Unification & Chaining

Outline

I. Unification

II. First-order definite clauses

III. Forward chaining

IV. Backward chaining

* Figures are from the textbook site.

I. Generalized Modus Ponens

$(p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q), \quad p_1', p_2', \ldots, p_n'$

Suppose there exists a substitution θ such that

$\text{SUBST}(\theta, p_i) = \text{SUBST}(\theta, p_i')$ for $1 \leq i \leq n$

Example $n = 1$, $p_1 \equiv$ *Dad*$(x,$ *John*$)$, $p_1' \equiv$ *Dad*$($*David*$, y)$, $\theta = \{x/$*David*, *y/John*$\}$

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$X_1 \neq \textit{In}(1, C_1)$

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- The process of finding substitutions that make different logical expressions look identical.
- Carried out by the algorithm UNIFY.

$\text{UNIFY}(p, q) = \theta$ where $\text{SUBST}(\theta, p) = \text{SUBST}(\theta, q)$

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UNIFY(*Knows*(*John*, x), *Knows*(*John*, *Jane*)) = {x/*Jane*}

UNIFY(*Knows*(*John*, x), *Knows*(y, *Bill*)) = {x/*Bill*, y/*John*}

UNIFY(*Knows*(*John*, x), *Knows*(y, *Mother*(y))) = {y/*John*, x/*Mother*(*John*)}

Unification (cont'd)

- Conflicting substitutions

$\text{UNIFY}(Knows(John, x), Knows(x, Elizabeth)) = failure$

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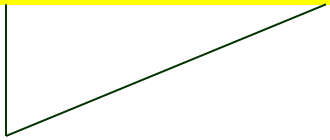
UNIFY(*Knows*(*John*, x), *Knows*(x, *Elizabeth*)) = *failure*



$\{x/John\}$

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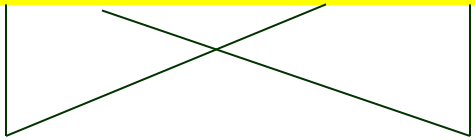

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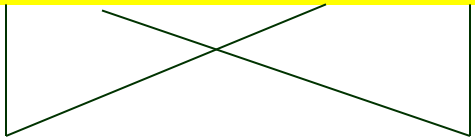

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x cannot take on the values *John* and *Elizabeth* at the same time!

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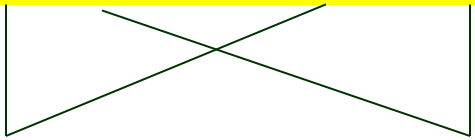

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- Multiple unifiers

UNIFY(*Knows*(*John*, x), *Knows*(y, z))

could return $\{y/John, x/z\}$

or $\{y/John, x/John, z/John\}$

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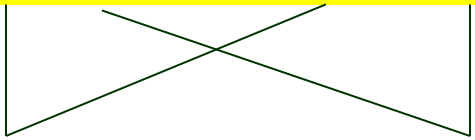

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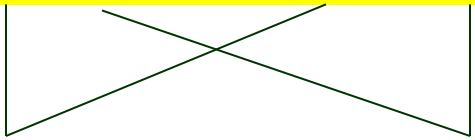

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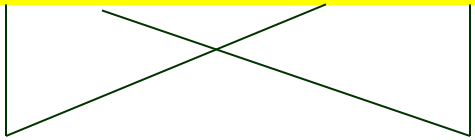

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more general unifier for fewer restriction on variable values

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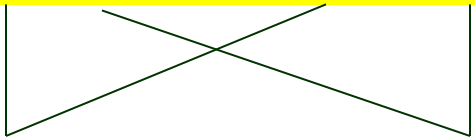

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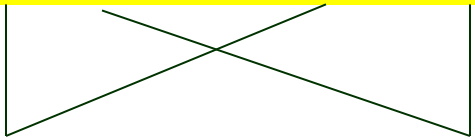

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Unification Algorithm

function UNIFY(x, y, θ=*empty*) **returns** a substitution to make x and y identical, or *failure*
  if θ = *failure* **then return** *failure*
  else if $x = y$ **then return** θ
  else if VARIABLE?(x) **then return** UNIFY-VAR(x, y, θ)
  else if VARIABLE?(y) **then return** UNIFY-VAR(y, x, θ)
  else if COMPOUND?(x) **and** COMPOUND?(y) **then**
    return UNIFY(ARGS(x), ARGS(y), UNIFY(OP(x), OP(y), θ))
  else if LIST?(x) **and** LIST?(y) **then**
    return UNIFY(REST(x), REST(y), UNIFY(FIRST(x), FIRST(y), θ))
  else return *failure*

function symbol of x

argument list of y

function UNIFY-VAR(var, x, θ) **returns** a substitution
  if $\{var/val\} \in \theta$ for some val **then return** UNIFY(val, x, θ)
  else if $\{x/val\} \in \theta$ for some val **then return** UNIFY(var, val, θ)
  else if OCCUR-CHECK?(var, x) **then return** *failure*
  else return add $\{var/x\}$ to θ

Recursively explore two expressions x and y "side by side" to build up a unifier.

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  else if $\{x/val\} \in \theta$ for some val **then return** UNIFY(var, val, θ)
  else if OCCUR-CHECK?(var, x) **then return** *failure* // check whether the variable *var* appears
  else return add $\{var/x\}$ to θ // inside the complex term x. match fails if so
// because no unifier can be constructed.

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Unification Example

UNIFY$\big(P(x, f(a, y), z), P\big(b, z, f(a, c)\big), \{\}\big)$ Variables: x, y, z

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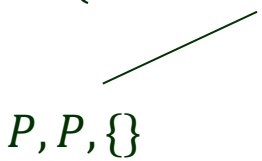

$P, P, \{\}$

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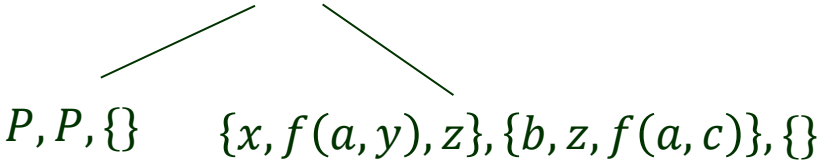

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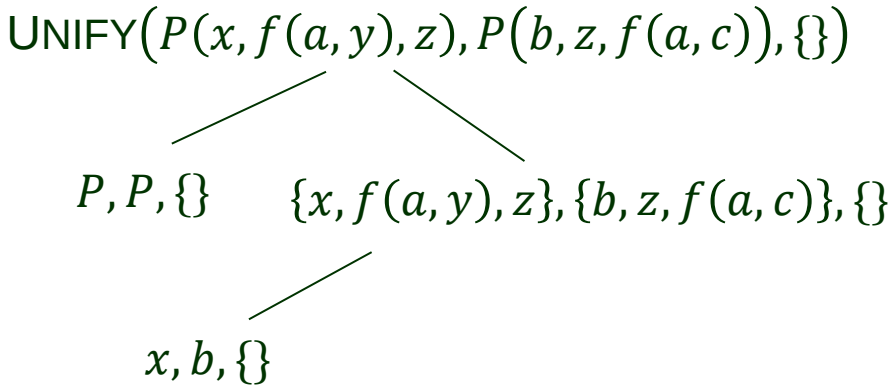

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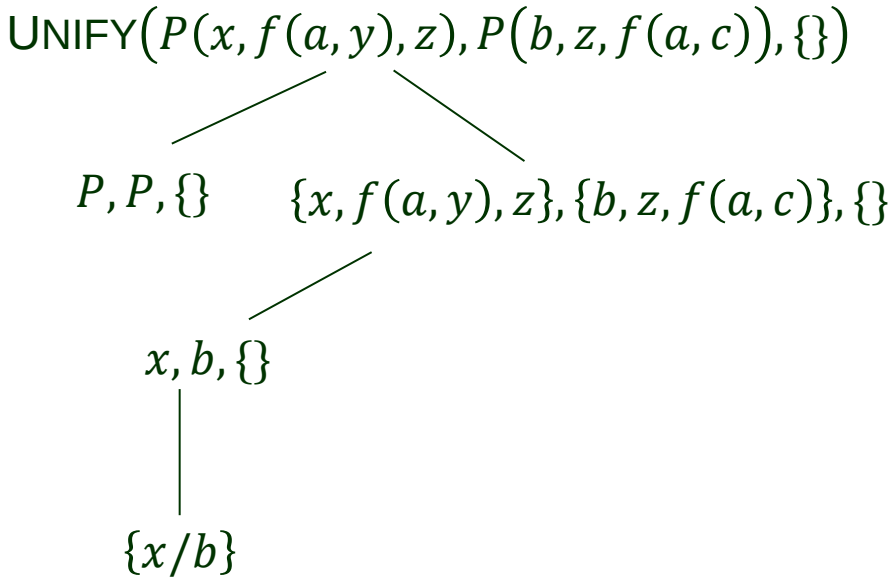

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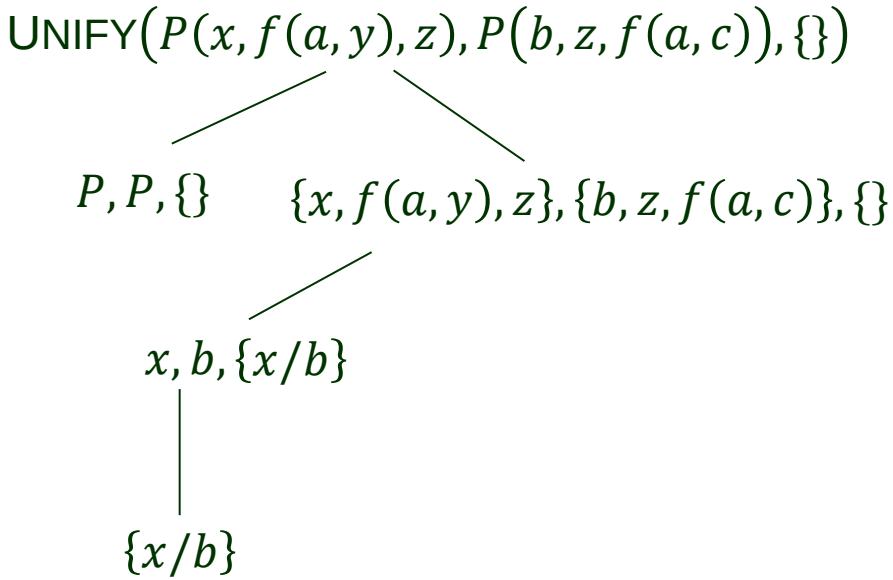

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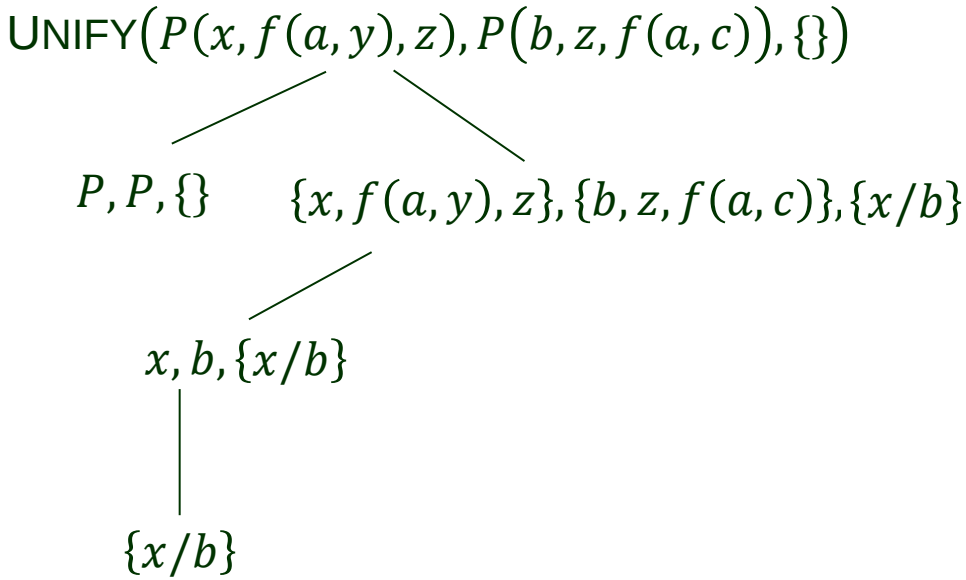

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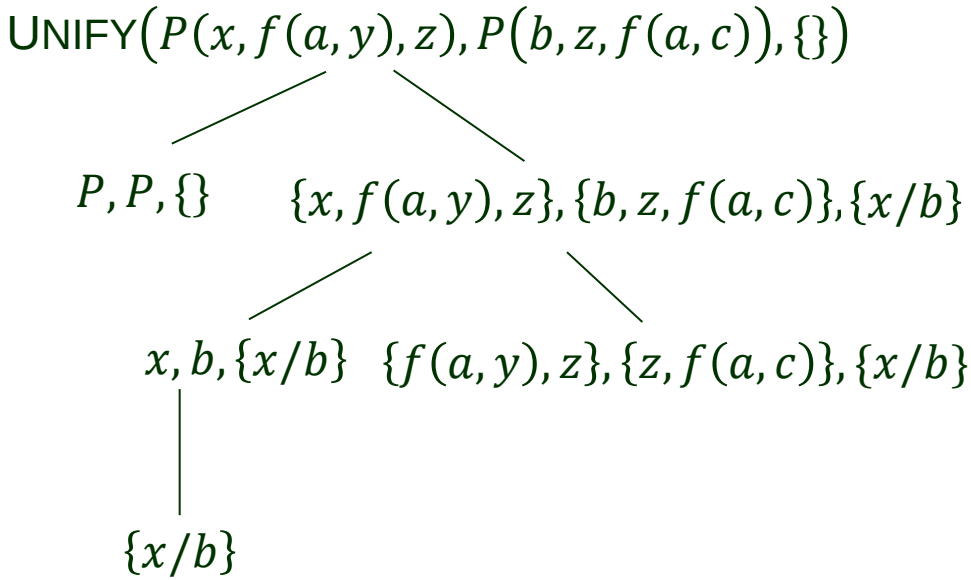

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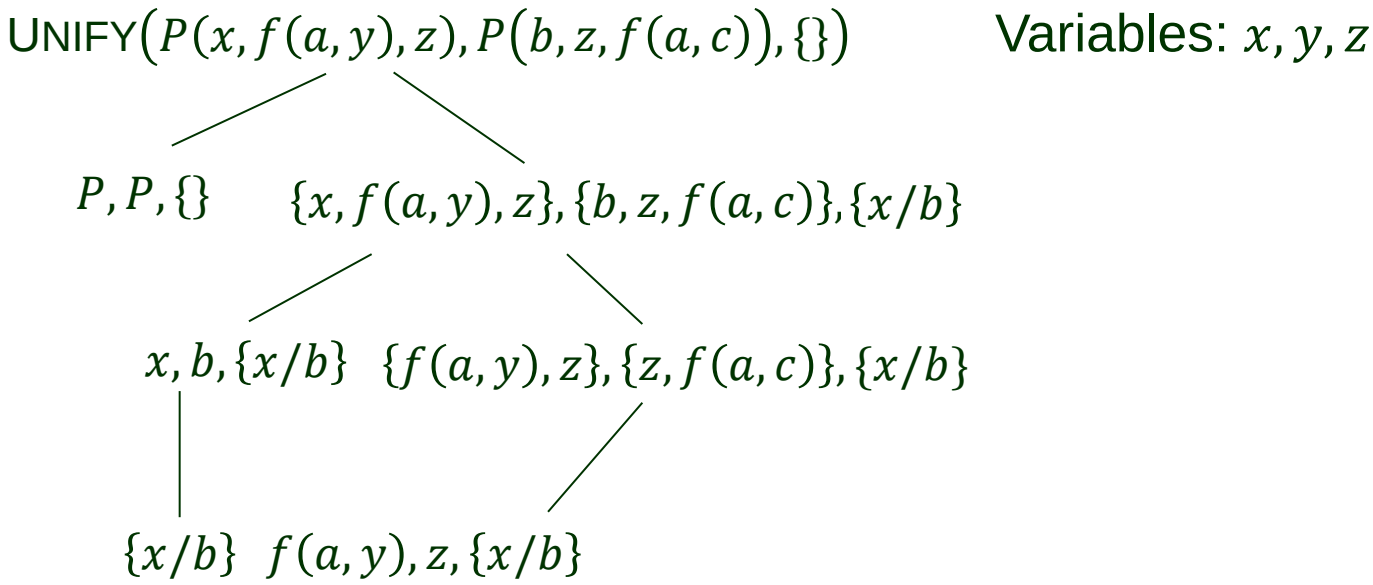

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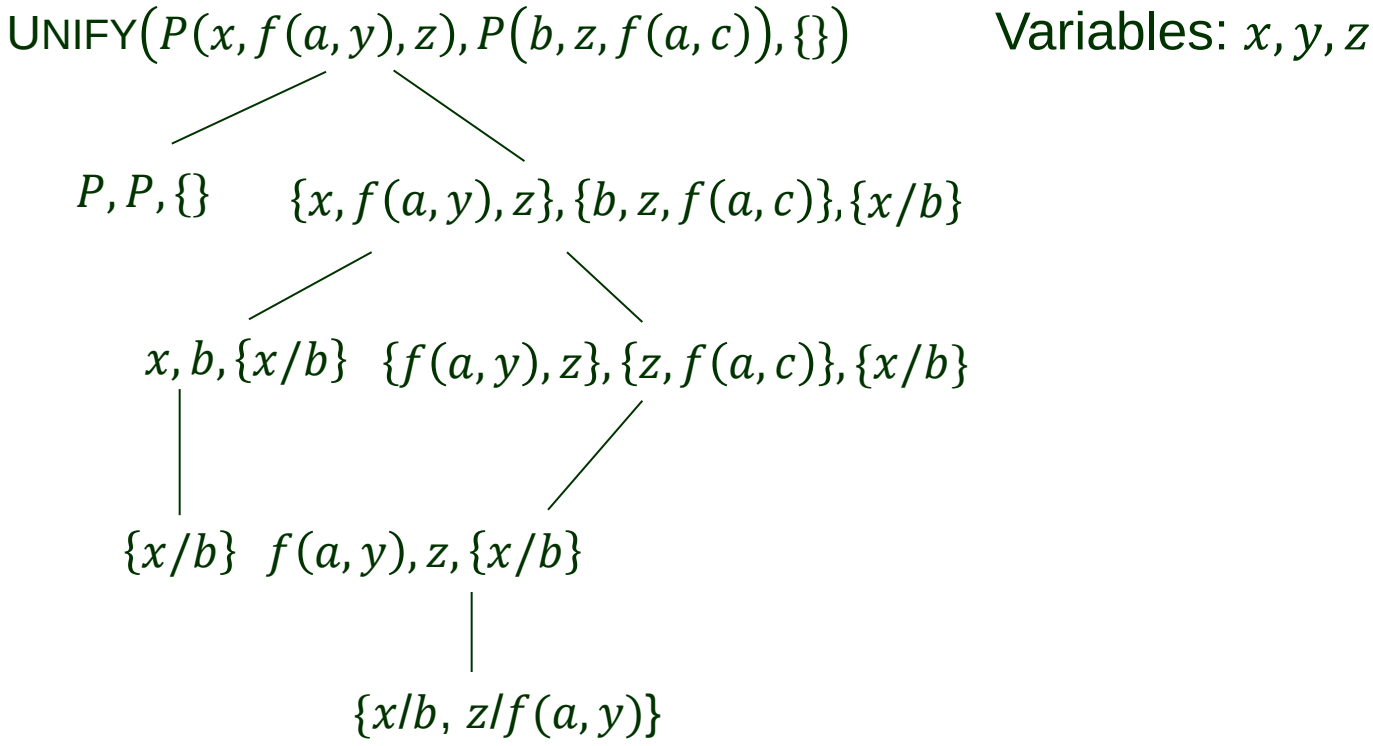

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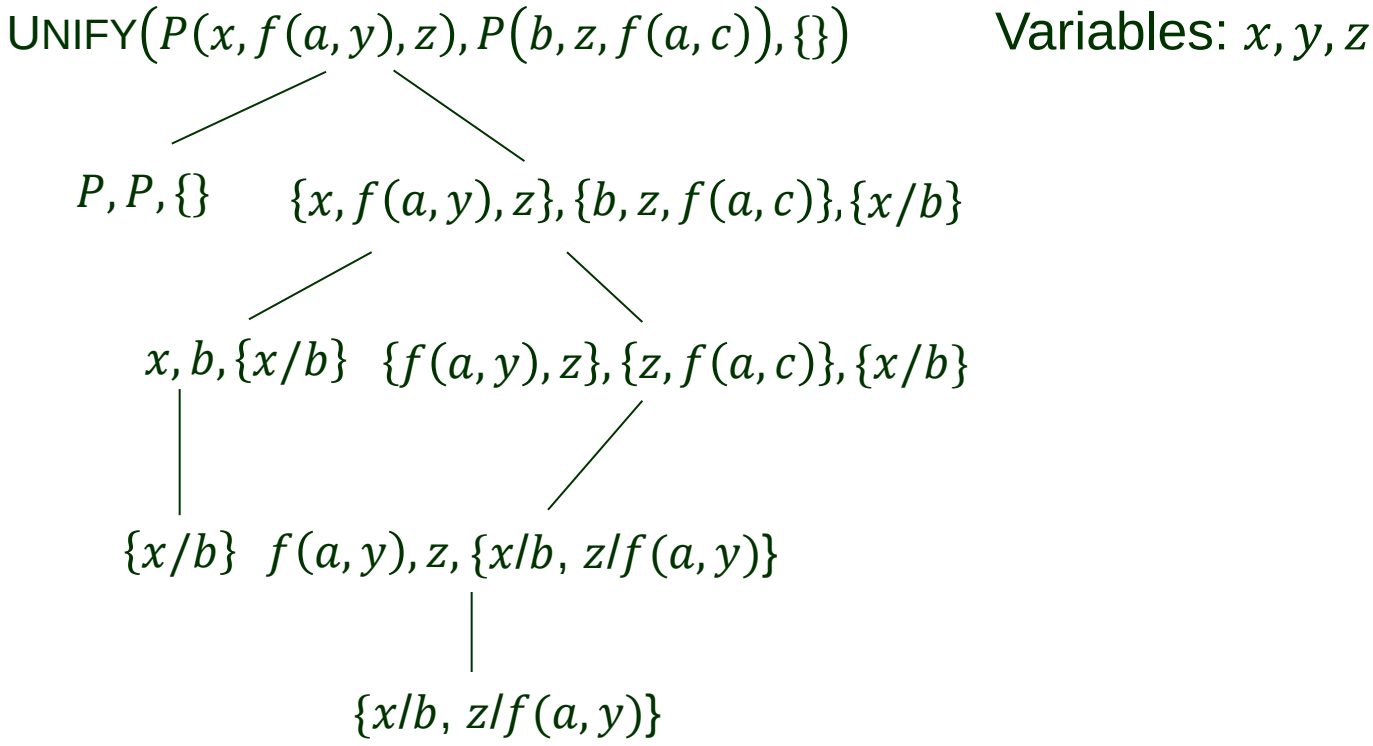

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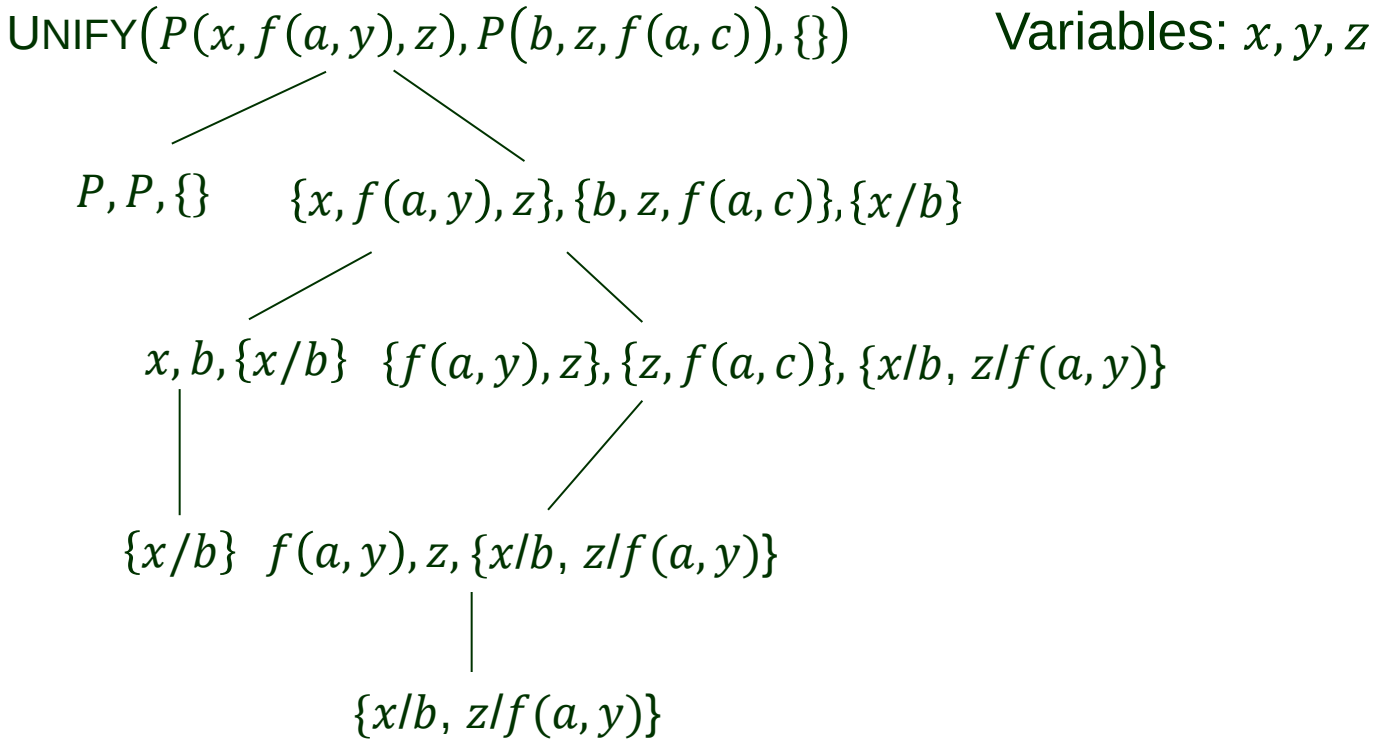

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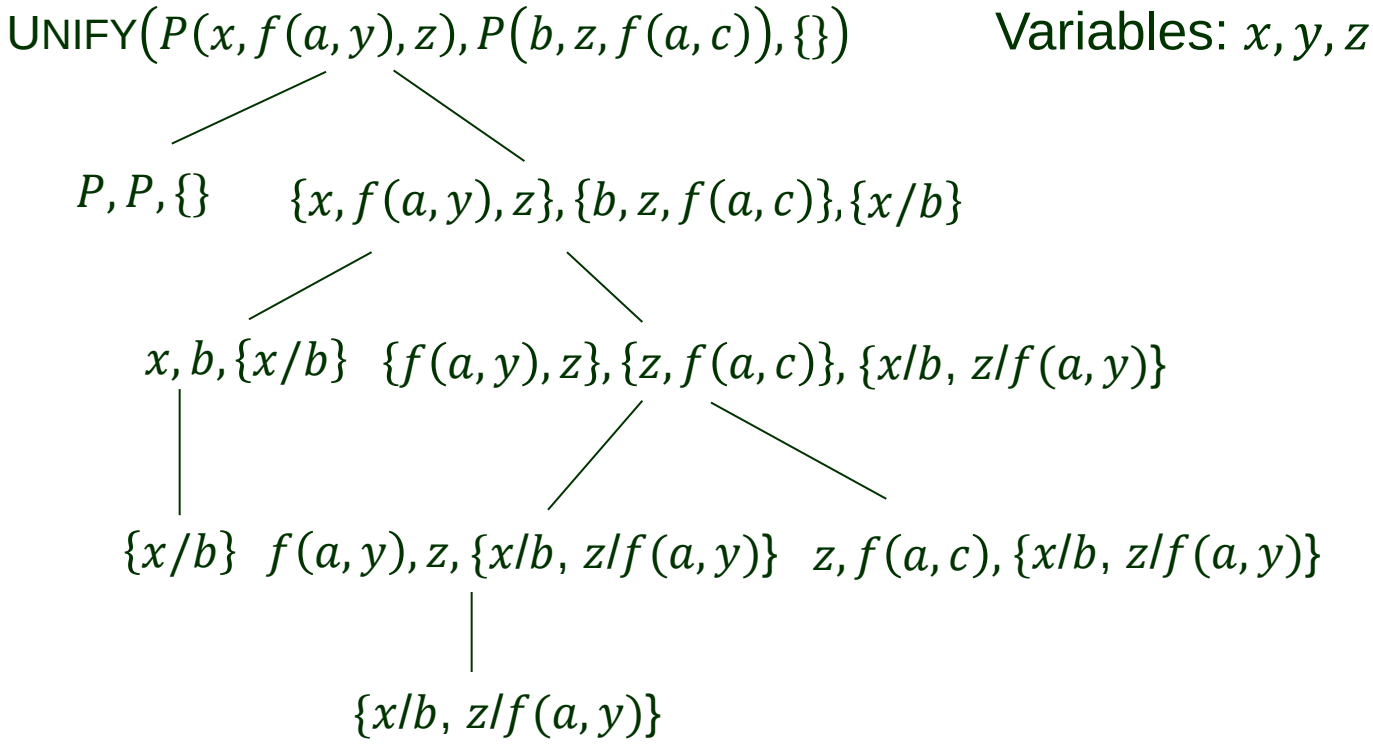

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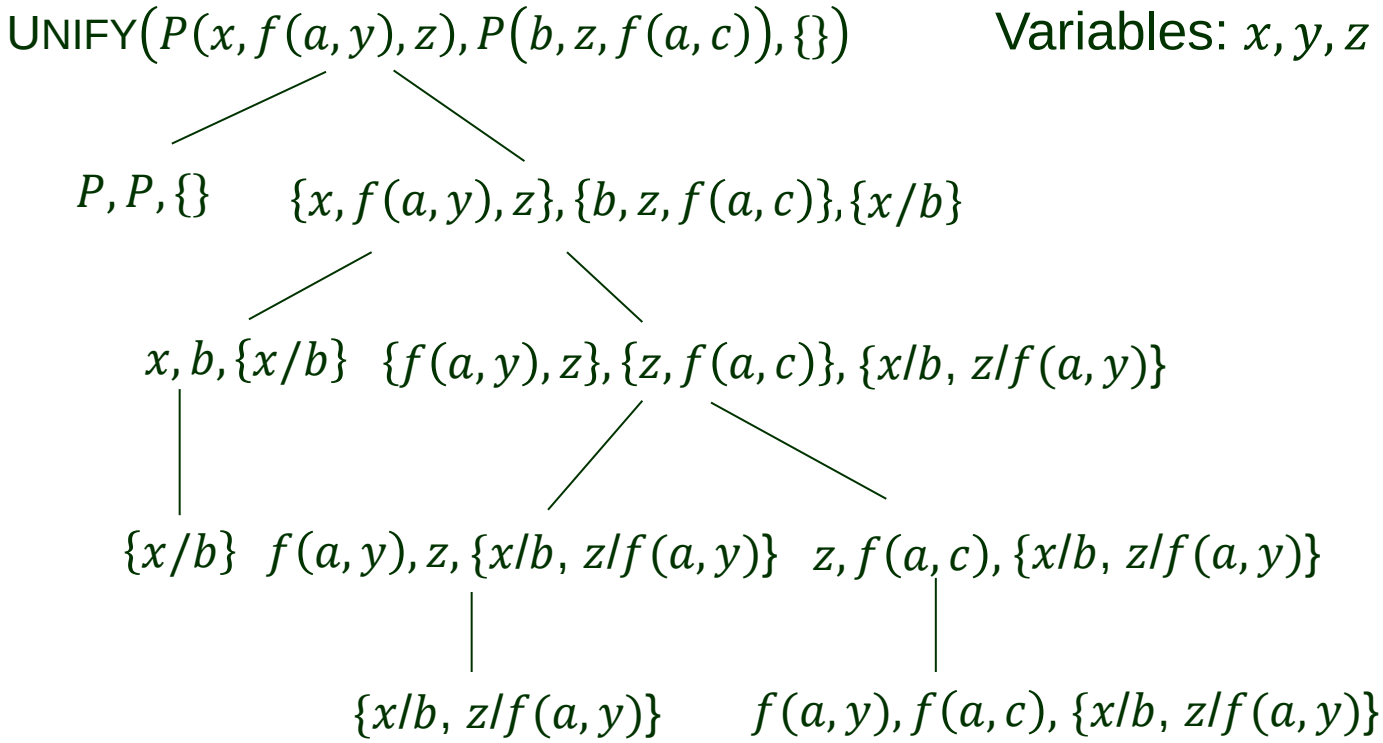

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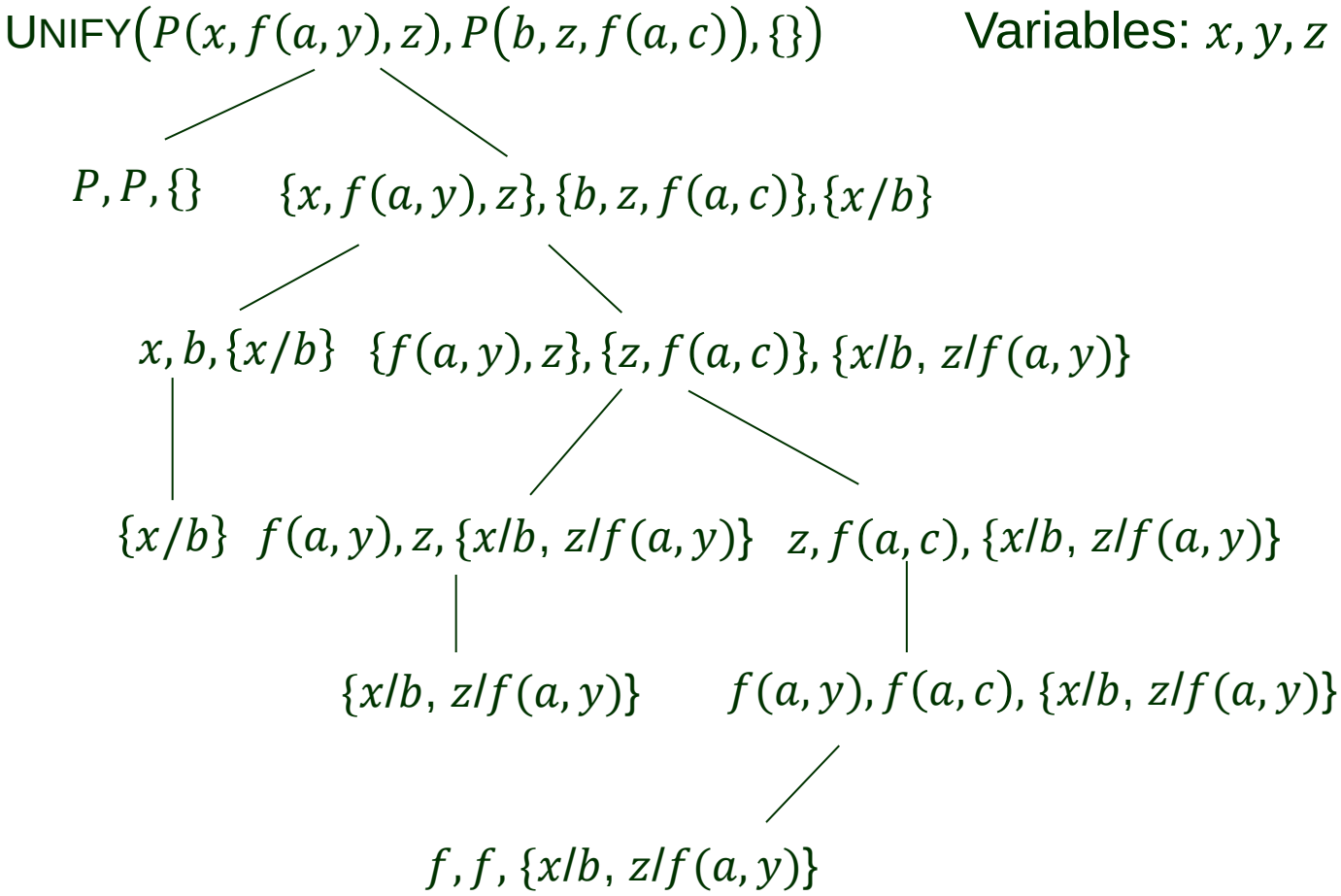

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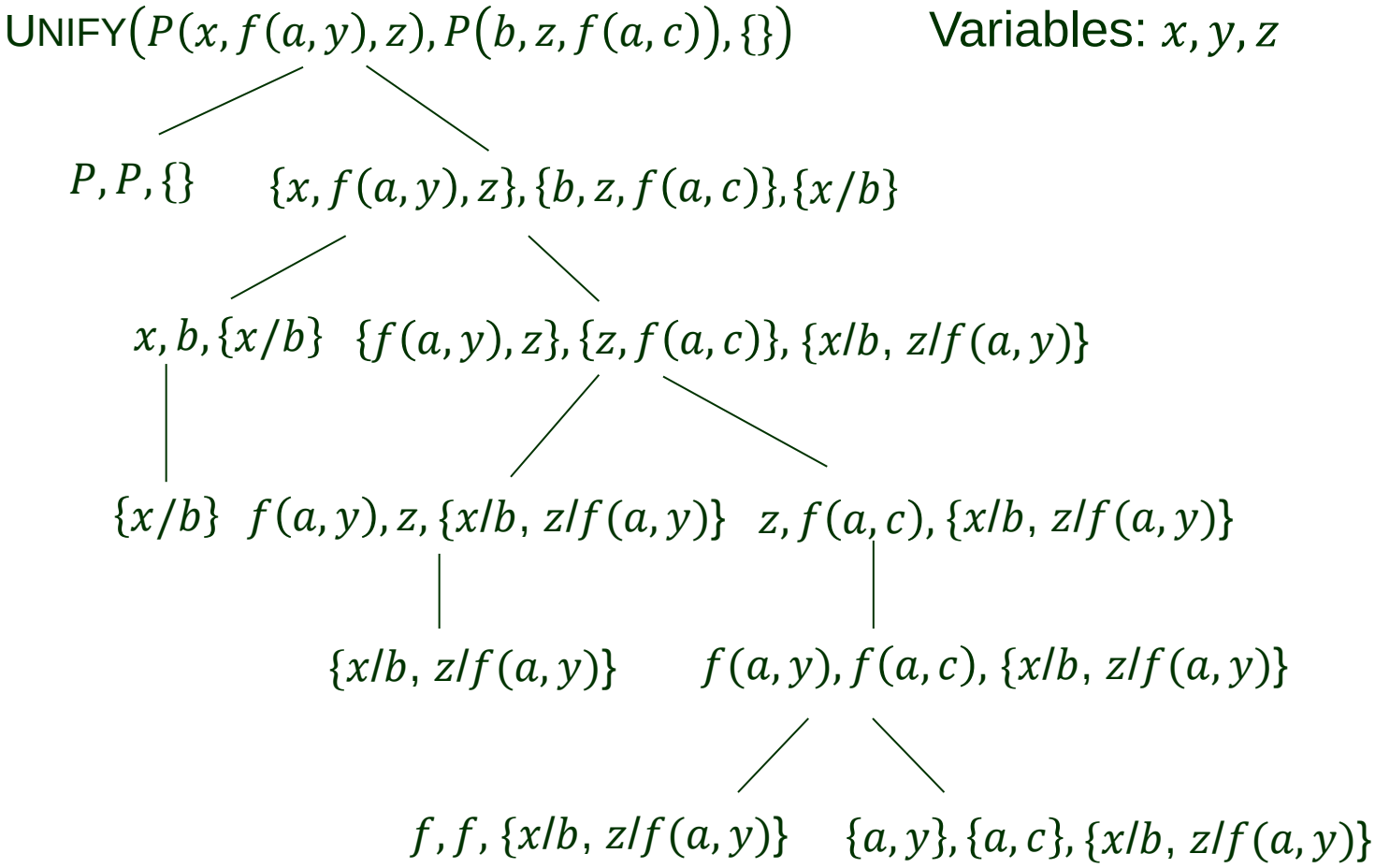

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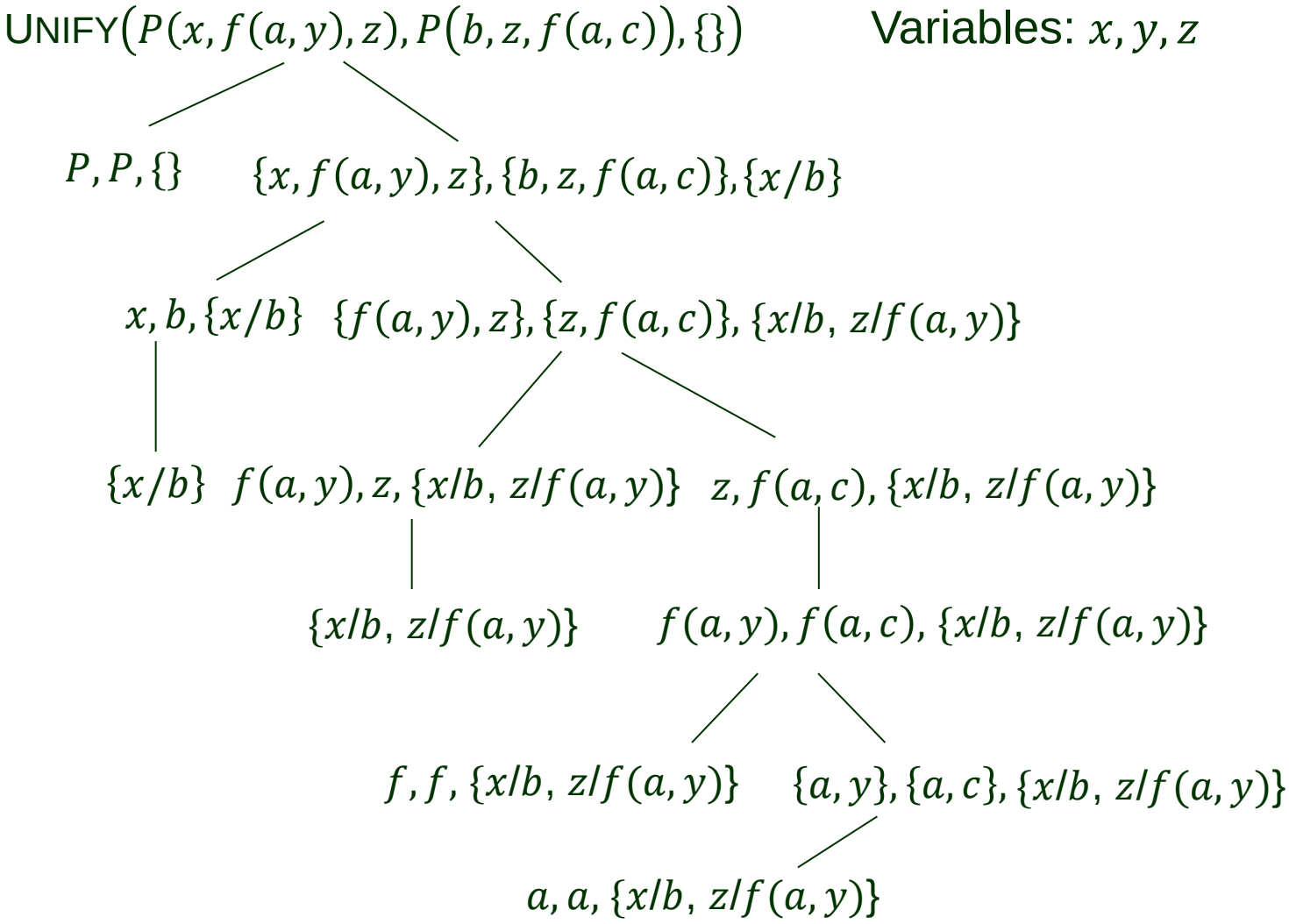

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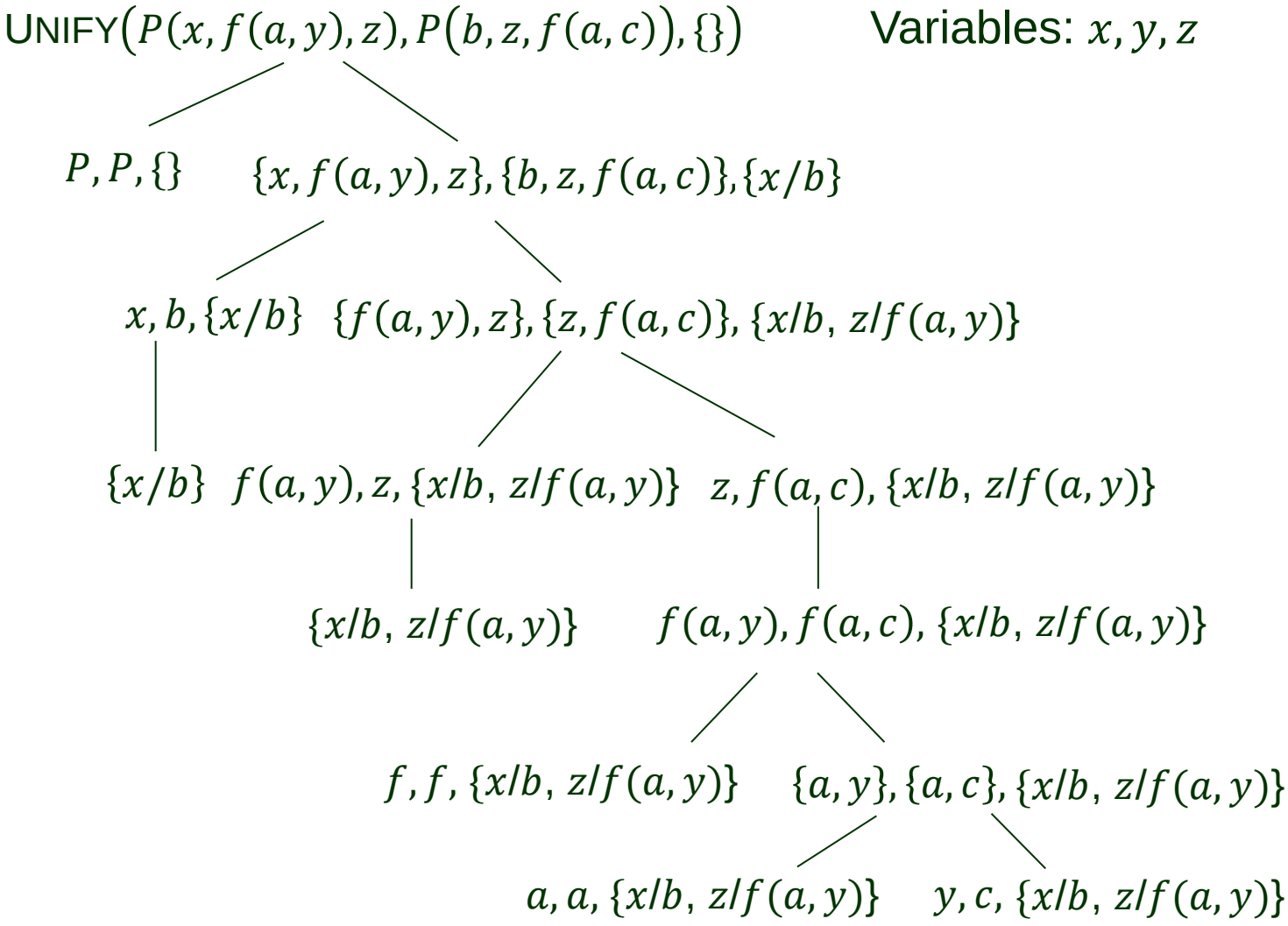

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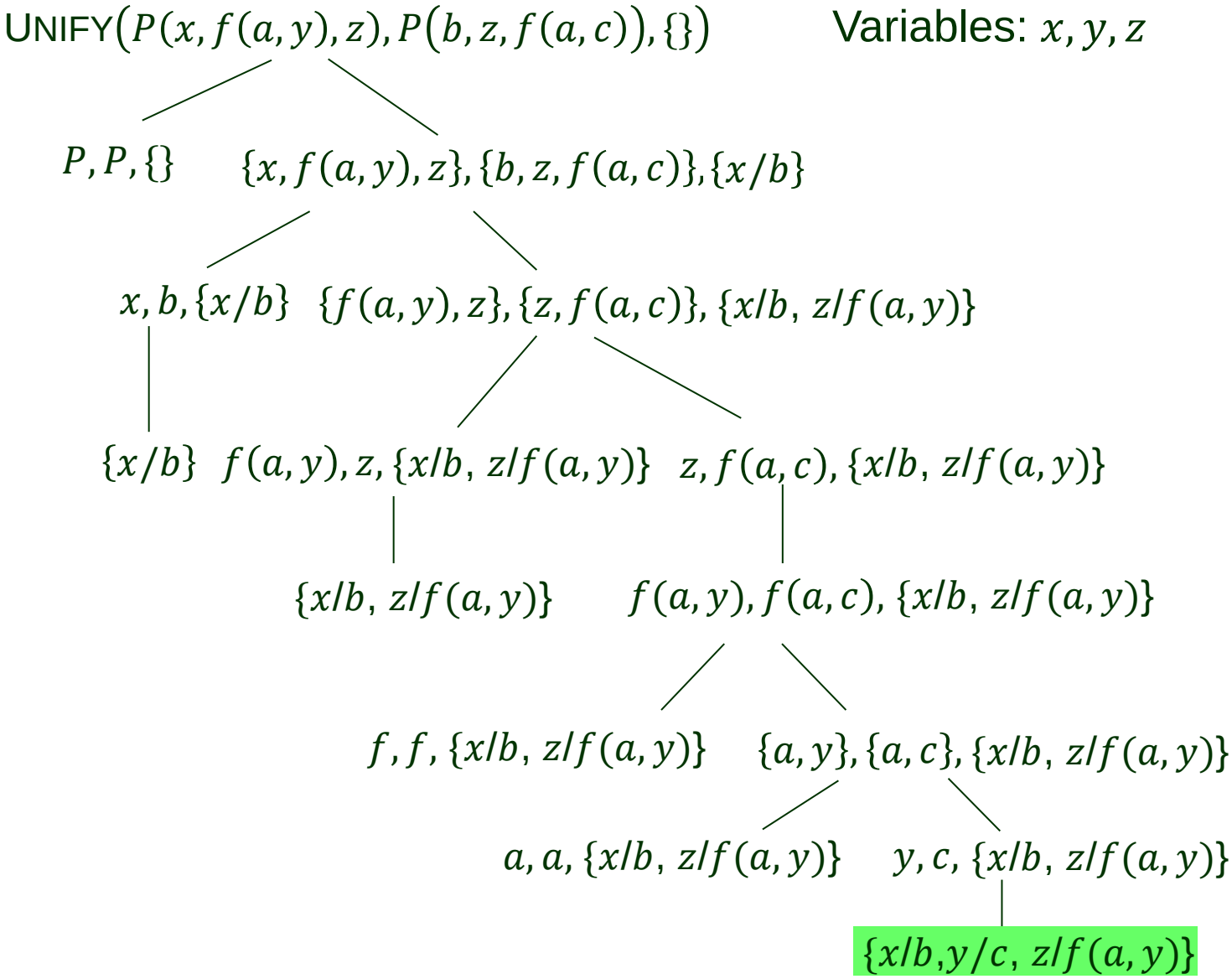

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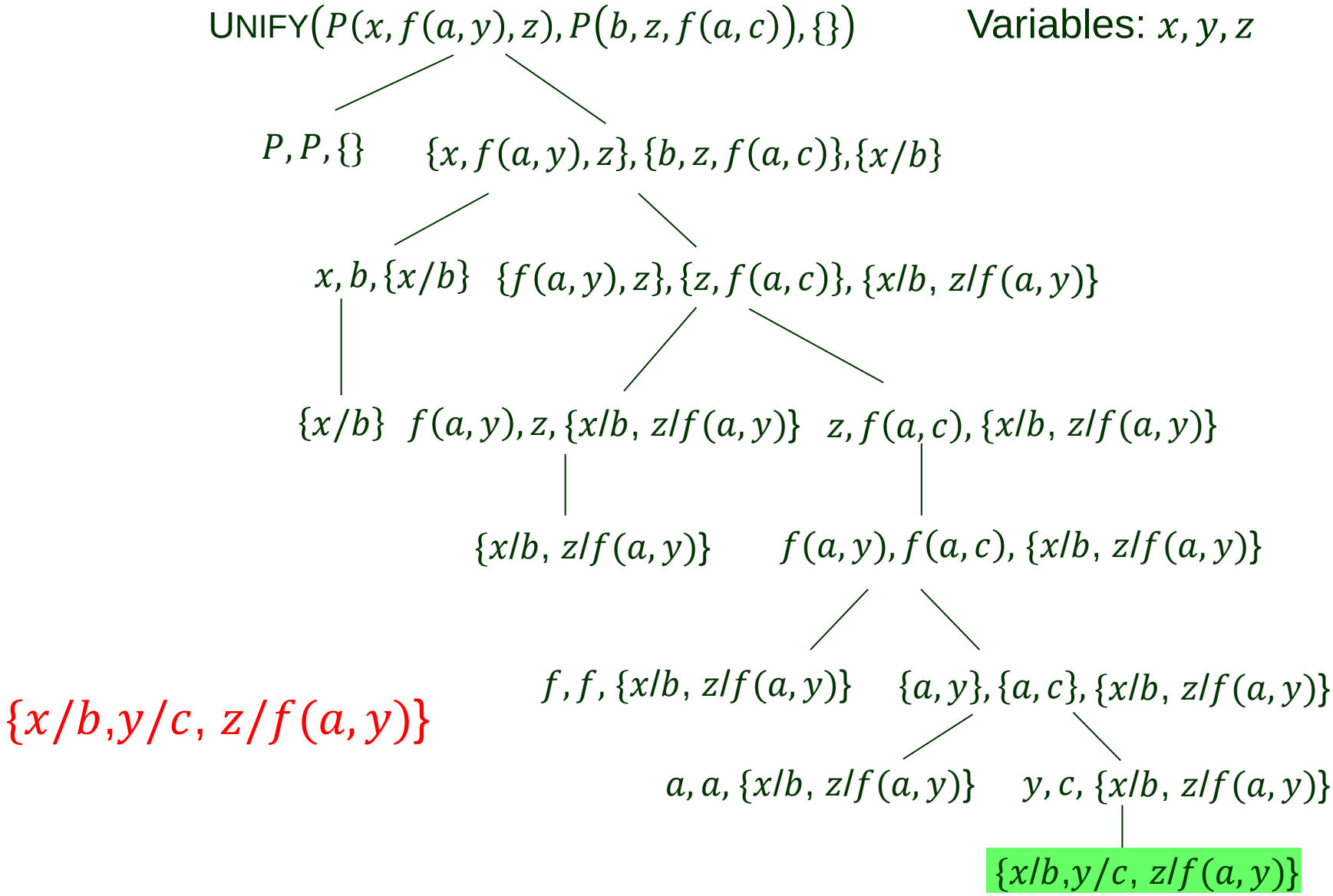

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$$Human(Socrates) \Rightarrow Fallible(Socrates)$$

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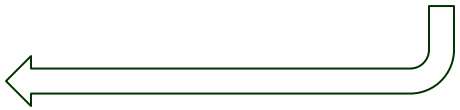

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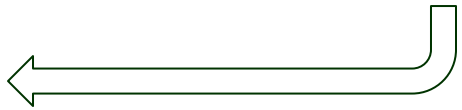

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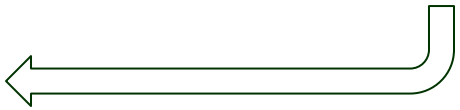

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The *KB* consists of first-order definite clauses with no function symbols.
It is called a *Datalog*.

III. Simple Forward Chaining

1. Start from the known facts.
2. Trigger all the rules whose premises are satisfied.
3. Add their conclusions to the known facts.
4. Repeat steps 2 and 3 until one of the following situations occurs:
    a. The query is answered.
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Likes(x,*IceCream*) is a renaming of *Likes*(y,*IceCream*).
Both have the meaning: “Everyone likes ice cream”.

Execution of Forward Chaining

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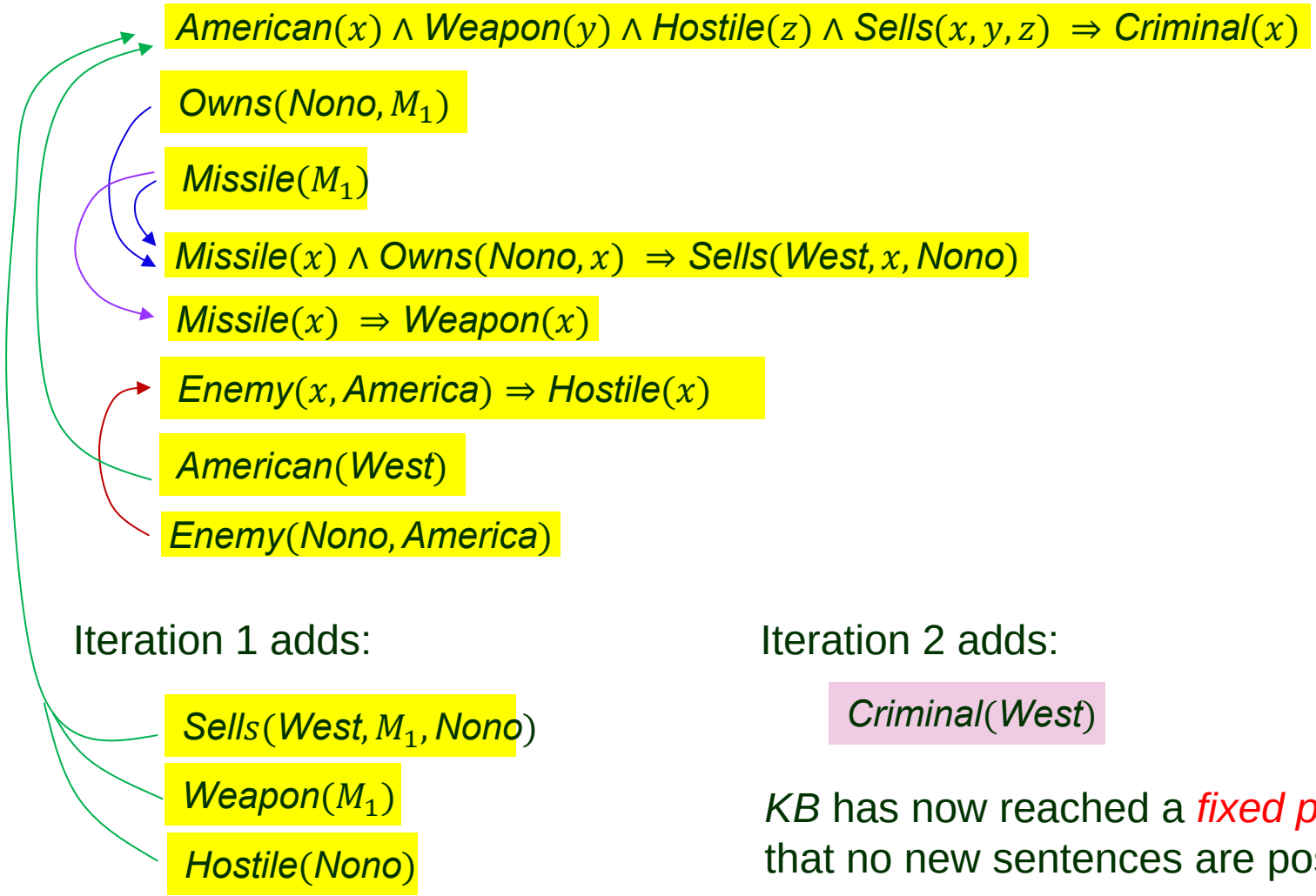


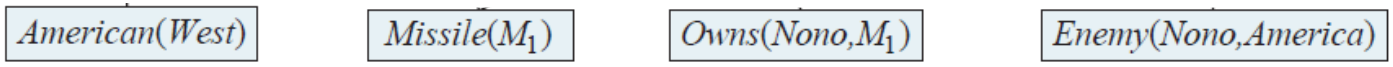
Iteration 1 adds:

Iteration 2 adds:

Criminal(*West*)

KB has now reached a *fixed point*, meaning that no new sentences are possible.

Proof Tree



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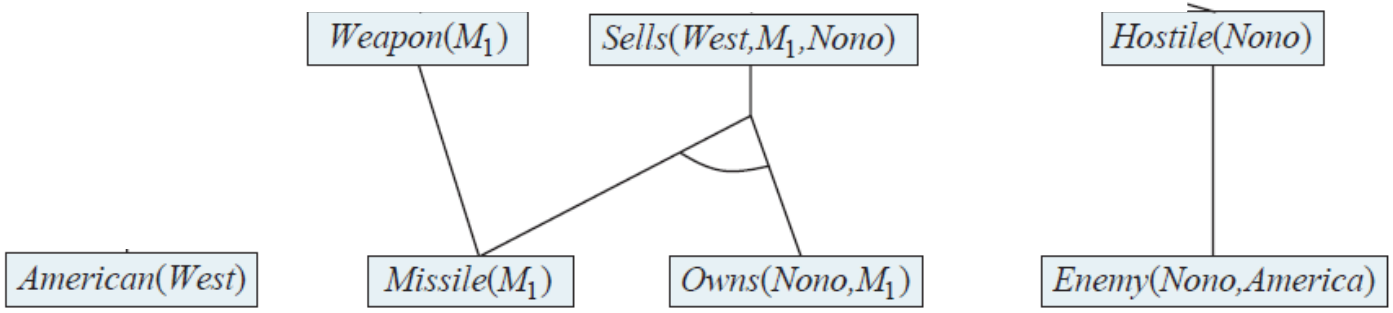
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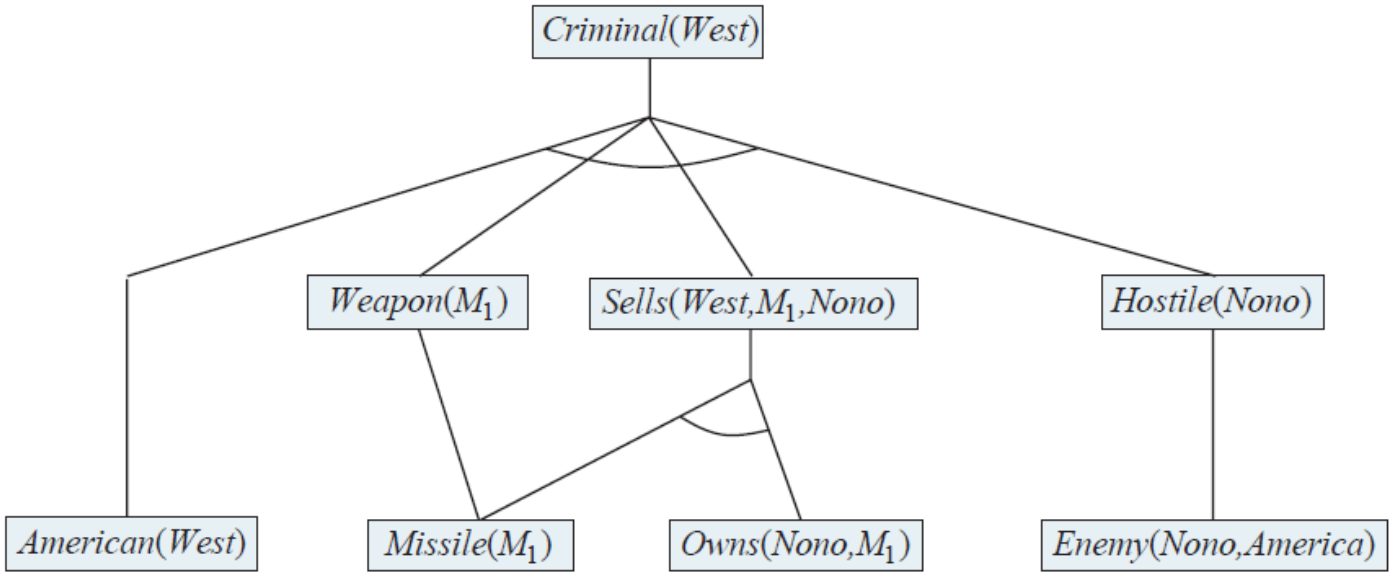
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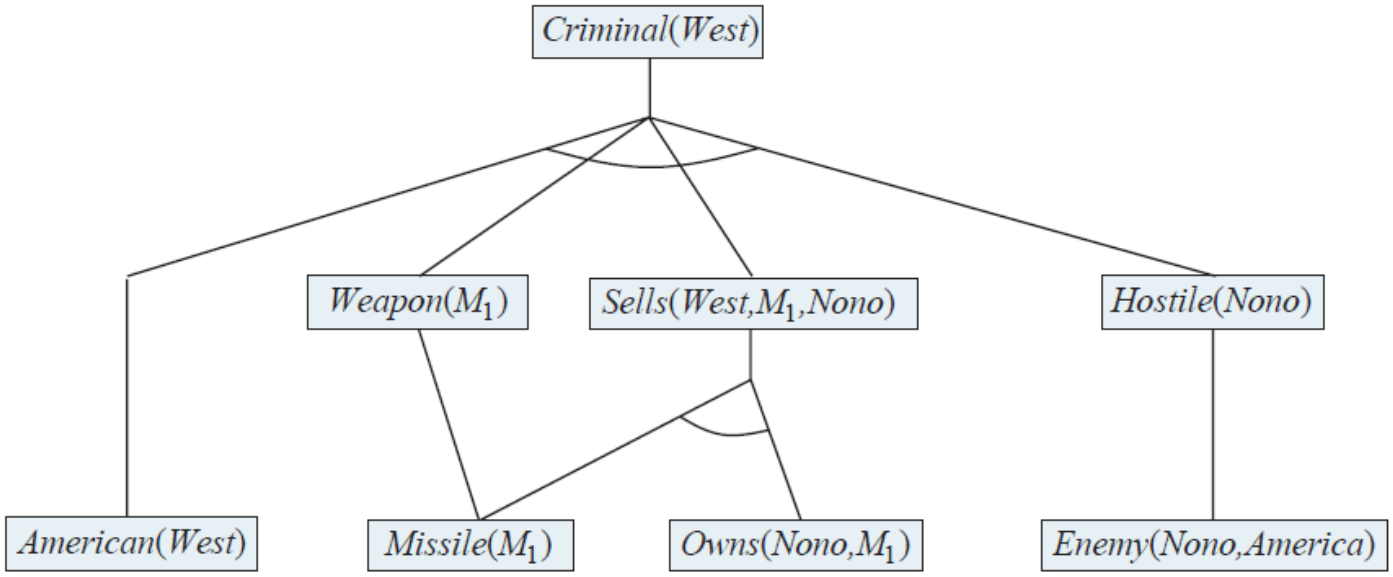

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  Every inference is an application of Generalized Modus Ponens.

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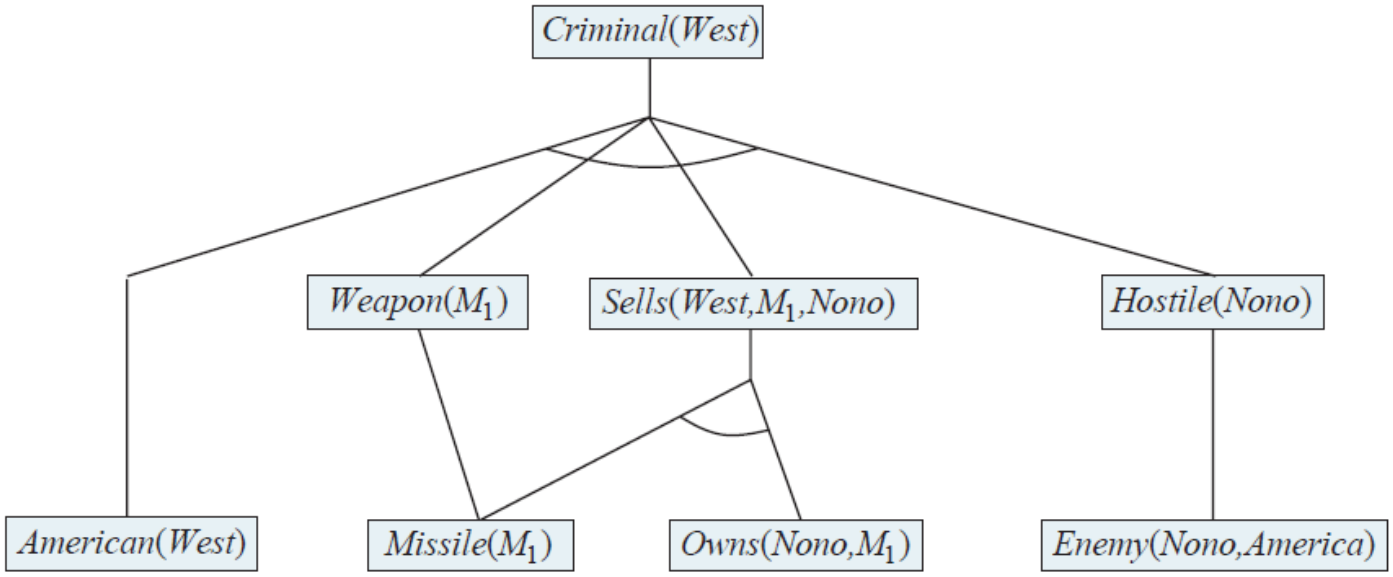

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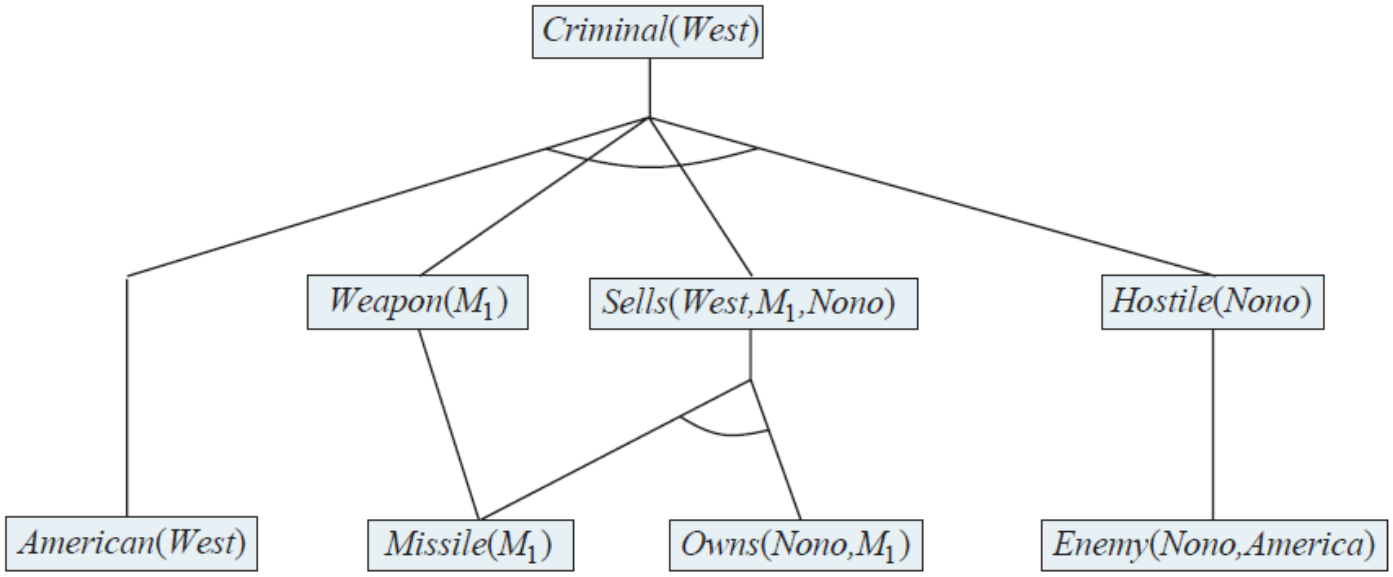

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  - Guaranteed except for a query with no answer, the algorithm could fail to terminate. E.g., $NatNum(0)$ and $NatNum(n) \Rightarrow NatNum(S(n))$.

Improvement 1: Matching Rules Against Known Facts

Inefficiency of simple forward chaining:

- ♠ Exhaustively matches every rule against every fact.
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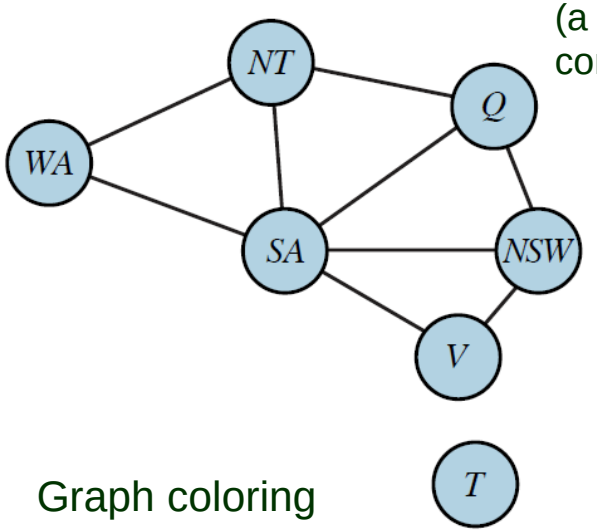
Use a heuristic, e.g., the minimum-remaining-values (MRV) heuristic for CSPs.

CSP as a Definite Clause

View every conjunct in the premise as a constraint on the variables it contains.



Graph coloring

color of WA (a Skolem constant)

Diff(*wa*, *nt*) ∧ *Diff*(*wa*, *sa*) ∧
Diff(*nt*, *q*) ∧ *Diff*(*nt*, *sa*) ∧
Diff(*q*, *nsw*) ∧ *Diff*(*q*, *sa*) ∧
Diff(*nsw*, *v*) ∧ *Diff*(*nsw*, *sa*) ∧
Diff(*v*, *sa*) ⇒ *Colorable*()

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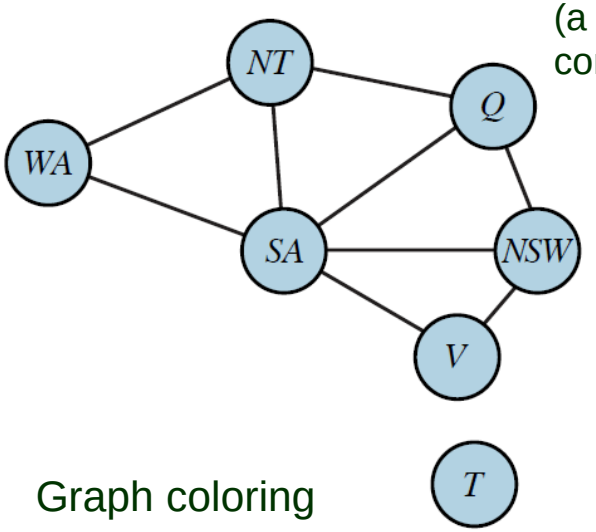

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Constraint satisfaction is NP-hard.

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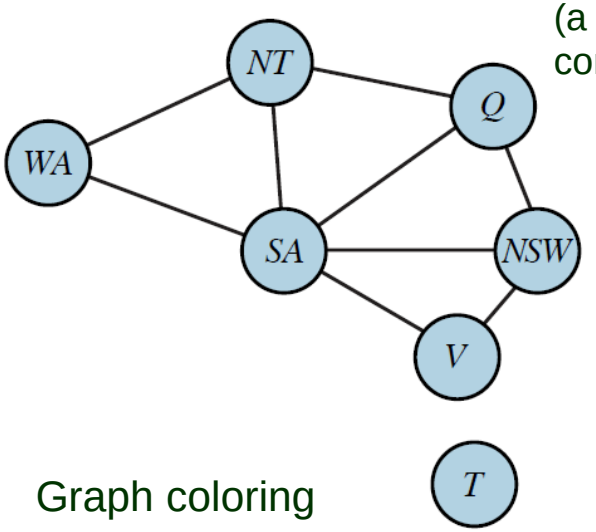

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Diff(*Green*, *Red*) *Diff*(*Green*, *Blue*)
Diff(*Blue*, *Red*) *Diff*(*Blue*, *Green*)

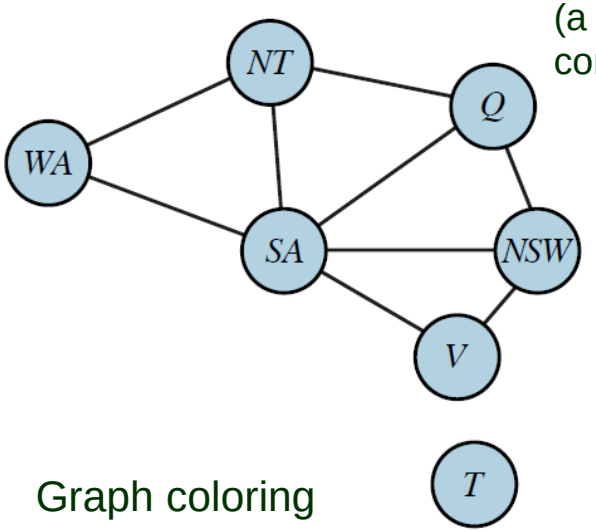
Constraint satisfaction is NP-hard.

⇩

Matching a definite clause against a set of facts (under unification) is NP-hard.

CSP as a Definite Clause

View every conjunct in the premise as a constraint on the variables it contains.



Graph coloring

color of WA (a Skolem constant)

Diff(*wa, nt*) ∧ *Diff*(*wa, sa*) ∧
Diff(*nt, q*) ∧ *Diff*(*nt, sa*) ∧
Diff(*q, nsw*) ∧ *Diff*(*q, sa*) ∧
Diff(*nsw, v*) ∧ *Diff*(*nsw, sa*) ∧
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Good news View every Datalog clause as a CSP and apply heuristics for CSPs (e.g., tree structure, cutset conditioning, etc.).

Improvement 2: Incremental FC

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E.g., the Rete algorithm

IV. Backward Chaining

Works like AND/OR search:

- OR
  - The goal query can be proved by any rule in the *KB*.
  - A query containing a variable, e.g., $Person(x)$ can be proved in multiple ways.
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How does it work?

- Fetch all clauses that unify with the goal.
- Rename the variables in every such clause to be brand-new.
- Prove every conjunct in the clause by keeping track of the expanded substitution as it goes.

Depth-First Proof Tree

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

$Missile(x) \Rightarrow Weapon(x)$

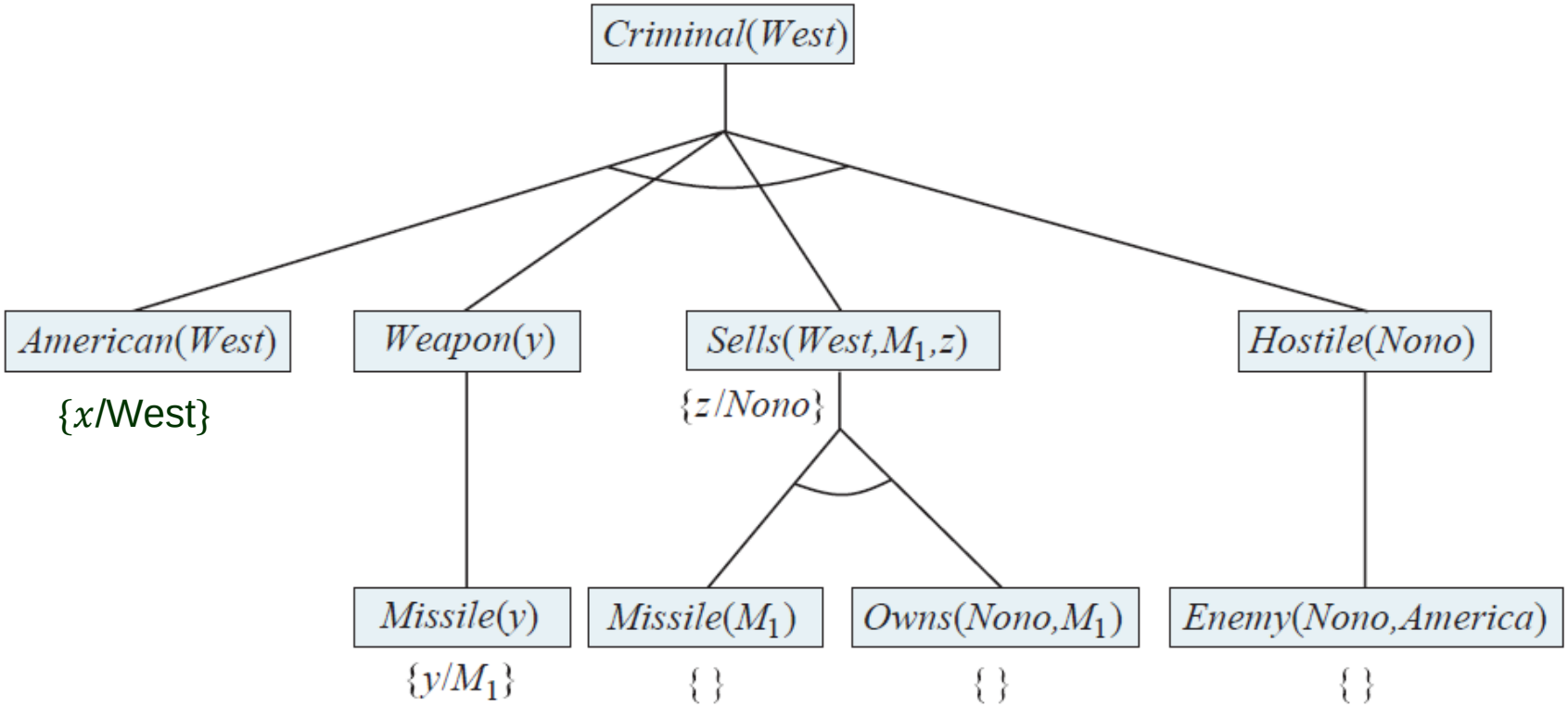
$Enemy(x, America) \Rightarrow Hostile(x)$

$Owns(Nono, M_1)$

$Missile(M_1)$

$American(West)$

$Enemy(Nono, America)$



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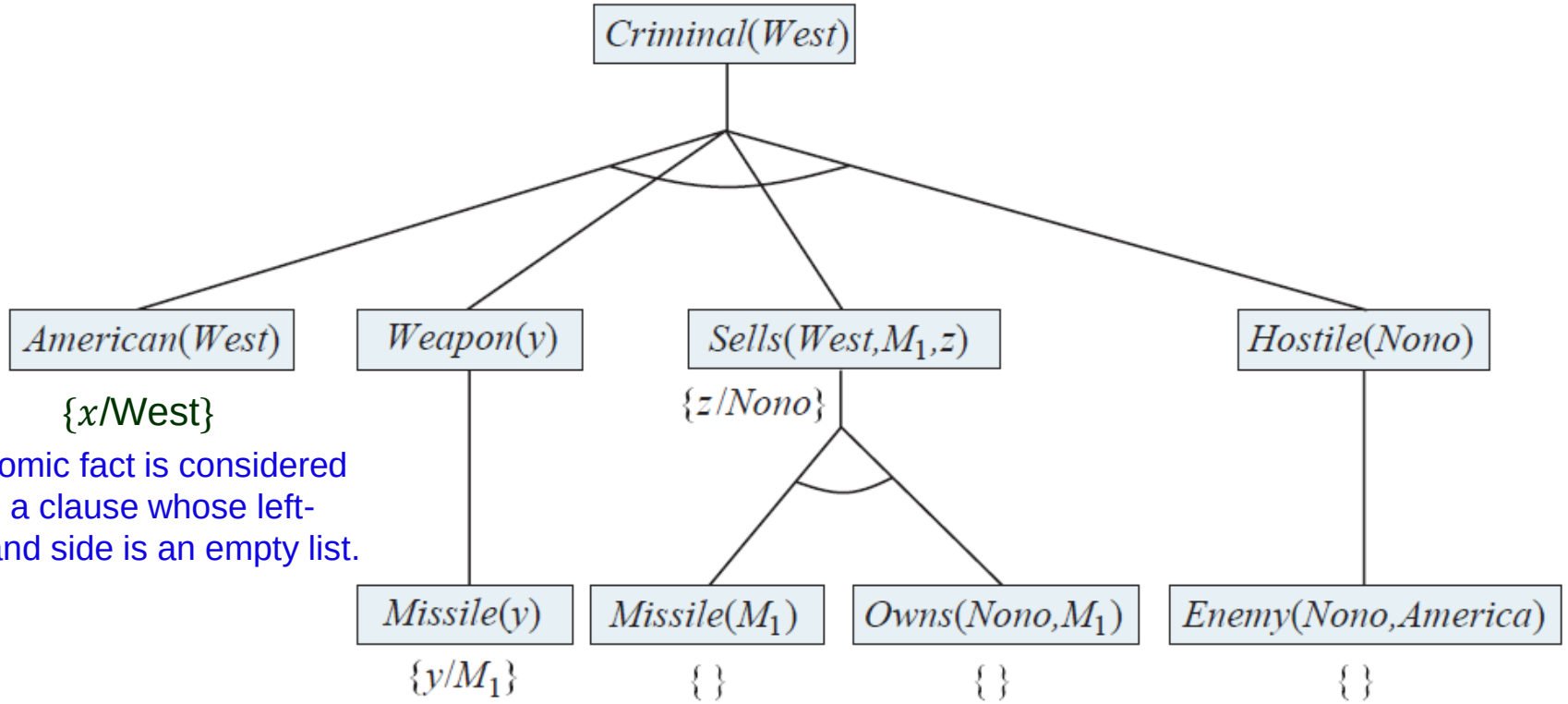
$Enemy(Nono, America)$



Atomic fact is considered as a clause whose left-hand side is an empty list.