Lecture notes for *Principles of Artificial Intelligence* (COMS 4720/5720)
Yan-Bin Jia, Iowa State University

Resolution in FOL

Outline

I. Logic programming

II. Resolution

* Figures are from the textbook site.

I. Logic Programming

Algorithm = Logic + Control (Robert Kowalski)

Prolog (1972) is the most widely used logic programming language.

- Rapid prototyping
- Symbolic manipulation (e.g., writing compliers, parsing natural languages)

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Describes the relations among the three arguments of append.

Query Example

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(2) `append( [A|X], Y, [A|Z]) :- append(X,Y,Z).`

Query: `append(X, Y, [1, 2, 3])`

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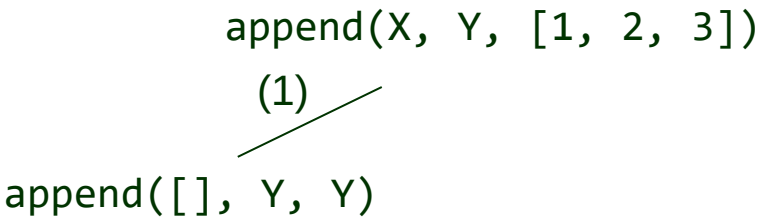

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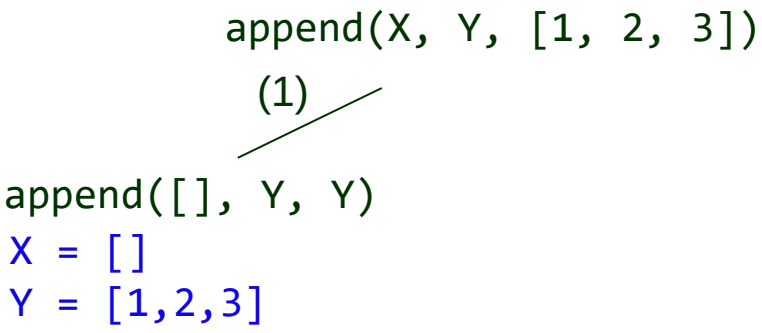

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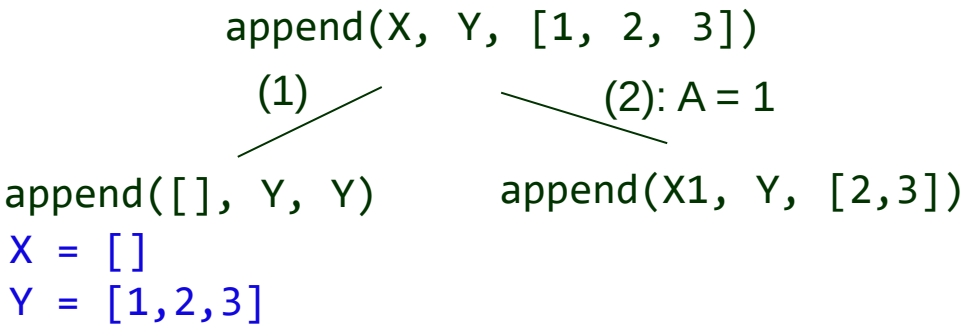

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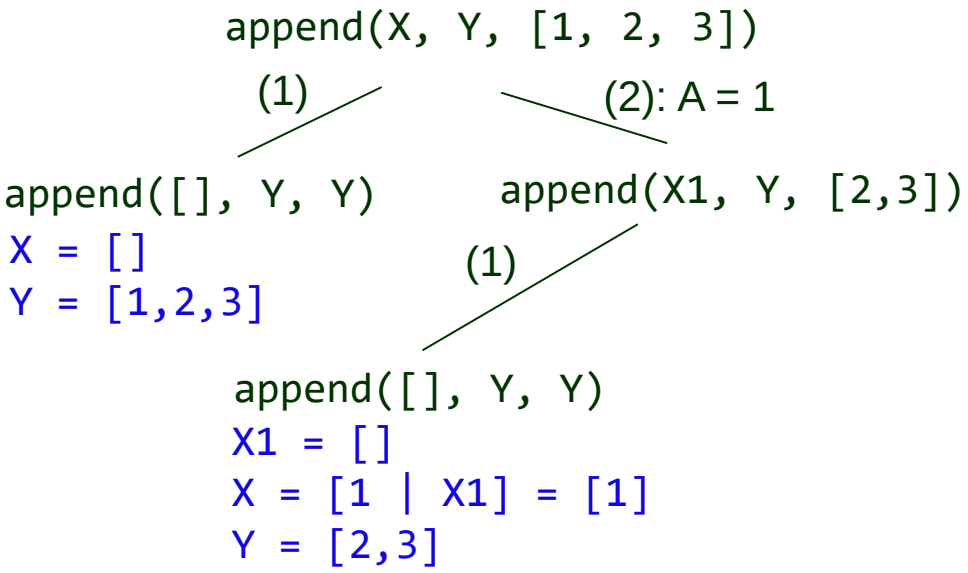

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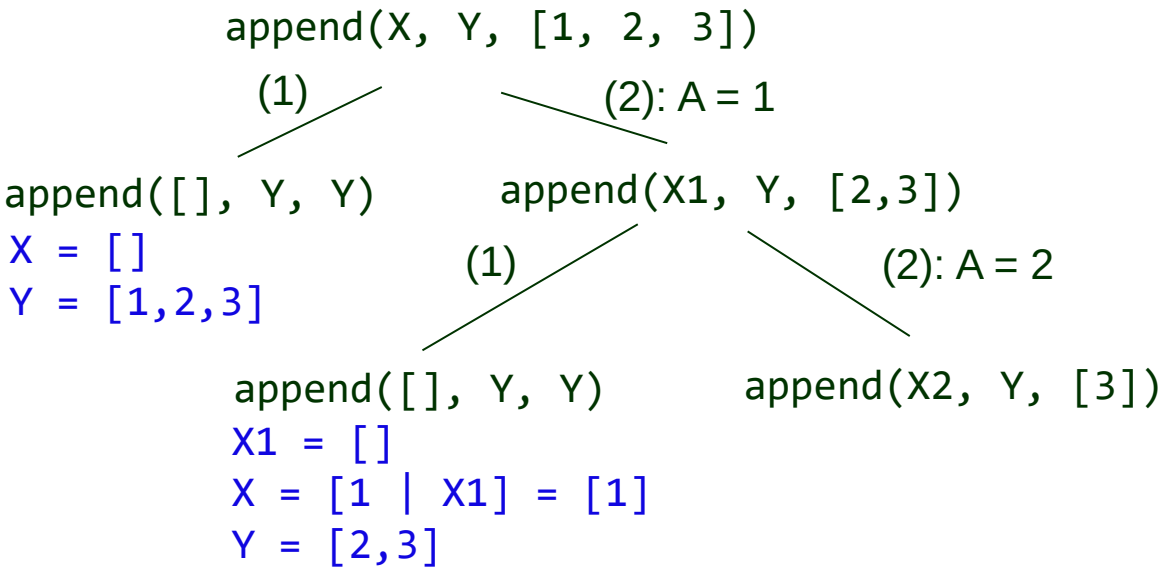

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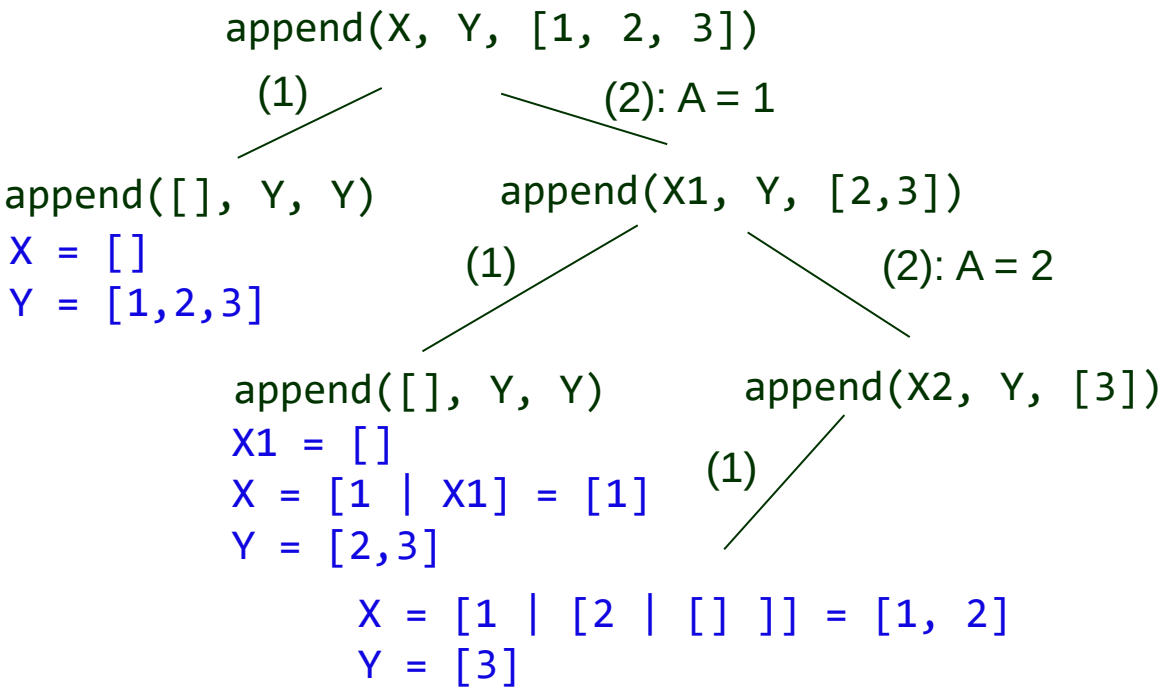

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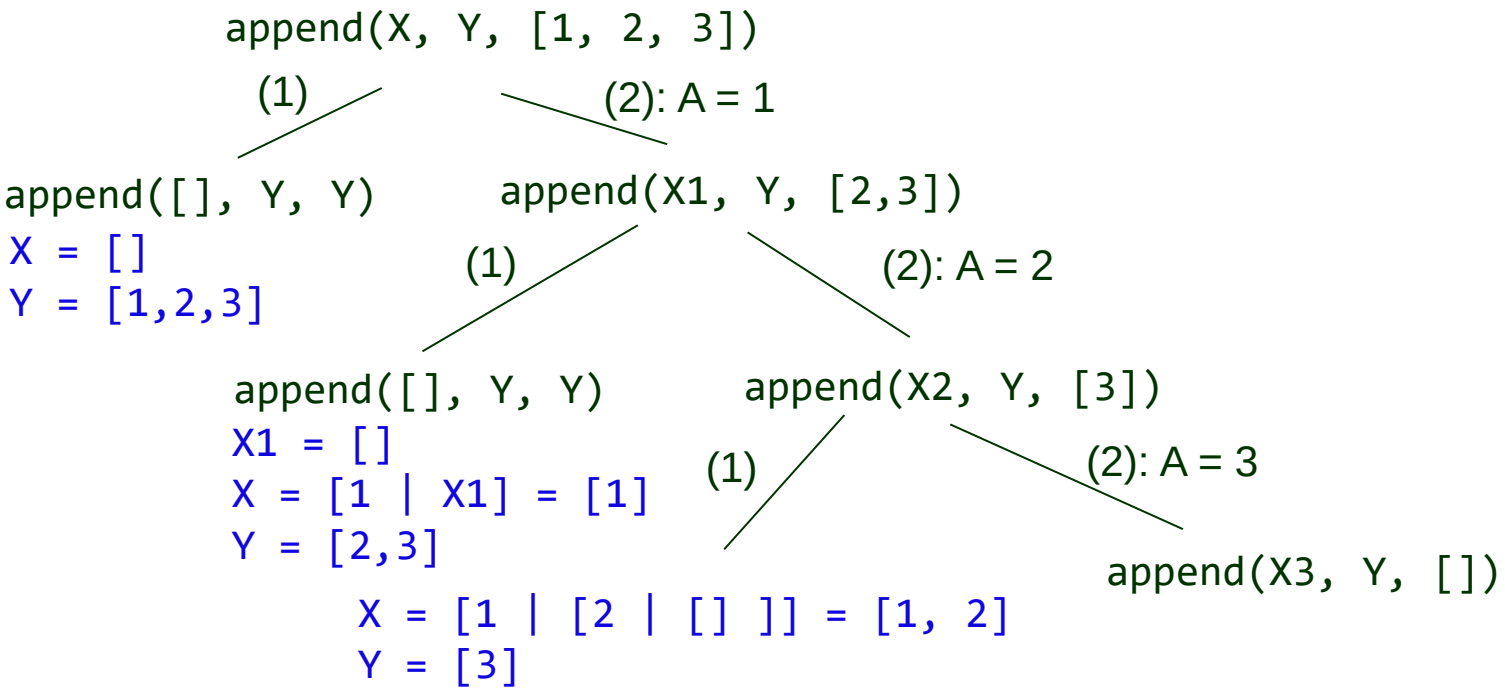

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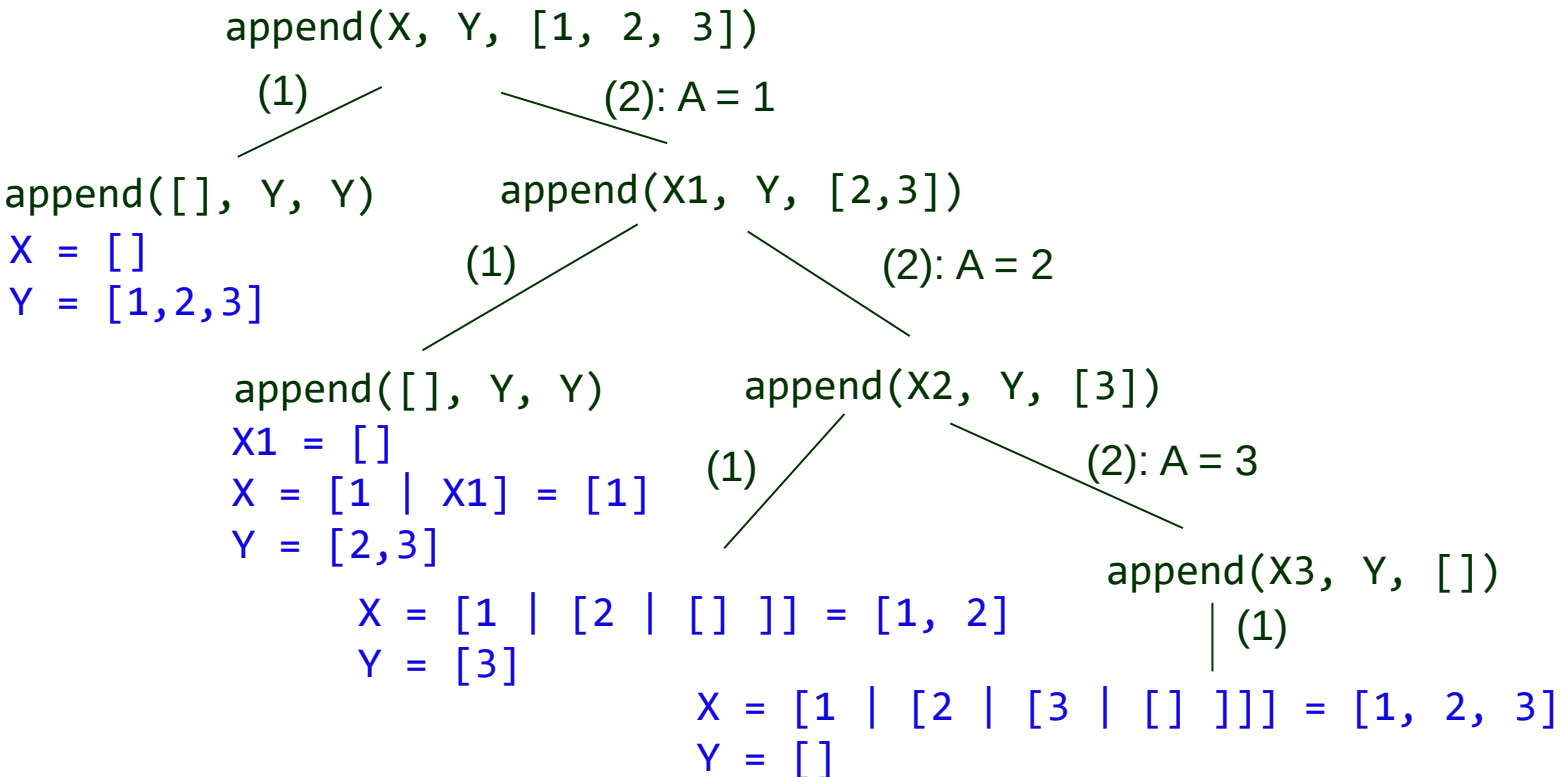

Infinite Loop

Finds if a path exists between two nodes in a directed graph.

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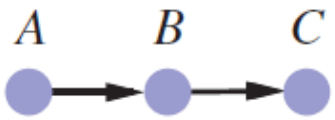

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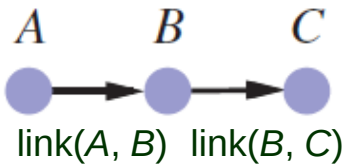

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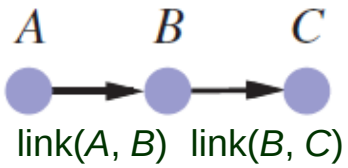

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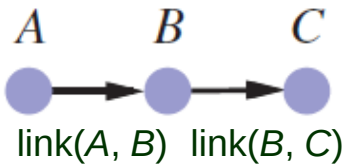

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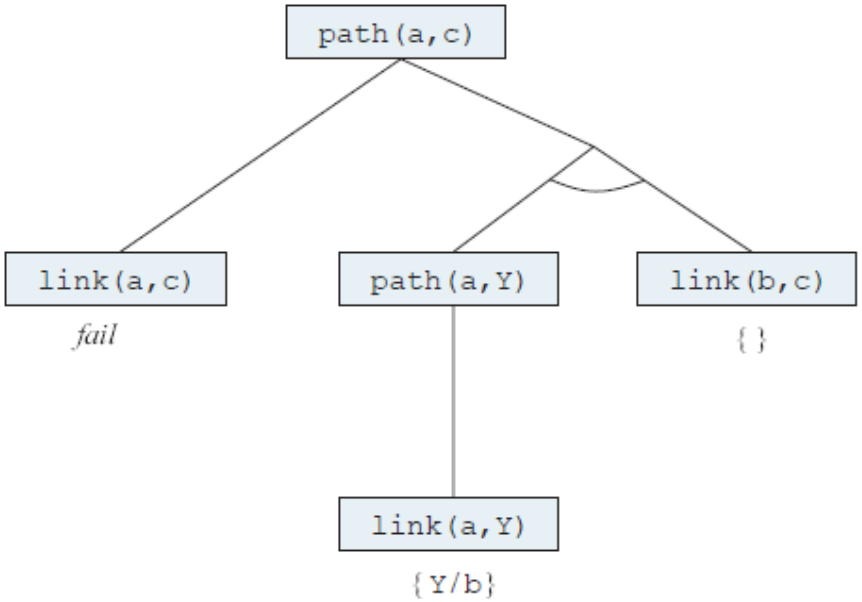

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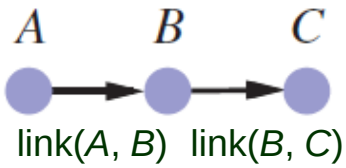

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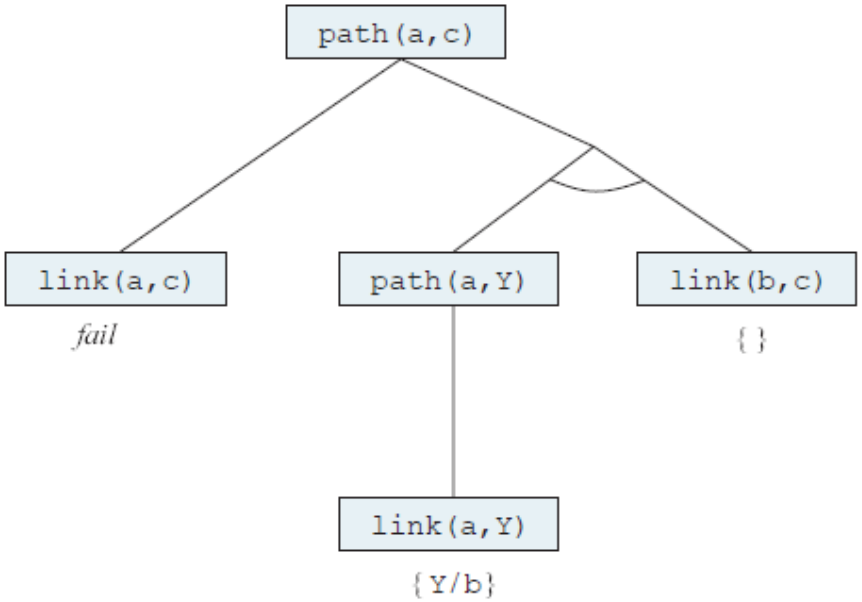

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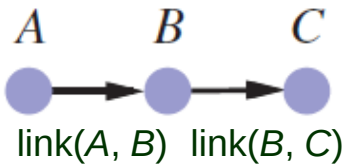

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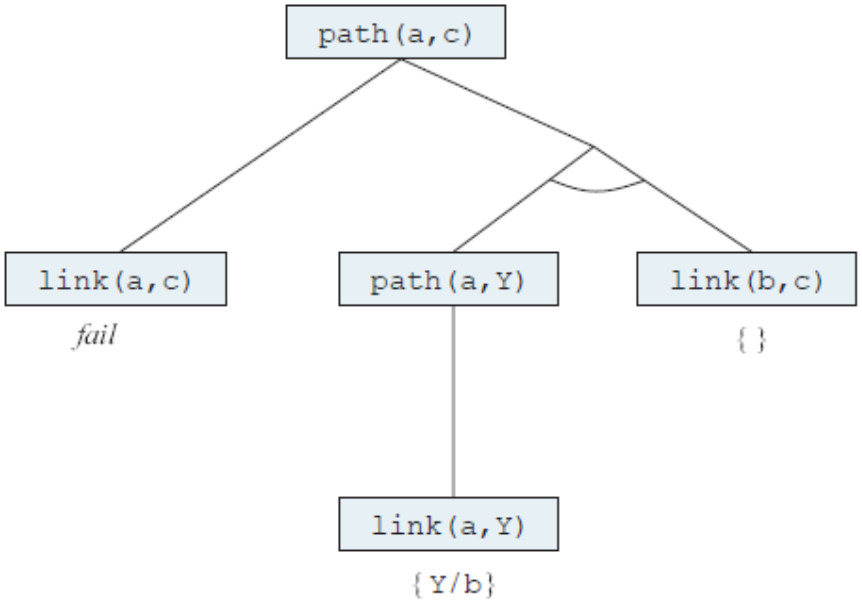

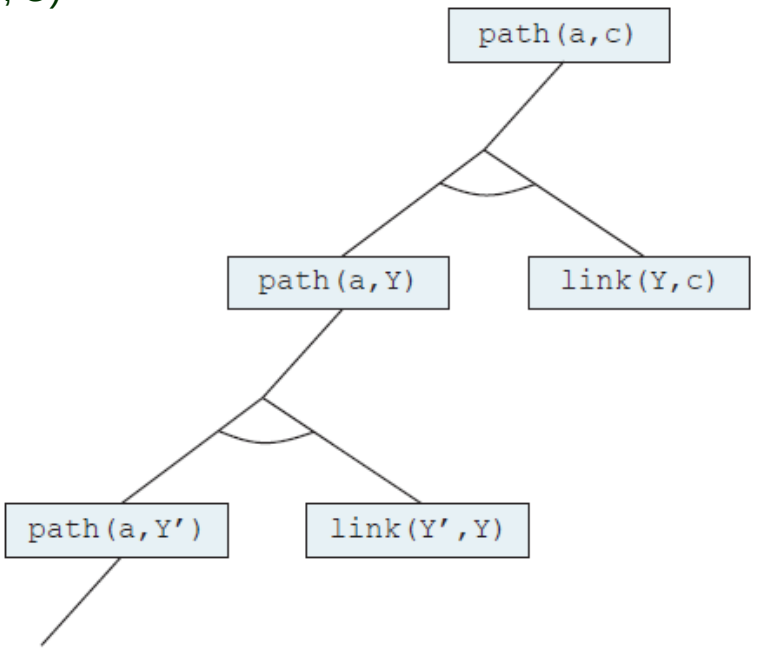

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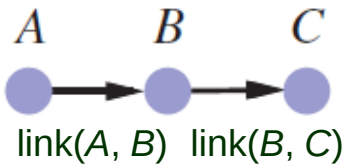

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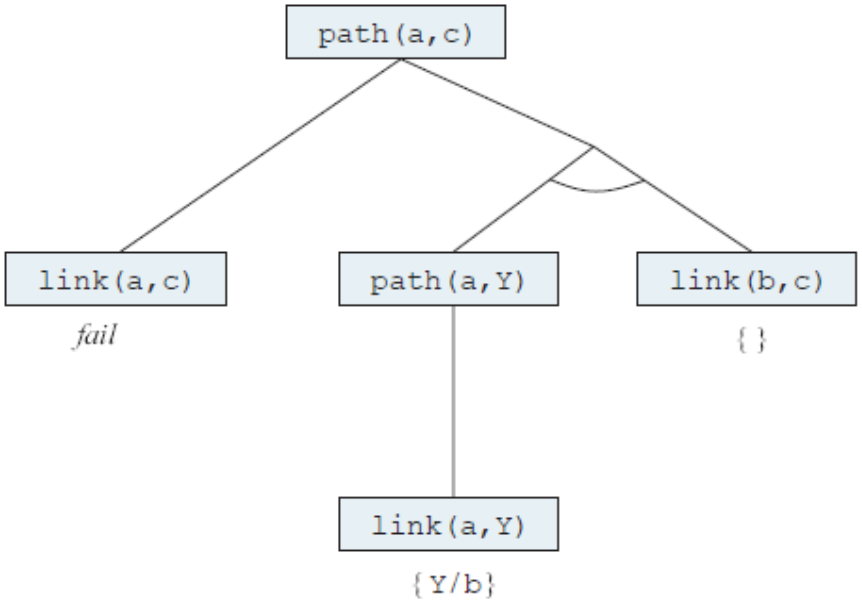
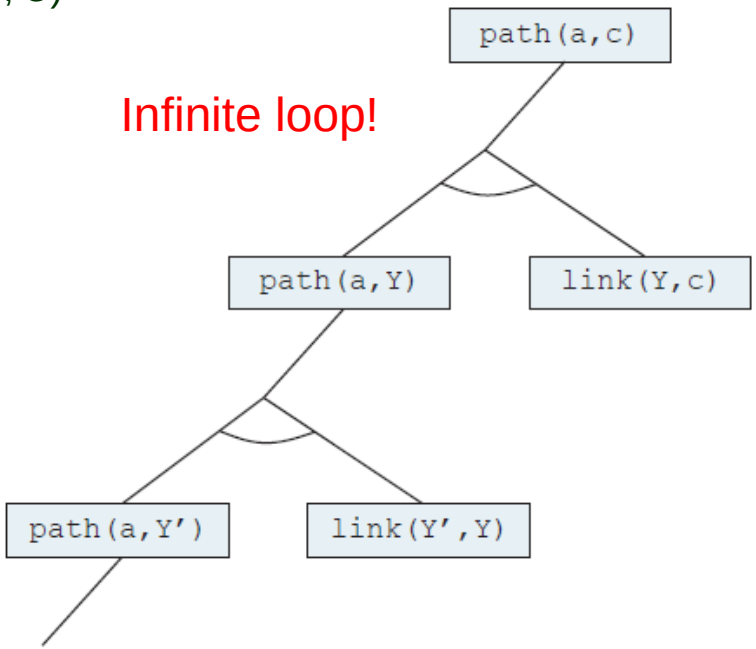


Infinite loop!



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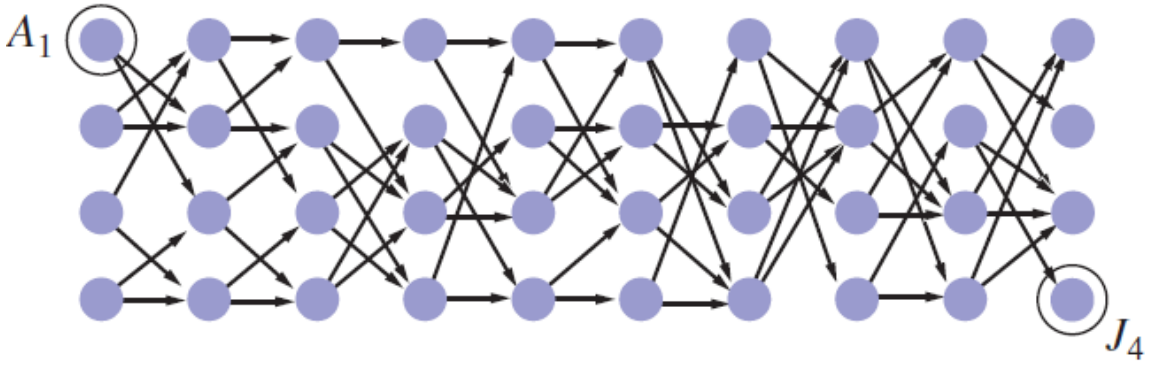

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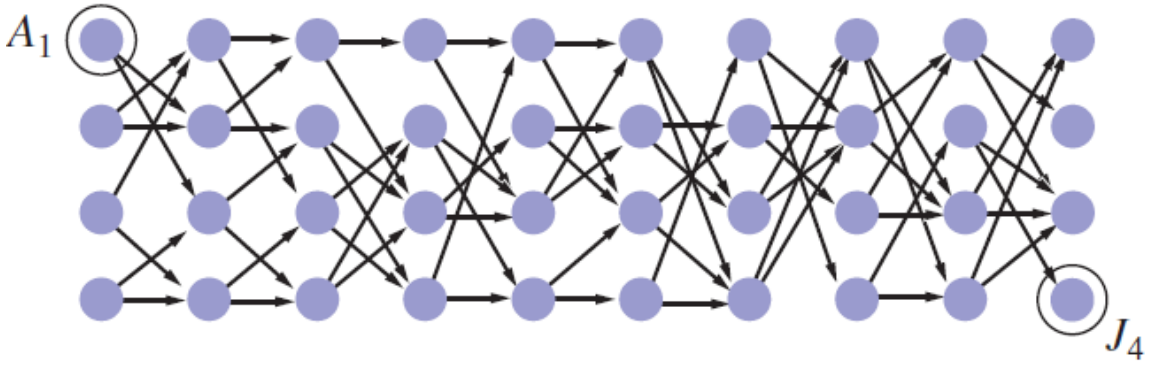

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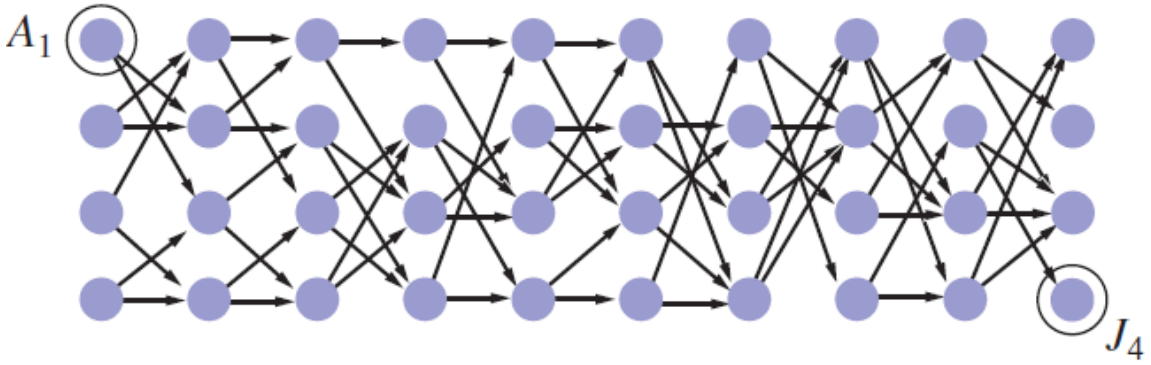

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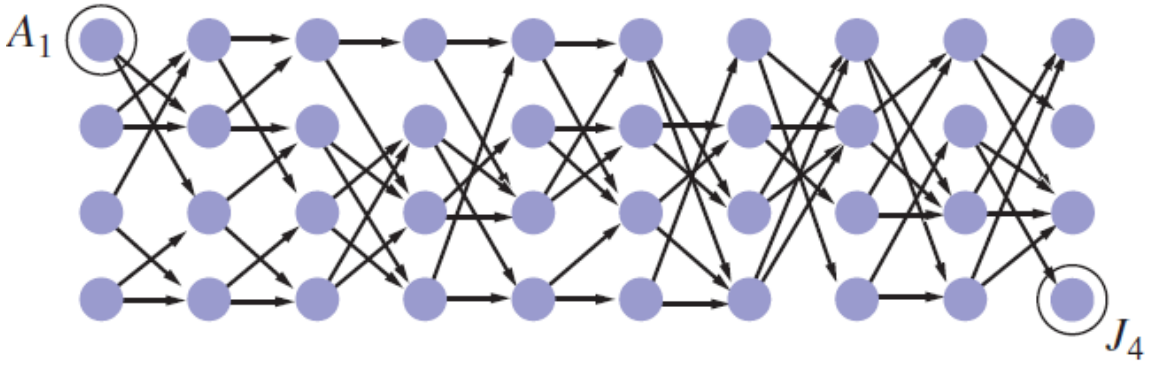

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- ♠ Prolog performs 877 inferences (most of which involve nodes from which the goal is unreachable).
- ♦ Forward chaining performs only 62 inferences.

II. Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\text{SUBST}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

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resolvent: $\textit{Animal}(F(x)) \vee \neg \textit{Kills}(G(x), x)$

- Binary resolution as given above does not yield a complete inference procedure.
- *Full resolution* does. It resolves subsets of literals in each clause that are unifiable.

Example Proof 1

The crime example:

$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x, y, z) \vee \neg Hostile(z) \vee Criminal(x)$

$\neg Missile(x) \vee \neg Owns(Nono, x) \vee Sells(West, x, Nono)$

$\neg Missile(x) \vee Weapon(x)$

$\neg Enemy(x, America) \vee Hostile(x)$

$Owns(Nono, M_1)$

$Missile(M_1)$

$American(West)$

$Enemy(Nono, America)$

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$\neg Missile(x) \vee Weapon(x)$

$\neg Enemy(x, America) \vee Hostile(x)$

$Owns(Nono, M_1)$

$Missile(M_1)$

$American(West)$

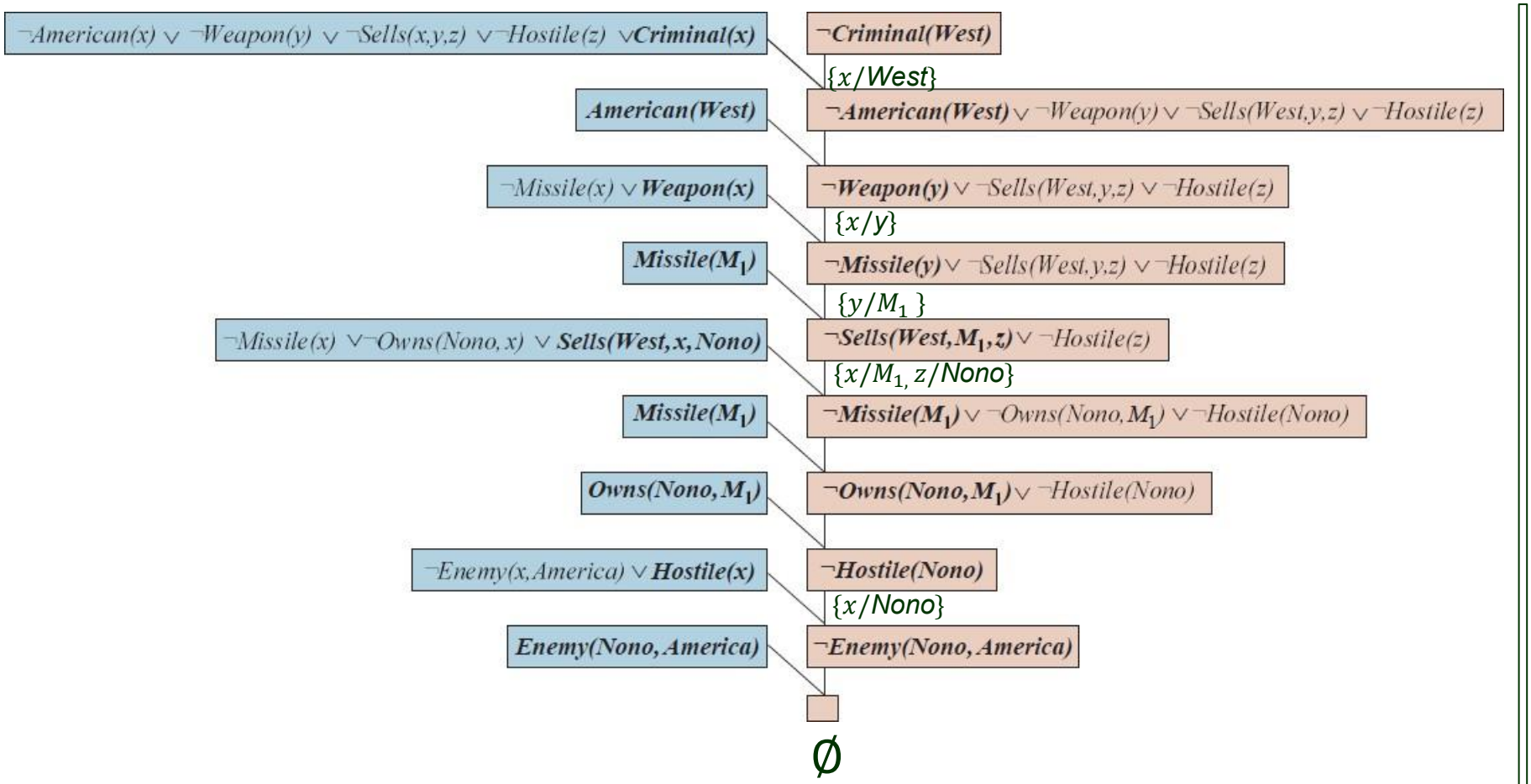
$Enemy(Nono, America)$

We prove *Criminal*(*West*) by adding

$\neg\ Criminal(West)$

and deriving the empty clause ∅.

Resolution Proof 1



- Like backward chaining on the main spine.
- Always resolve with a clause containing a positive literal that unifies with the leftmost literal of the "current" clause on the spine.

Example Proof 2 (with Skolemization)

Everyone who loves all animals is loved by someone.
Anyone who kills an animal is loved by no one.
Jack loves all animals.
Either Jack or Curiosity killed the cat, who is named Tuna.
Did Curiosity kill the cat?

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A. $\forall x \ \left(\forall y \ \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x, y)\right) \Longrightarrow \exists y \ \mathit{Loves}(y, x)$

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D. $Kills(Jack, Tuna) \vee Kills(Curiosity, Tuna)$

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Background knowledge

E. $Cat(Tuna)$

F. $\forall x \ \ Cat(x) \Rightarrow Animal(x)$

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Background knowledge

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Negated goal

$\neg$ G. $\neg Kills(Curiosity, Tuna)$

Converting to CNF

A. $\forall x \quad (\forall y \ \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x, y)) \Rightarrow (\exists y \ \mathit{Loves}(x, y))$

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$F(x)$ $\qquad$ $G(x)$

Converting to CNF

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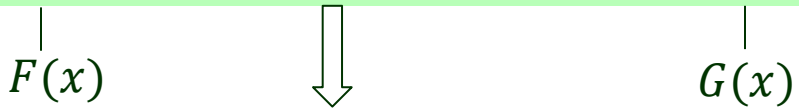


A1. $Animal(F(x)) \vee Loves(G(x), x)$

A2. $\neg Loves(x, F(x)) \vee Loves(G(x), x)$

Converting to CNF

A. $\forall x \quad (\forall y \ Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \ Loves(x, y))$

$F(x)$ $G(x)$

A1. $Animal(F(x)) \vee Loves(G(x), x)$

A2. $\neg Loves(x, F(x)) \vee Loves(G(x), x)$

B. $\neg Animal(z) \vee \neg Kills(x, z) \vee \neg Loves(y, x)$

C. $\neg Animal(x) \vee Loves(Jack, x)$

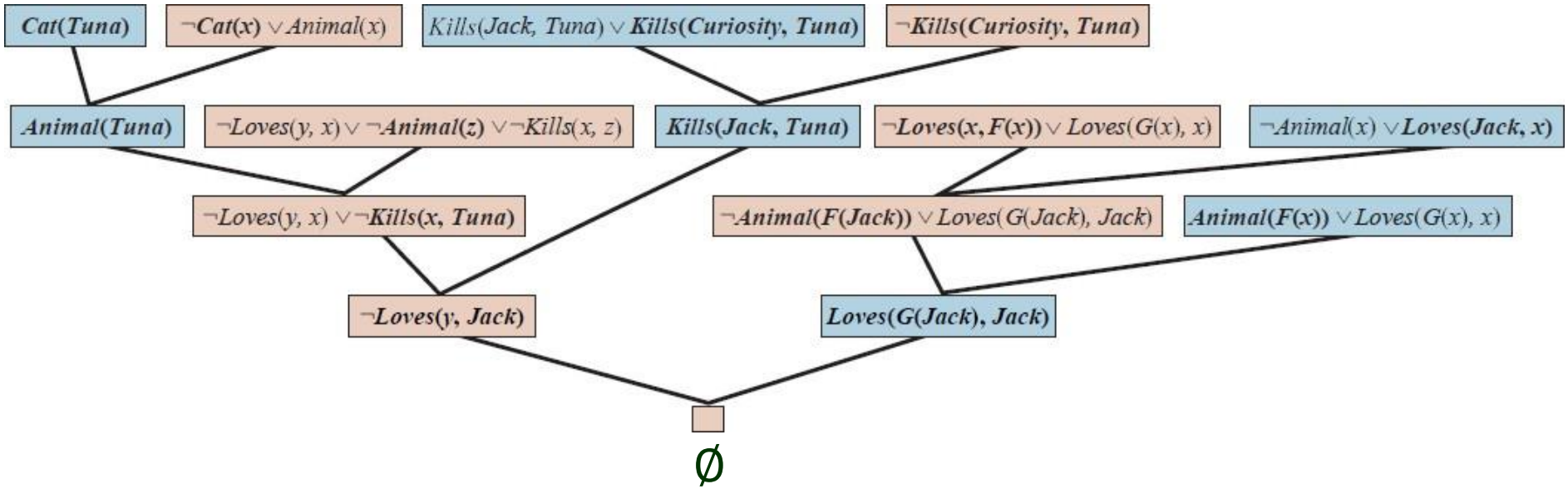
D. $Kills(Jack, Tuna) \vee Kills(Curiosity, Tuna)$

E. $Cat(Tuna)$

F. $\neg Cat(x) \vee Animal(x)$

¬ G. $\neg Kills(Curiosity, Tuna)$

Resolution Proof 2



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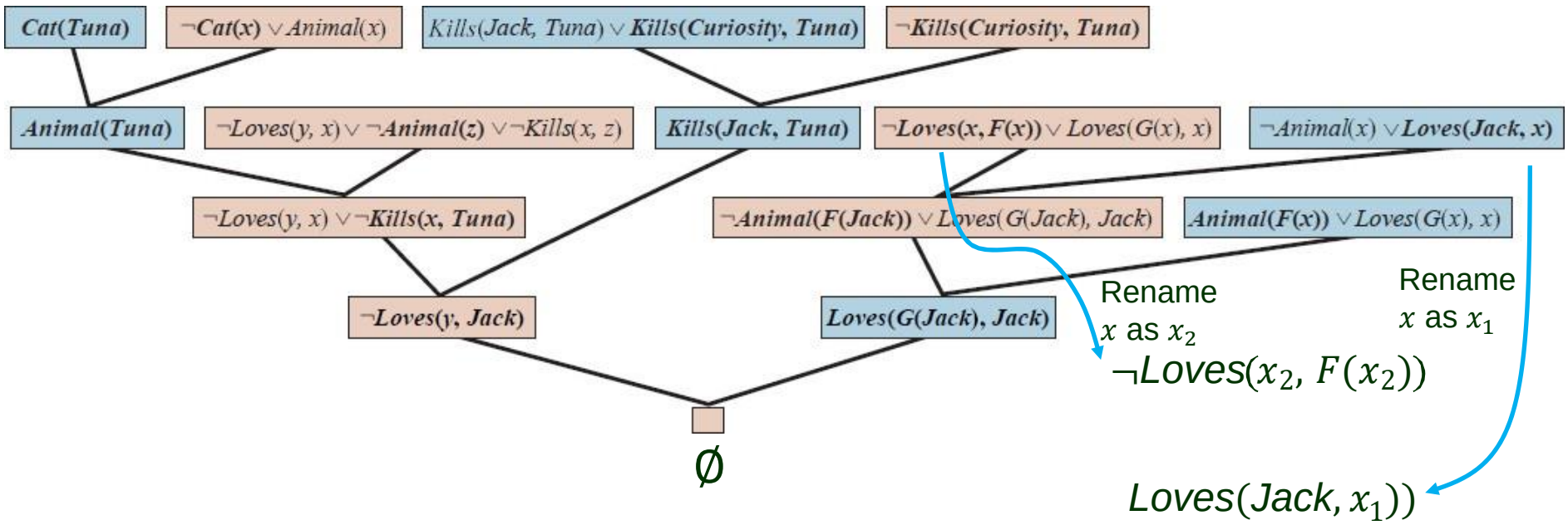

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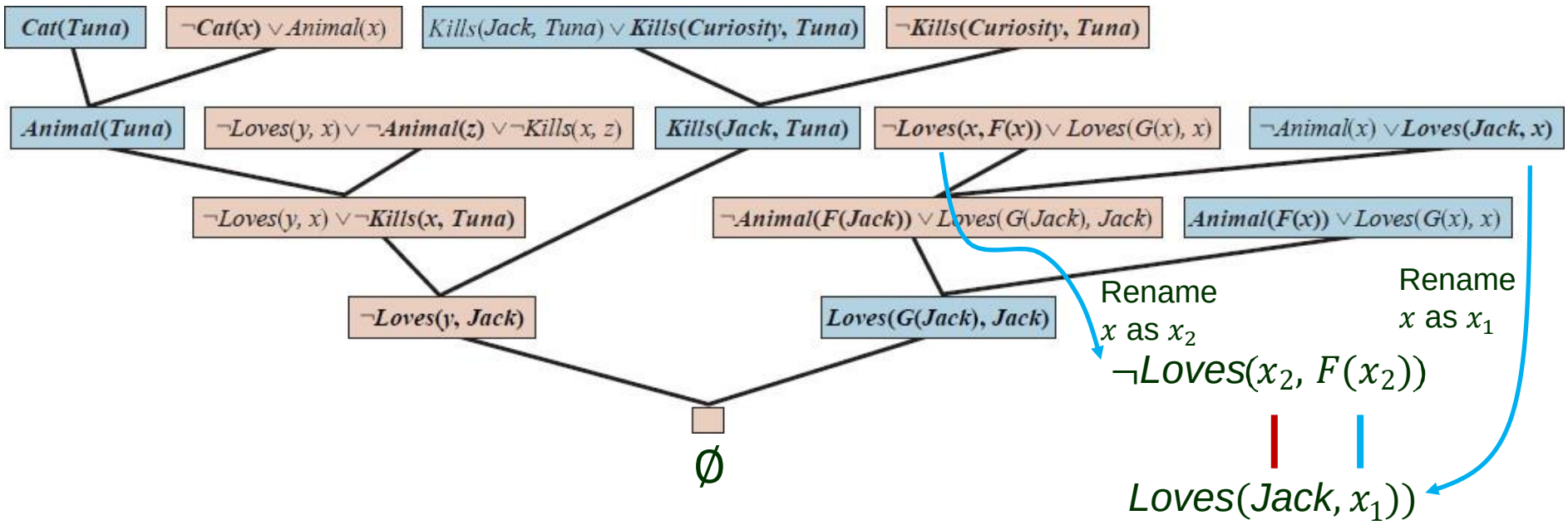

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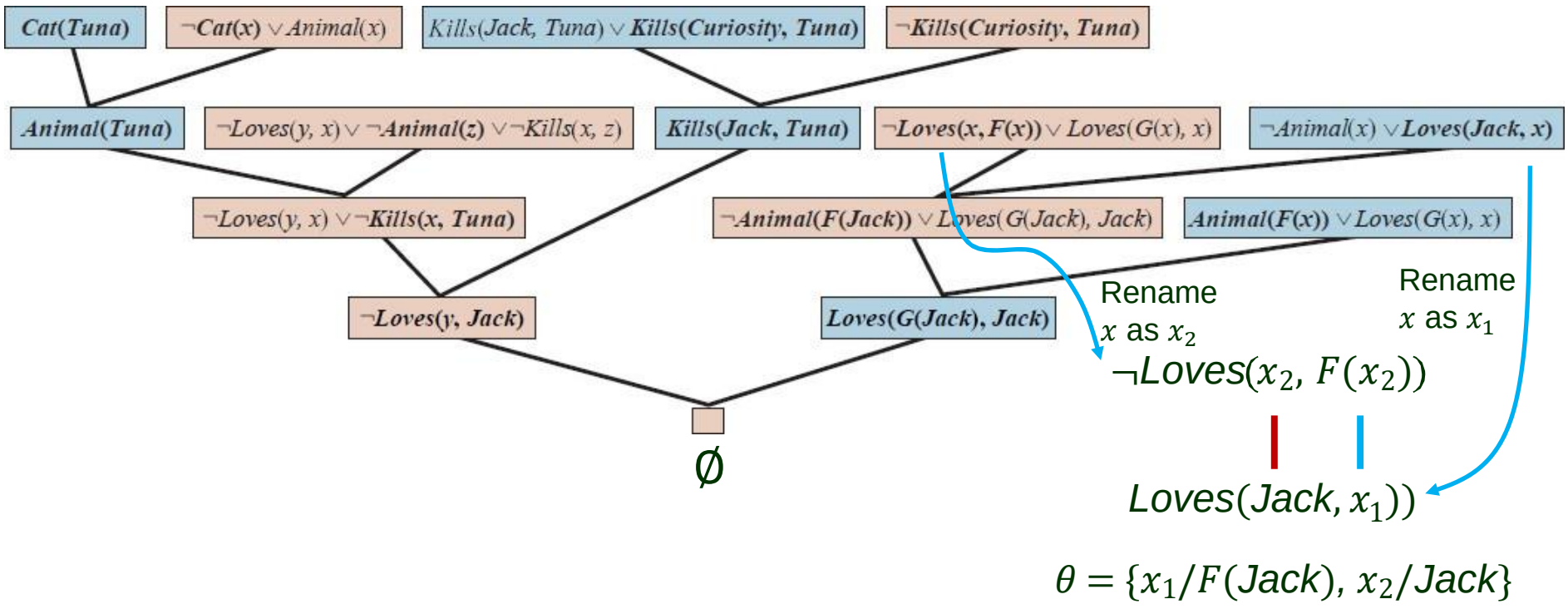

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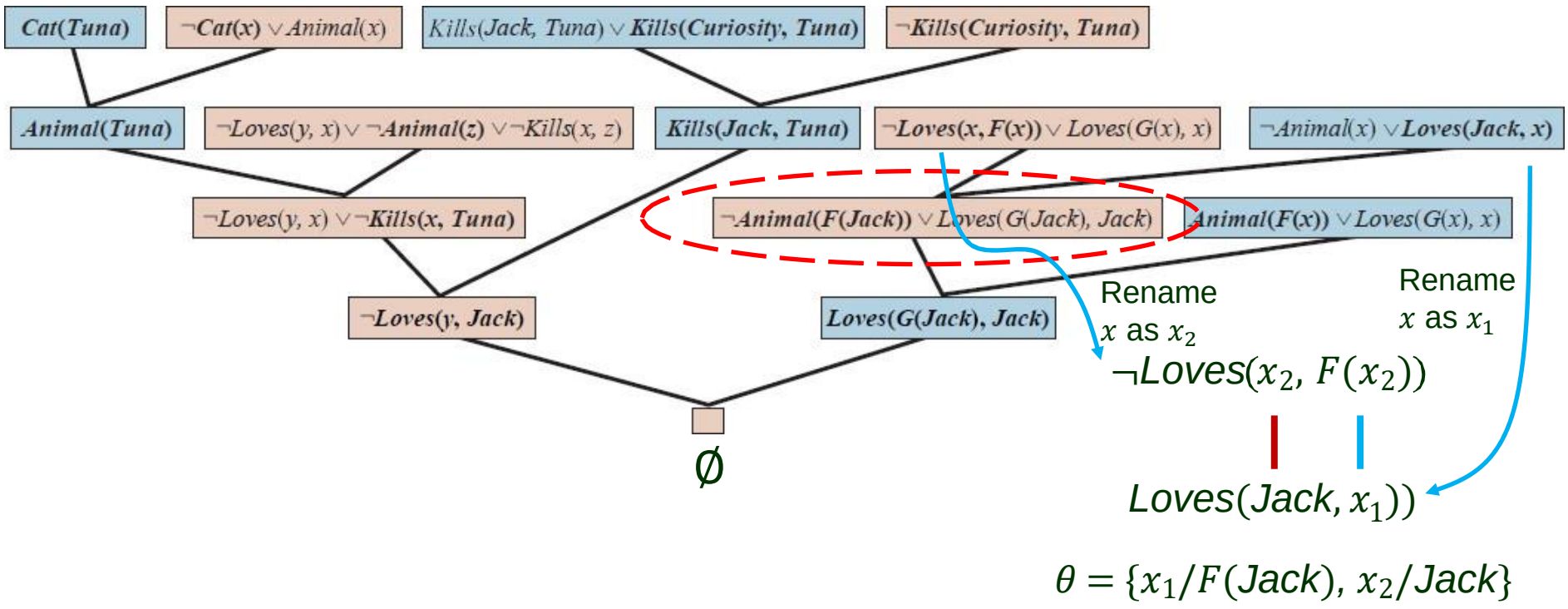

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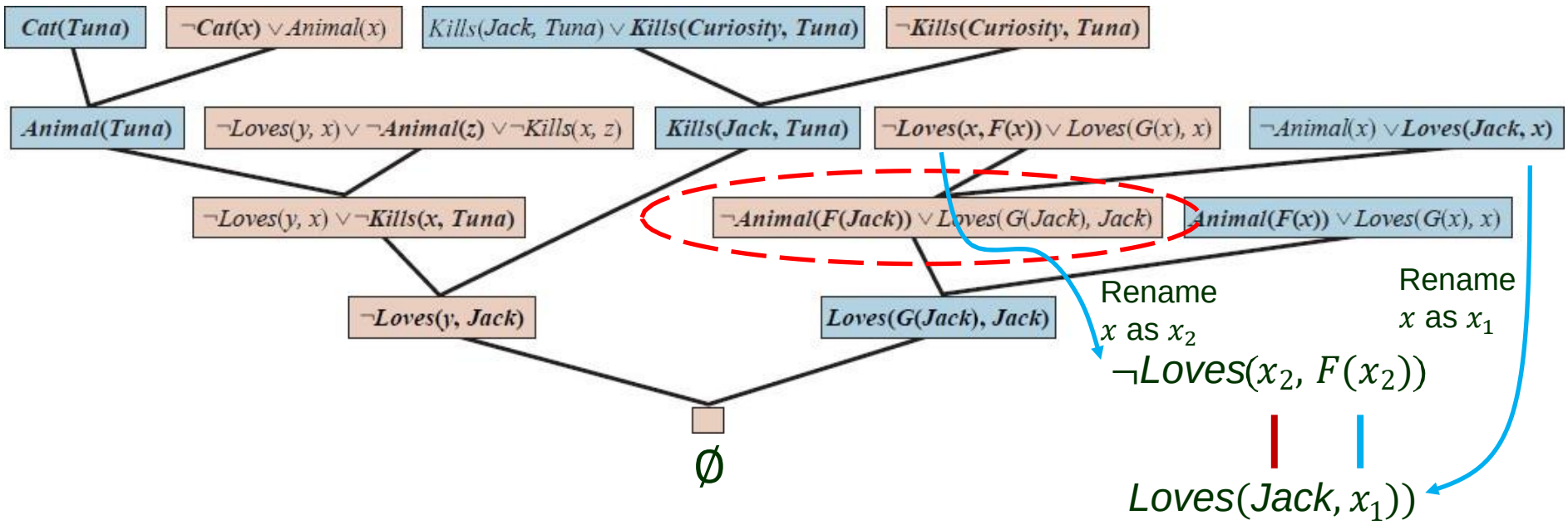

Paraphrased in English:

$\theta = \{x_1/F(Jack), x_2/Jack\}$

Suppose Curiosity did not kill Tuna. We know that either Jack or Curiosity did; thus Jack must have. Now, Tuna is a cat and cats are animals, so Tuna is an animal. Because anyone who kills an animal is loved by no one, we know that no one loves Jack. On the other hand, Jack loves all animals, so someone loves him; so we have a contradiction. Therefore, Curiosity killed the cat.

Completeness of Resolution

Theorem If a set S of sentences is unsatisfiable, then resolution will always be able to derive a contradiction.

Not all logical consequences of S can be generated using resolution.

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We can use resolution to find all answers to a question $Q(x)$ by proving that $KB \wedge \neg Q(x)$ is unsatisfiable.