Yan-Bin Jia, Iowa State University

Segment Trees

Outline:

I. General windowing queries

II. Inapplicability of an interval tree

III. Locus approach

IV. Structure of a segment tree

V. Query and construction

VI. Solution of a general windowing query

VII. Analysis

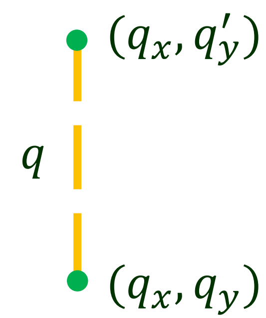
Textbook: *Computational Geometry: Algorithms and Applications (*3rd ed.) by M. de Berg et al., Springer-Verlag, 2008.

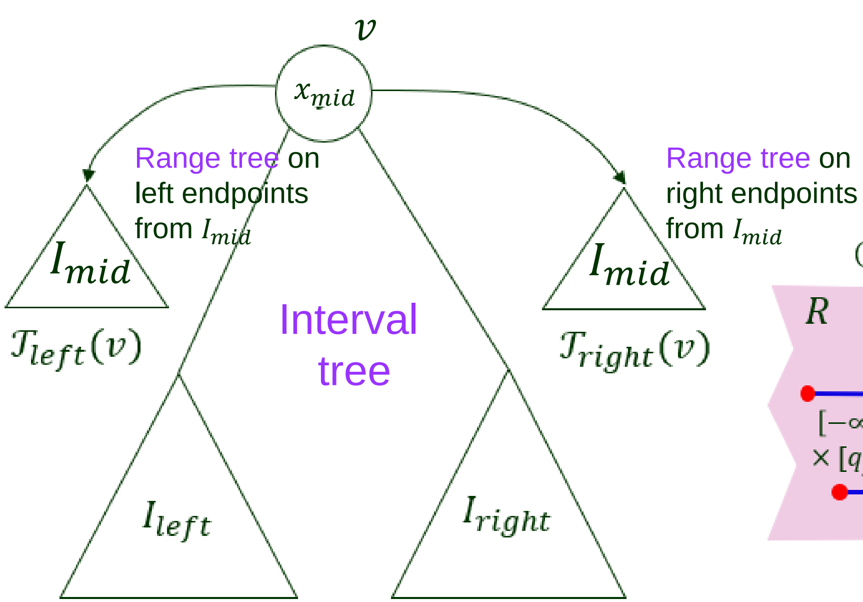
I. Windowing Query Over Horizontal Segments

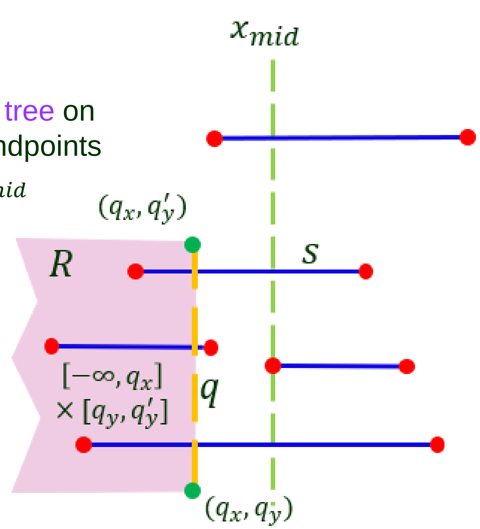
Interval tree + range trees

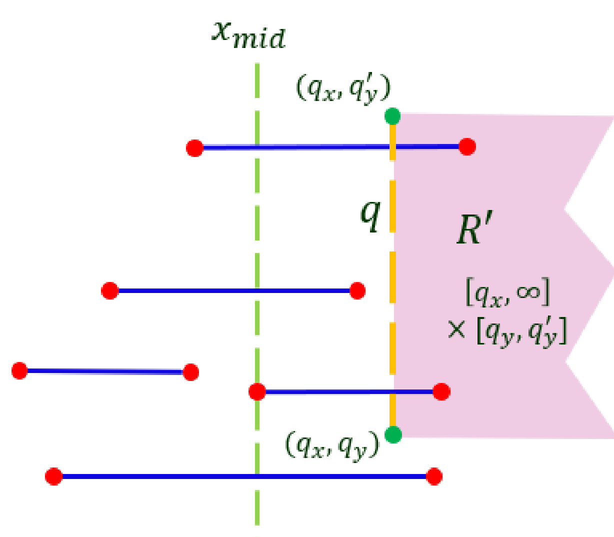
Query object: vertical segment $q_x \times [q_y, q'_y]$

Set: n horizontal line segments









Interval Tree + Priority Search Trees

Applicable to axis-parallel segments only.

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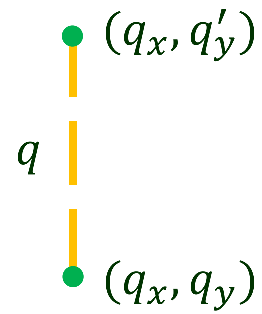
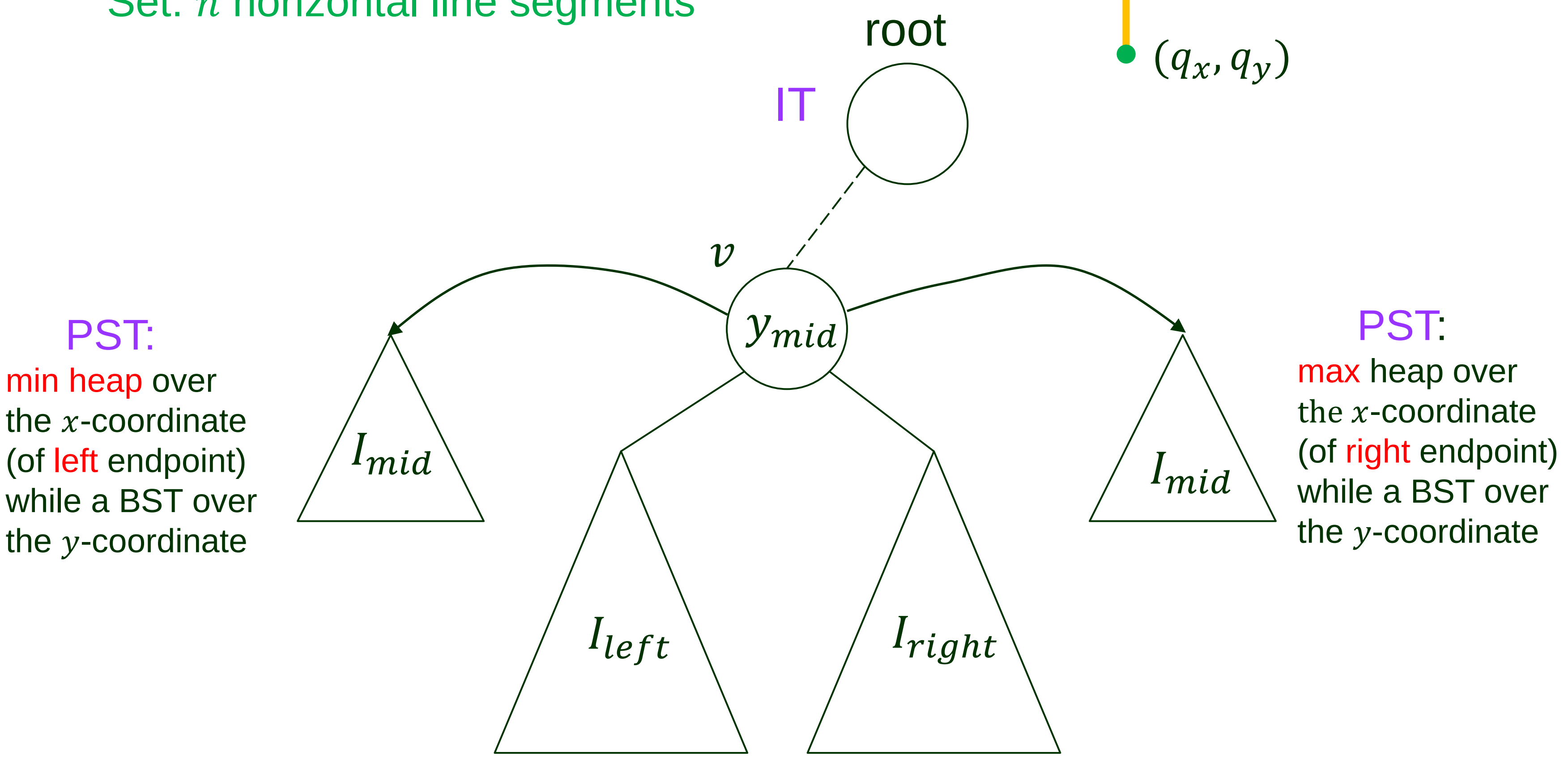
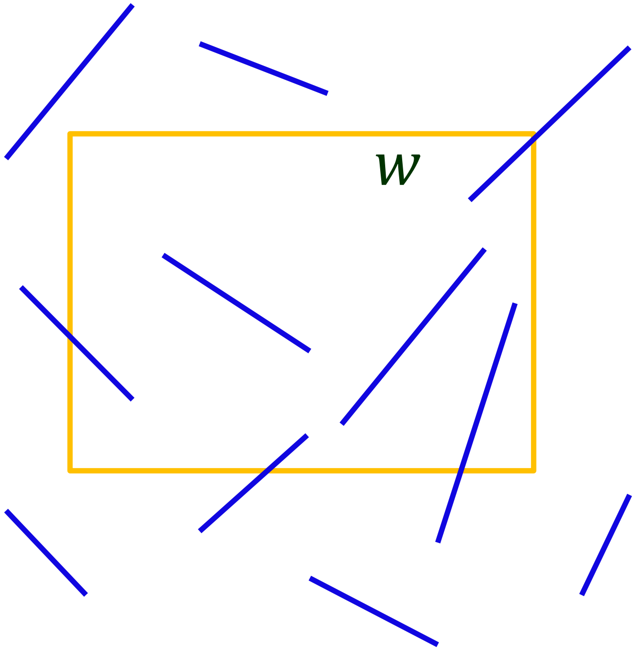




General Windowing Queries

Line segments can have arbitrary orientations.



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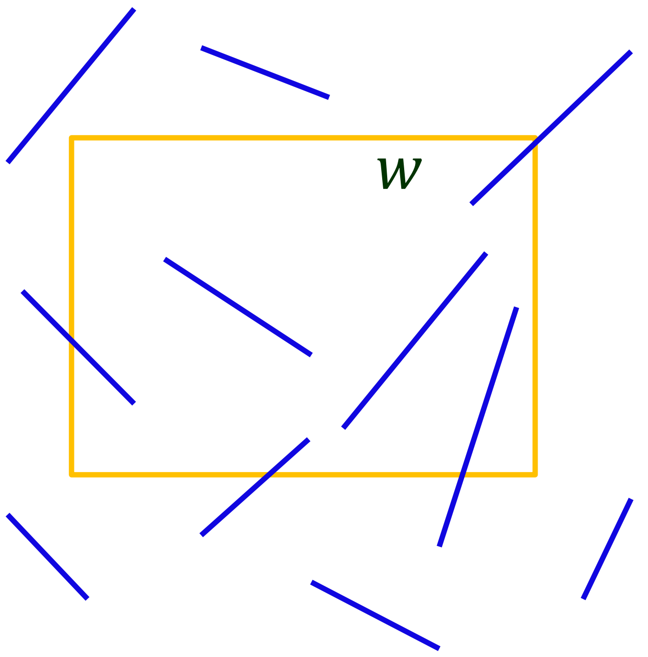

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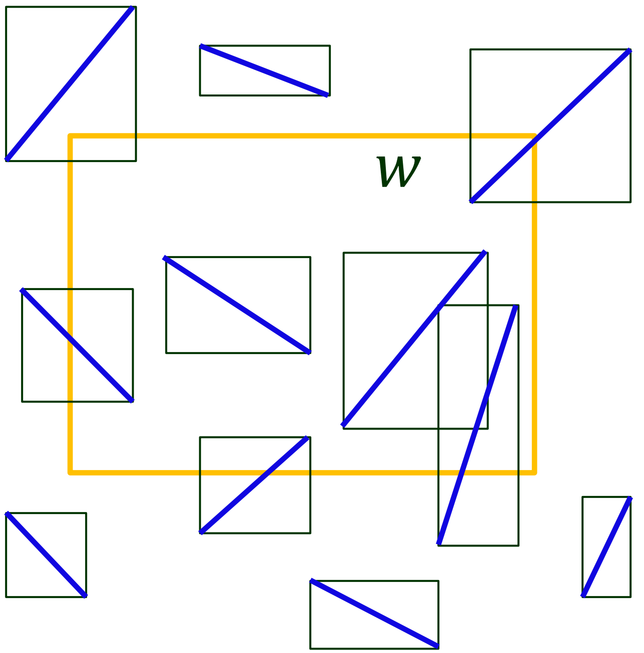

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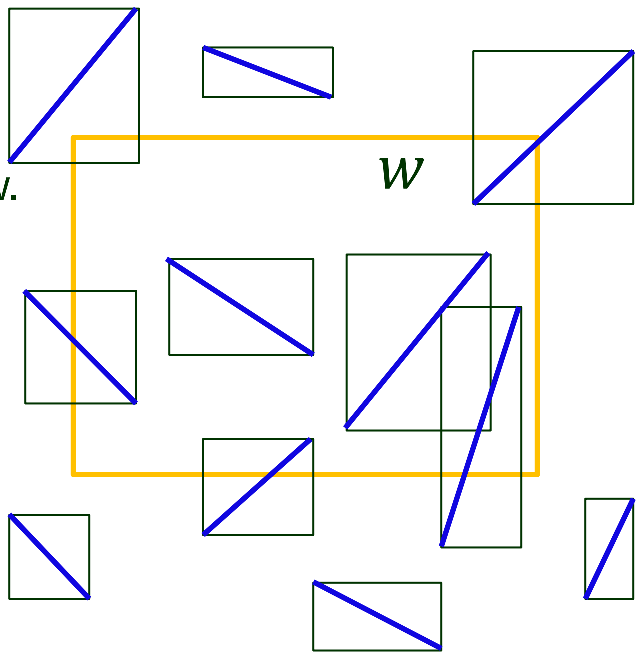

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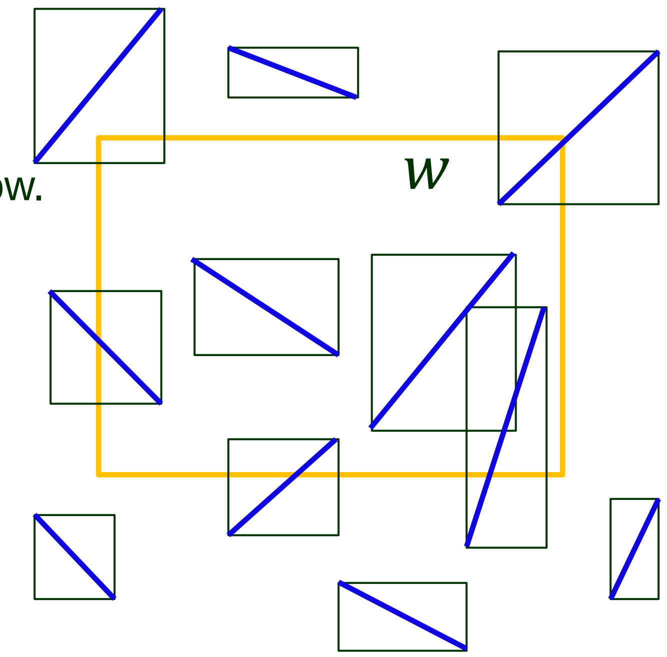

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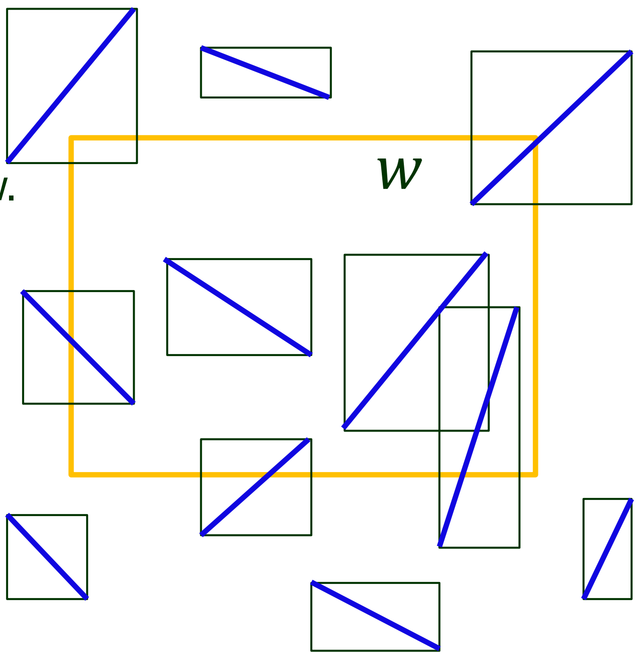

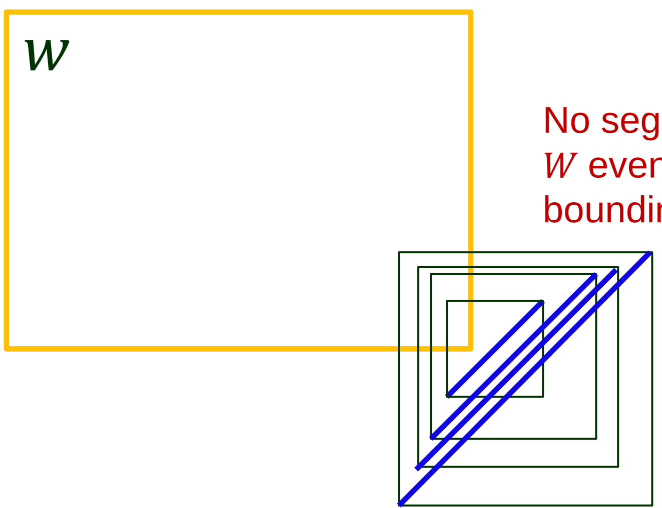

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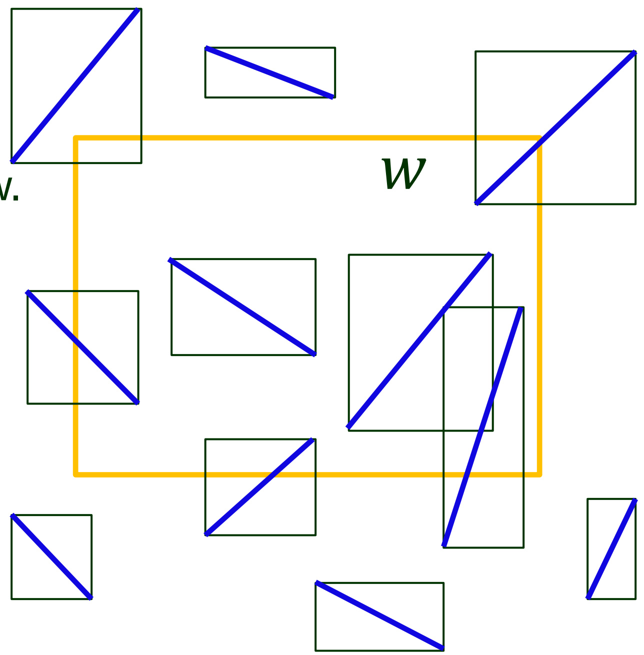

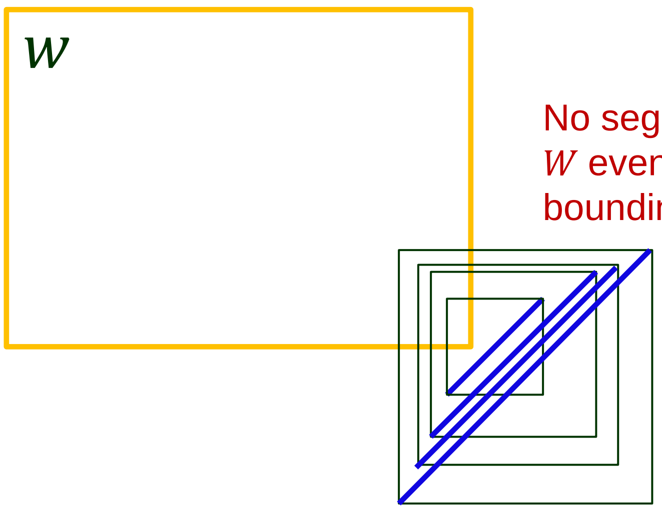

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♠ No guarantee on a fast query time.

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Set: $S = \{s_1, s_2, \ldots, s_n\}$ of n segments

- arbitrarily oriented
- non-intersecting in the interior
- possibly sharing endpoints

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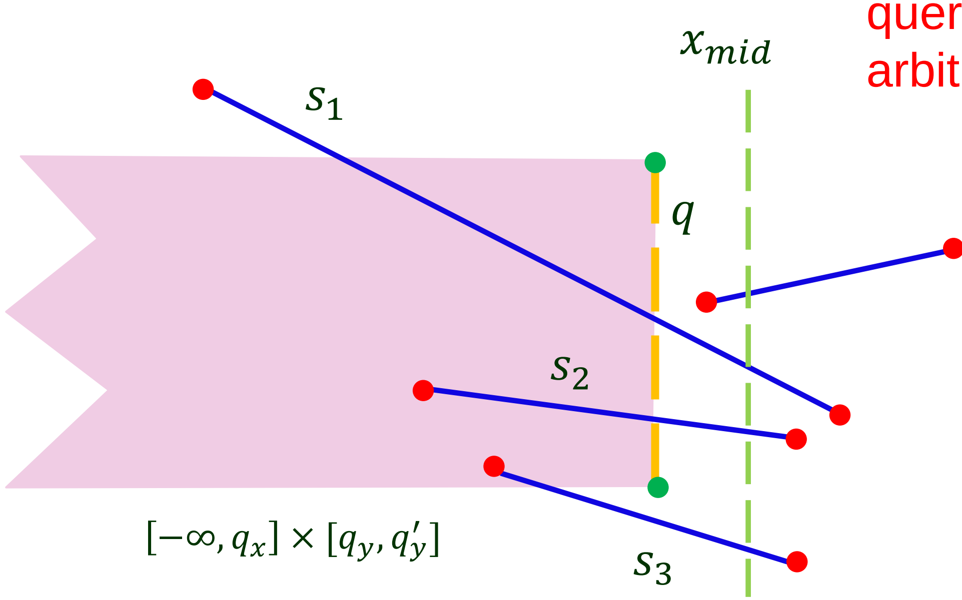

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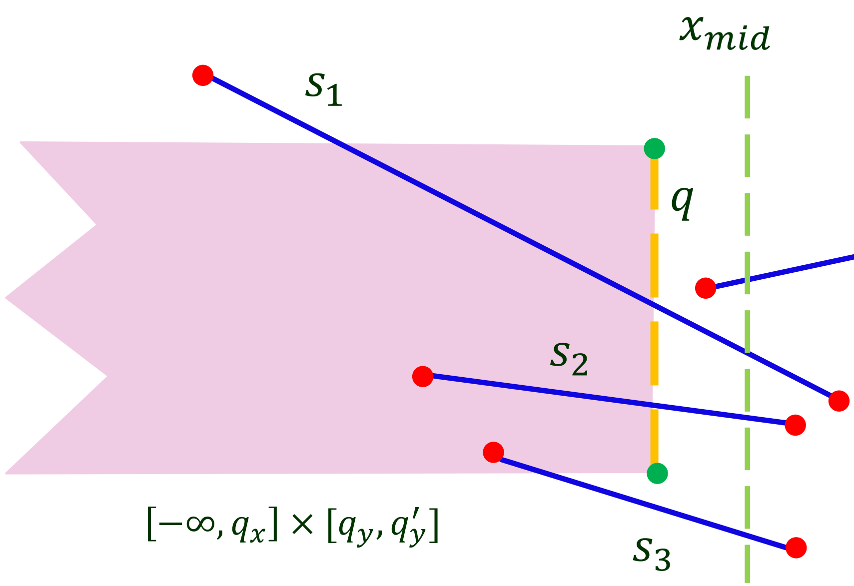

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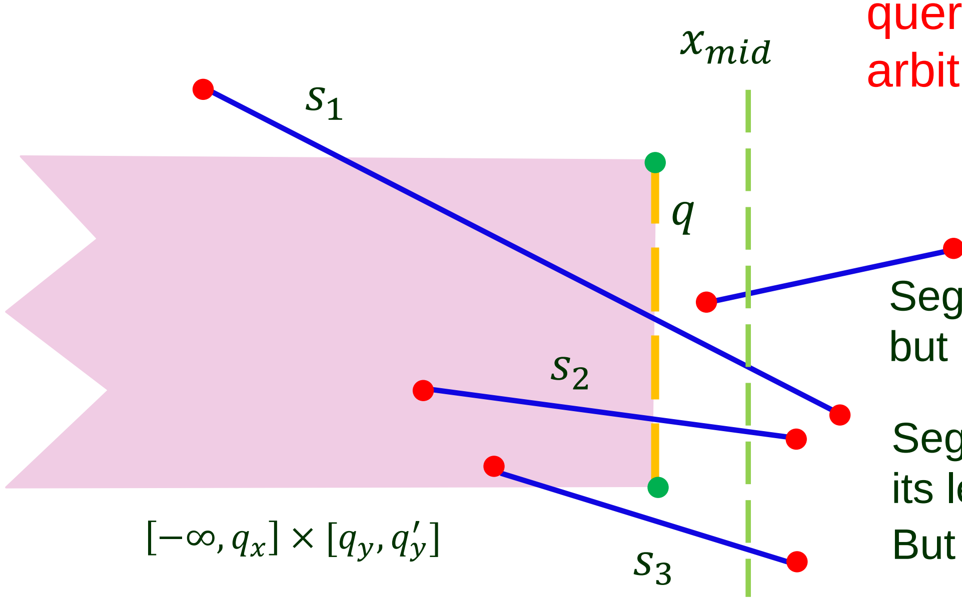

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Window W: $[q_x, q'_x] \times [q_y, q'_y]$ defined by four parameters.

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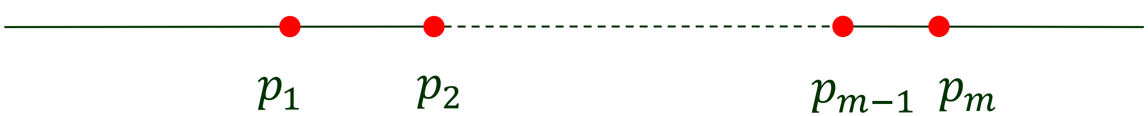

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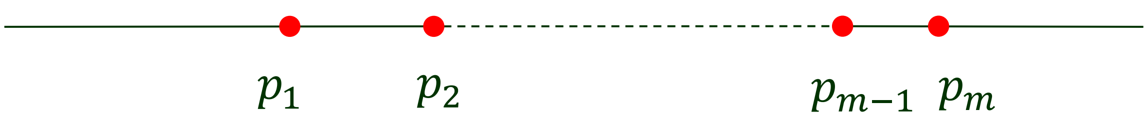

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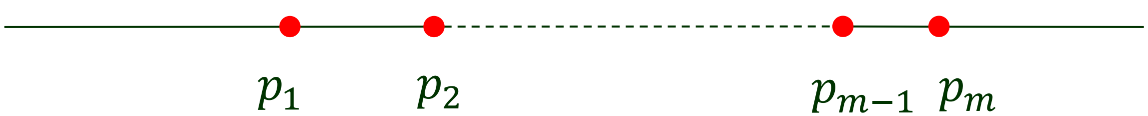

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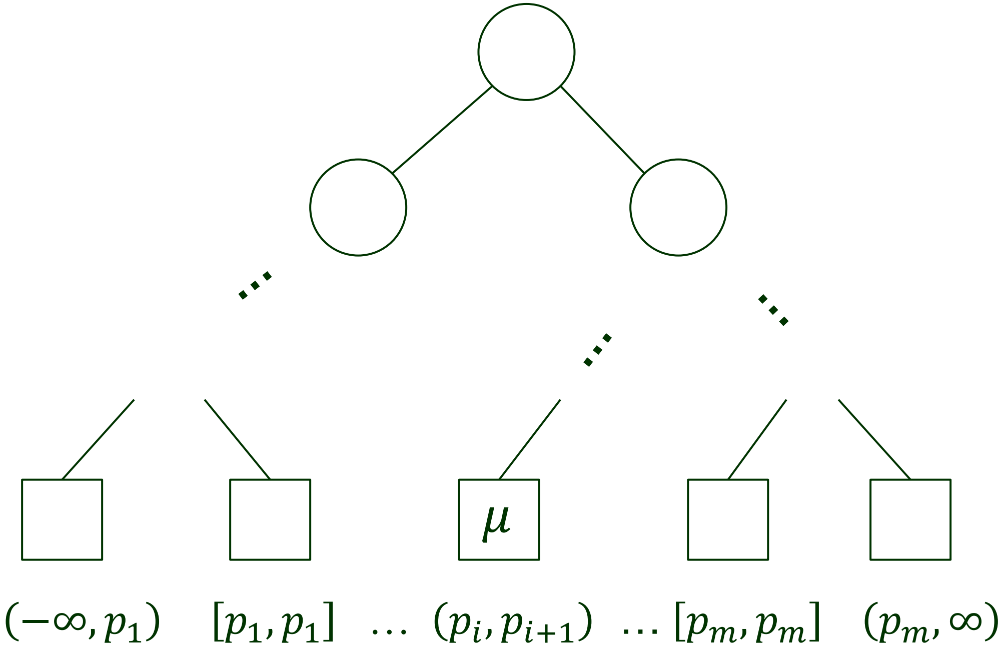
- Every open interval has two consecutive endpoints.
- Every closed interval consists of a single endpoint.
- Open intervals alternate with closed intervals.

Using a Binary Search Tree?



μ : a leaf

Int(μ): elementary interval corresponding to μ.

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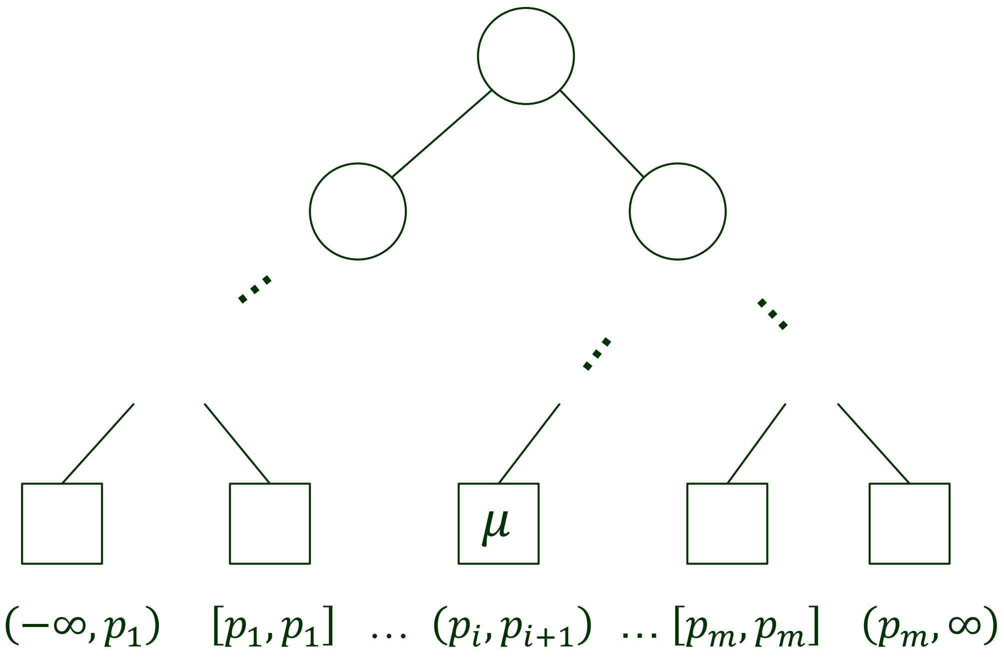

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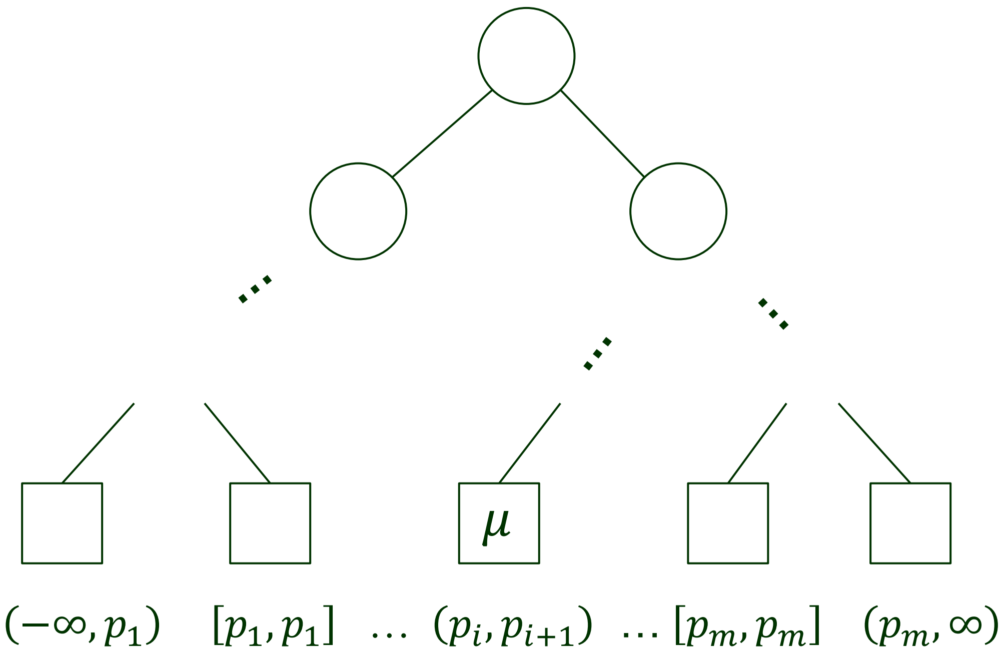

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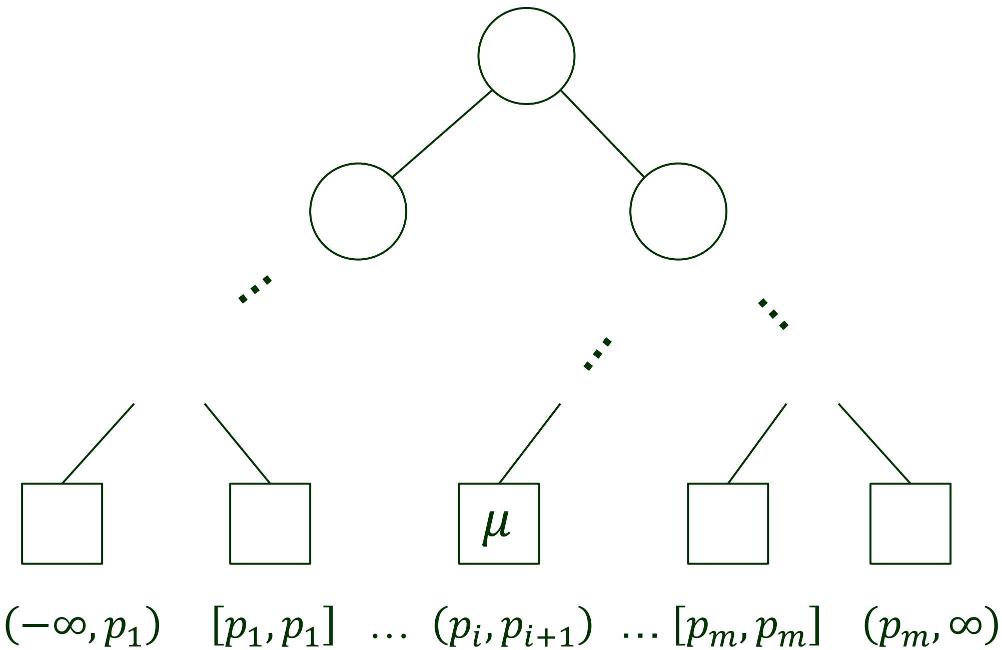

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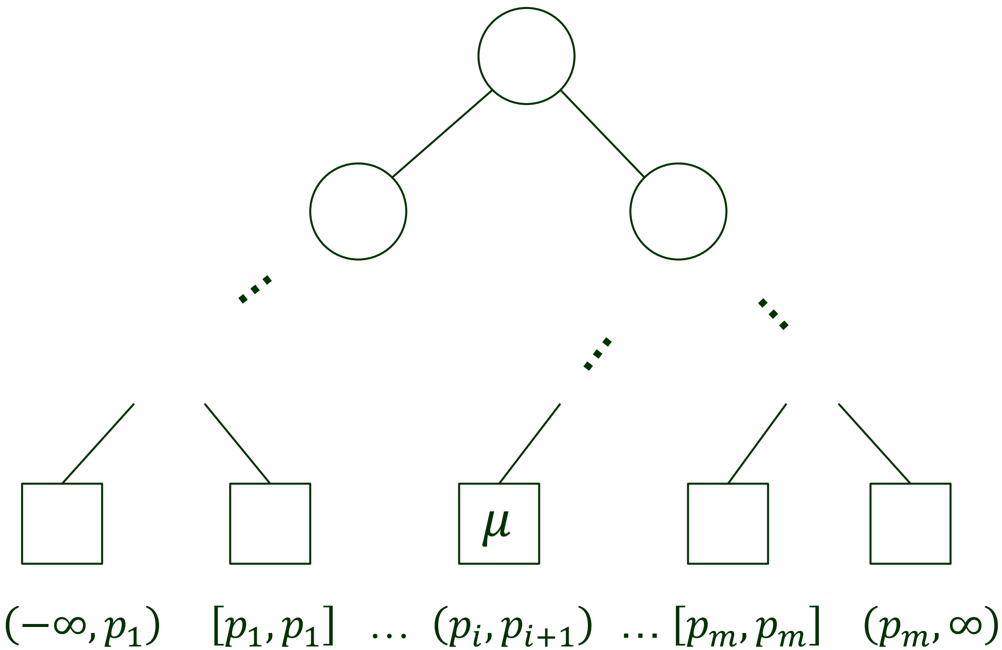

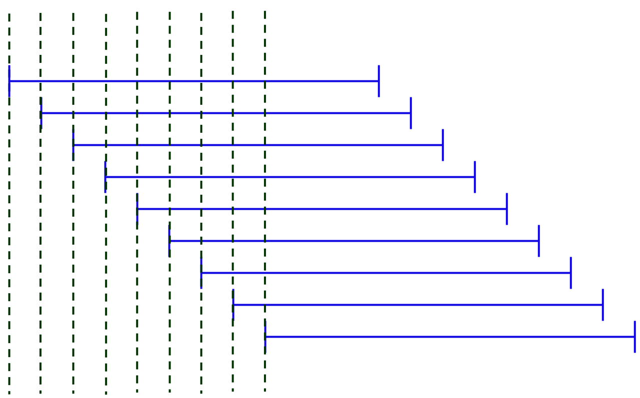

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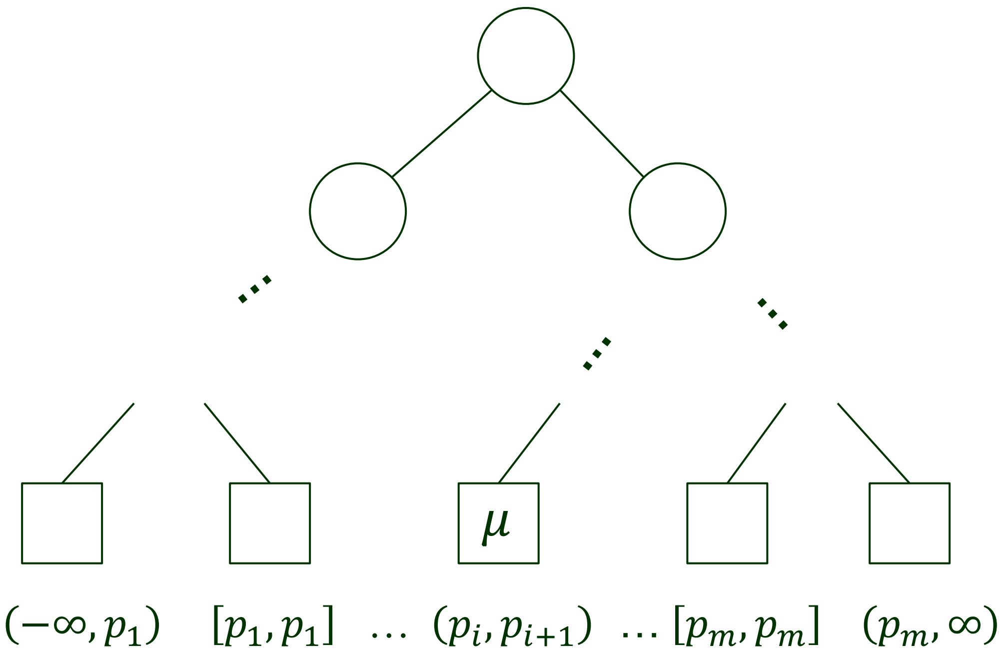

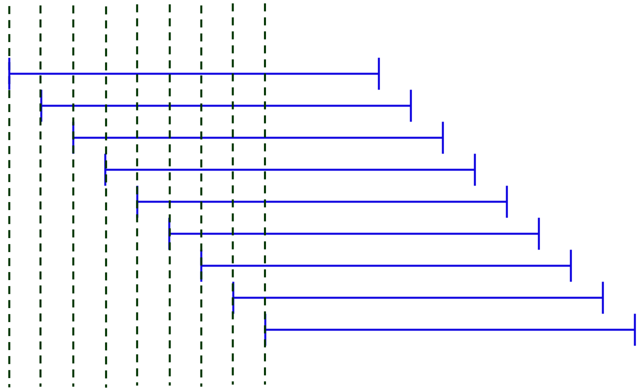

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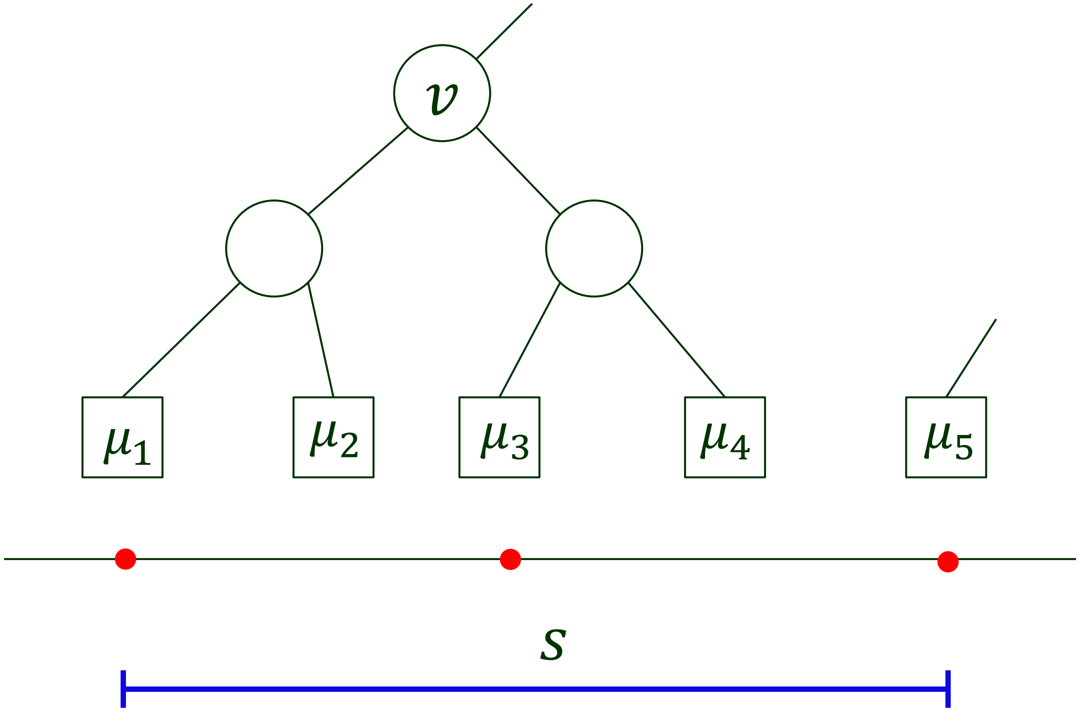
$O(n^2)$ possible!

Store an Interval As High as Possible



Interval s is stored five times at $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$.

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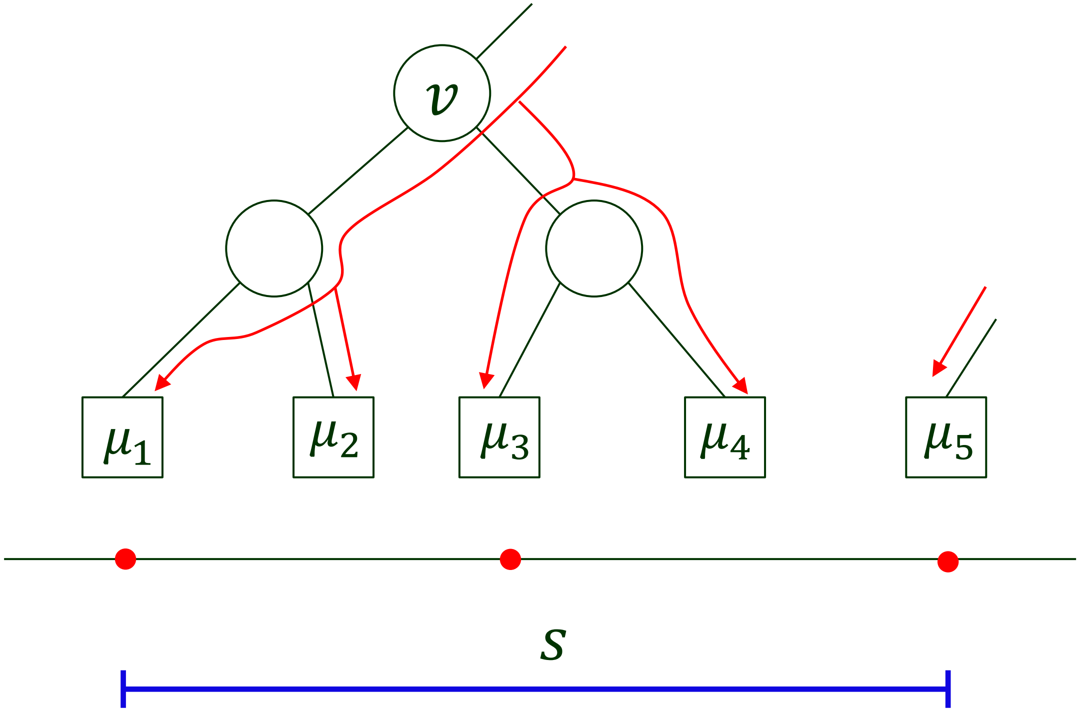

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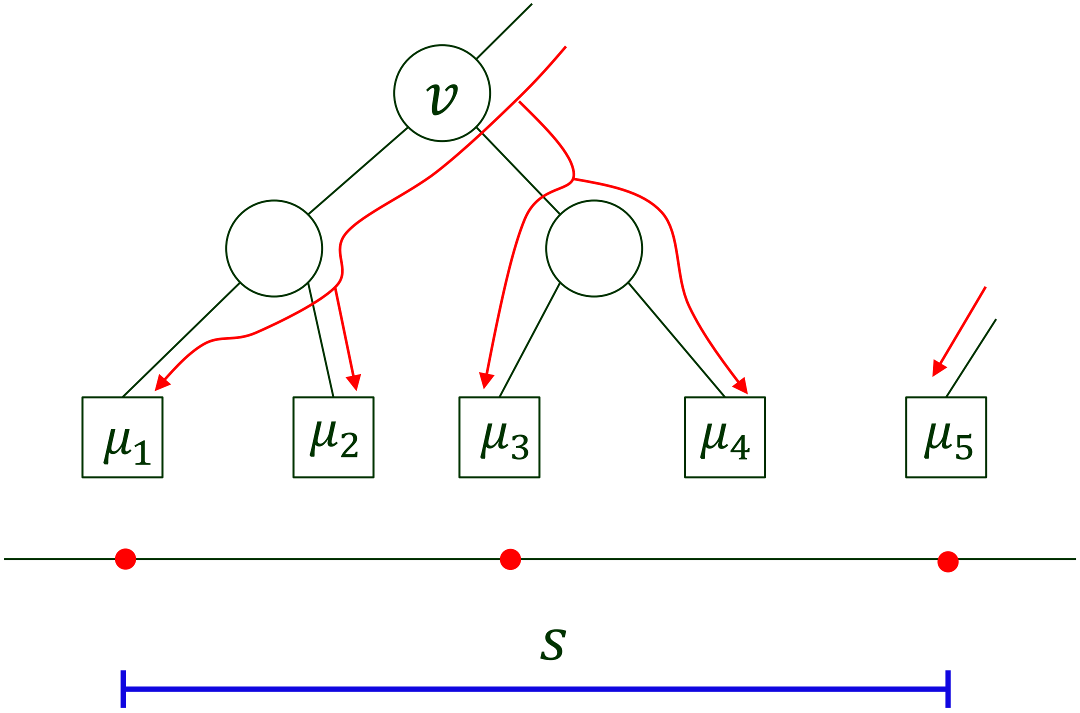

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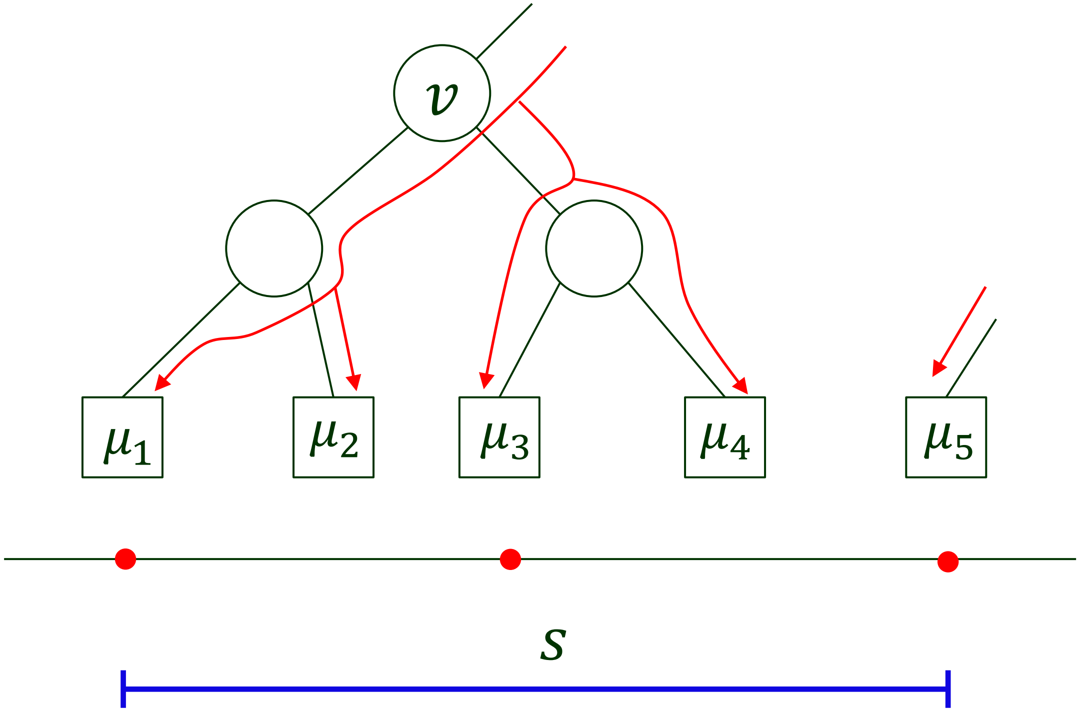

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Why not store s only two times at v and μ_5?

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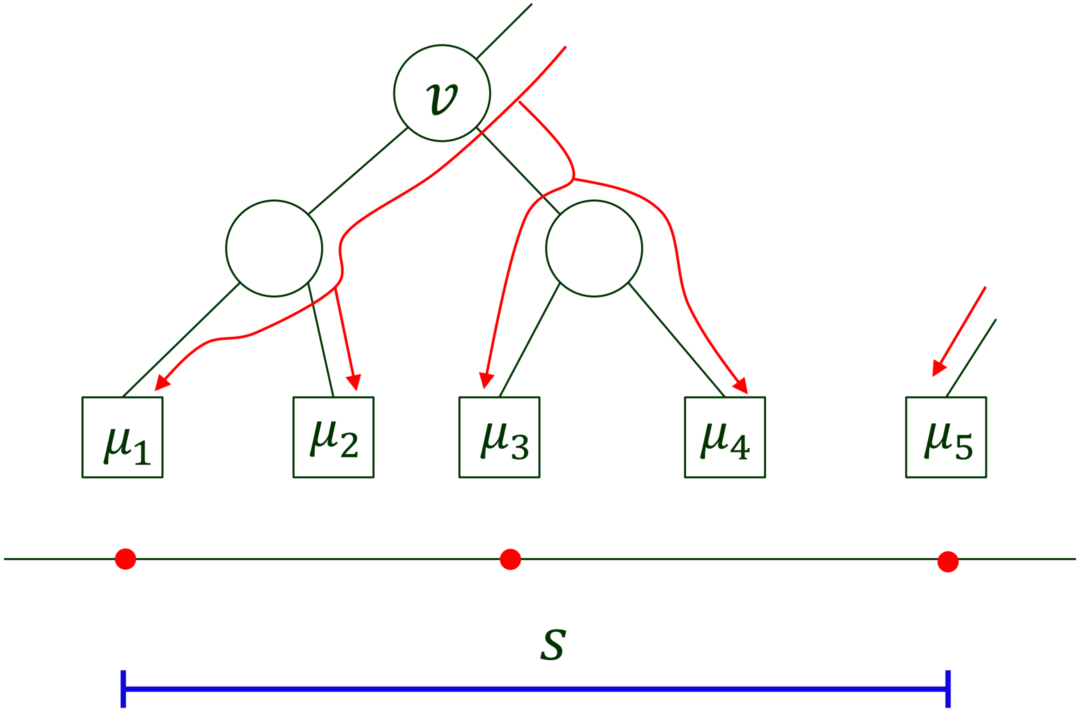

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Idea: Store a segment s at the *fewest nodes* whose corresponding intervals form a partitioning of s.

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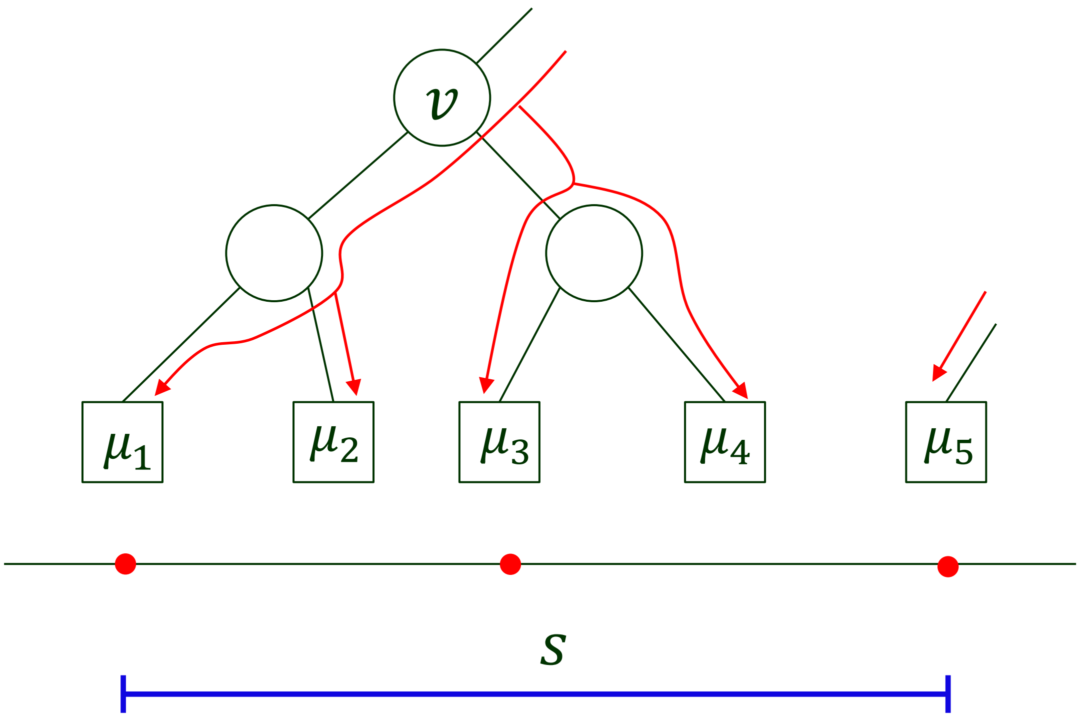

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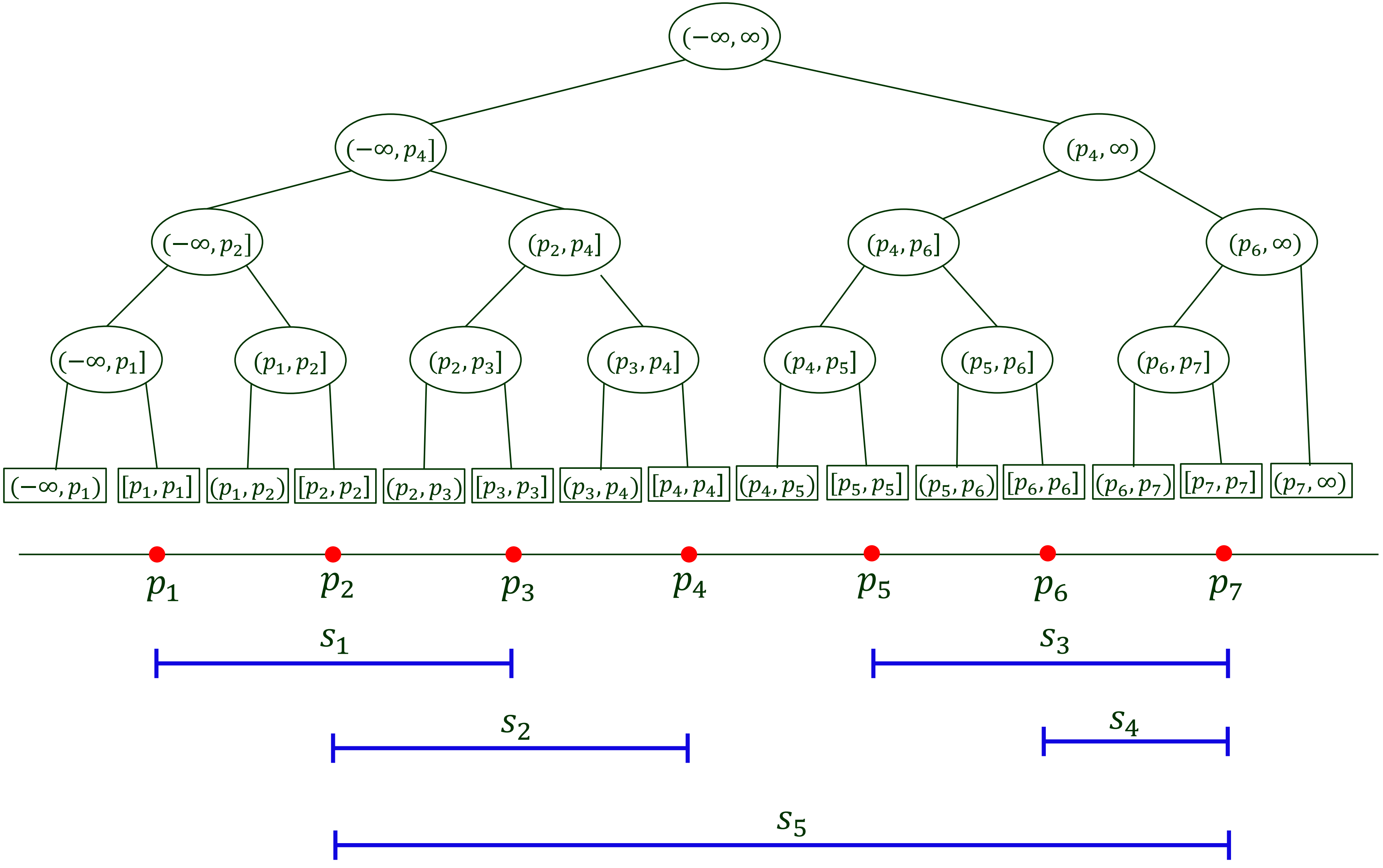
$\Downarrow$

These nodes must be as high as possible in the tree.

IV. Segment Tree



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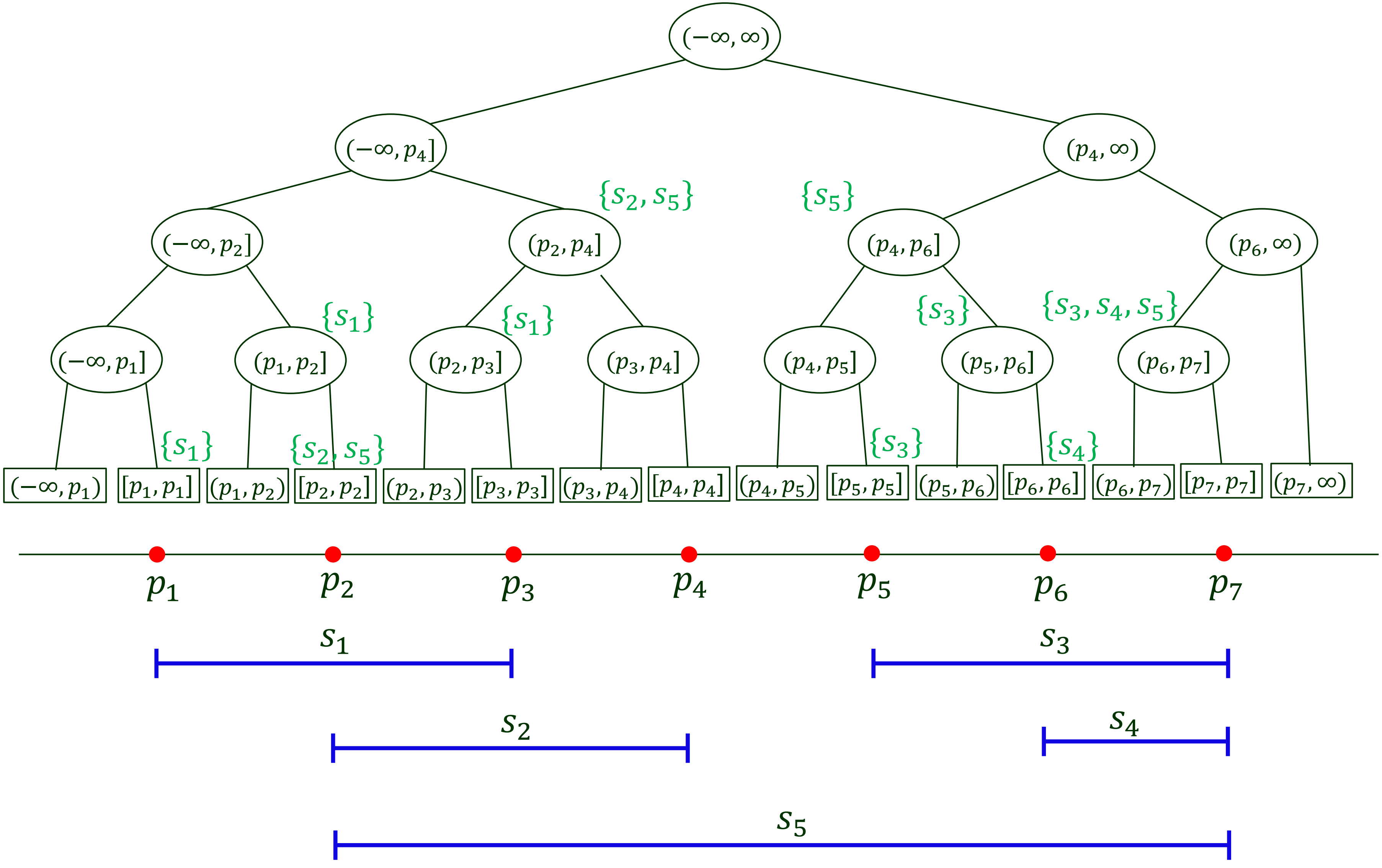
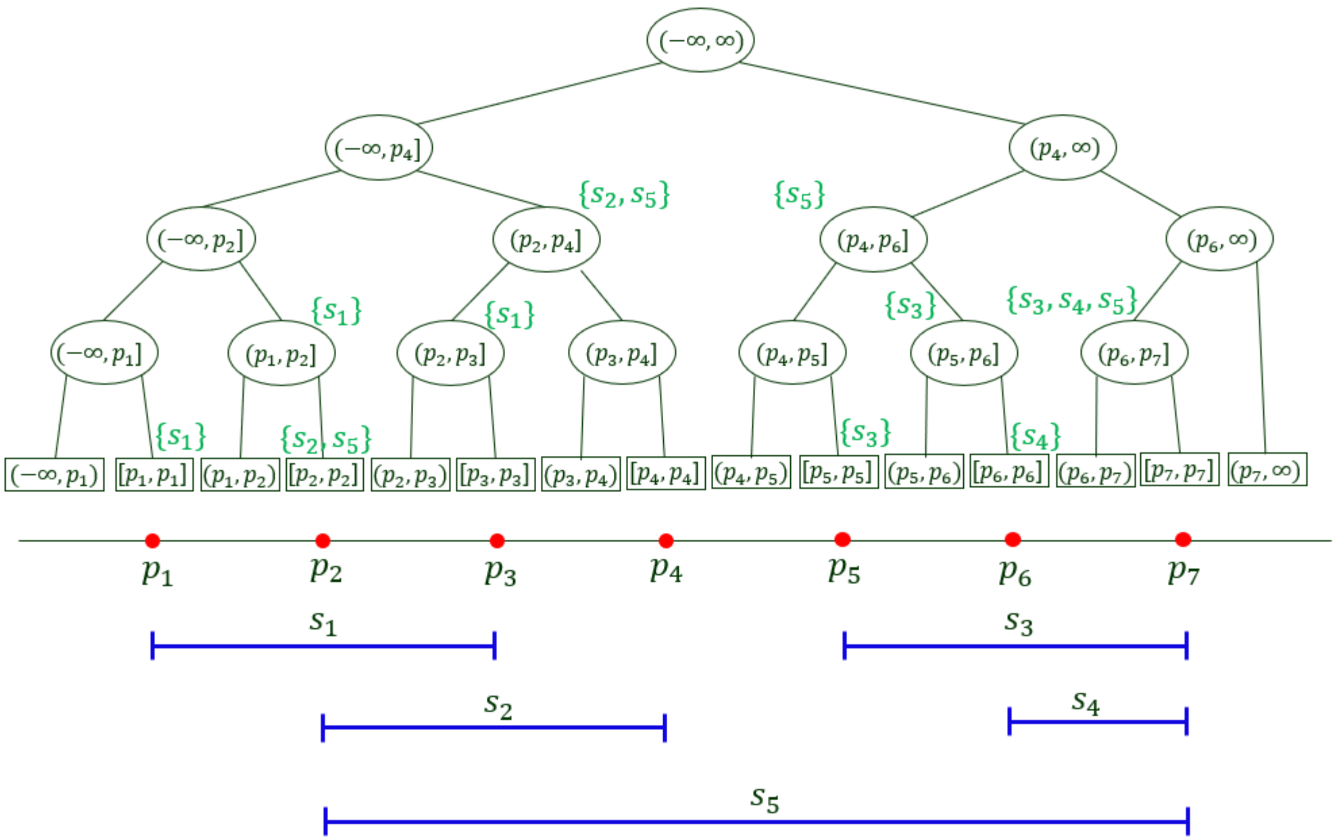


Tree Structure

- Leaves ↔ elementary intervals (left-to-right)



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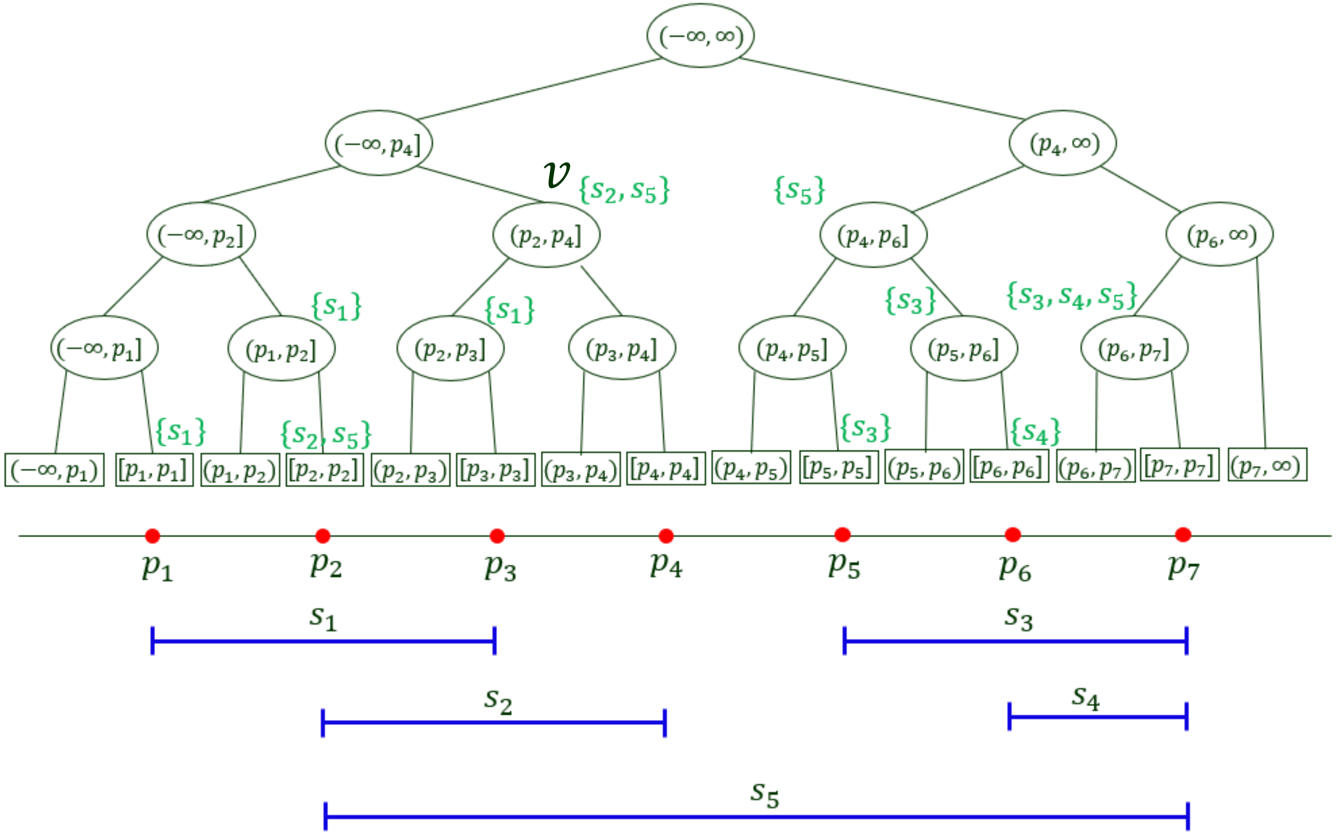

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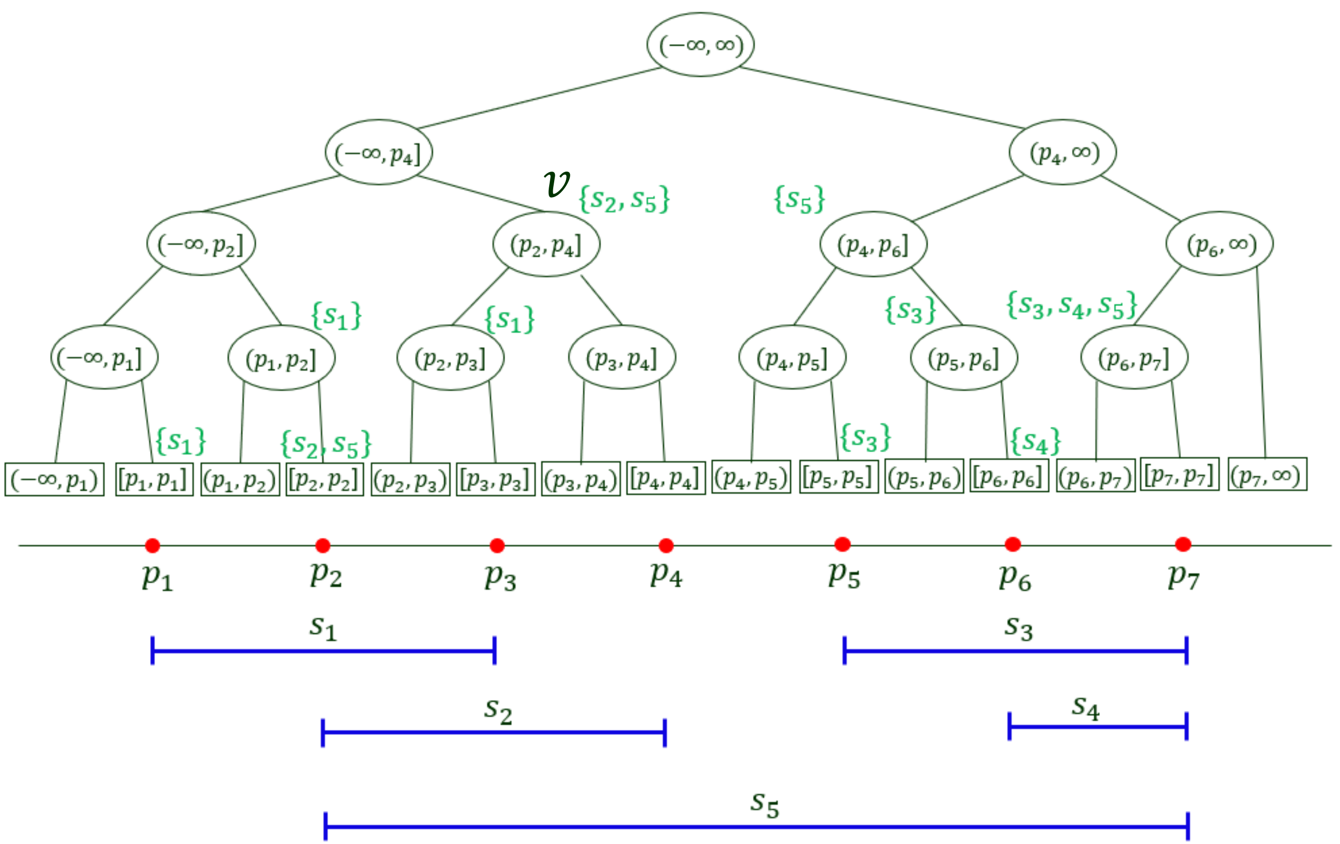

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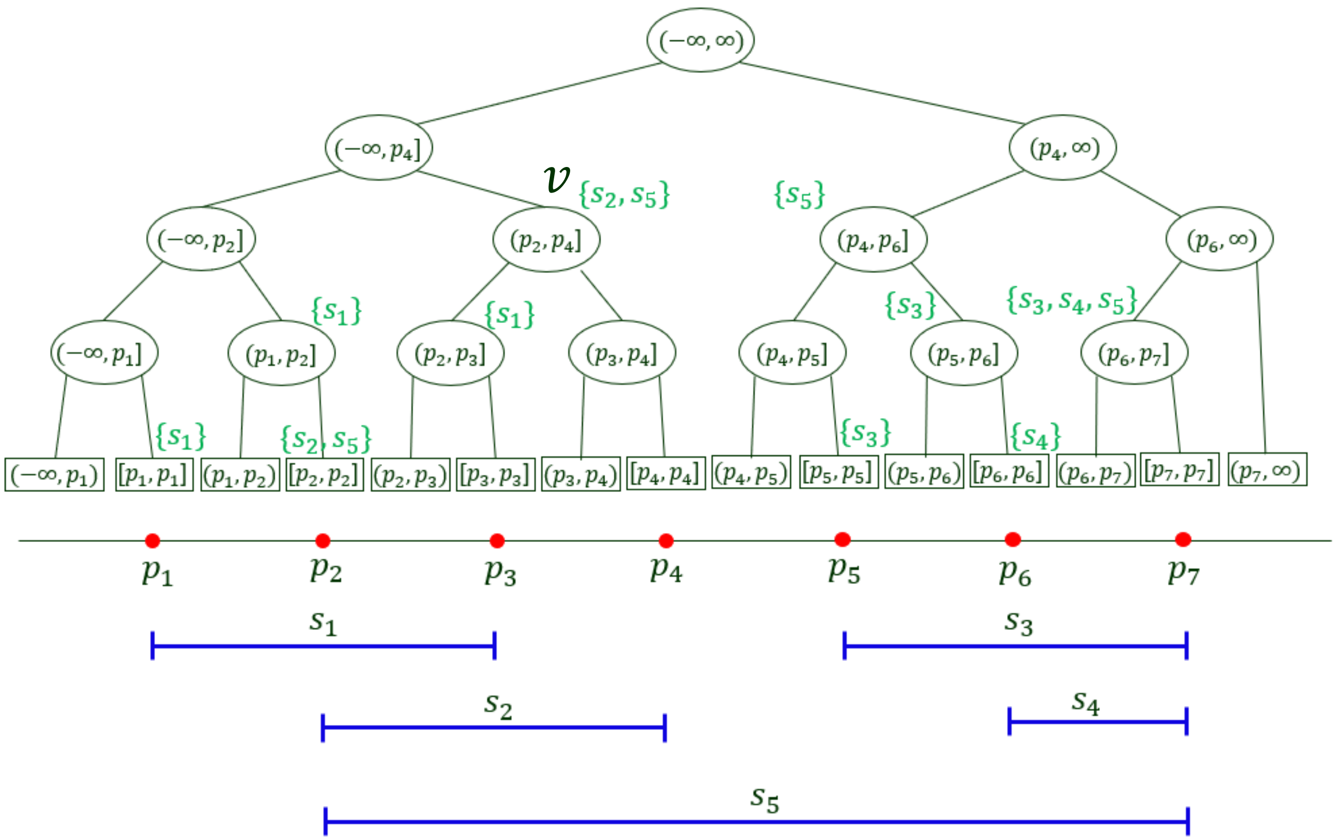

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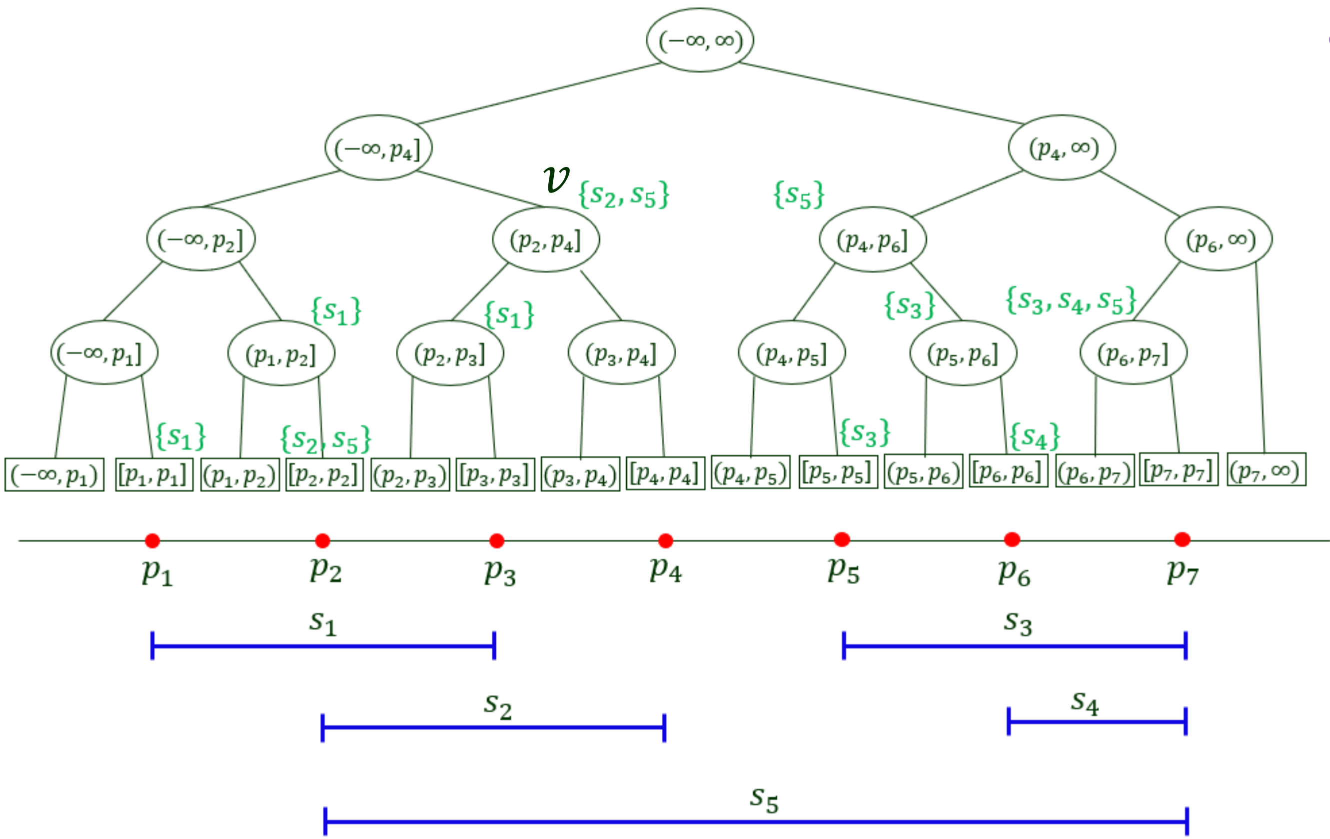

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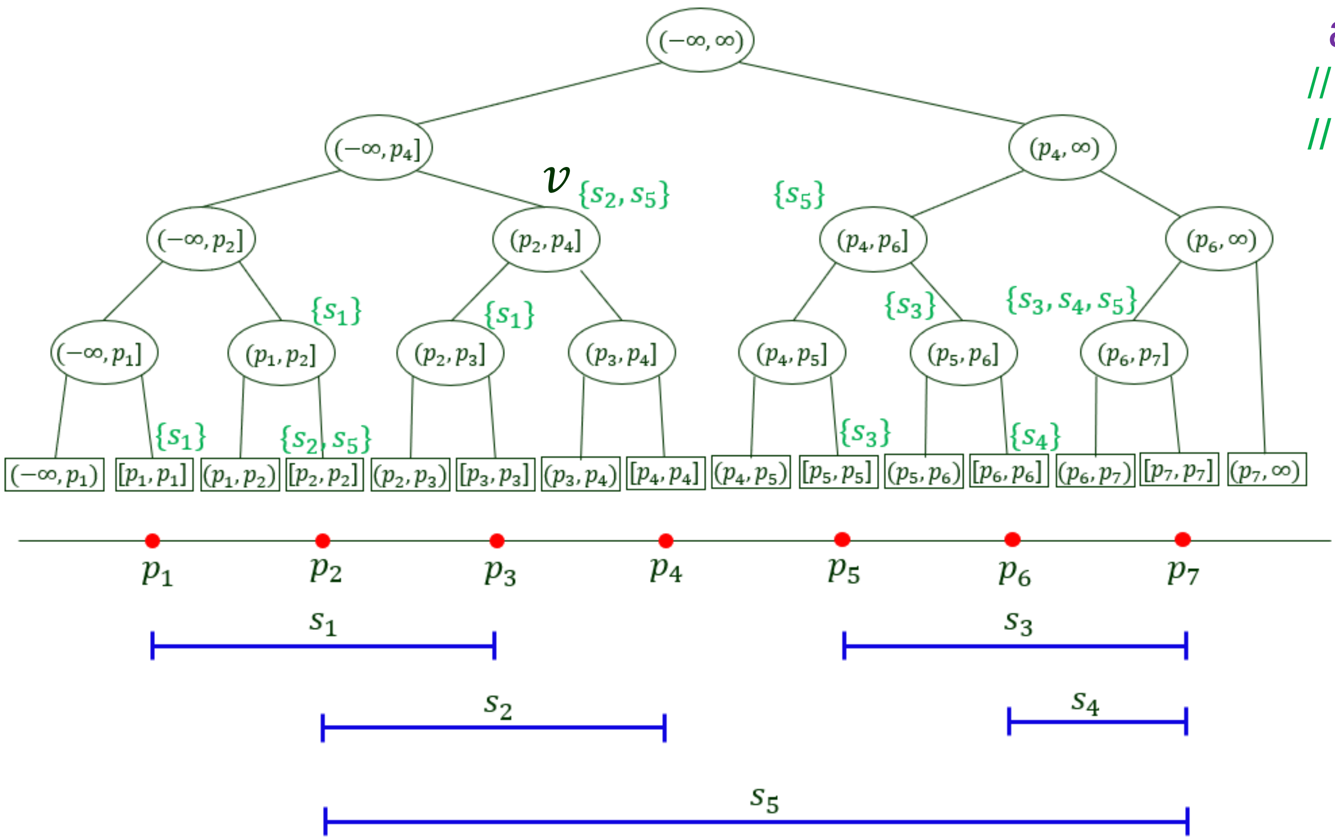

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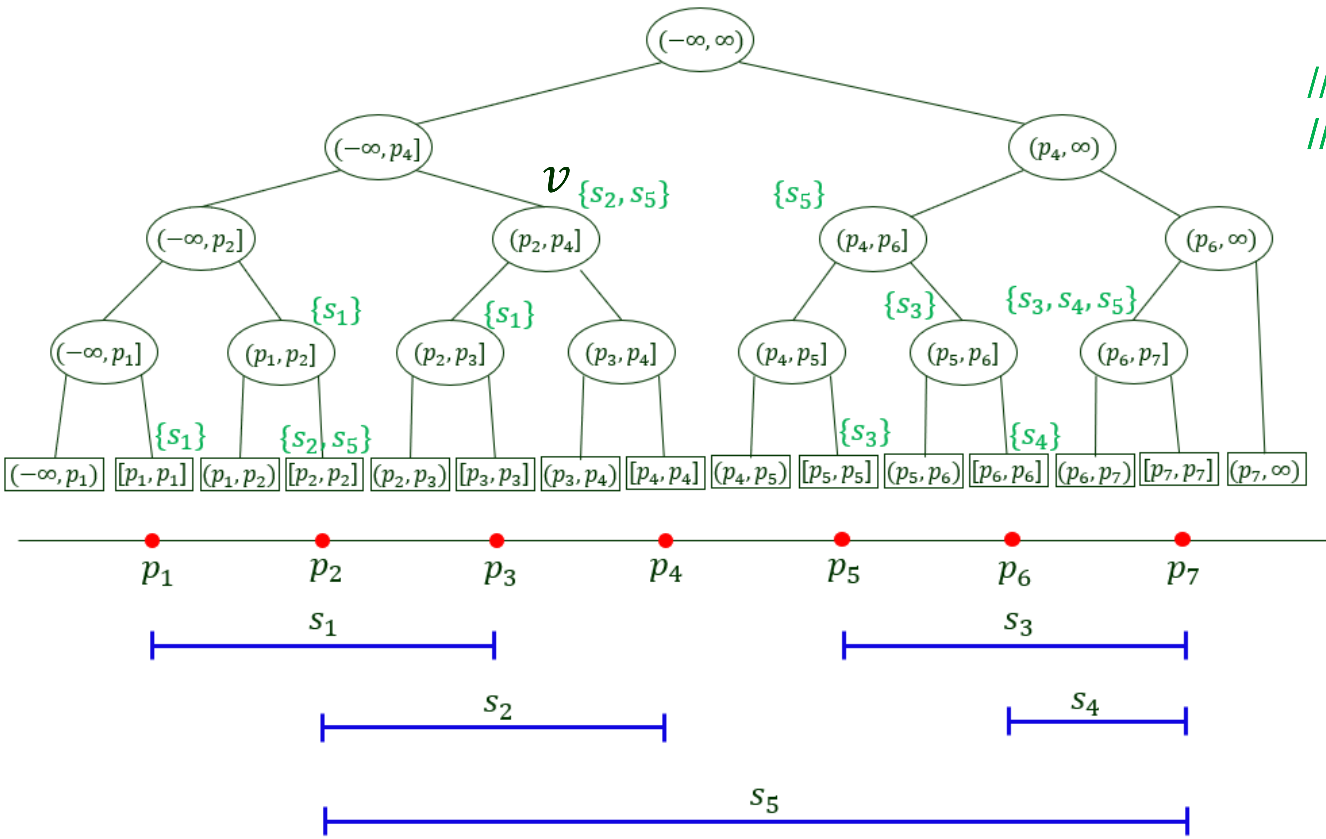

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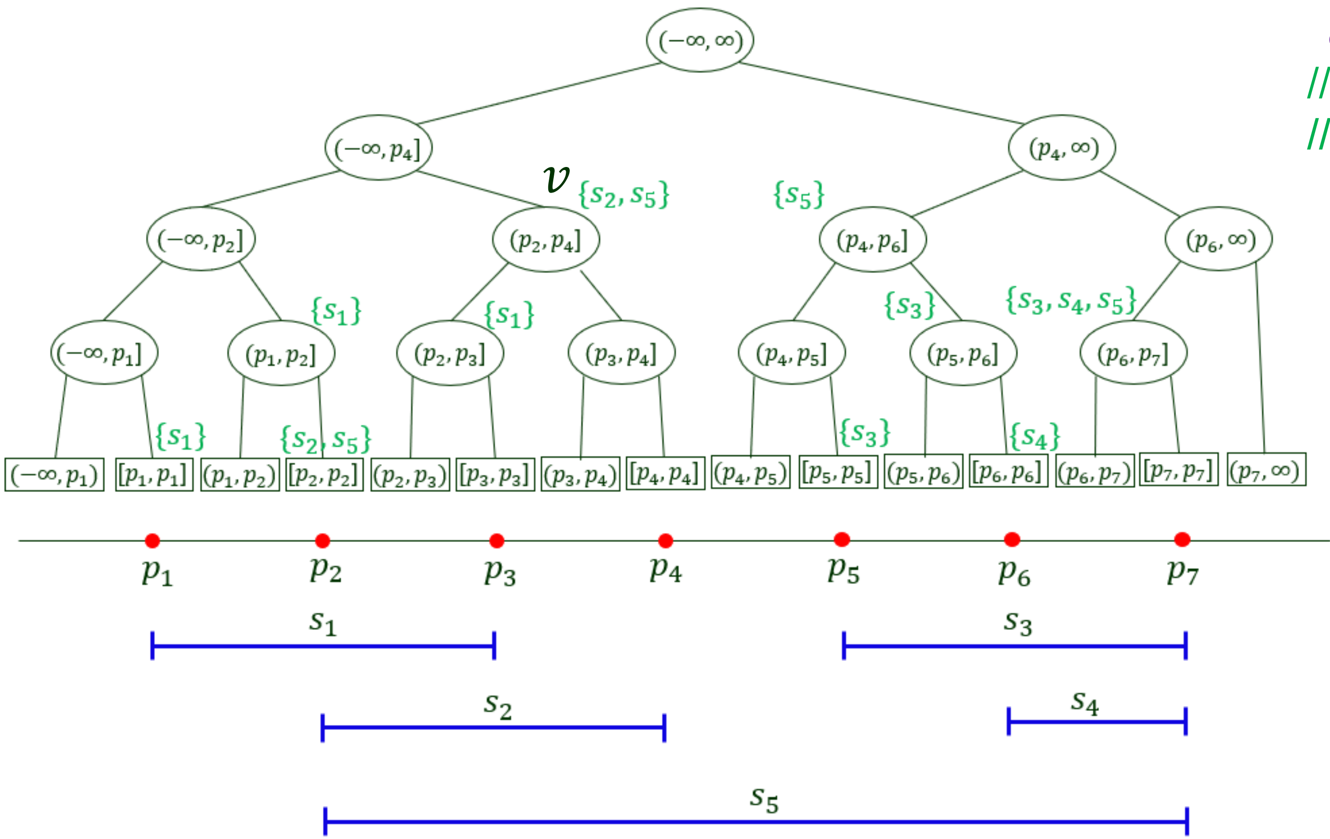

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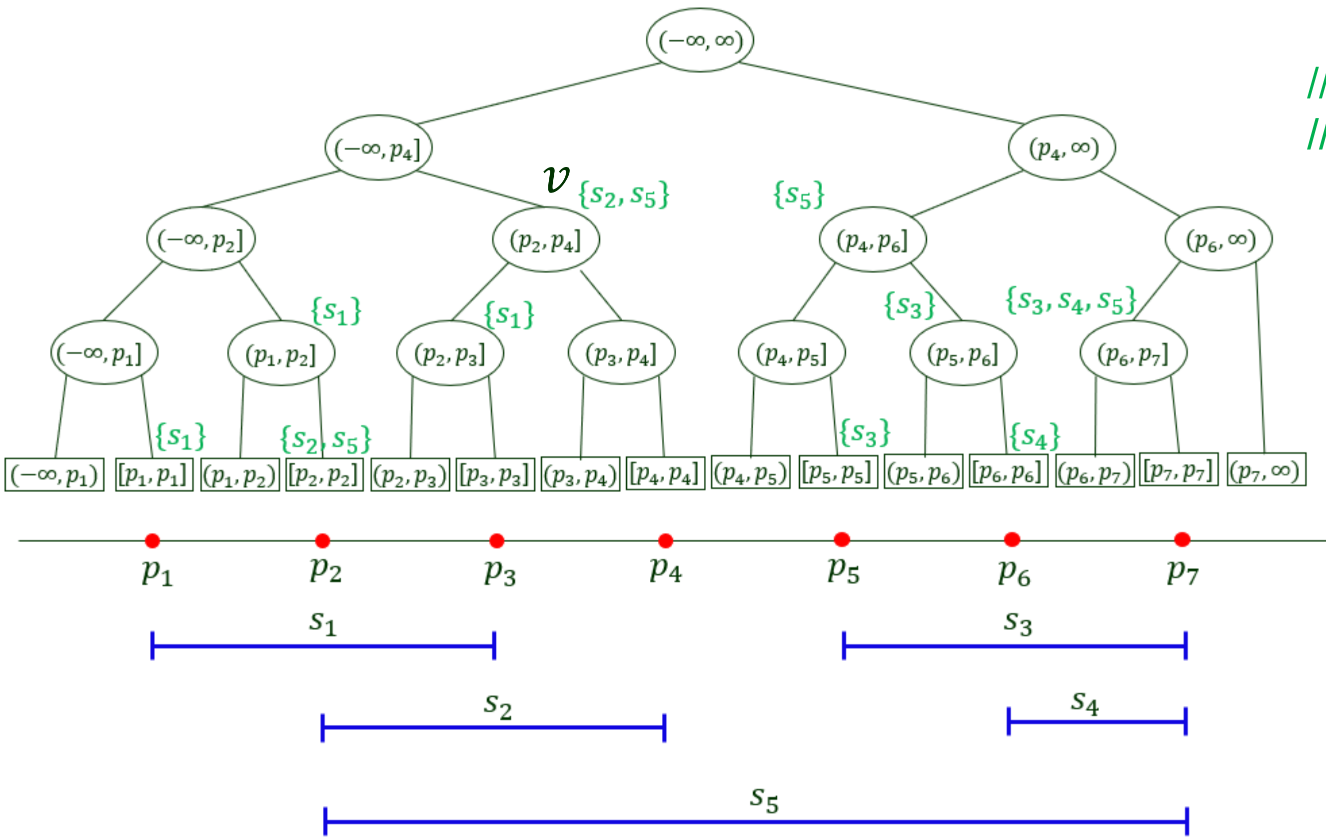

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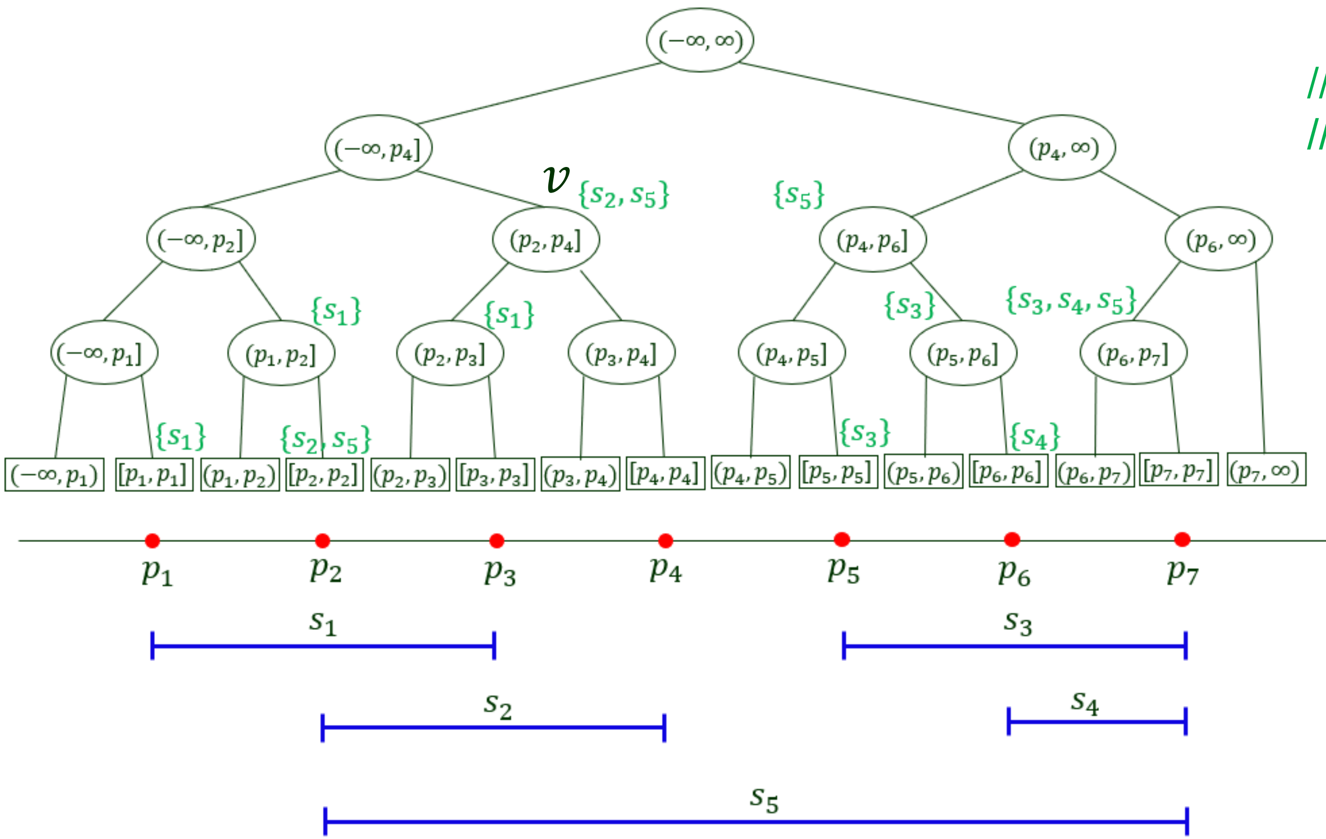

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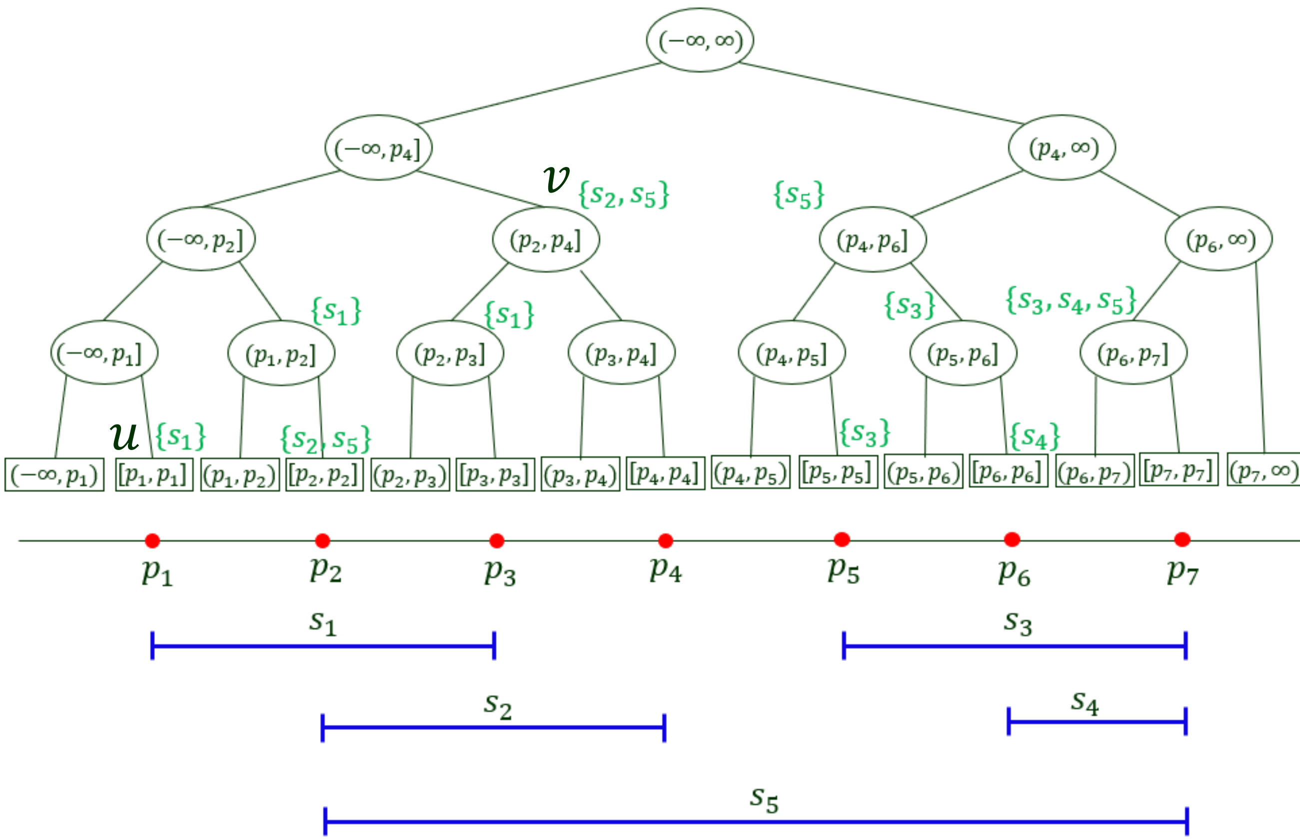

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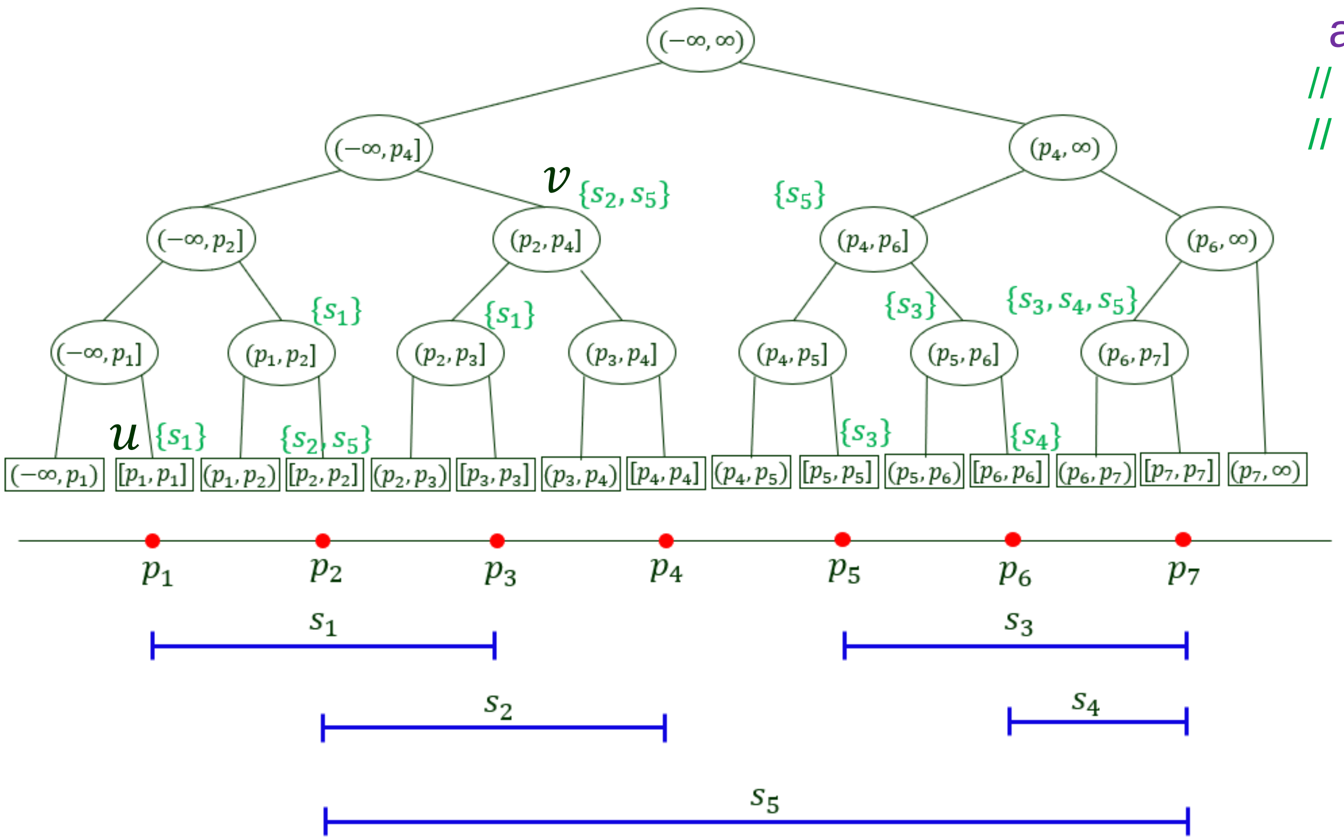

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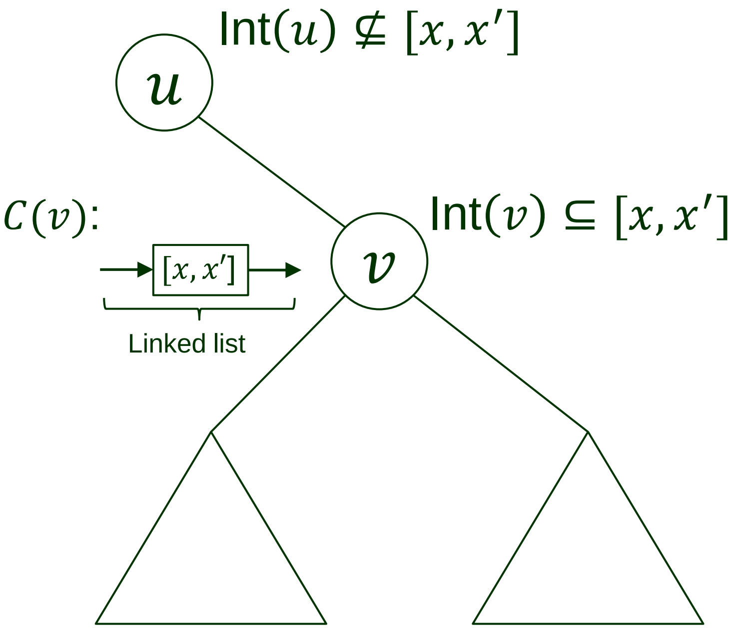

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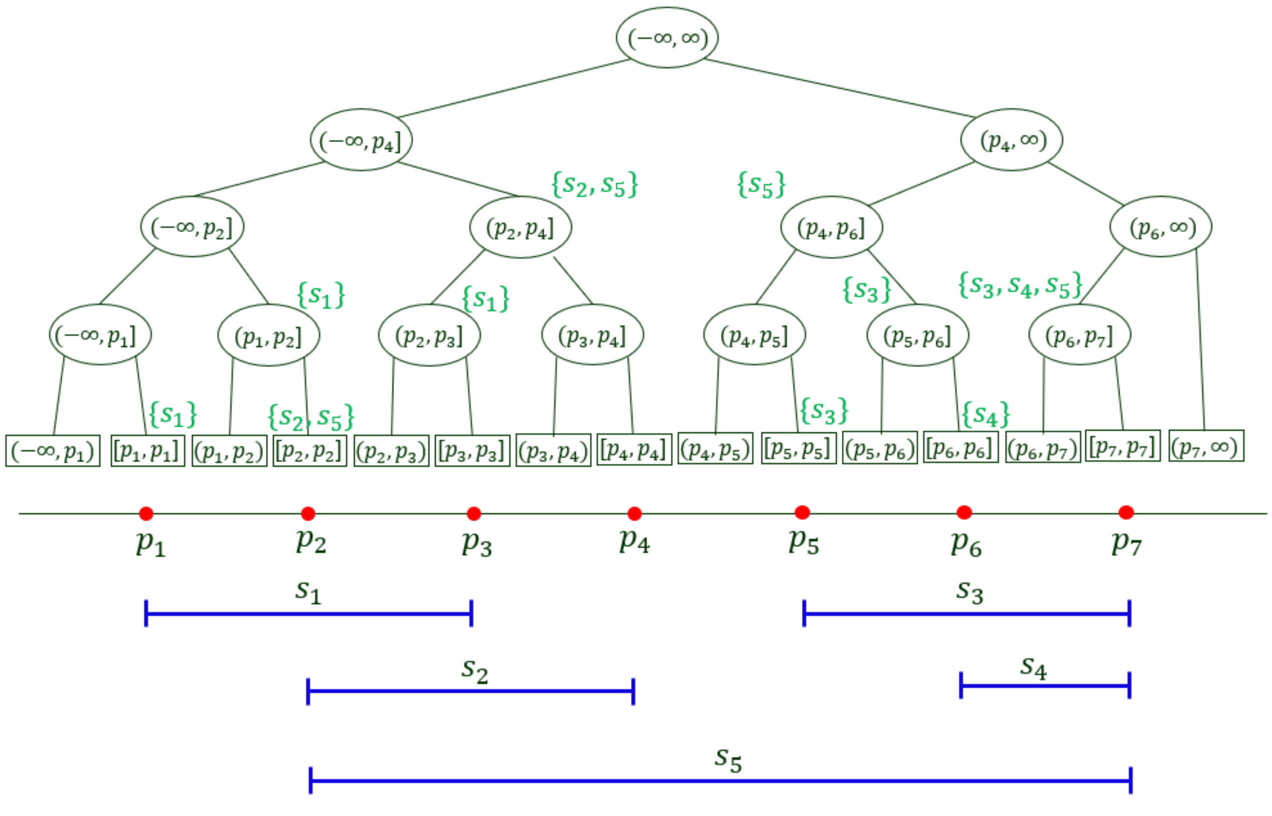

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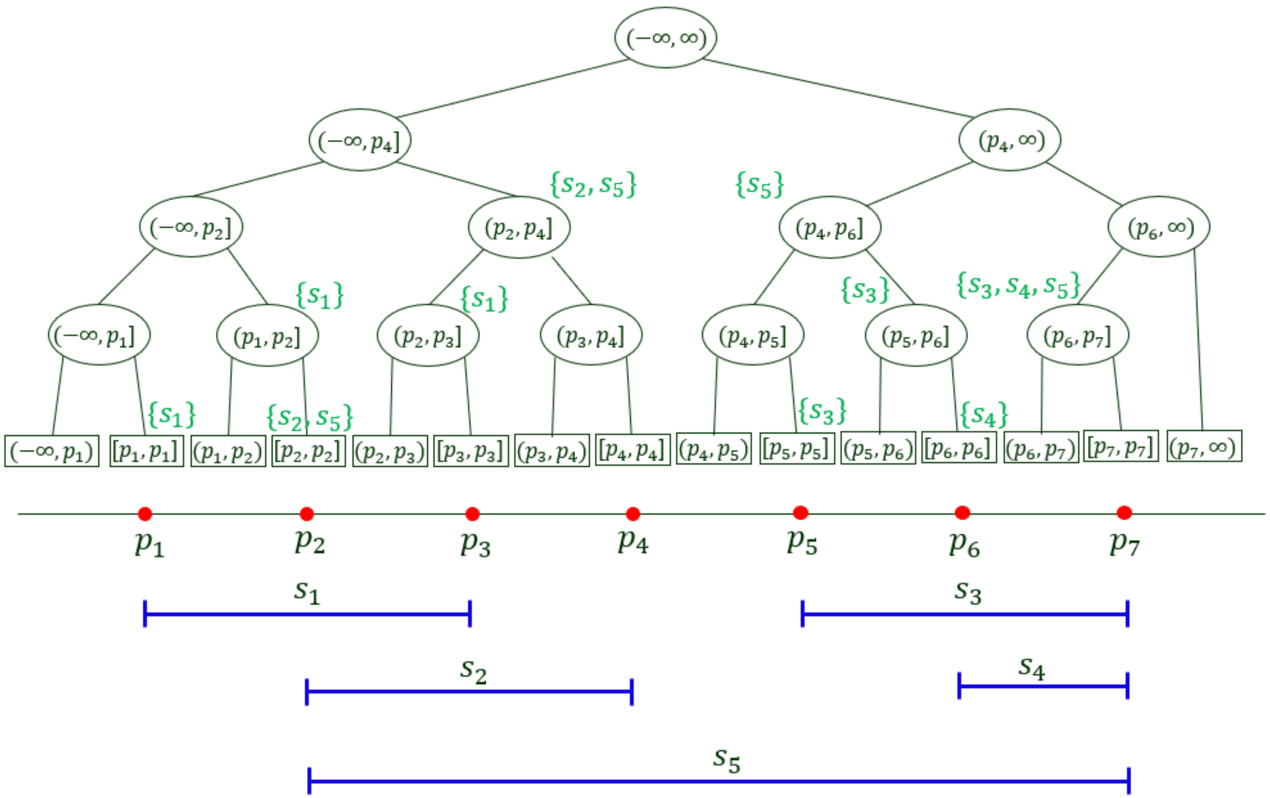

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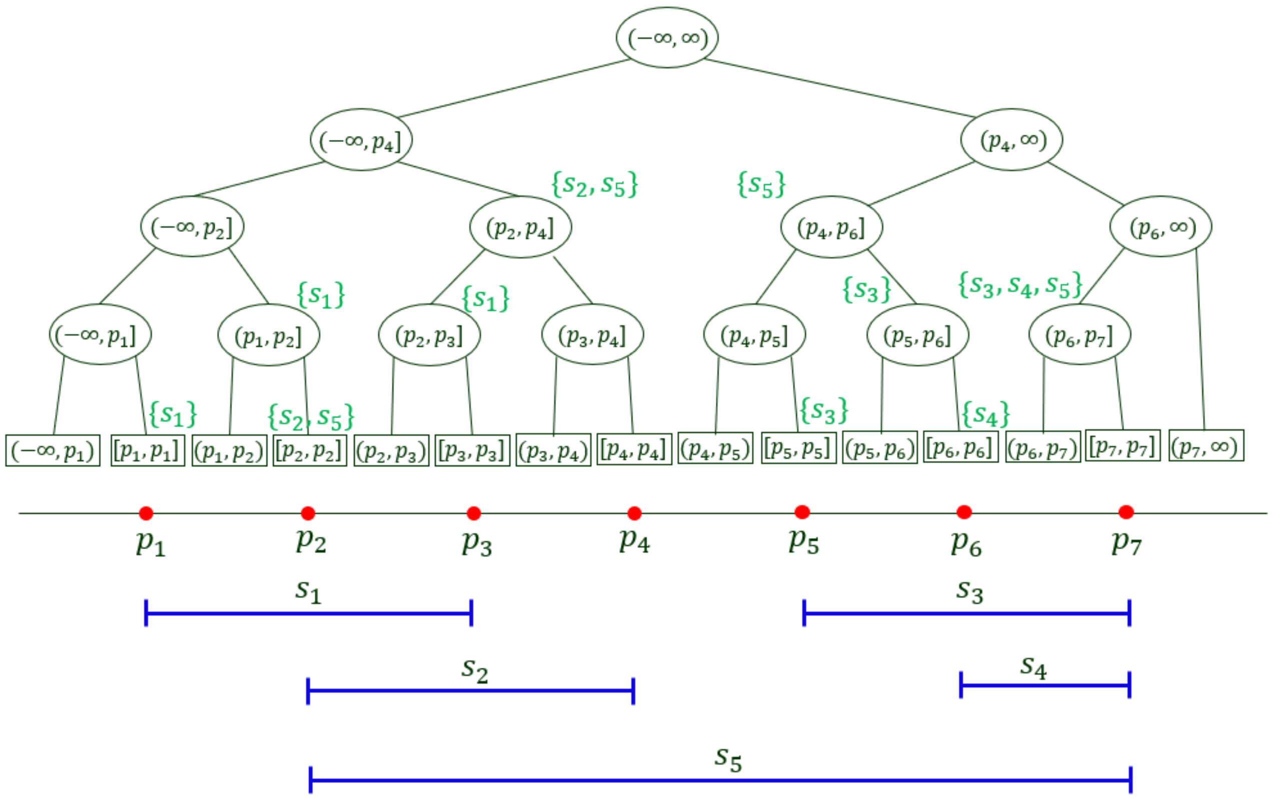

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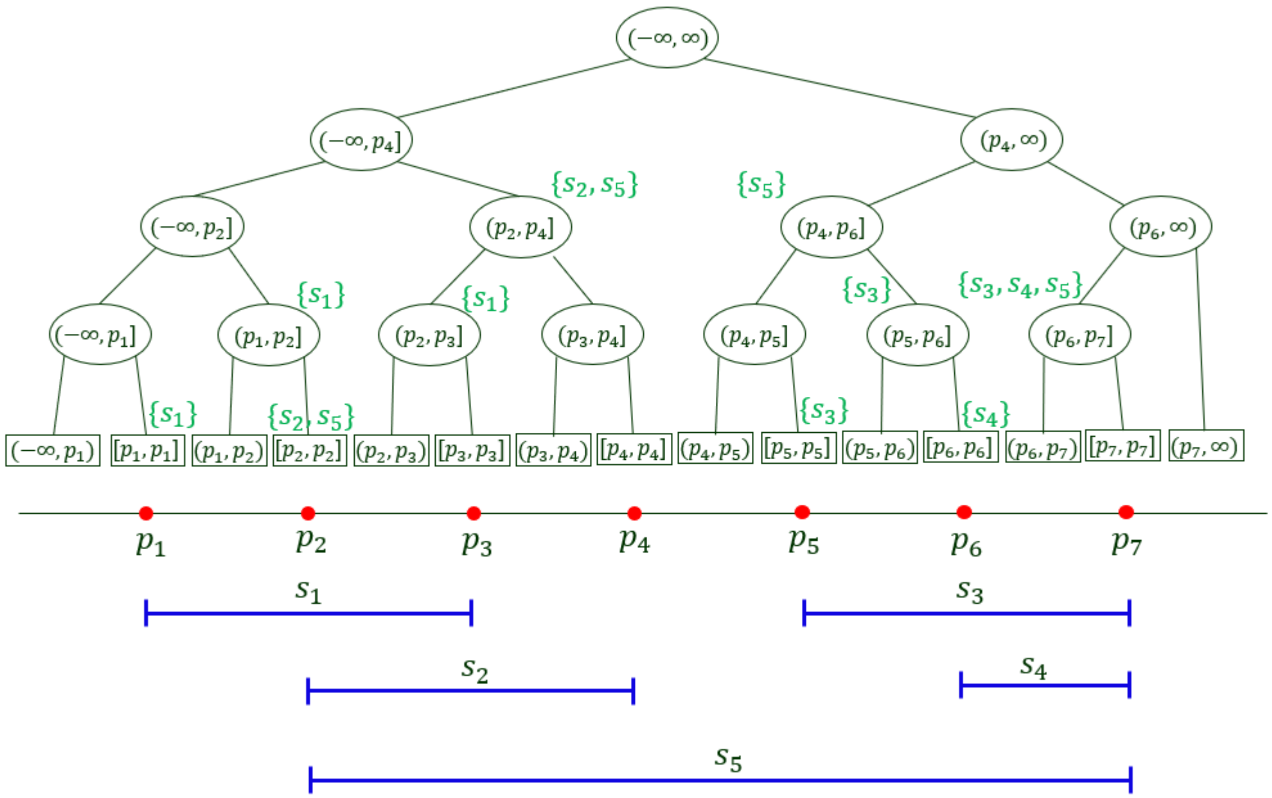

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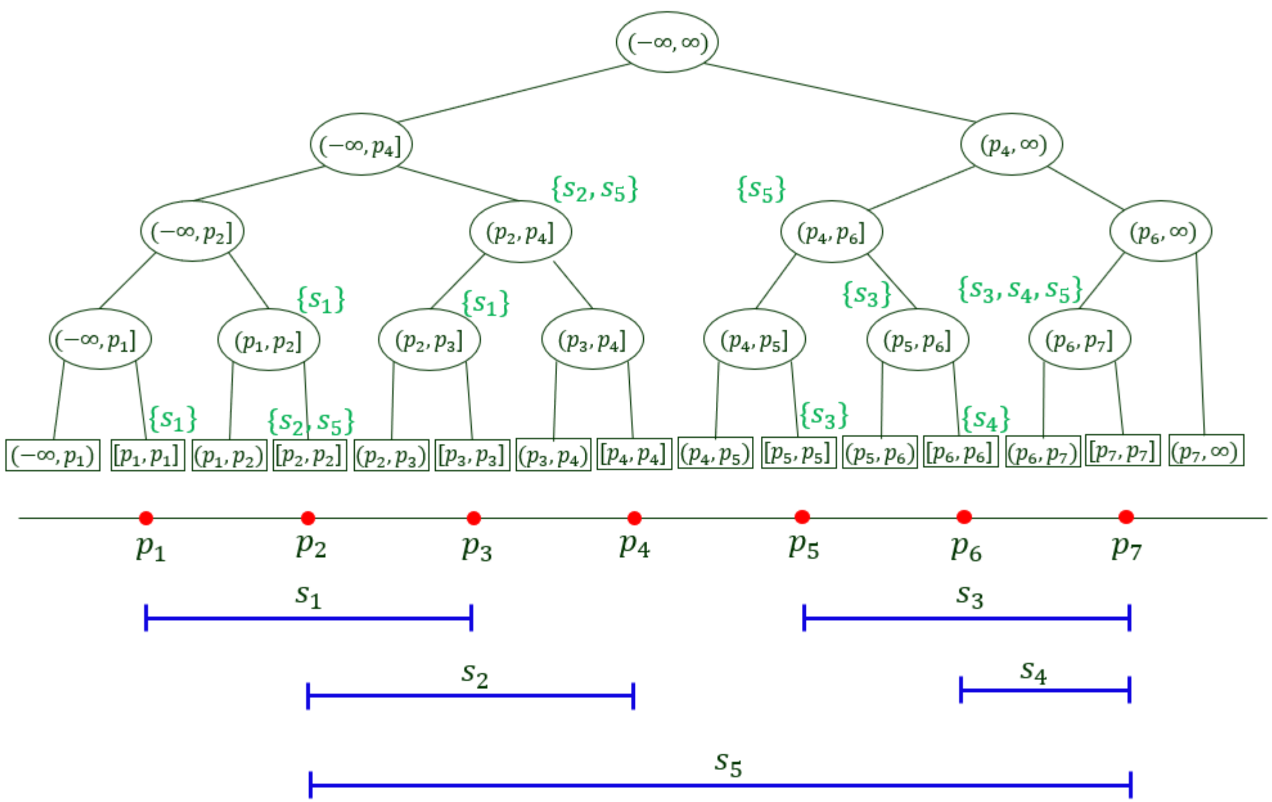

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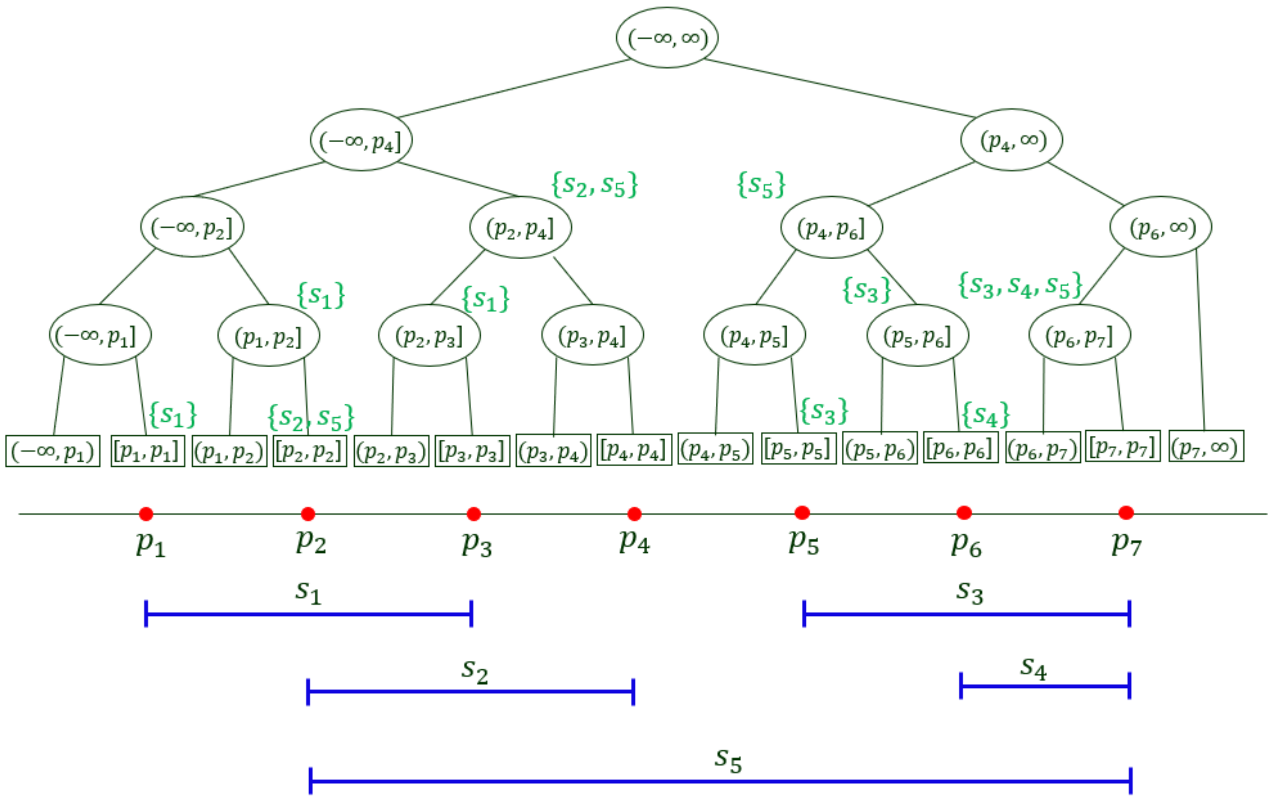

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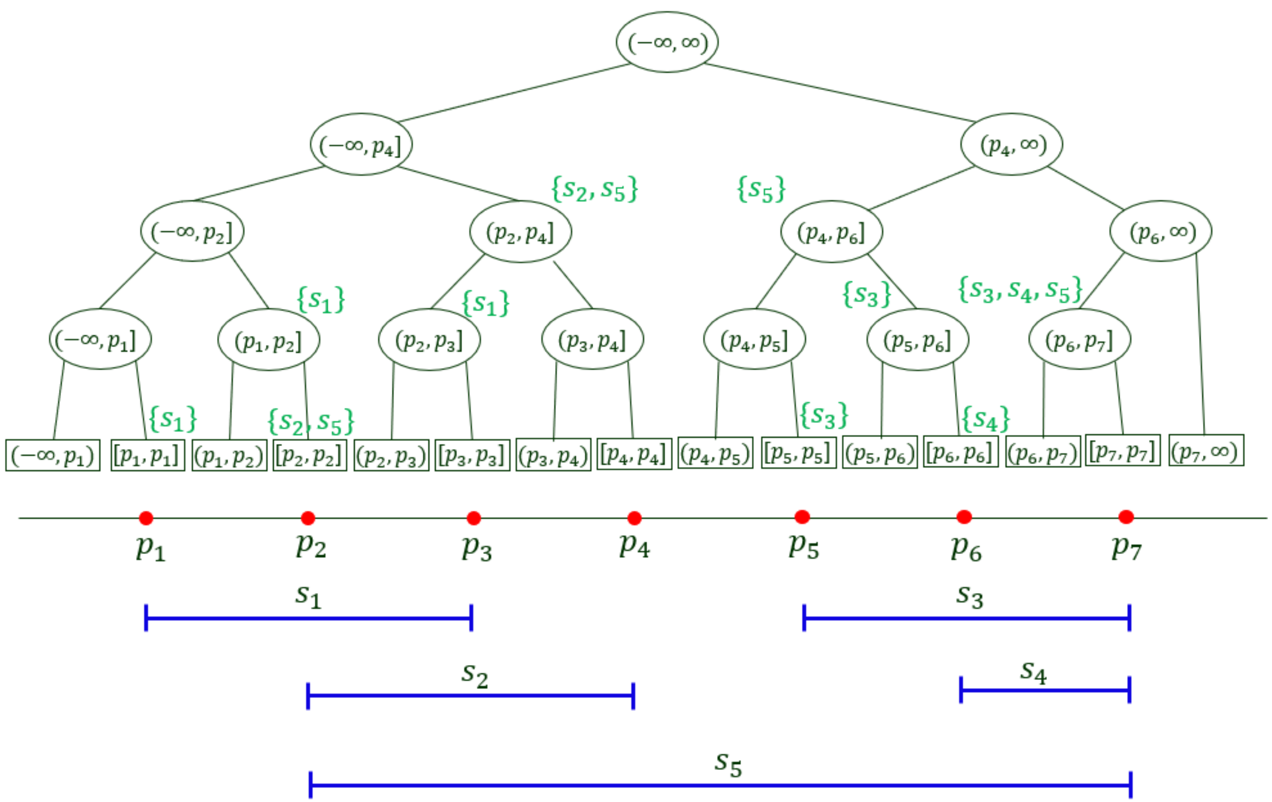

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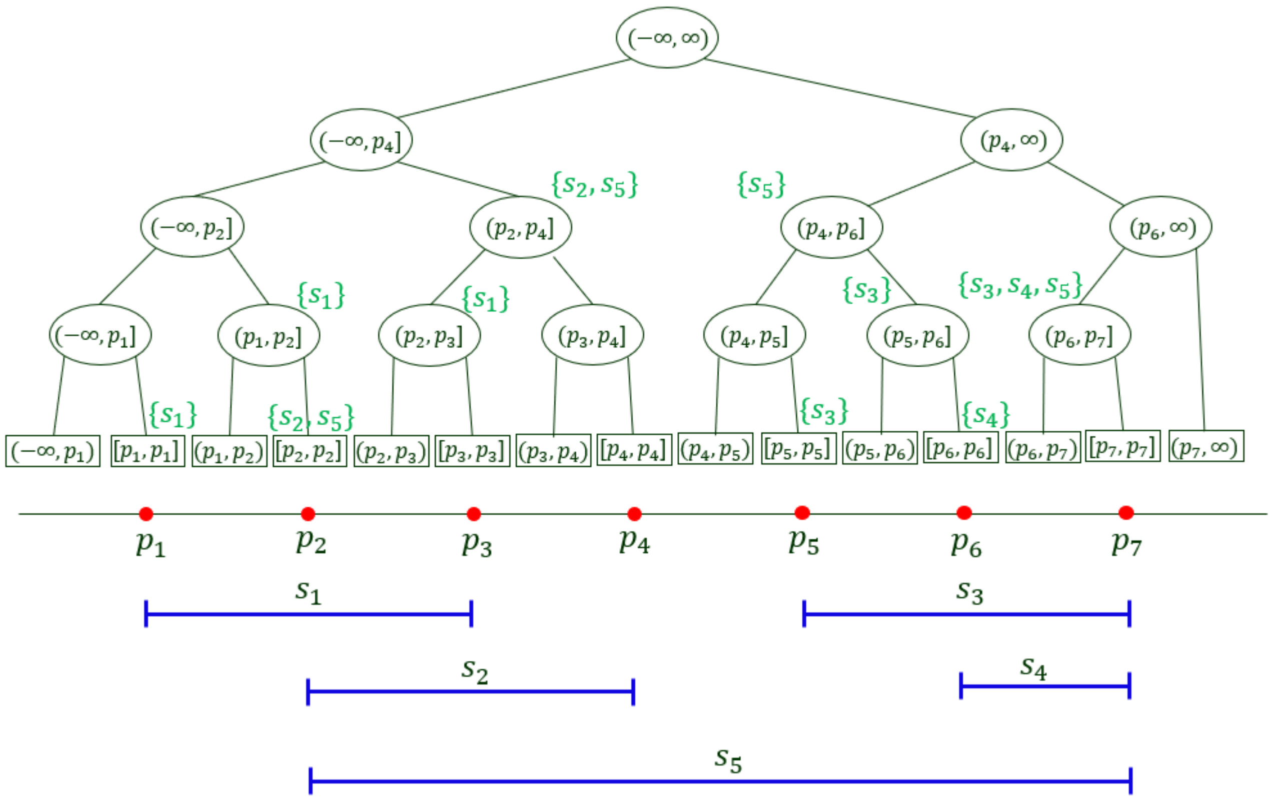

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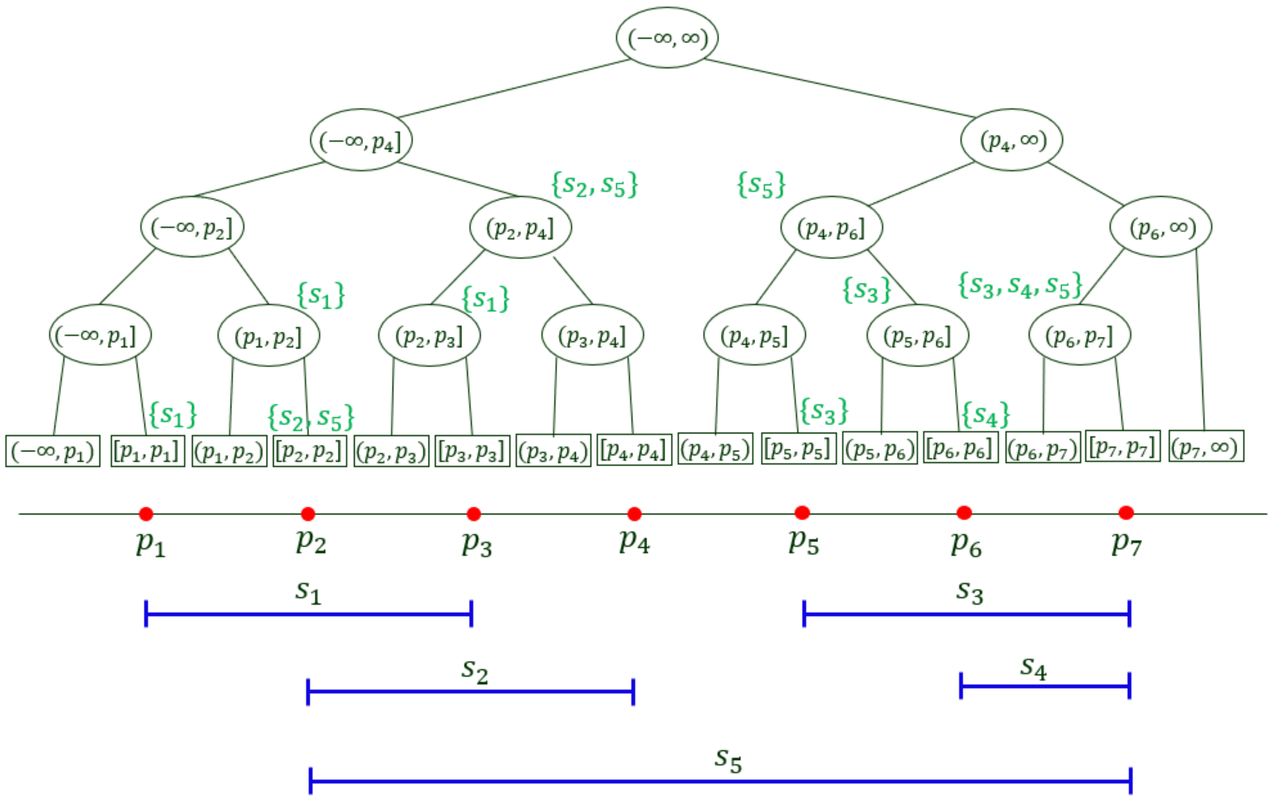

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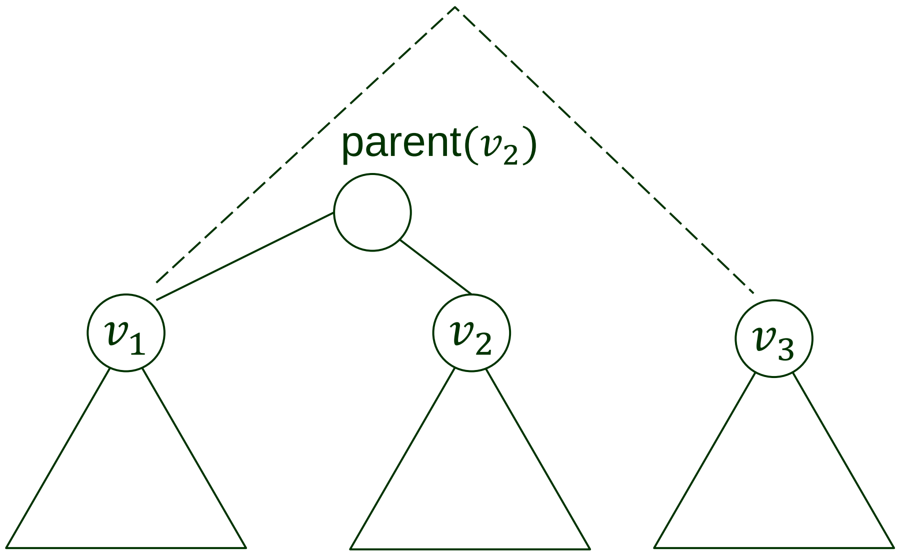

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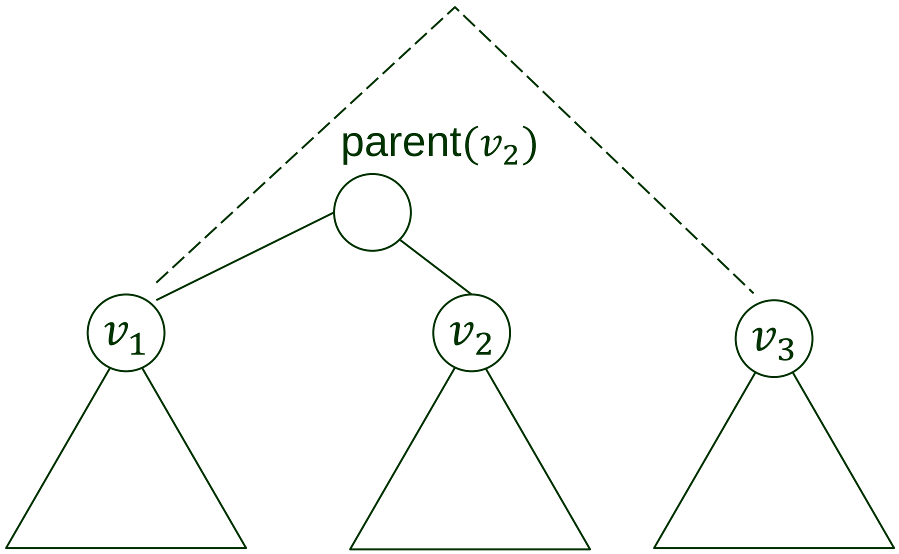

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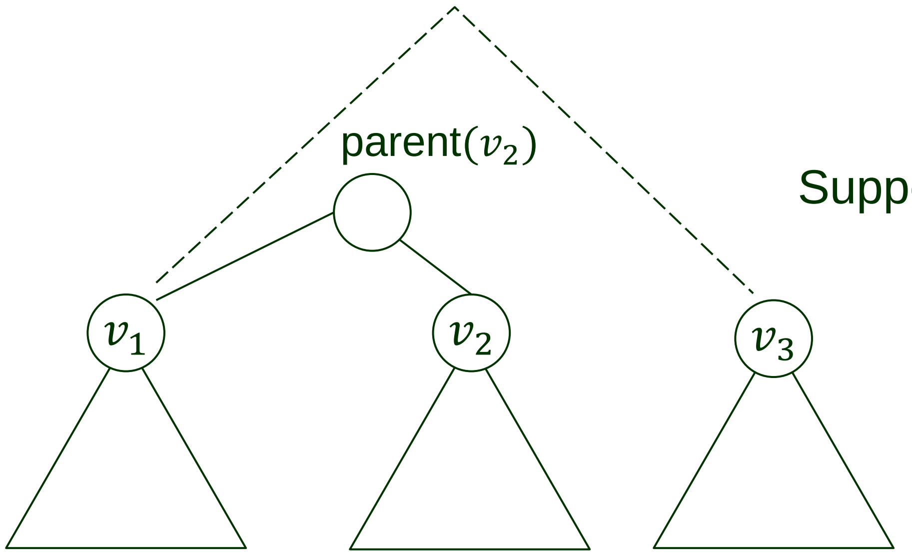

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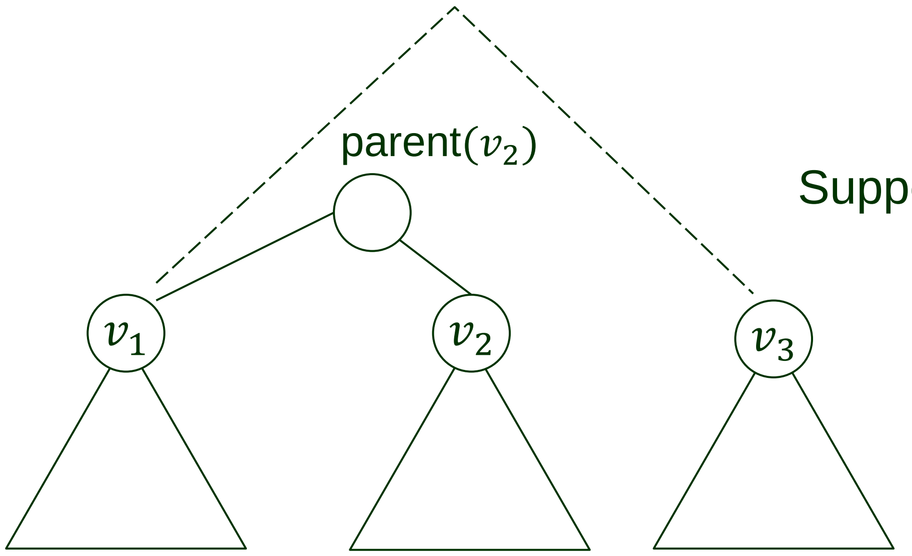

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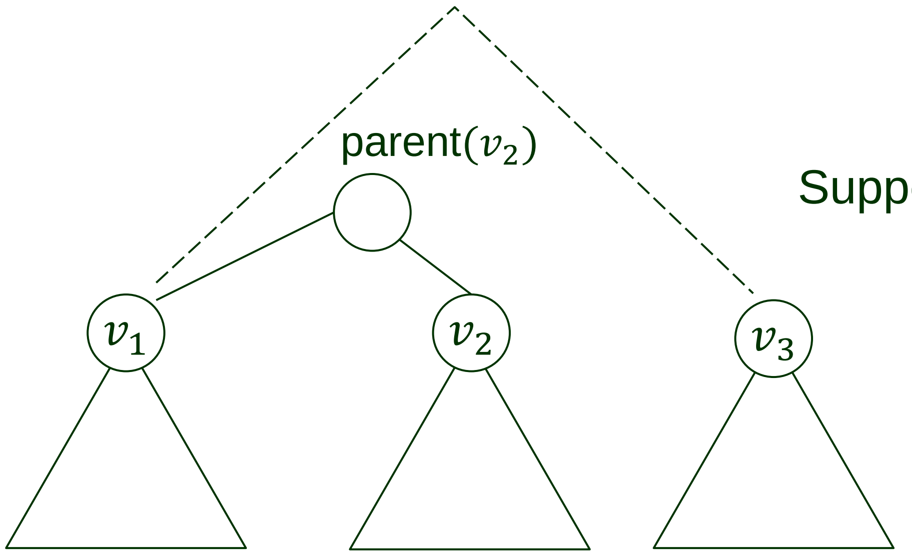

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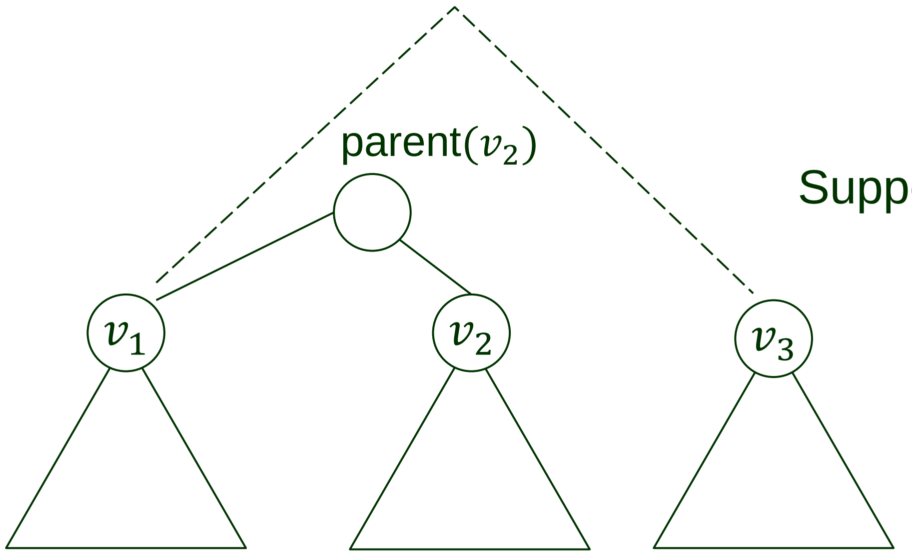

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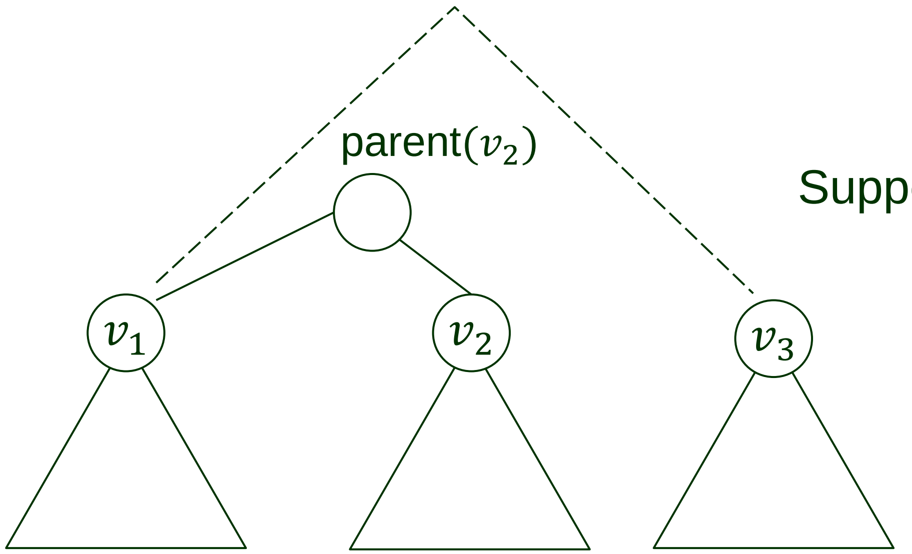

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$$O(n \log n)$$

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// precondition: $q_x \in \mathrm{Int}(v)$
report all the intervals in $C(v)$ // canonical subset
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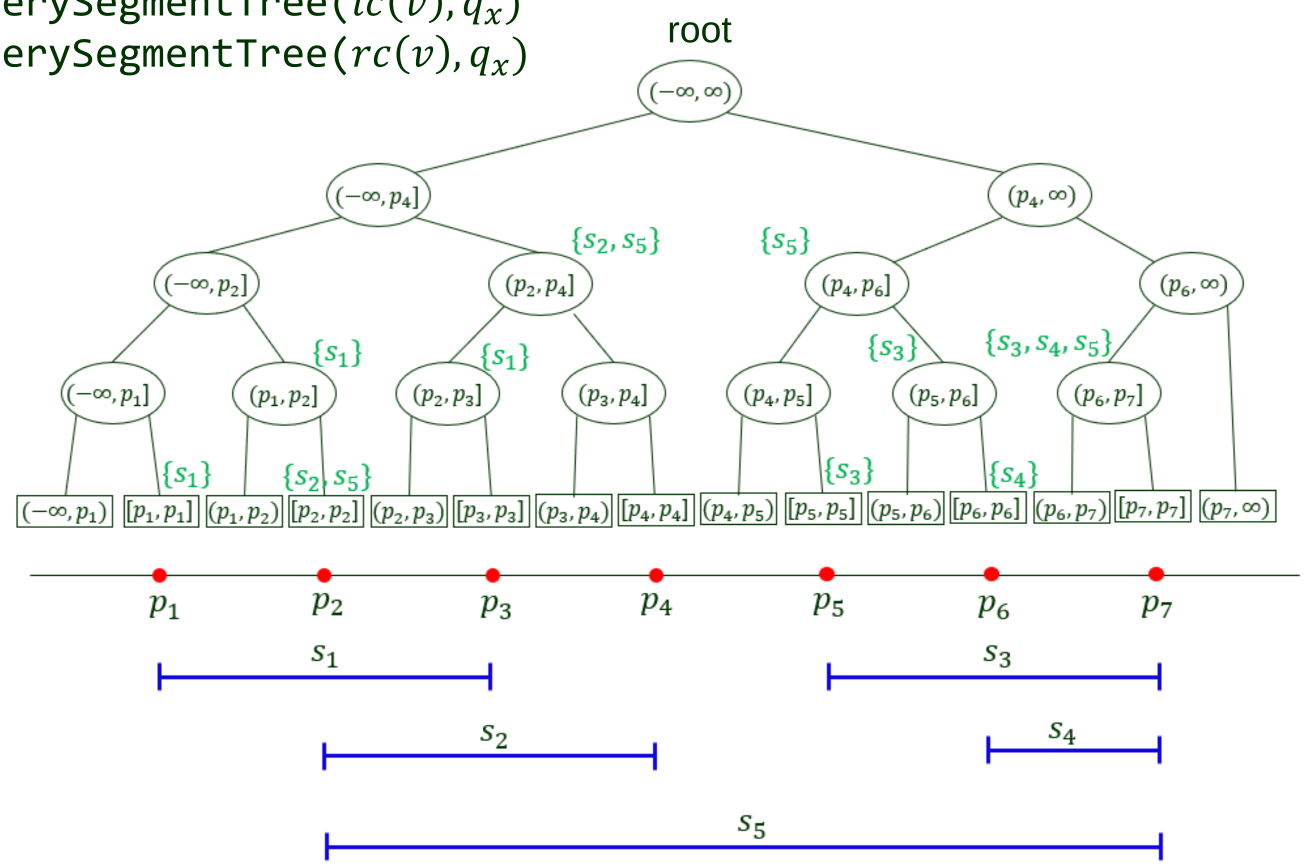

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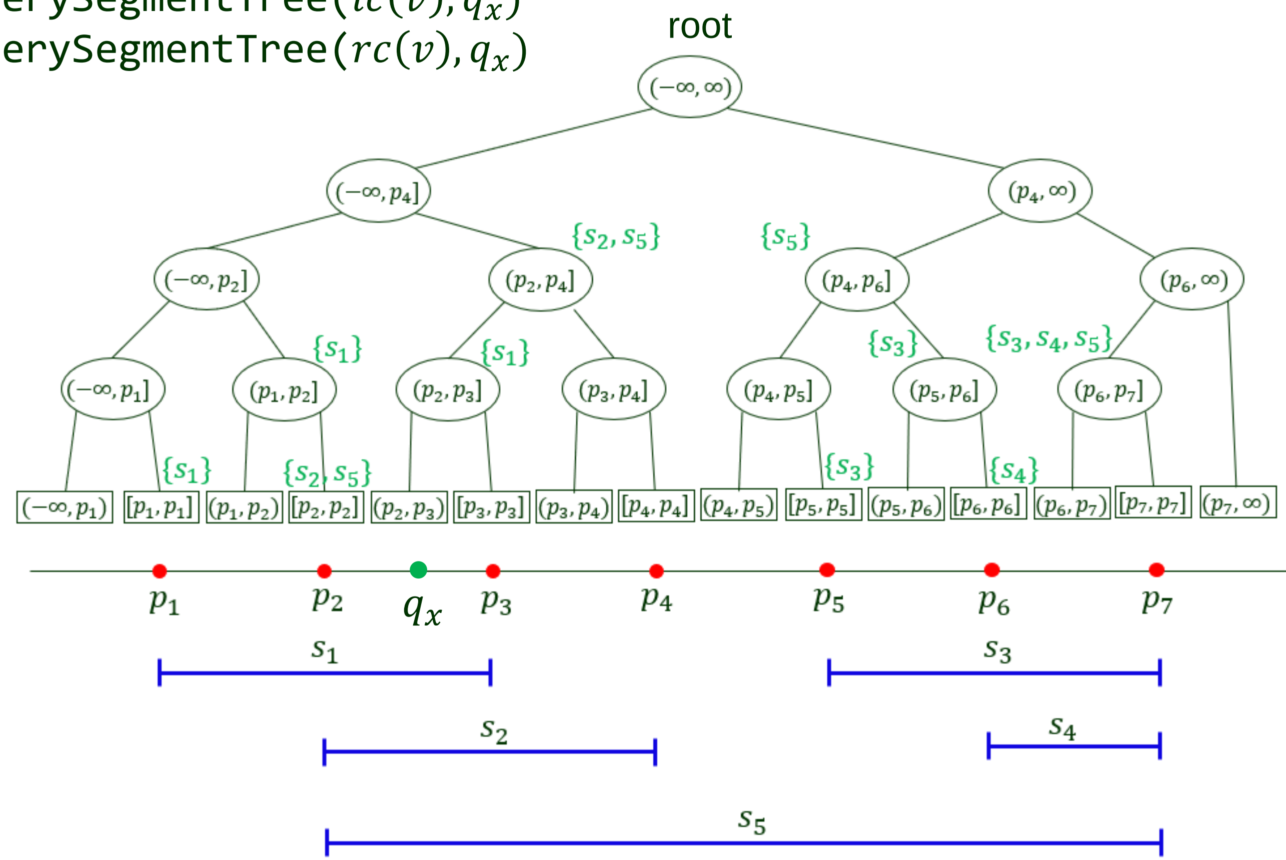
Example: q_x



V. Query Algorithm

```
QuerySegmentTree(v, q_x)
// precondition: q_x ∈ Int(v)
report all the intervals in C(v) // canonical subset
if v is not a leaf
    then if q_x ∈ Int(lc(v))
            then QuerySegmentTree(lc(v), q_x)
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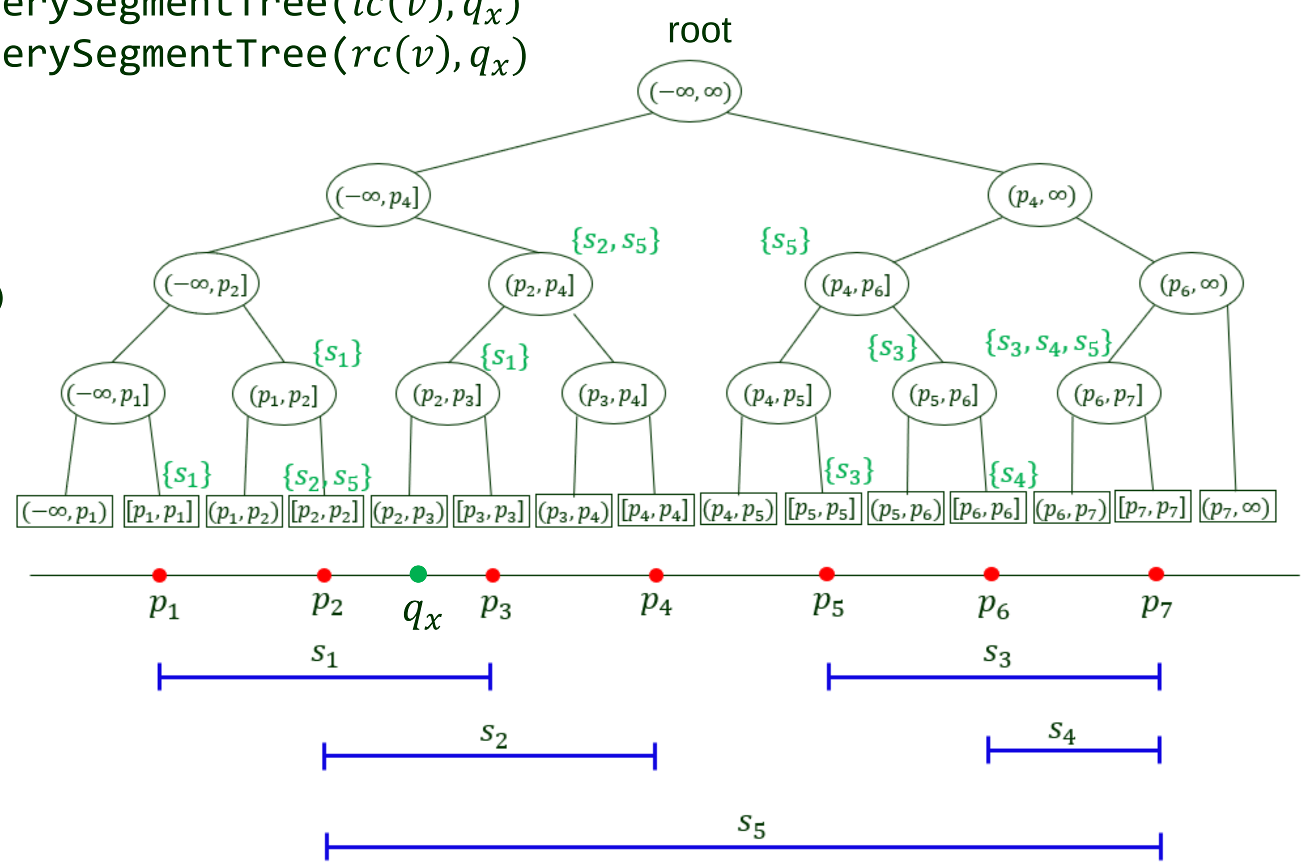
```

Example: q_x

QuerySegmentTree(root, q_x)



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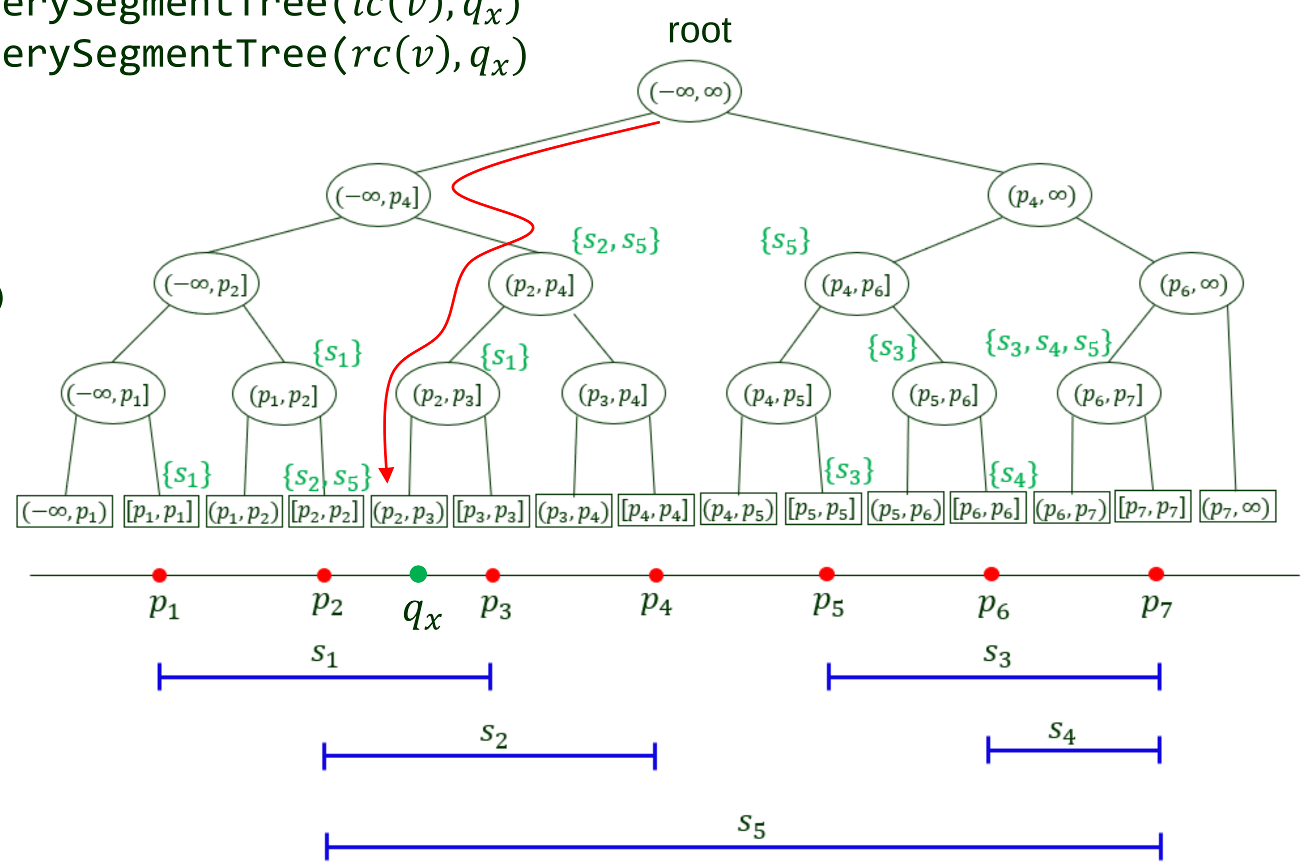

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    if v is not a leaf
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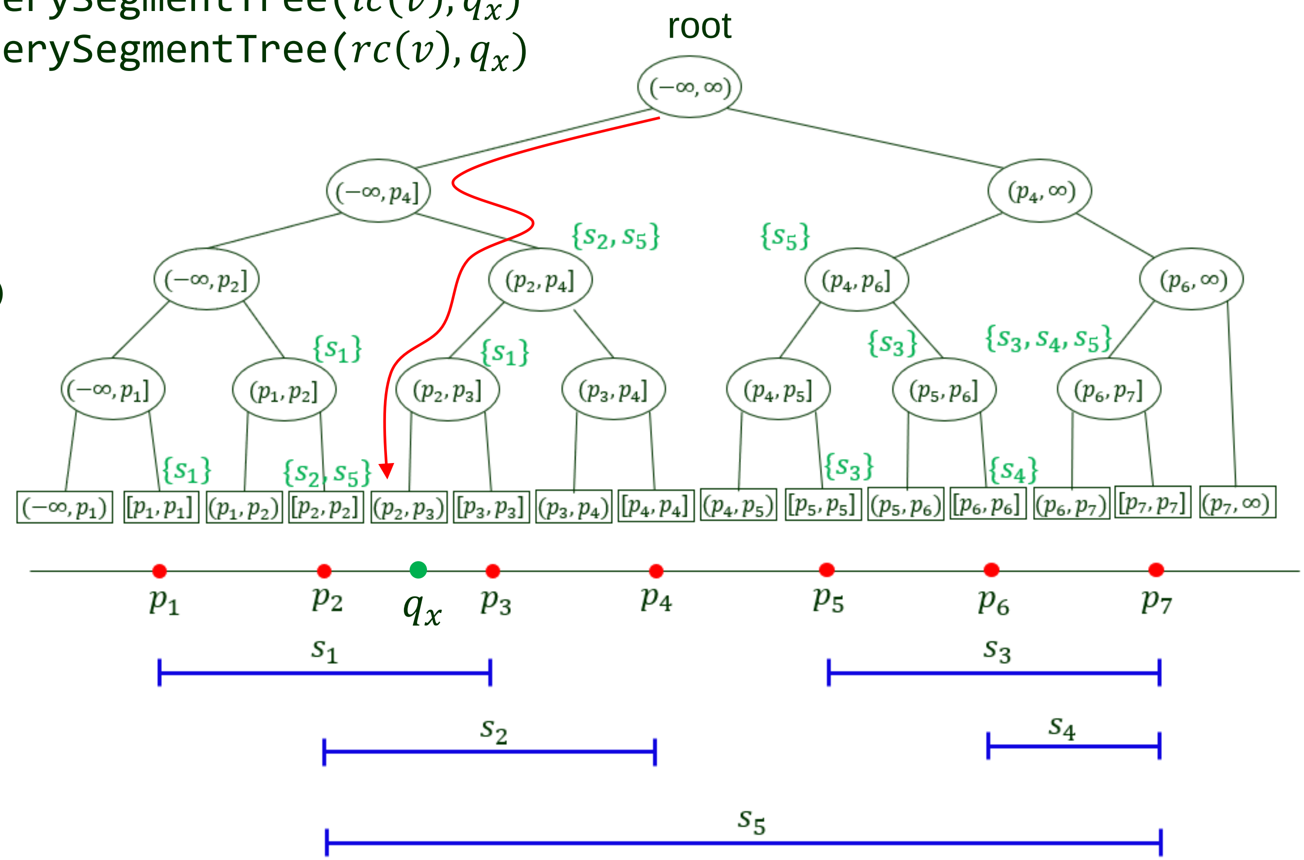
Example: q_x

QuerySegmentTree(root, q_x)

Output in order:

s_2, s_5, s_1



V. Query Algorithm

QuerySegmentTree(v, q_x)
// precondition: $q_x \in \mathrm{Int}(v)$
report all the intervals in $C(v)$ // canonical subset
if v is not a leaf
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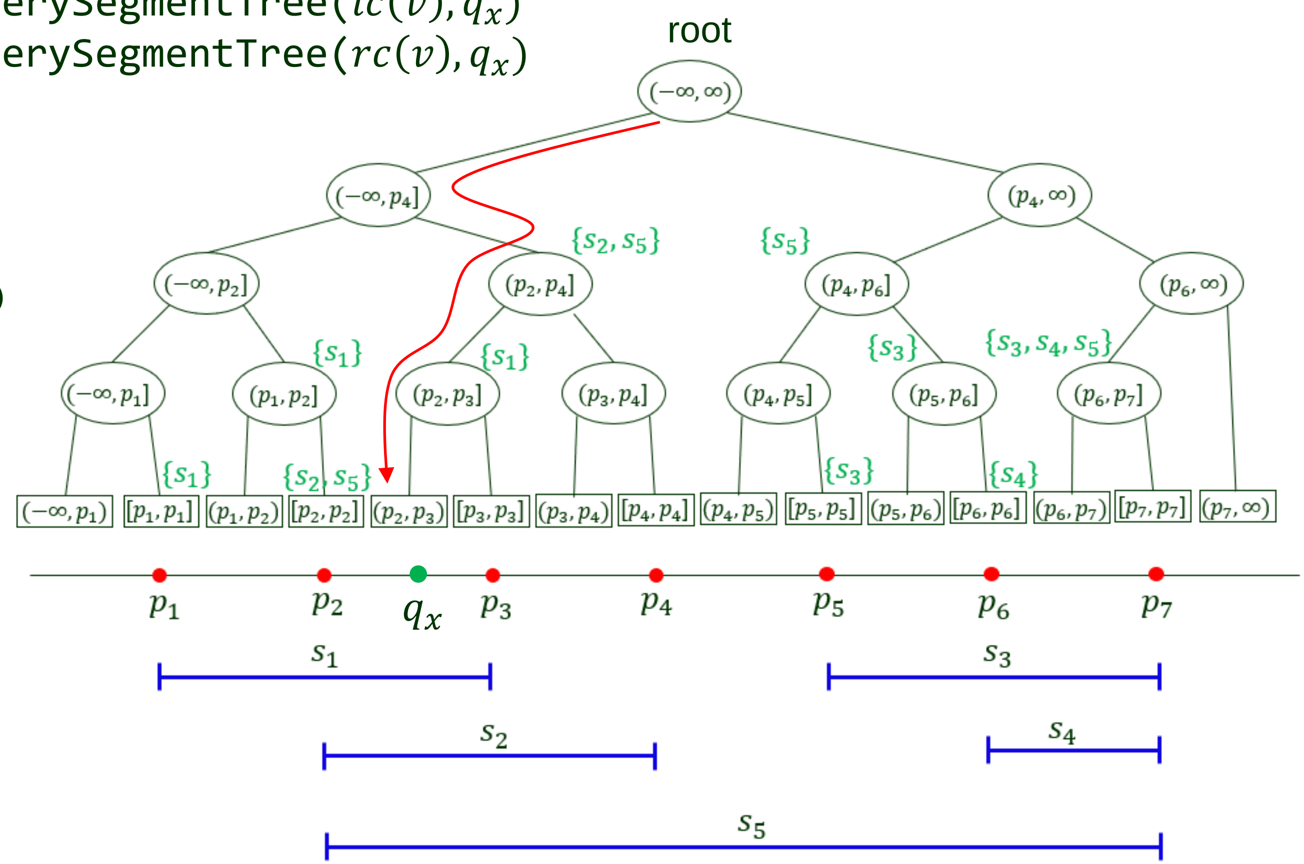

Query time:

$O(\log n + k)$

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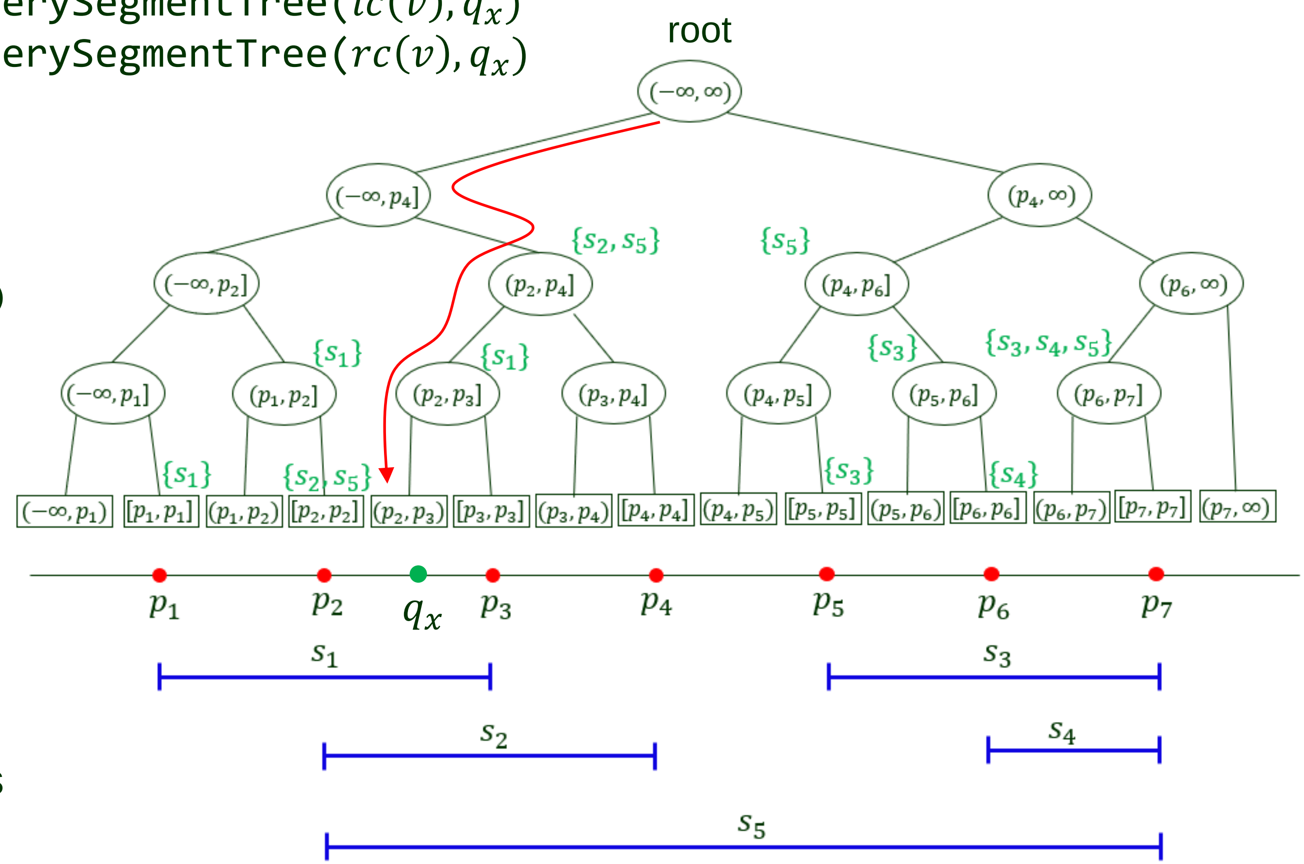

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#reported intervals

Segment Tree Construction

- Sort endpoints from I to yield elementary intervals.

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- Sort endpoints from I to yield elementary intervals.

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- Compute canonical subsets by inserting original intervals $[x, x']$ from I one by one.

Segment Tree Insertion

InsertSegmentTree($v, [x, x']$)

1. **if** Int(v)$\subseteq [x, x']$ // Int(parent(v))$\not\subseteq [x, x']$ holds
2.   **then** store $[x, x']$ at v
3.   **else if** Int($lc(v)$)$\cap [x, x'] \neq \emptyset$
4.   **then** InsertSegmentTree($lc(v), [x, x']$)
5.   **if** Int($rc(v)$)$\cap [x, x'] \neq \emptyset$
6.   **then** InsertSegmentTree($rc(v), [x, x']$)

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    then store $[x, x']$ at v
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- At each visited node v, either $[x, x']$ is stored or Int(v) contains an endpoint of $[x, x']$.
  - An interval is stored ≤ 2 times at each depth.

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```
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if Int(v) ⊆ [x, x'] // Int(parent(v)) ⊈ [x, x'] holds
    then store [x, x'] at v
    else if Int(lc(v)) ∩ [x, x'] ≠ ∅
             then InsertSegmentTree(lc(v), [x, x'])
         if Int(rc(v)) ∩ [x, x'] ≠ ∅
             then InsertSegmentTree(rc(v), [x, x'])
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- ≤ 2 storage actions + ≤ 2 containments

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Compared with an interval tree, a segment tree has

♣ the same query time $O(\log n + k)$

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Compared with an interval tree, a segment tree has

♣ the same query time $O(\log n + k)$

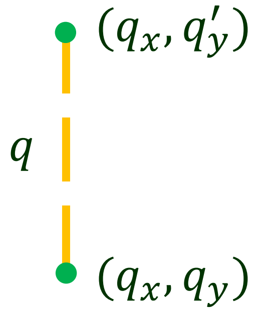
♣ a larger storage $O(n \log n)$ than $O(n)$,

but it allows to answer more complicated queries.

VI. Back to Windowing

Query segment: $q = q_x \times [q_y, q_y']$

n *arbitrarily oriented* segments



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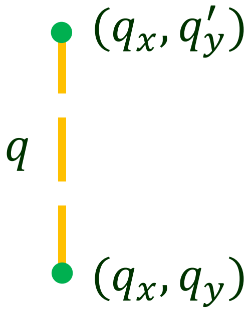
Construct a segment tree $\mathcal{T}$.



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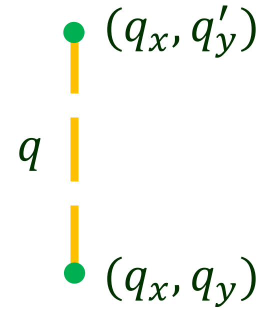

Construct a segment tree $\mathcal{T}$.

- On the x-intervals of the segments in S.
- Canonical subset $C(v)$ at a vertex v store segments rather than their x-intervals.

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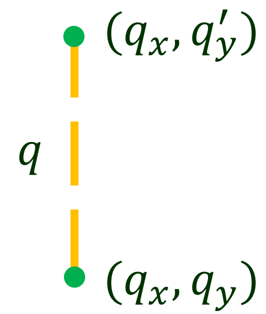

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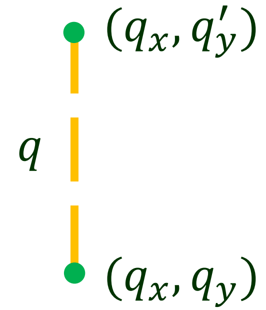

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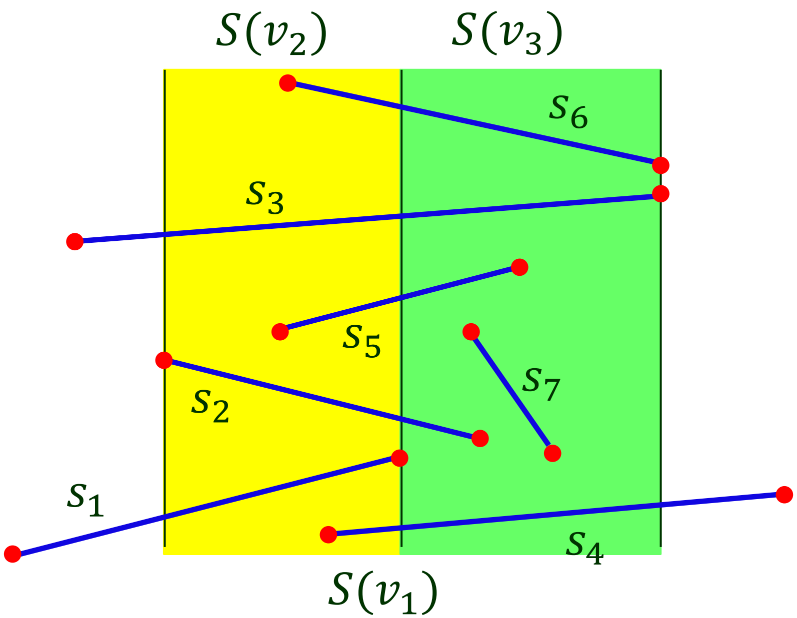
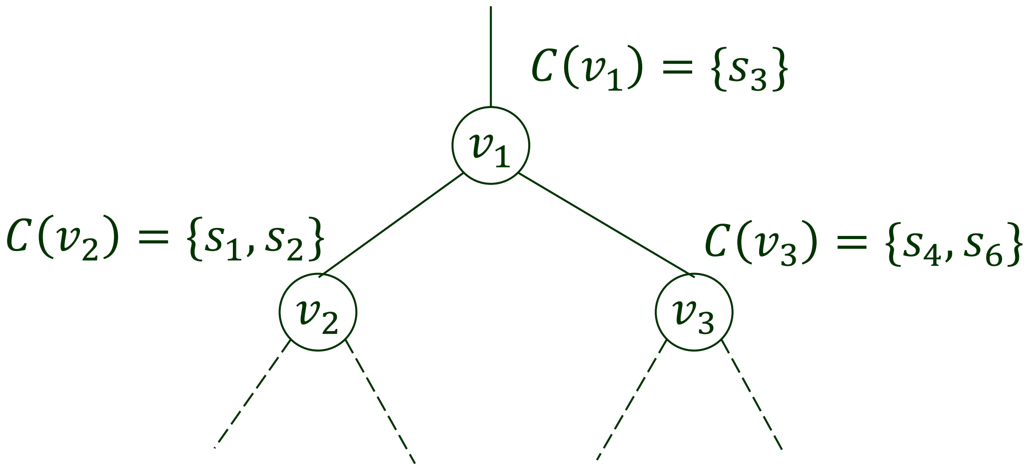
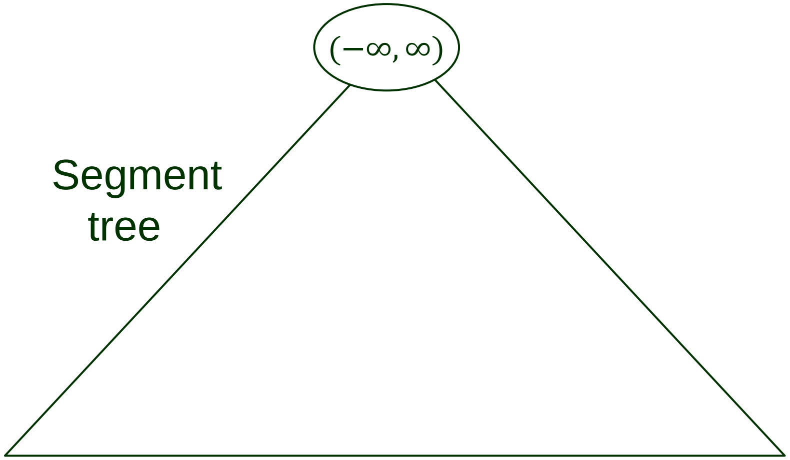




Segments on the Search Path



- Search with q_x in $\mathcal{T}$.

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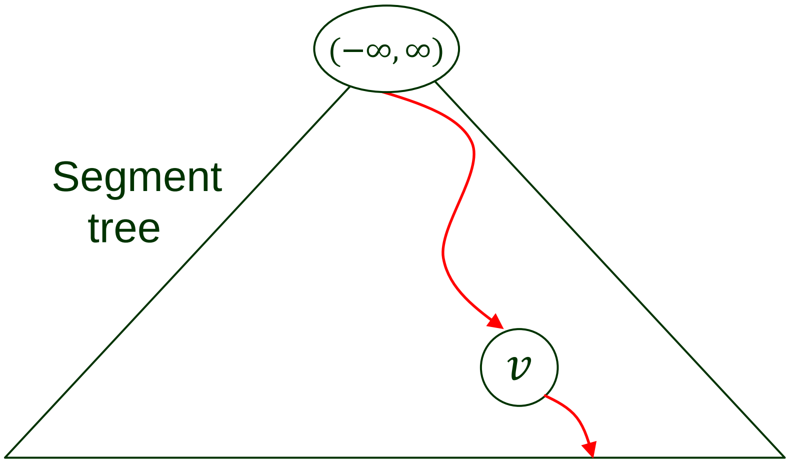

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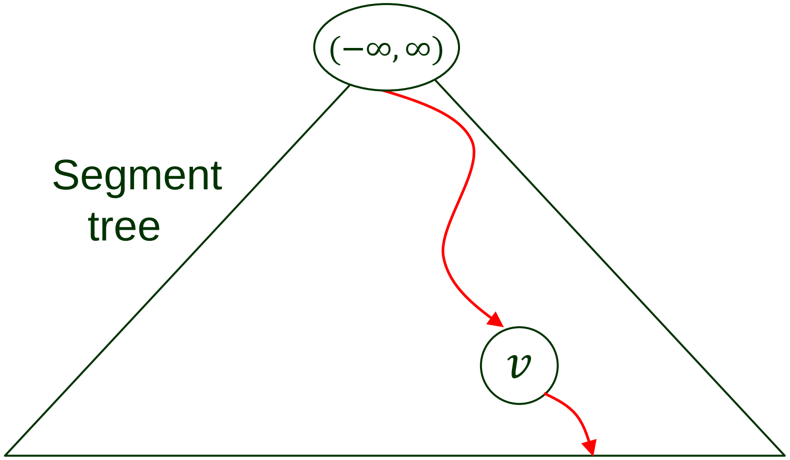
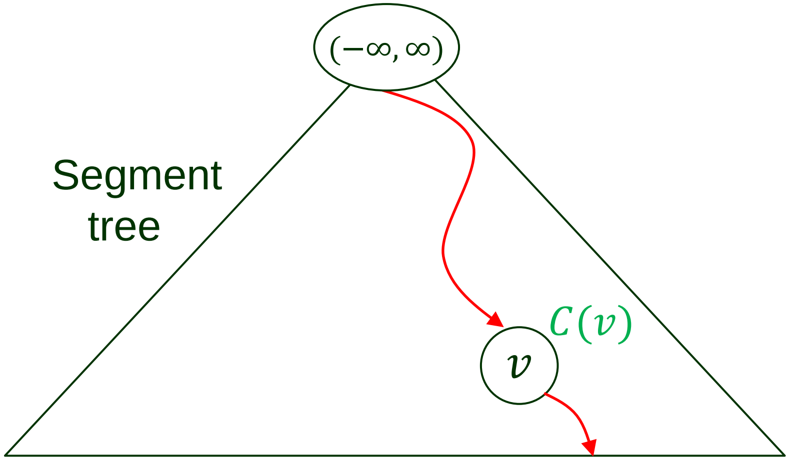


- Search with q_x in $\mathcal{T}$.
- $O(\log n)$ canonical sets together include all the segments intersected by the vertical line $x = q_x$.

Segments on the Search Path



- Search with q_x in $\mathcal{T}$.
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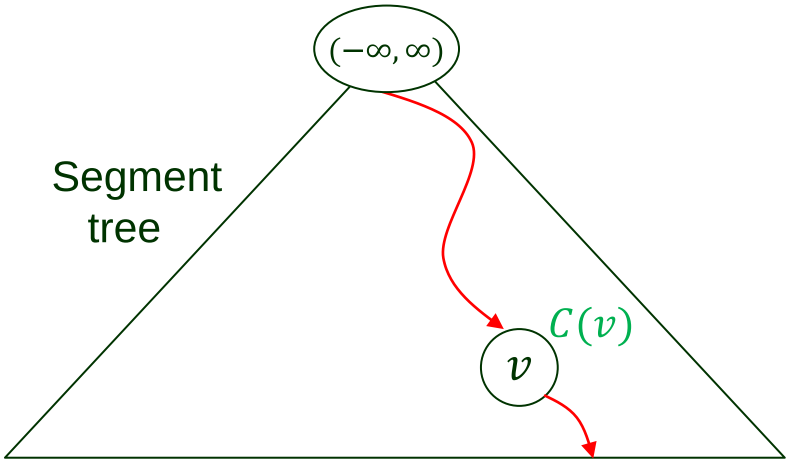
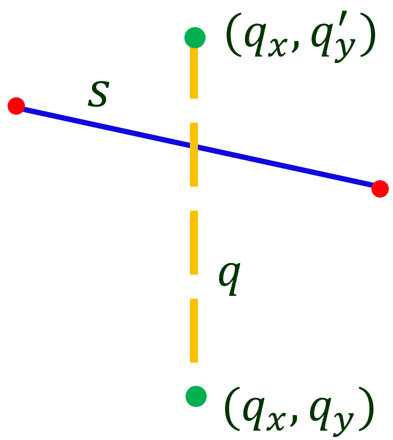
- v is a node on the search path.

Segments on the Search Path





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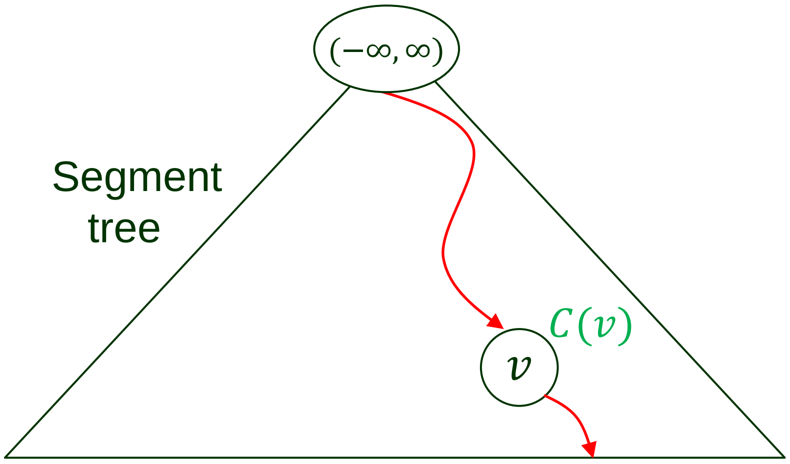

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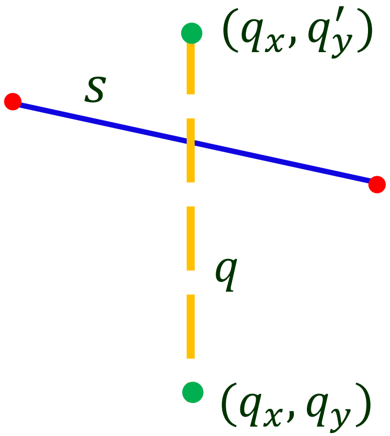
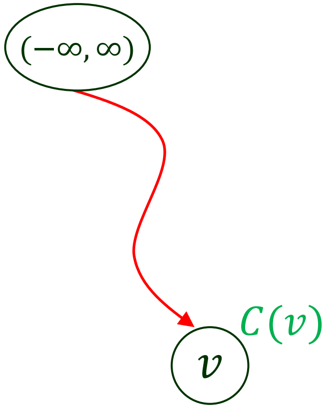


- Segment $s \in C(v)$ is intersected by q iff

(q_x, q_y) below s and (q_x, q'_y) above s

VII. Storage of a Canonical Set



- Segments in $C(v)$ do not intersect each other.
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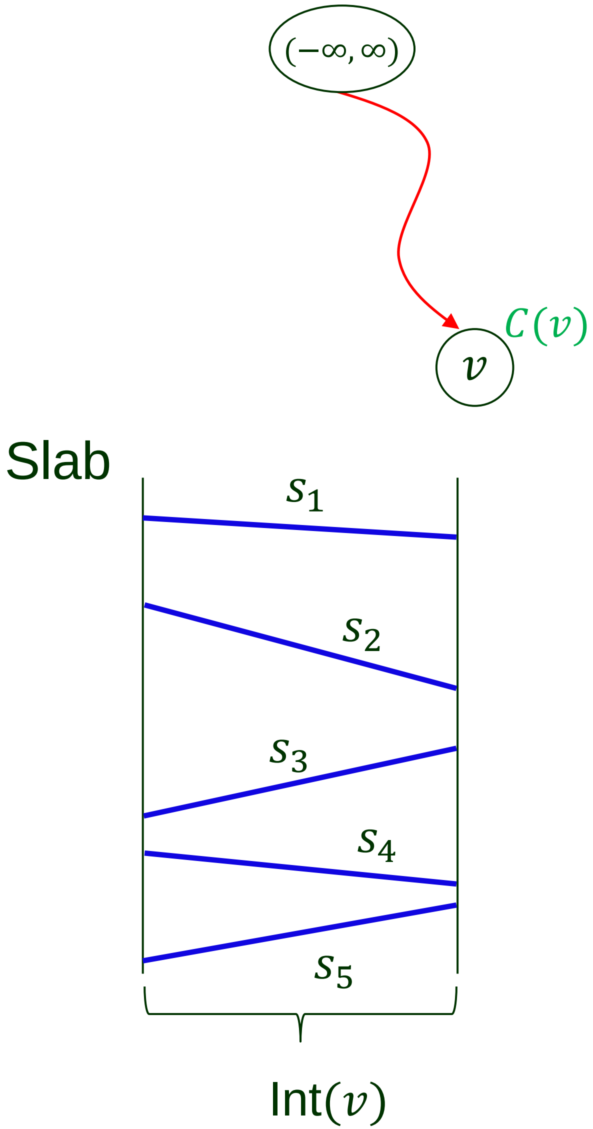

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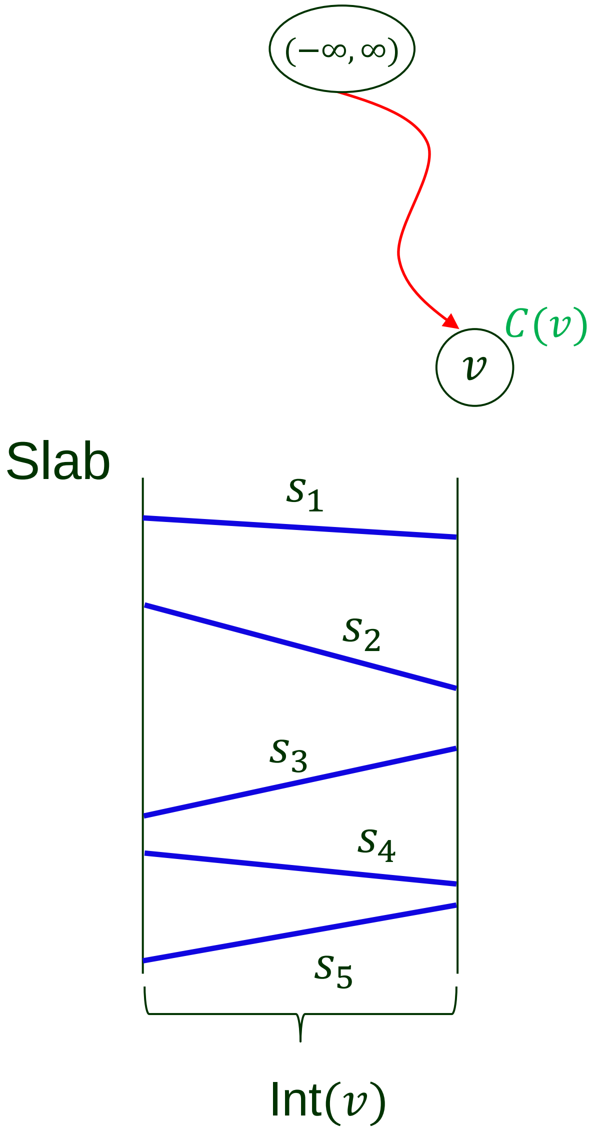

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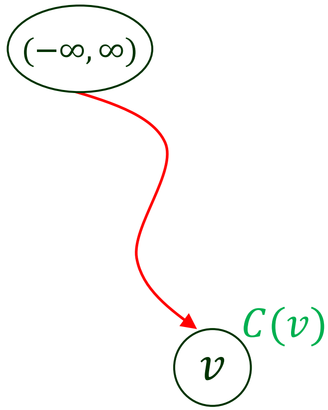

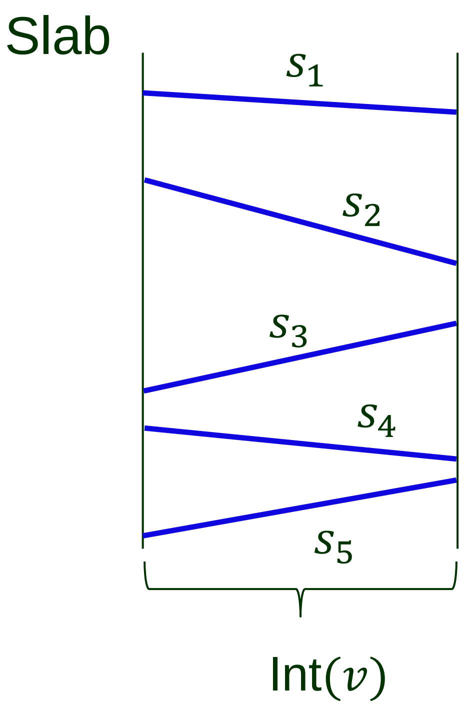

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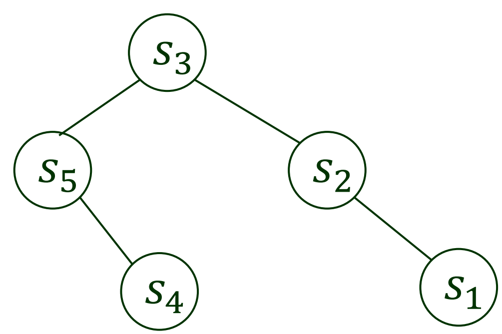

associate structure $\mathcal{B}(v)$

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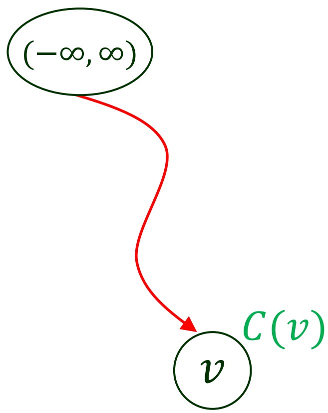

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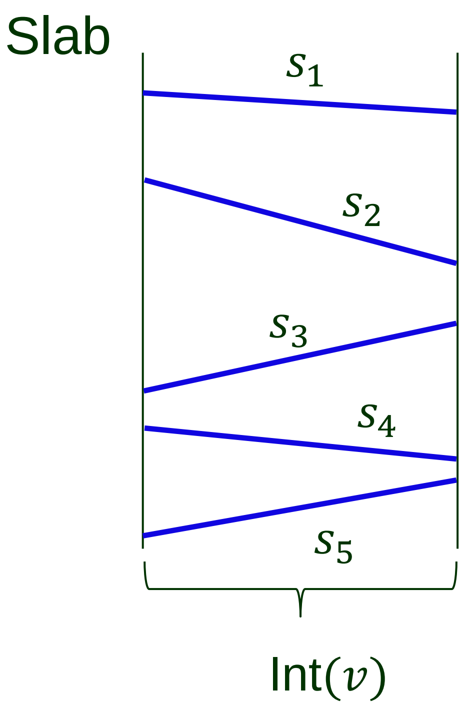

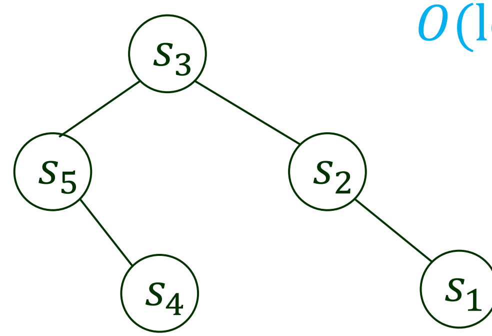

associate structure $\mathcal{B}(v)$

$O(\log n + k_v)$ query time within $\mathcal{B}(v)$

#intersected segments in $C(v)$

Segment Tree Storage Summary

- The set S of segments is stored in a segment tree $\mathcal{T}$ based on

  their x-intervals.

- The canonical subset $C(v)$ of every internal node v in $\mathcal{T}$ is stored in a BST $\mathcal{B}(v)$ based on

  the vertical order within the slab $\mathrm{Int}(v) \times [-\infty, \infty]$.

Total storage: $O(n \log n)$

Construction time: $O(n \log n)$

Query Algorithm

Query object: a vertical line segment q: $q_x \times [q_y, q'_y]$

Set $S = \{s_1, s_2, \ldots, s_n\}$ of arbitrarily oriented segments

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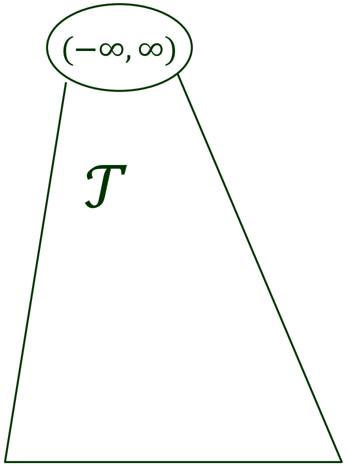

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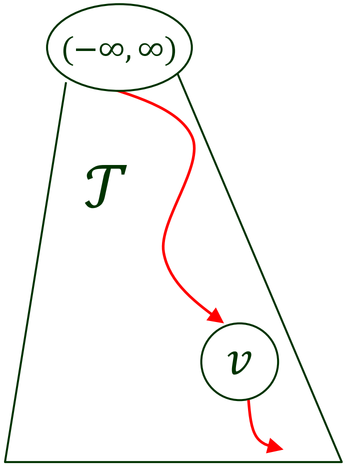

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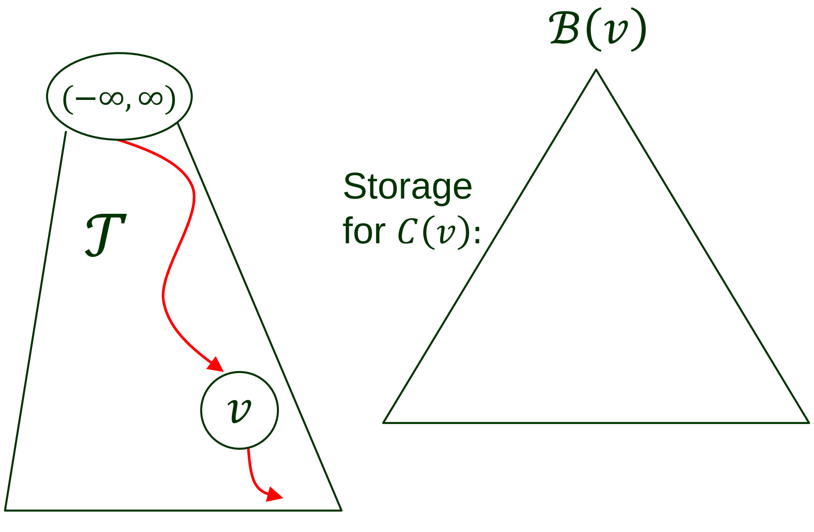

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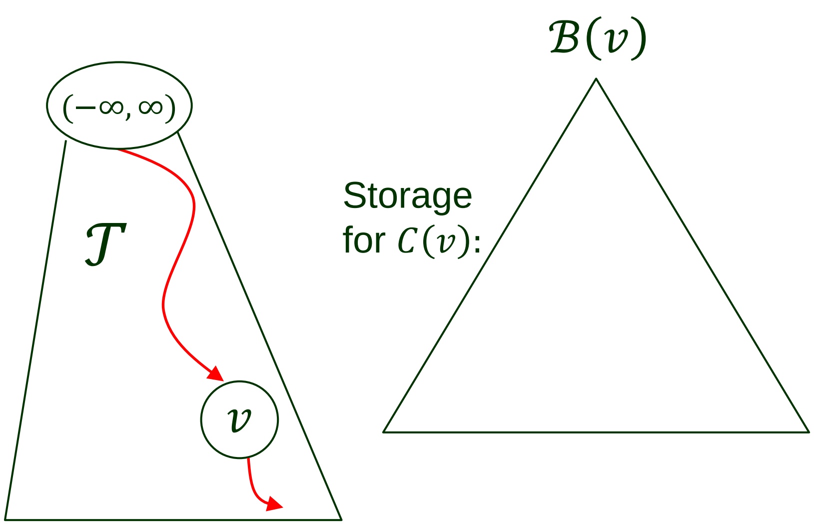
Set $S = \{s_1, s_2, \ldots, s_n\}$ of arbitrarily oriented segments

- Search with q_x in a segment tree $\mathcal{T}$.
- At every node v on the search path, search with the two endpoints of q (essentially q_y and q'_y) to report segments in $C(v)$ intersected by q.



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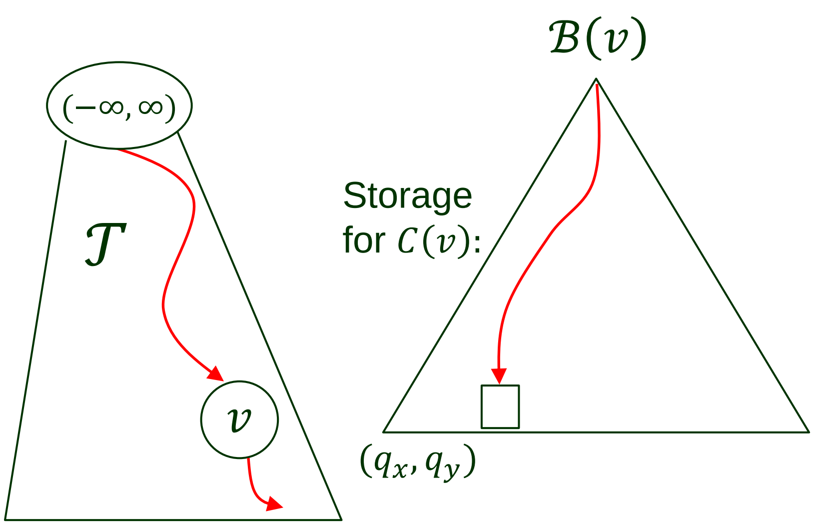
- At every node v on the search path, search with the two endpoints of q (essentially q_y and q_y') to report segments in $C(v)$ intersected by q.



Query Algorithm

Query object: a vertical line segment q: $q_x \times [q_y, q'_y]$

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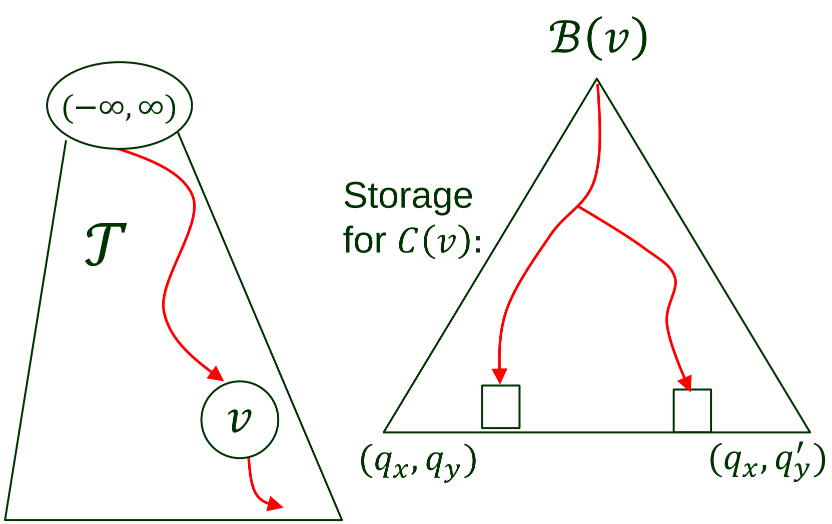

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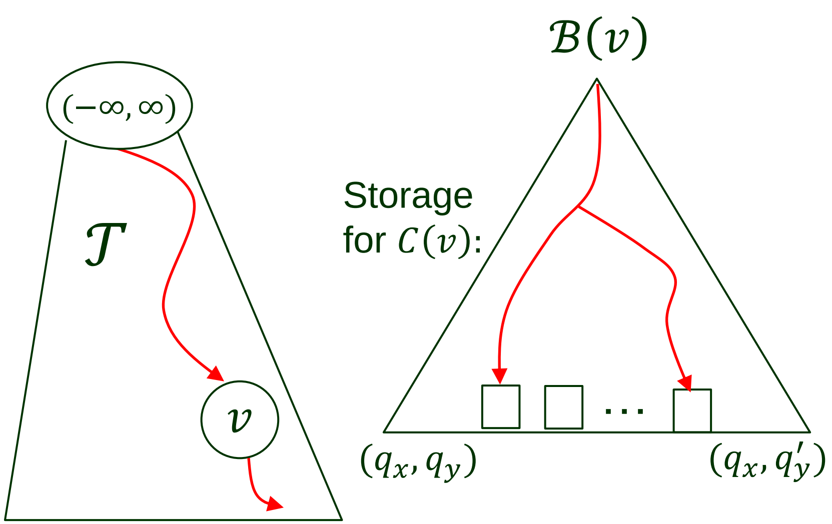

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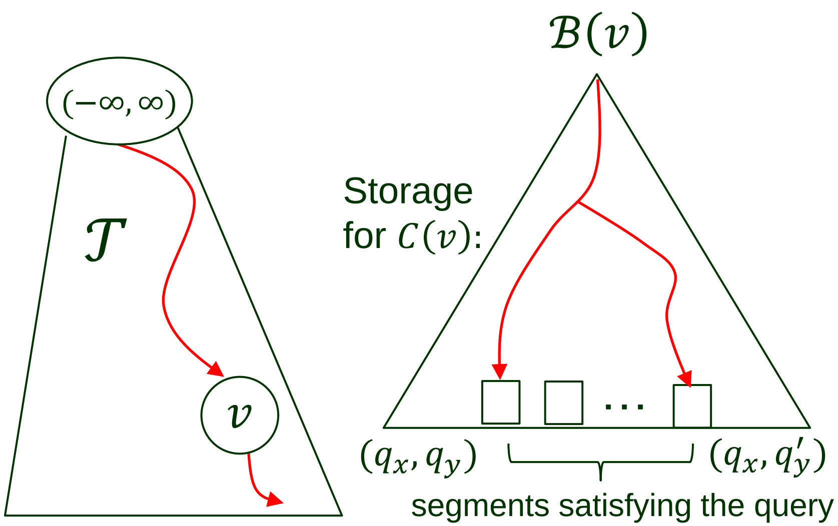

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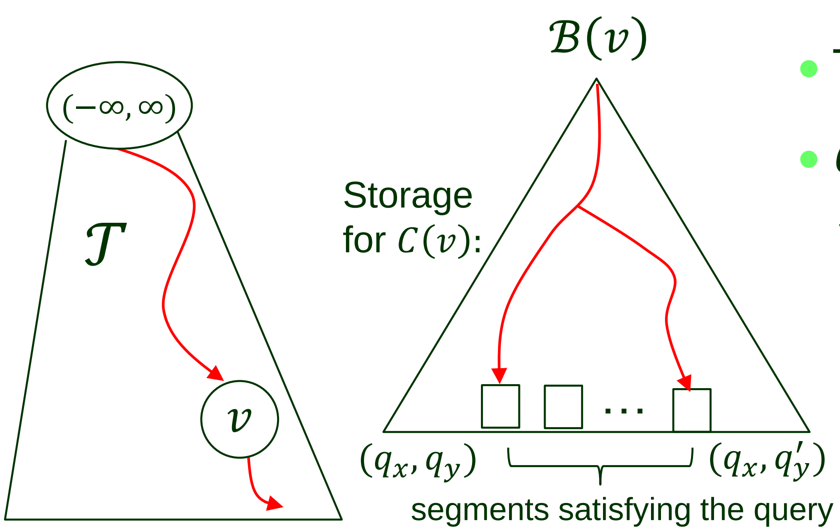

- The search takes $O(\log n + k_v)$ time.
- $O(\log n)$ nodes on the search path, each requiring such a search.

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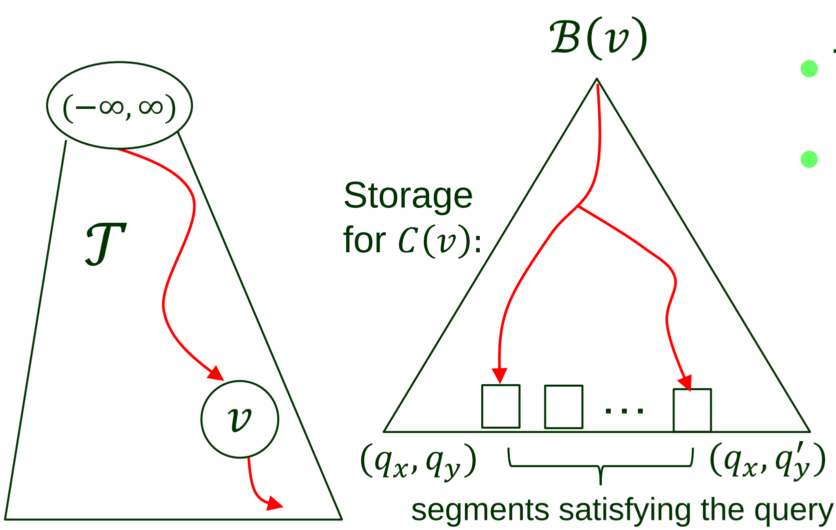

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$\Downarrow$

Query time: $O(\log^2 n + k)$