

Basics of Probability

Outline:

I. Acting under uncertainty

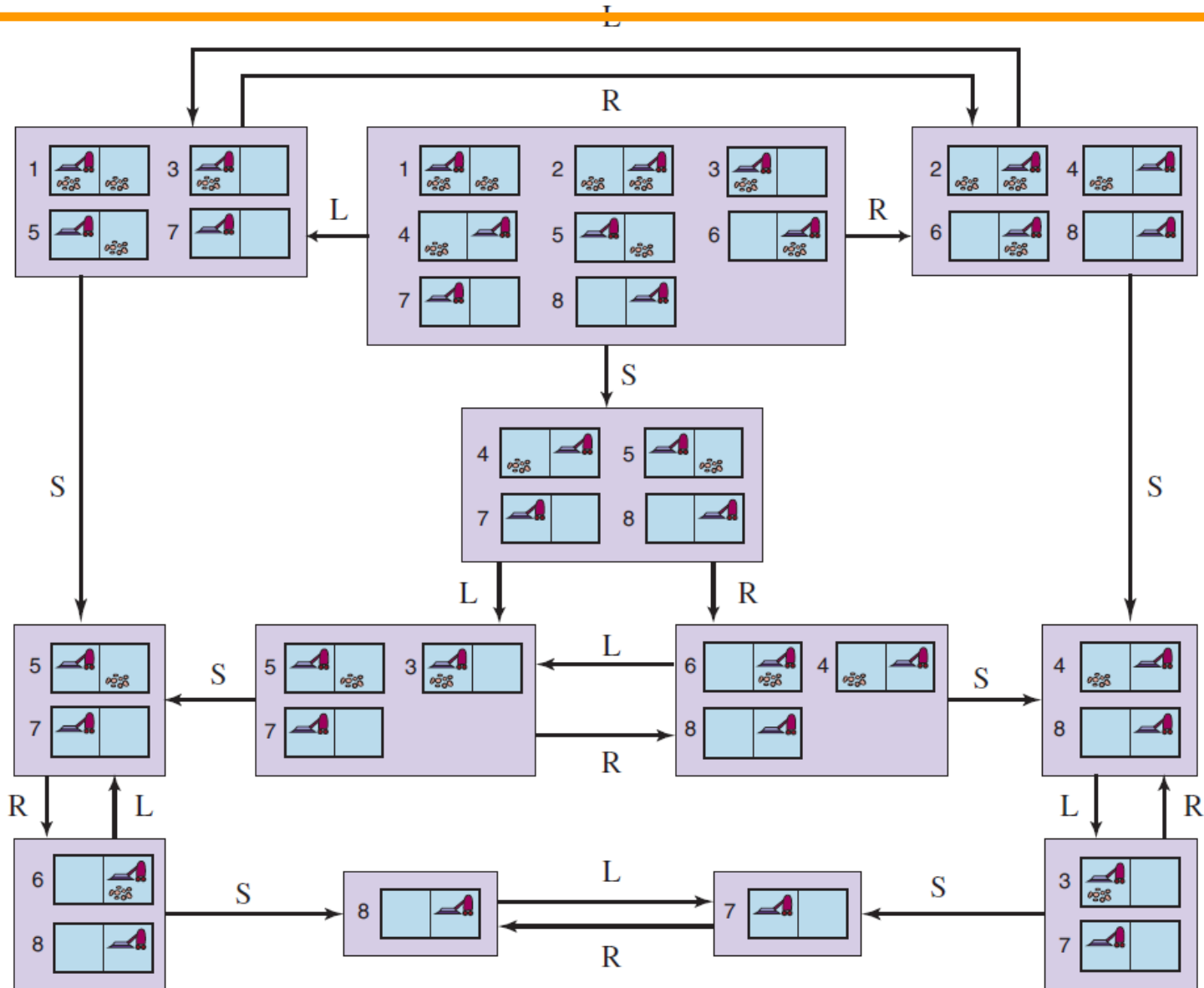
II. Probability model

III. Probability distribution

* Figures are from the [textbook site](#) or by the instructor.

** A small portion of the notes is adapted from those by Dr. Jin Tian.

I. Acting Under Uncertainty



Drawbacks of Using Belief States

Keeping track of a belief state has several drawbacks:

- ♣ **Large belief state** full of unlikely possibilities because every possible explanation of the percept needs to be considered.
- ♣ **Correct contingent plan** growing arbitrarily large to handle every eventuality.
- ♣ **No successful plan** guaranteed sometimes yet action required.

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Make a rational decision depending on

- ♦ relative importance of various goals, and
- ♦ likelihood that, and degree to which, they will be achieved.

From Categorical to Uncertain Beliefs

- ◆ Intelligent behavior requires knowledge about the world.
- ◆ **Propositional and first-order logics** are effective for representing and reasoning with categorical beliefs about the world.
- ♣ Due to uncertainty, any logical sentence could be true, false or unknown.
- ♣ Logical approach can break down.

Dental Diagnosis

Toothache ⇒ Cavity

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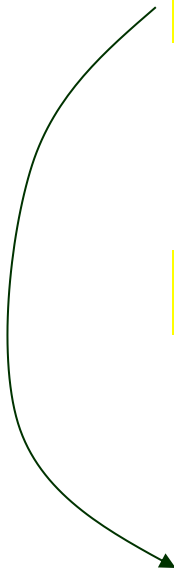
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Fix the rule by augment the LHS with **all qualifications** required for a cavity to cause a toothache!

- ♠ Too much work.
- ♠ Medical science has *no complete* domain theory.
- ♠ Not all the necessary tests have been or can be run on a patient.

Example of Reasoning Under Uncertainty

Beliefs

- If a patient has lung cancer, there is a 60% chance that an X-ray test will come back positive; and a 40% chance negative.
- If a patient does not have lung cancer, there is 2% percent chance that an X-ray test will come back positive ; and a 98% percent chance negative.
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Probability theory provides a framework for representing and reasoning under uncertainty.

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$\omega \in \Omega$ is a sample point, possible world, or atomic event.

- ♣ mutually exclusive
- ♣ exhaustive

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- b is the event that the results of rolling two dice summing to 10.

$$P(\text{Total} = 10) = P(b) = P((4,6)) + P((5,5)) + P((6,4)) = 1/12$$

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The prior of a is $1/6$, and the posterior of a given b is $1/3$.

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Random Variable

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$$P(\text{Even} = \textit{true}) = P(2) + P(4) + P(6) = 1/2$$

Use of Proportional Logic

For a Boolean random variable A , we abbreviate proportions $A = \text{true}$ and $A = \text{false}$ as a and $\neg a$, respectively.

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Example 4 distinct atomic events (or 4 possible worlds):

$\text{cavity} \wedge \text{toothache}$

$\neg \text{cavity} \wedge \text{toothache}$

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III. Probability Distribution

- Probabilities of all the possible values of a random variable.

$$P(\textit{Weather} = \textit{sun}) = 0.6$$

$$P(\textit{Weather} = \textit{rain}) = 0.1$$

$$P(\textit{Weather} = \textit{cloud}) = 0.29$$

$$P(\textit{Weather} = \textit{snow}) = 0.01$$

// The distribution is represented as a two-dimensional array.

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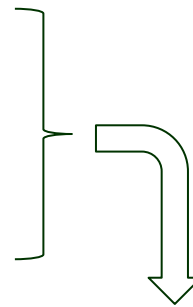
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$$\mathbf{P(\textit{Weather})} = \langle 0.6, 0.1, 0.29, 0.01 \rangle$$

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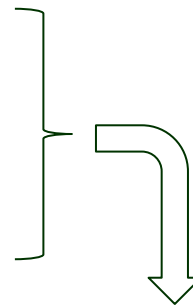
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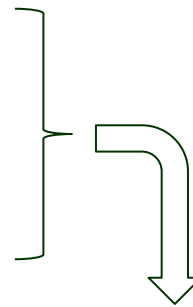
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Probability distribution: $\mathbf{P}(\textit{Weather}) = \langle 0.6, 0.1, 0.29, 0.01 \rangle$

$\mathbf{P}(X | Y)$ gives the values of $P(X = x_i | Y = y_j)$ for all i, j .

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Joint Probability Distribution

Joint probability distribution gives the probability of every atomic event on a set of RVs (i.e., every combination of their values).

$P(\text{Weather}, \text{Cavity})$ is a 4×2 matrix.

Weather Cavity	<i>sun</i>	<i>rain</i>	<i>cloud</i>	<i>snow</i>
true	0.144	0.02	0.016	0.02
false	0.576	0.08	0.064	0.08

Concise P Notation

$$P(\text{Weather}, \text{Cavity}) = P(\text{Weather} \mid \text{Cavity}) P(\text{Cavity})$$

{ *sun, rain, cloud, snow* } { *true, false* }

replaces 8 equations (corresponding to 8 possible worlds)

$$P(W = \text{sun} \wedge C = \text{true}) = P(W = \text{sun} \mid C = \text{true})P(C = \text{true})$$

$$P(W = \text{rain} \wedge C = \text{true}) = P(W = \text{rain} \mid C = \text{true})P(C = \text{true})$$

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Kolmogorov's Axioms

1. $0 \leq P(\omega) \leq 1$ for every world $\omega \in \Omega$.

2. $\sum_{\omega \in \Omega} P(\omega) = 1$

3. $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$ for any two propositions a, b .

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counts $a \wedge b$ twice.

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Inconsistent Beliefs

Proposition	Agent 1's belief	Agent 2 bets	Agent 1 bets	Agent 1 payoffs for each outcome			
				a, b	$a, \neg b$	$\neg a, b$	$\neg a, \neg b$
a	0.4	\$4 on a	\$6 on $\neg a$	-\$6	-\$6	\$4	\$4
b	0.3	\$3 on b	\$7 on $\neg b$	-\$7	\$3	-\$7	\$3
$a \vee b$	0.8	\$2 on $\neg(a \vee b)$	\$8 on $a \vee b$	\$2	\$2	\$2	-\$8
				-\$11	-\$1	-\$1	-\$1

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Agent 2 will
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Inconsistent Beliefs

Agent 2 will
lose \$4 if $\neg a$

Agent 1 will lose
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				a, b	$a, \neg b$	$\neg a, b$	$\neg a, \neg b$
a	0.4	\$4 on a	\$6 on $\neg a$	-\$6	-\$6	\$4	\$4
b	0.3	\$3 on b	\$7 on $\neg b$	-\$7	\$3	-\$7	\$3
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