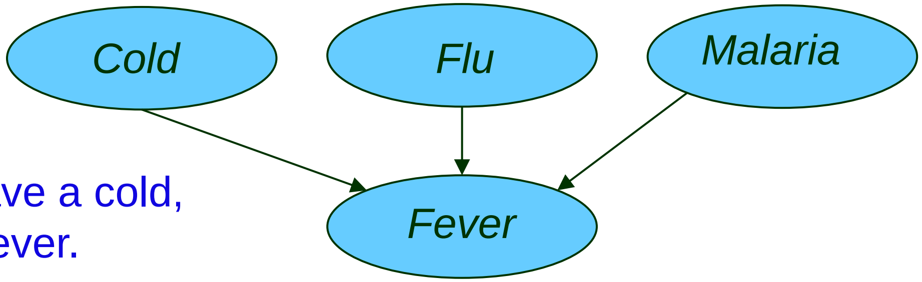

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Noisy-OR allows for uncertainty about each of *Cold*, *Flu*, *Malaria* to cause Fever.

- The causal relationship between a node and its parents may be *inhibited*.
  - List all the possible causes.
  - Inhibition of each parent is independent of inhibition of any other parents.

Causes of Fever

Cold	*Flu*	*Malaria*	$P(fever \mid \cdot)$	$P(\neg fever \mid \cdot)$
f	f	f	0.0	1.0
f	f	t	0.9	**0.1**
f	t	f	0.8	**0.2**
f	t	t	0.98	$0.02 = 0.2 \times 0.1$
t	f	f	0.4	**0.6**
t	f	t	0.94	$0.06 = 0.6 \times 0.1$
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$$q_j \equiv P\left(\neg x_i \mid X_j \in Parents(X_i) \wedge X_j = true \wedge \forall Y \left(Y \in parents(X_i) \wedge Y \neq X_j \Rightarrow Y = false\right)\right)$$

Independence of Inhibition

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$Cold$	Flu	$Malaria$	$P(fever \mid \cdot)$	$P(\neg fever \mid \cdot)$
f	f	f	0.0	1.0
f	f	t	0.9	**0.1**
f	t	f	0.8	**0.2**
f	t	t	0.98	$0.02 = 0.2 \times 0.1$
t	f	f	0.4	**0.6**
t	f	t	0.94	$0.06 = 0.6 \times 0.1$
t	t	f	0.88	$0.12 = 0.6 \times 0.2$
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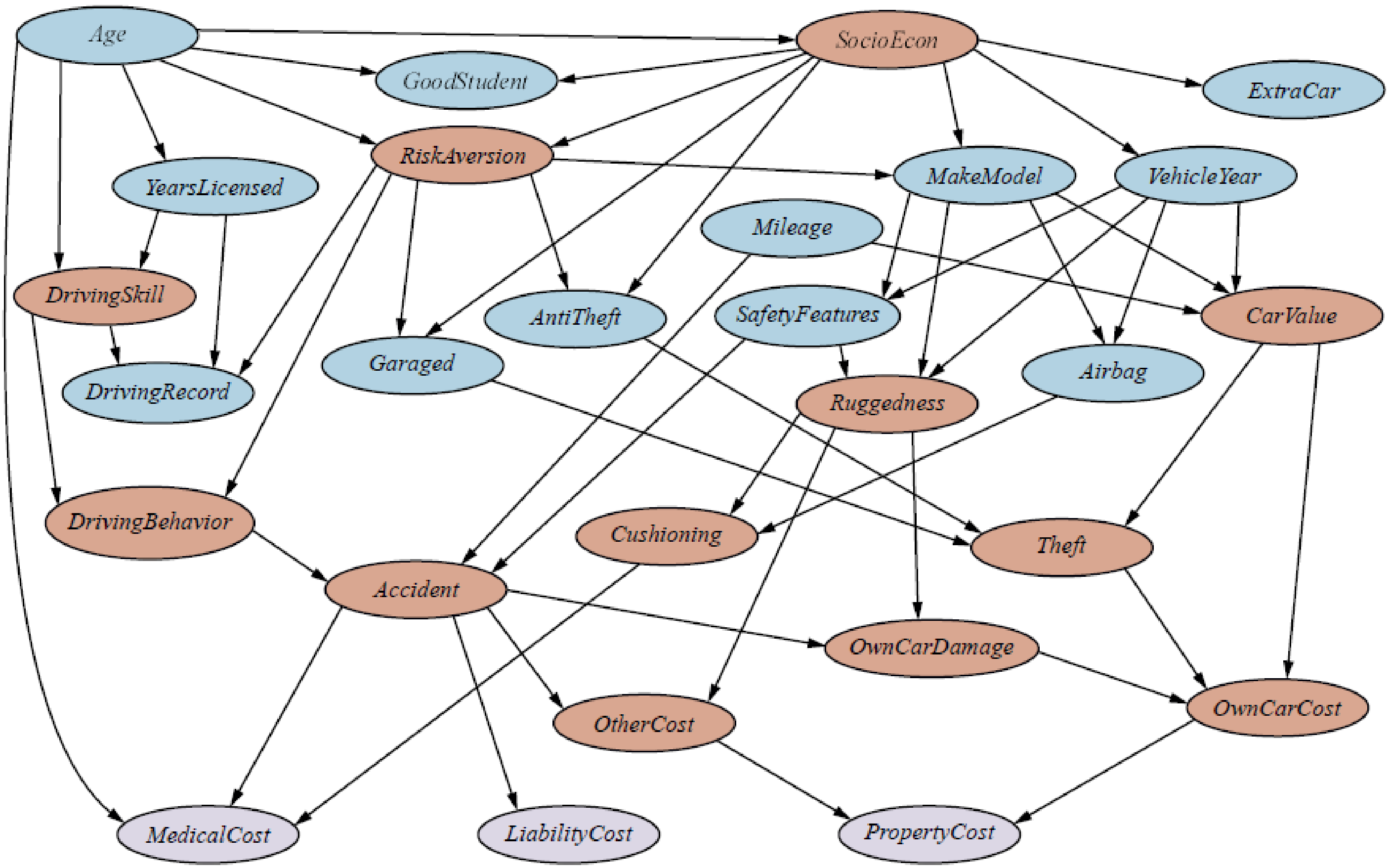
The entire CPT can be constructed using q_1, q_2, q_3.

$$P(x_i \mid parents(X_i)) = 1 - \prod_{X_j \in Parents(X_i) \wedge X_j = true} q_j$$

III. Case Study 1: Car Insurance

A car insurance company processes a car insurance application to decide on the annual premium based on the anticipated claims it will pay out for the applicant.



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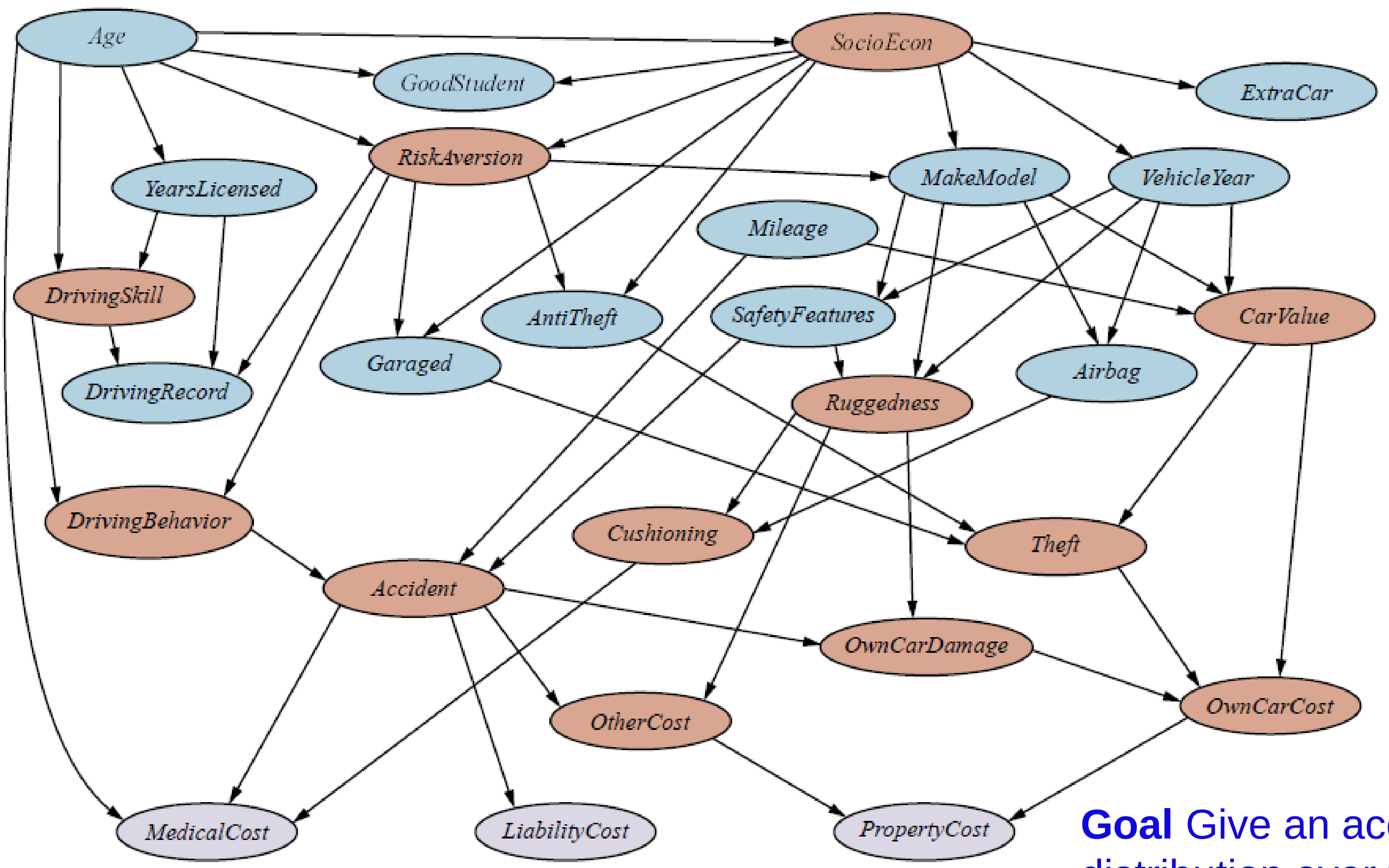

Goal Give an accurate, well-calibrated distribution over the output variables given the evidence available form the input.

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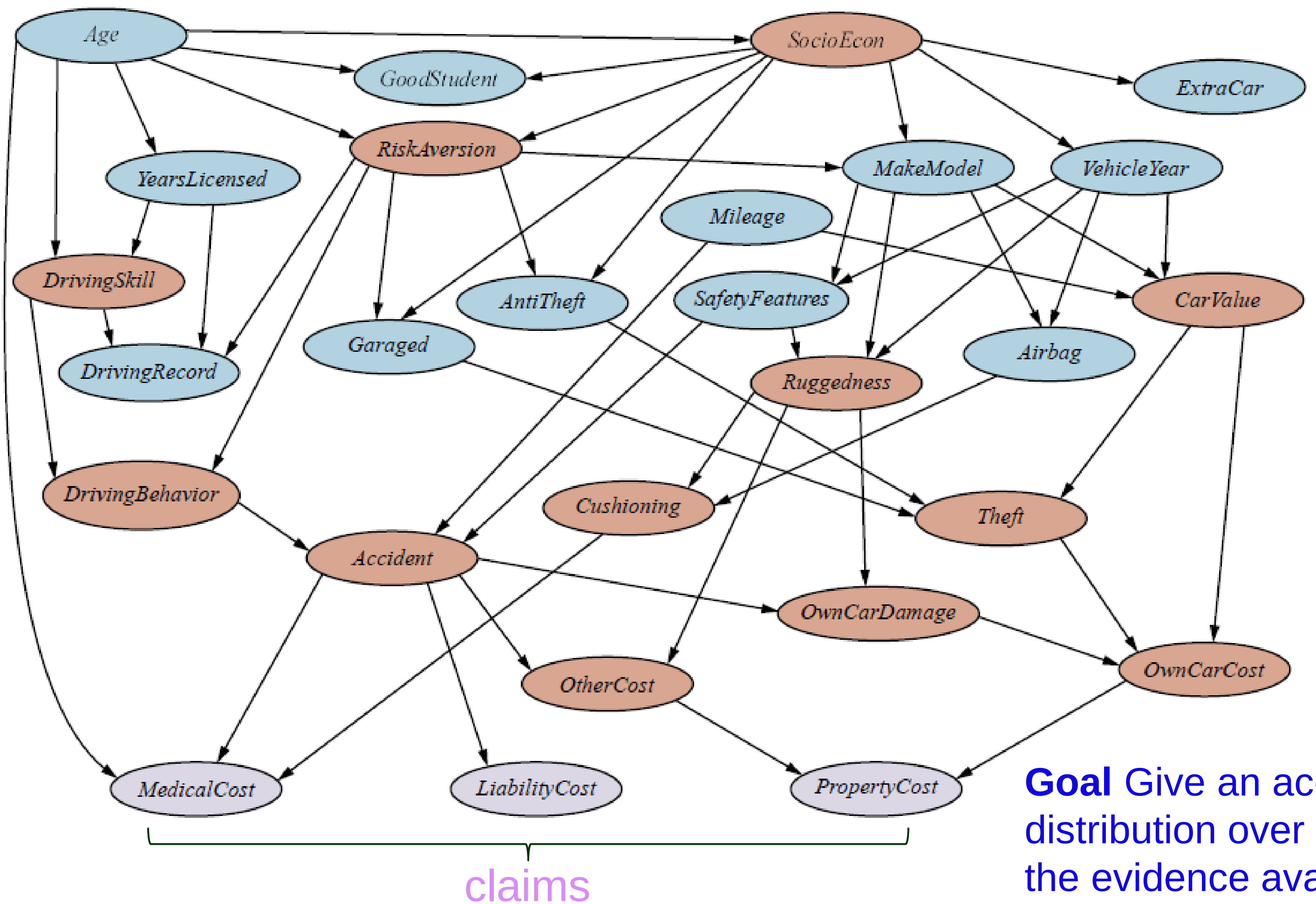

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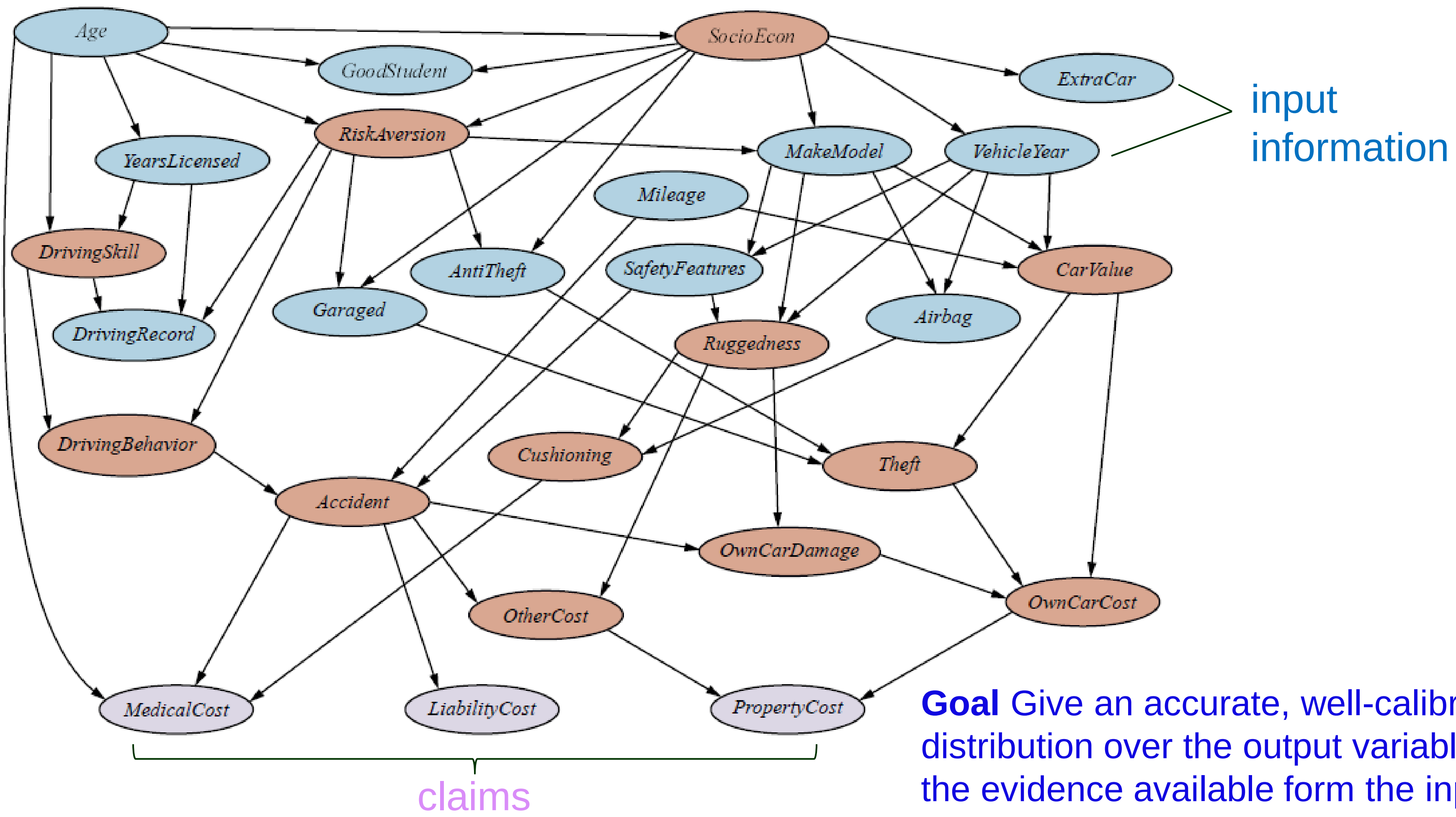

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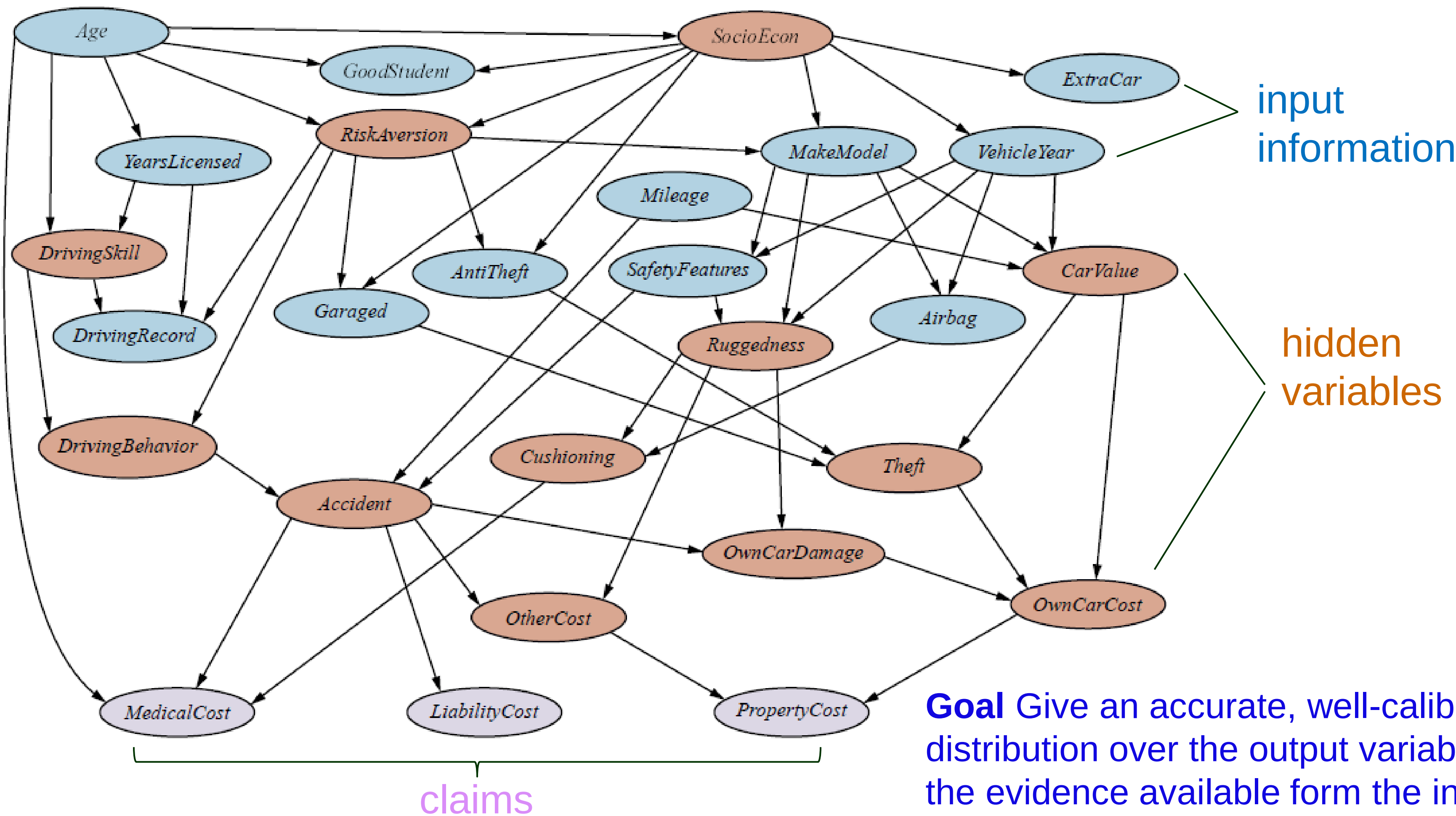

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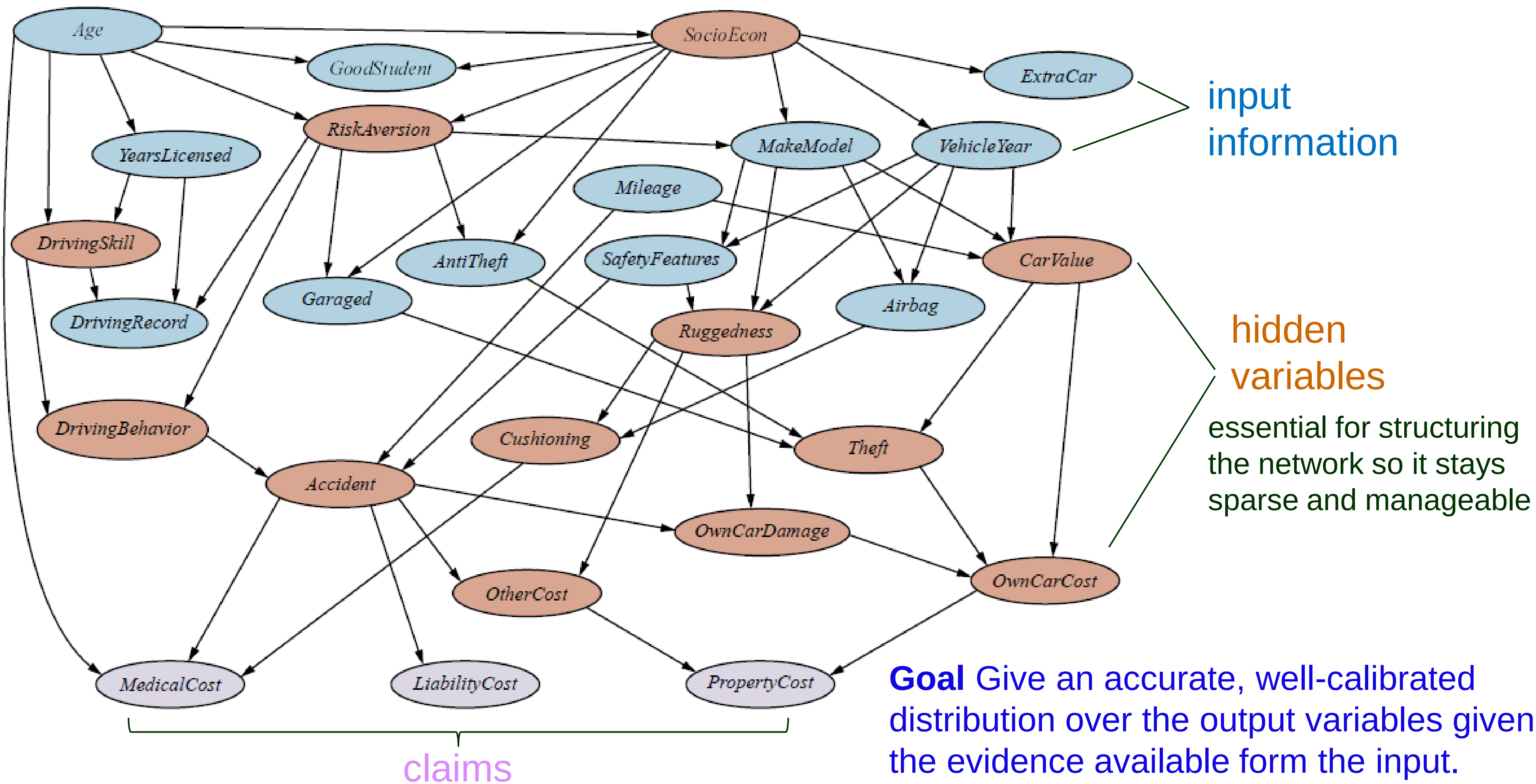

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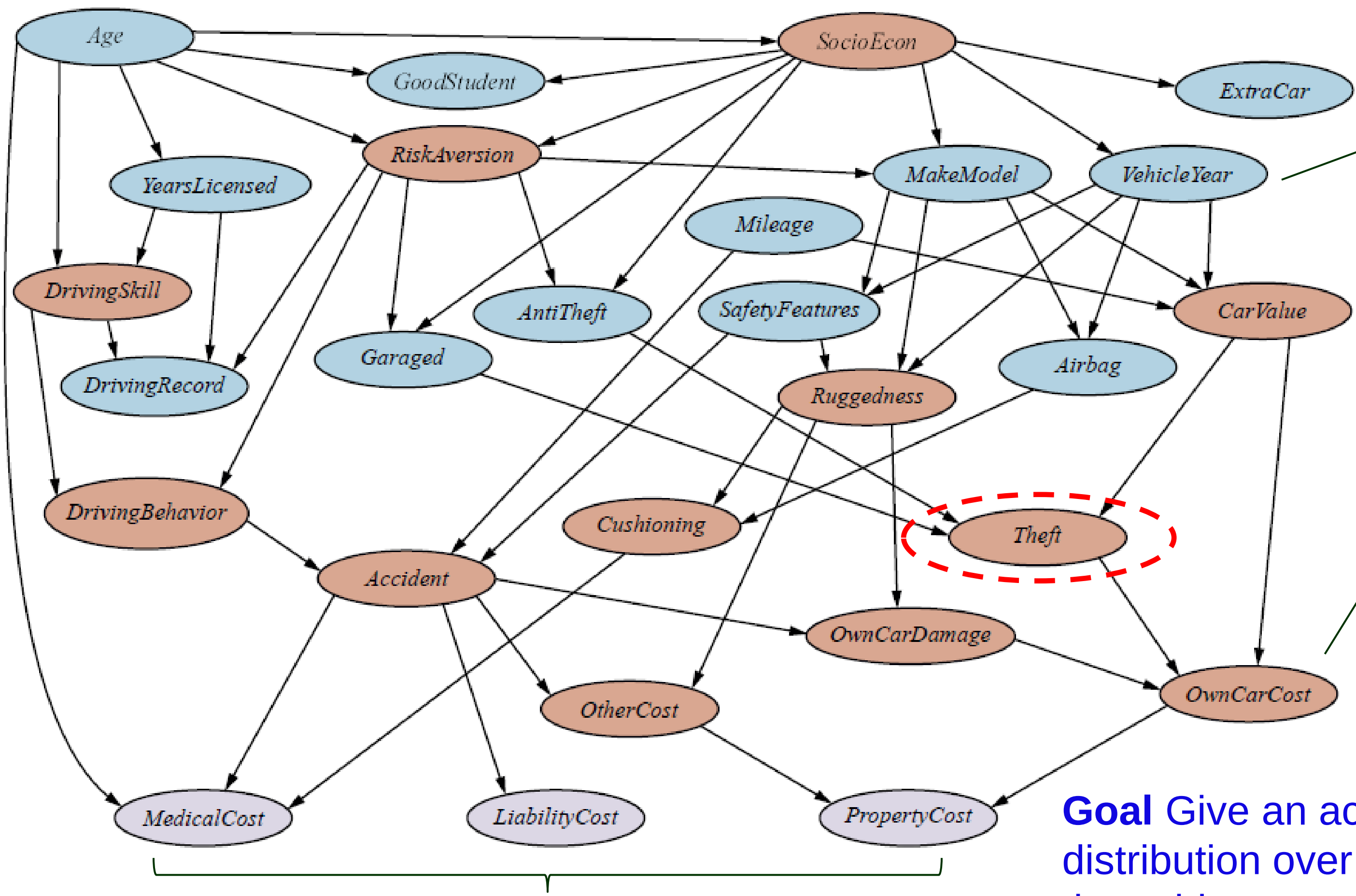

input information

hidden variables

essential for structuring the network so it stays sparse and manageable

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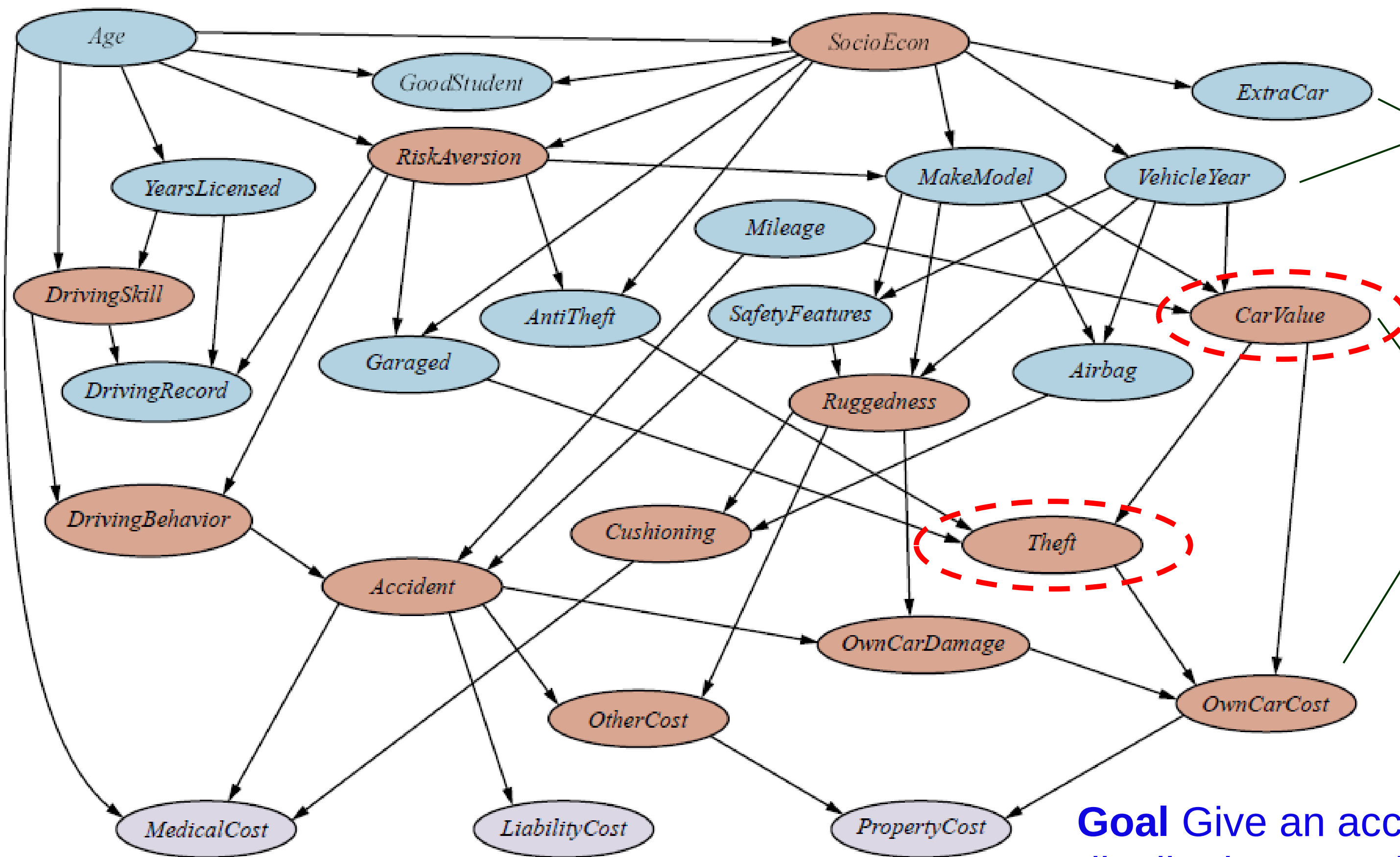

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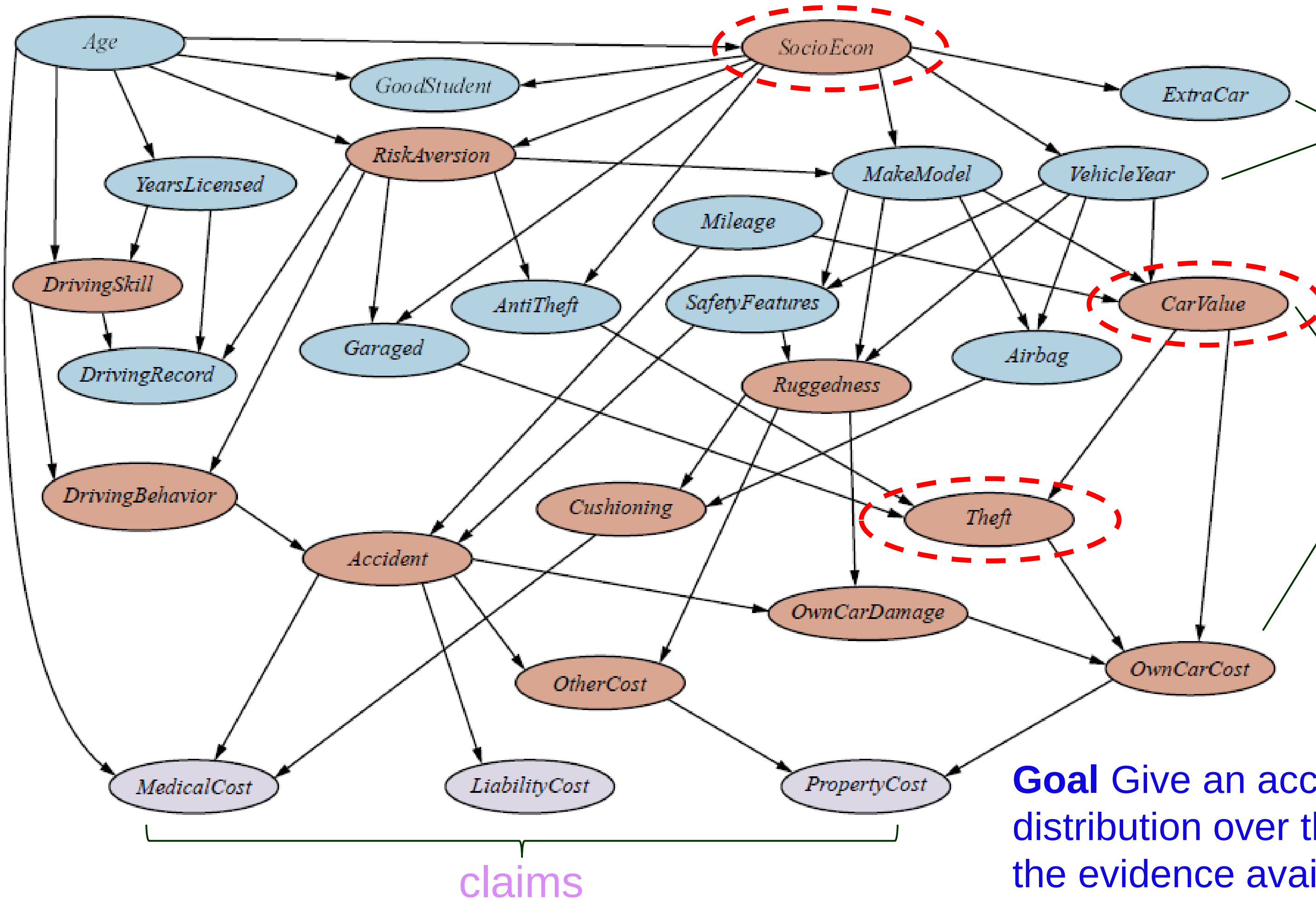

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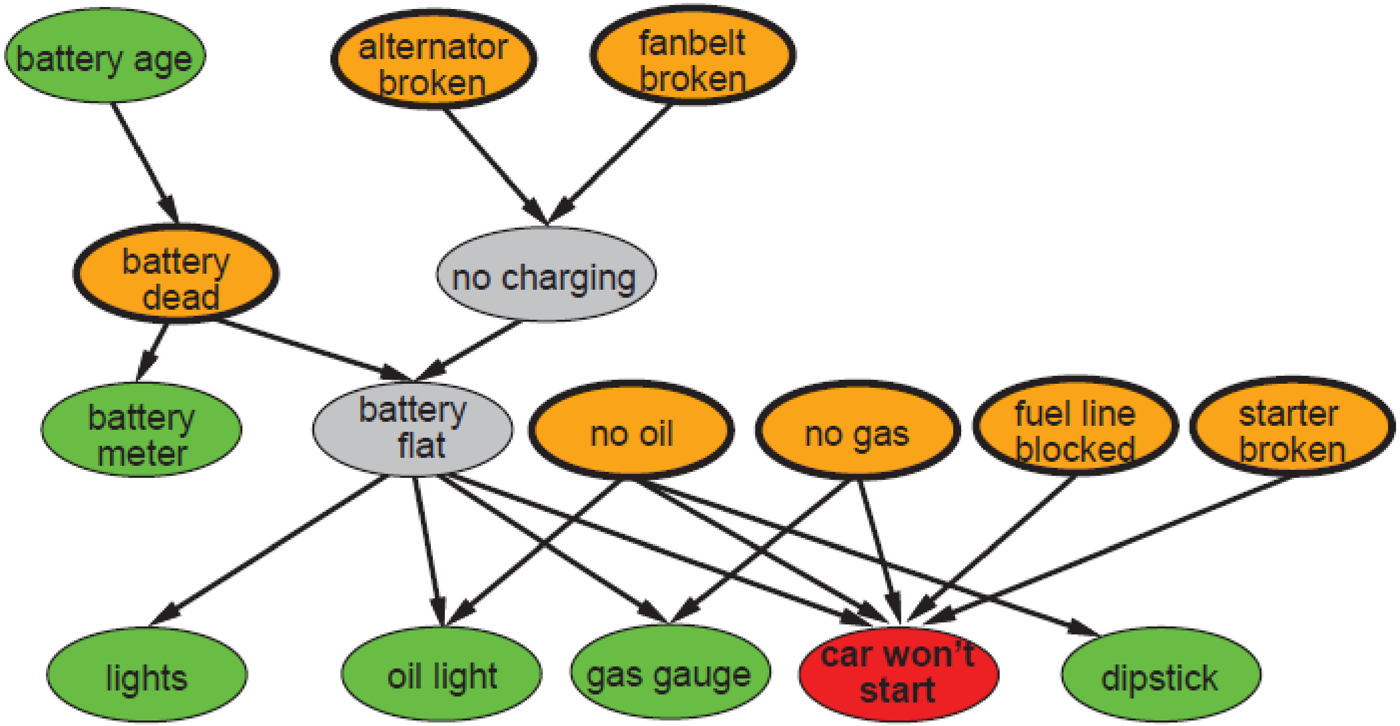
Case Study 2: Car Diagnosis



* Courtesy of Dr. Jin Tian