Exact Inference in Bayesian Networks

Outline

I. Efficient representation of conditional distributions

II. Probabilistic query using a BN

III. Variable Elimination

IV. Variable ordering and relevance

* Figures are from the textbook site.

I. Efficient Representations

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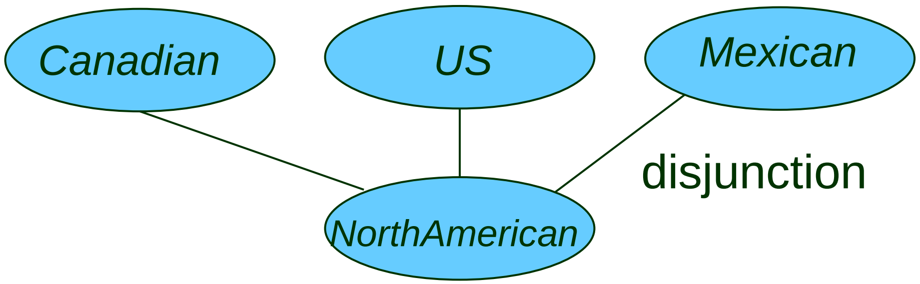

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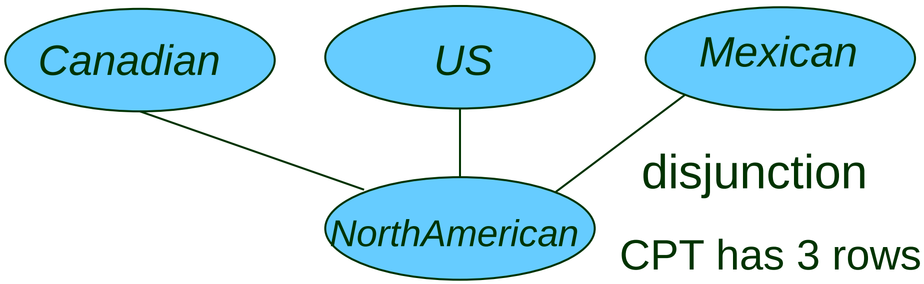

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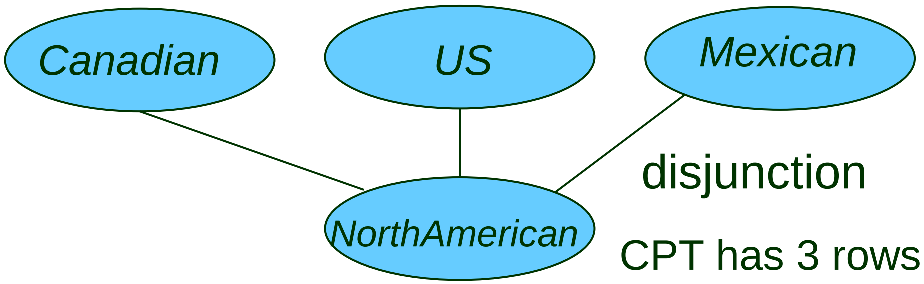

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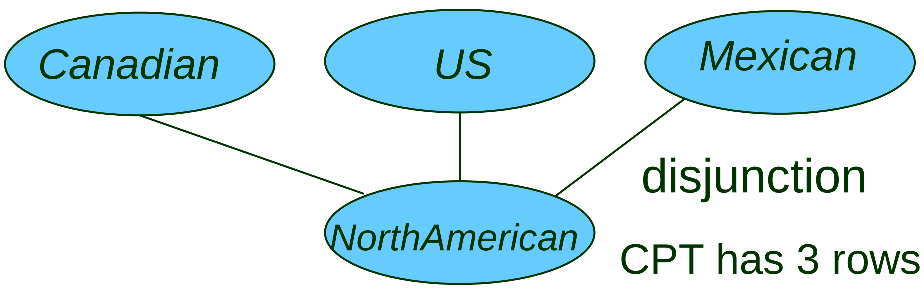

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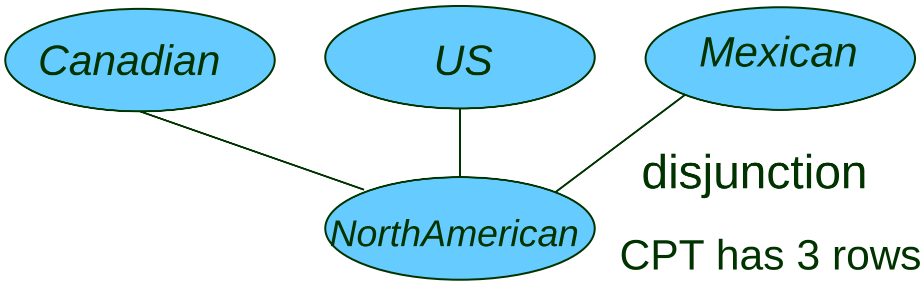

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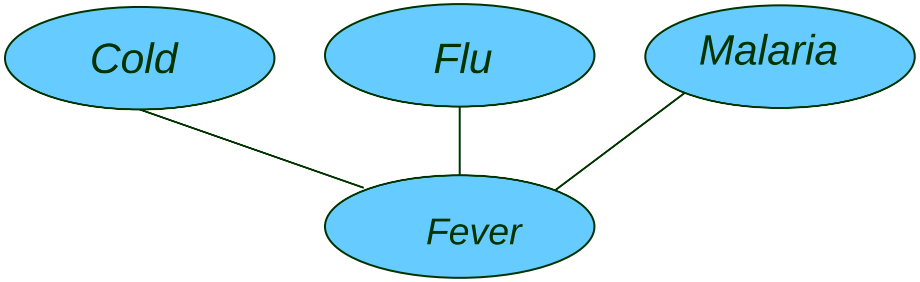

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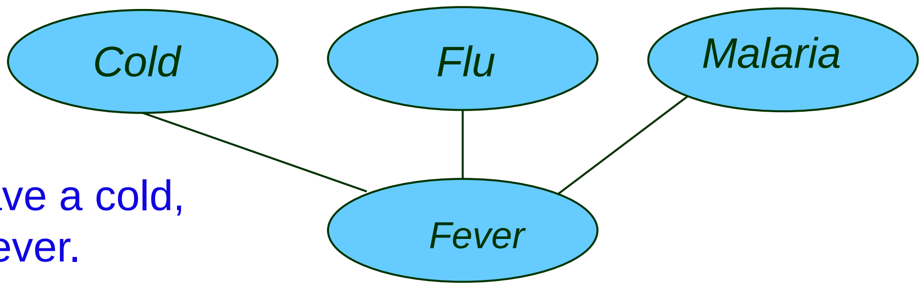

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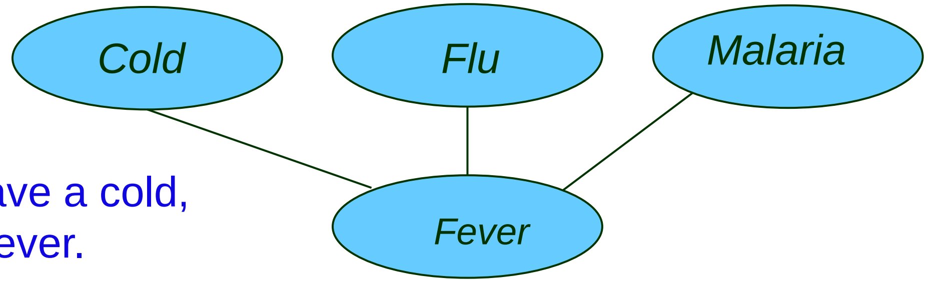

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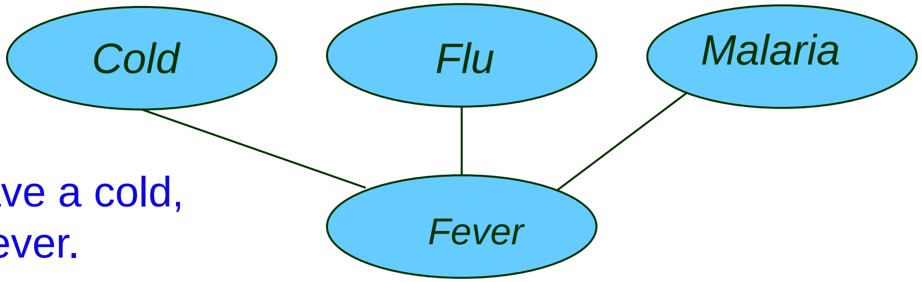

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- The causal relationship between a node and its parents may be *inhibited*.
  - List all the possible causes.
  - Inhibition of each parent is independent of inhibition of any other parents.

Causes of Fever

$Cold$	Flu	$Malaria$	$P(fever \mid \cdot)$	$P(\neg fever \mid \cdot)$
f	f	f	0.0	1.0
f	f	t	0.9	**0.1**
f	t	f	0.8	**0.2**
f	t	t	0.98	$0.02 = 0.2 \times 0.1$
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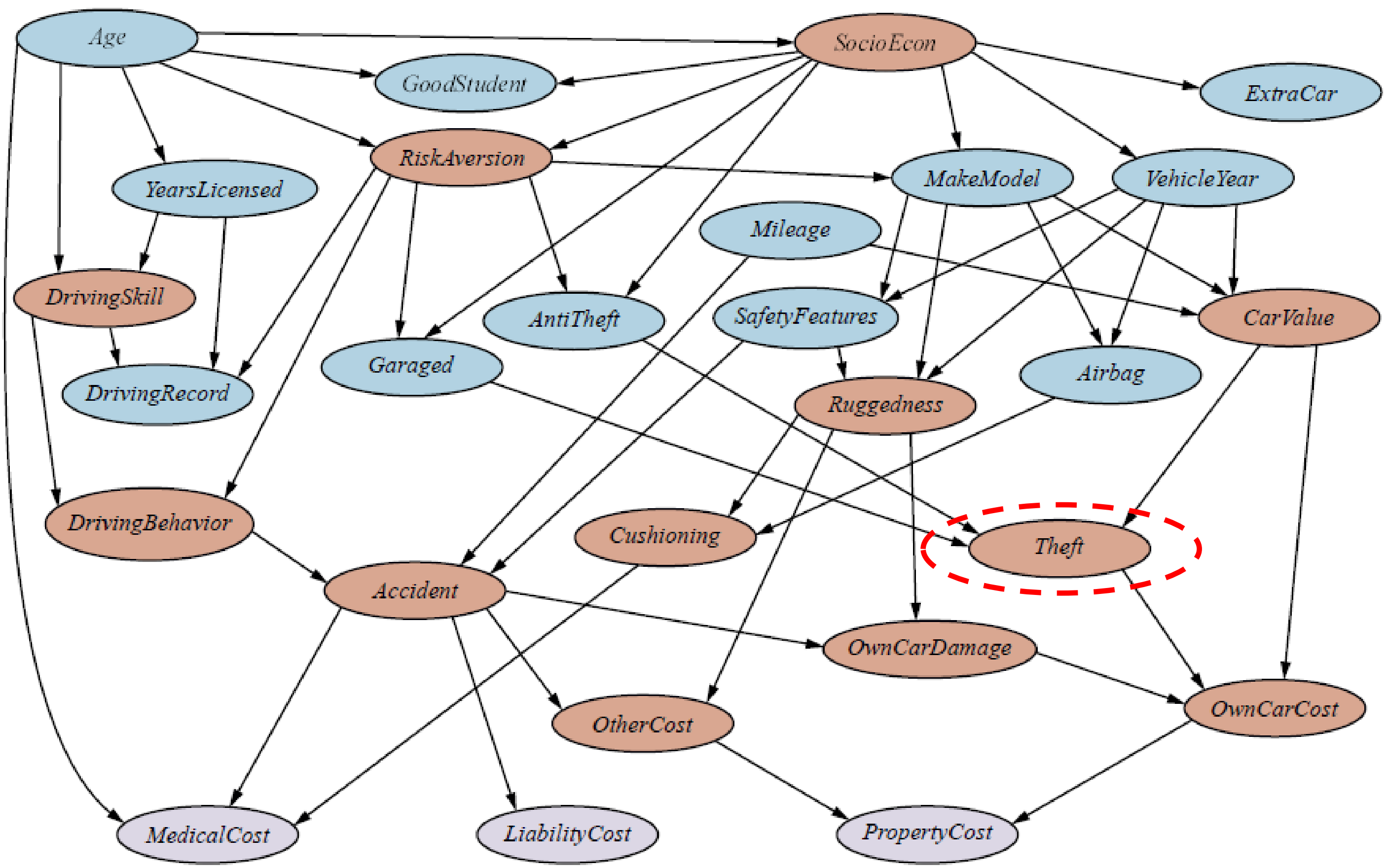
$$P(x_i \mid parents(X_i)) = 1 - \prod_{\{j : X_j = true\}} q_j$$

Case Study: Car Insurance

A car insurance company processes a car insurance application to decide on the annual premium based on the anticipated claims it will pay out for the applicant.



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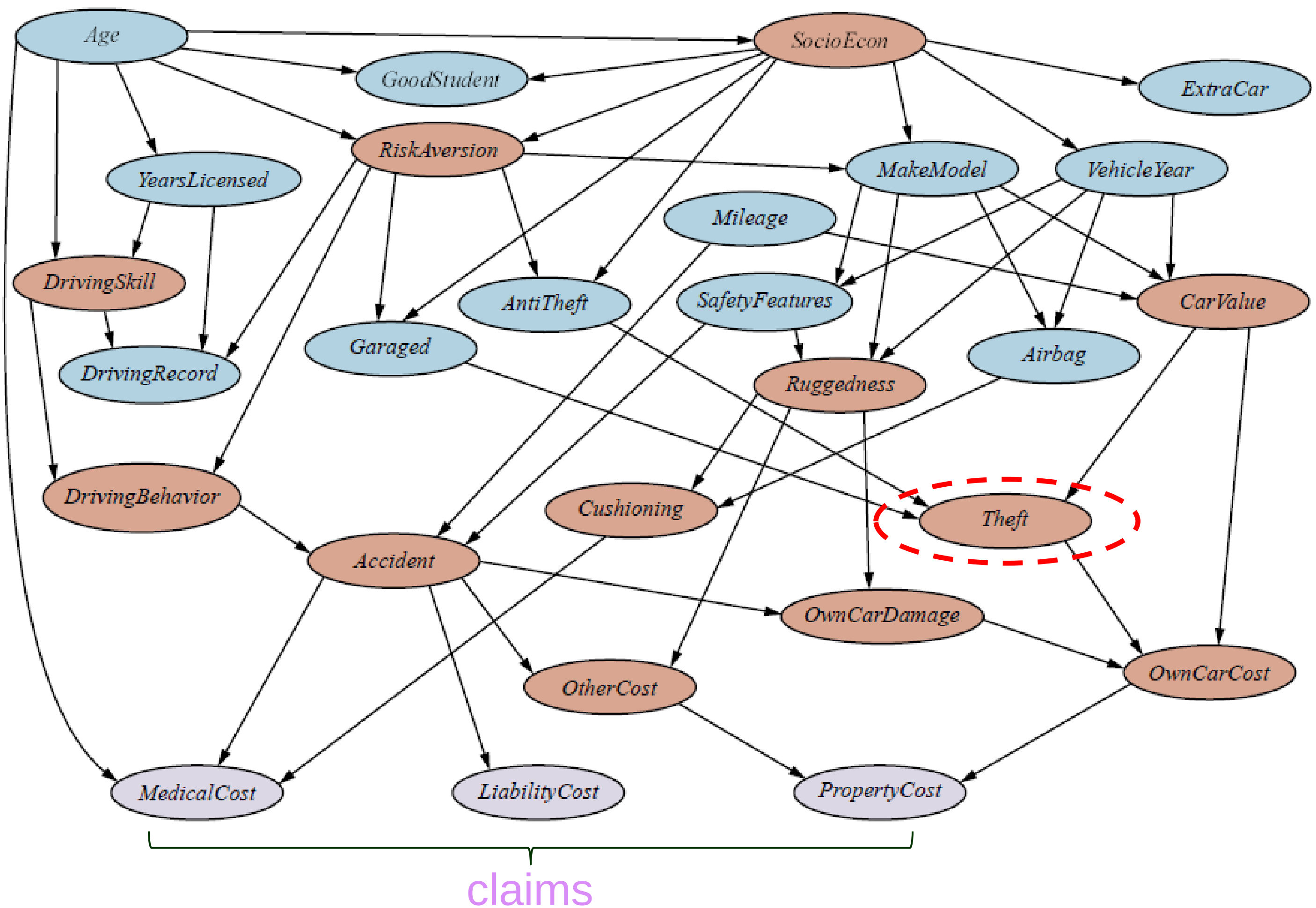

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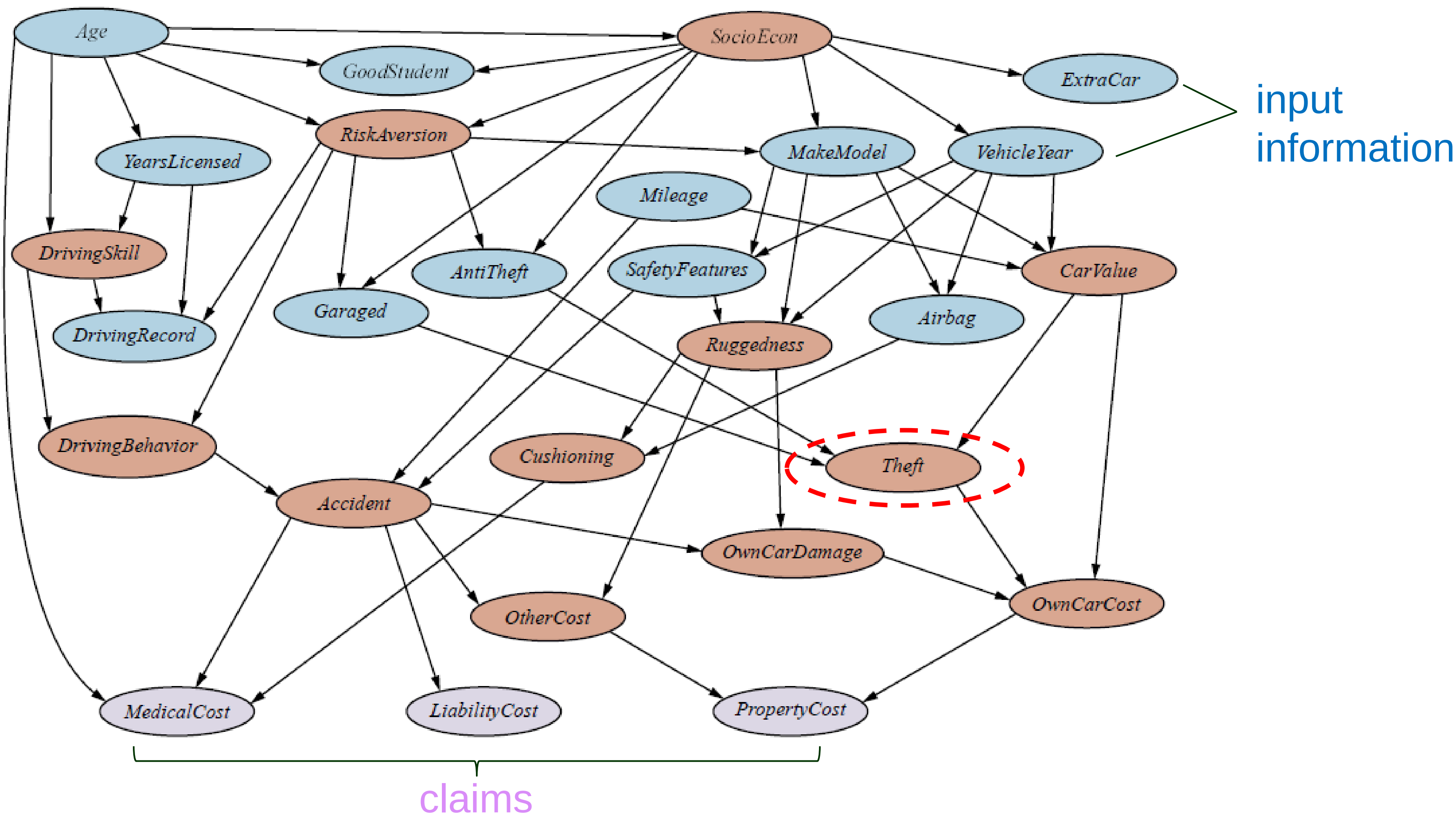

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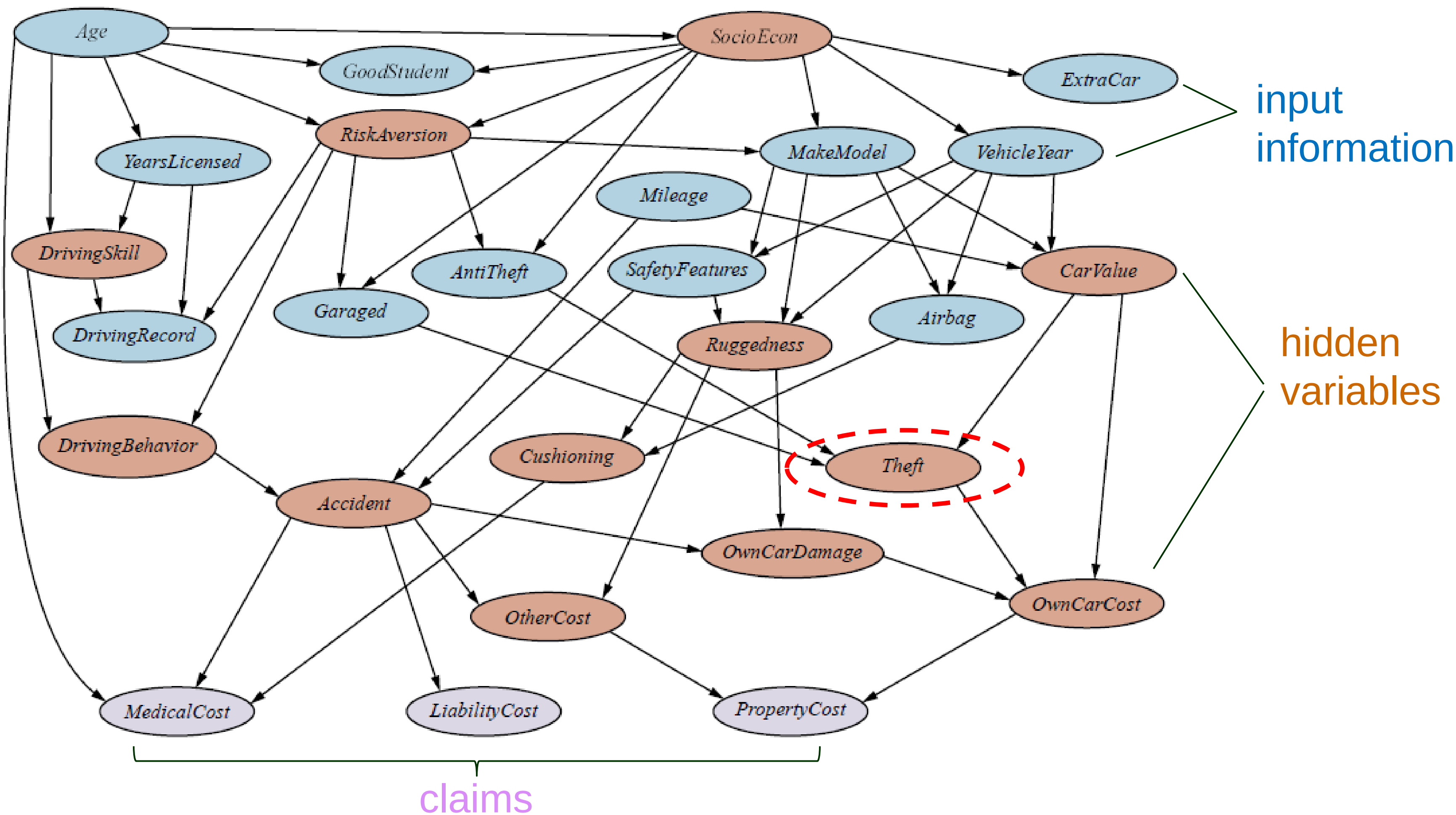

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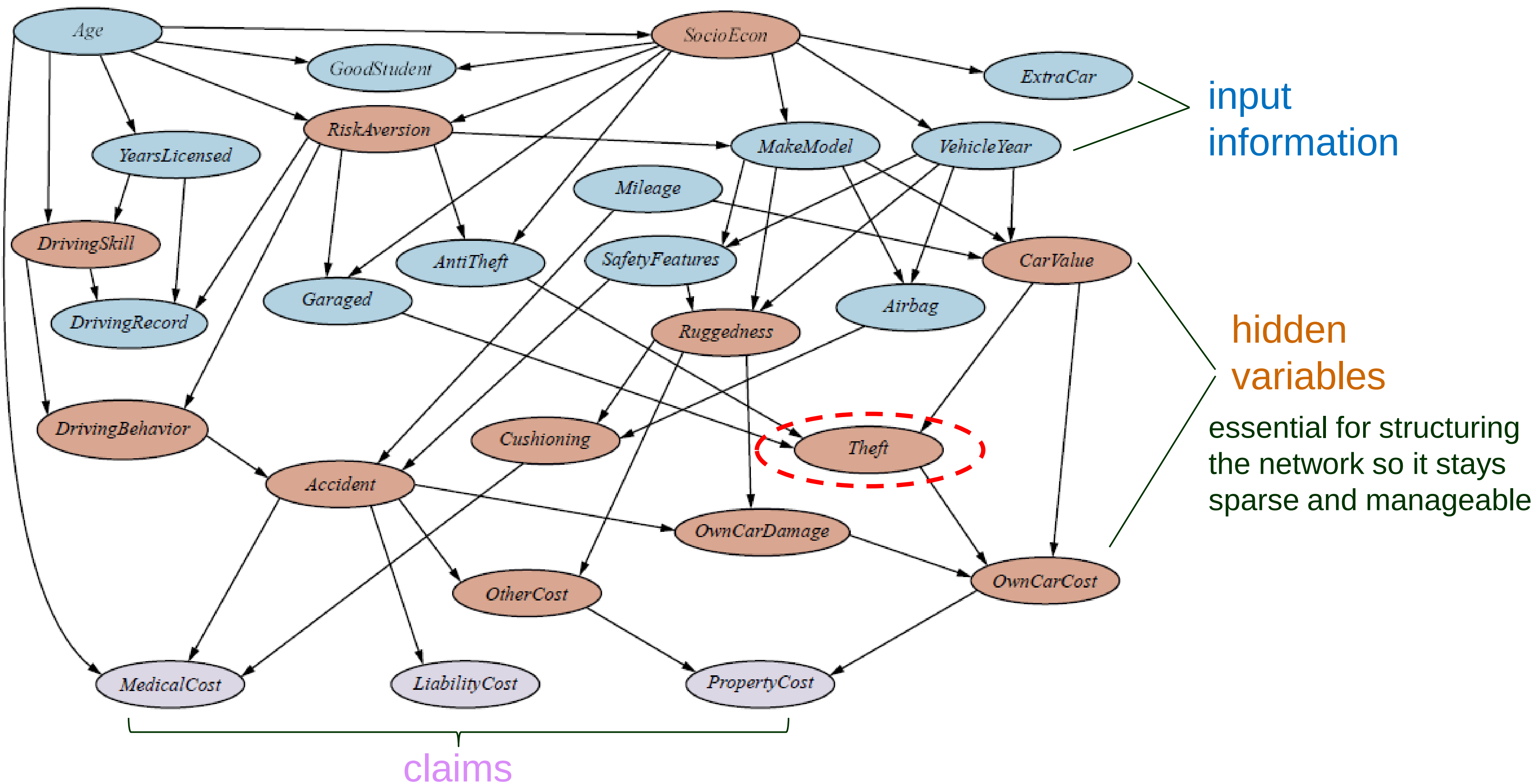

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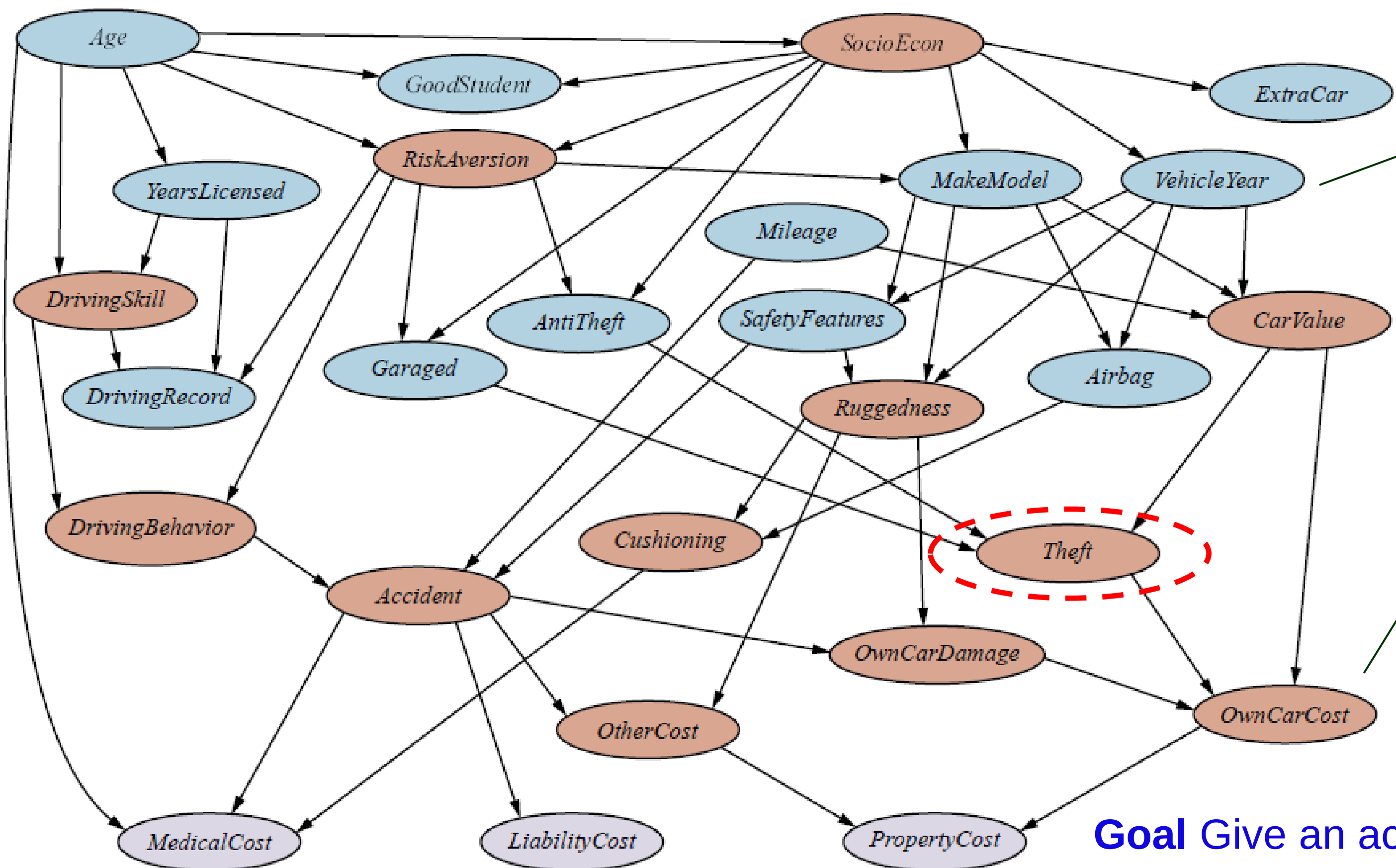

input information

hidden variables

essential for structuring the network so it stays sparse and manageable

claims

Goal Give an accurate, well-calibrated distribution over the output variables given the evidence available form the input.

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X: query variable

$\boldsymbol{E} = \{E_1, \ldots. E_m\}$: evidence variables

$\boldsymbol{e} = \{e_1, \ldots. e_m\}$: an observed event

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We will discuss exact algorithms for posterior probability computation.

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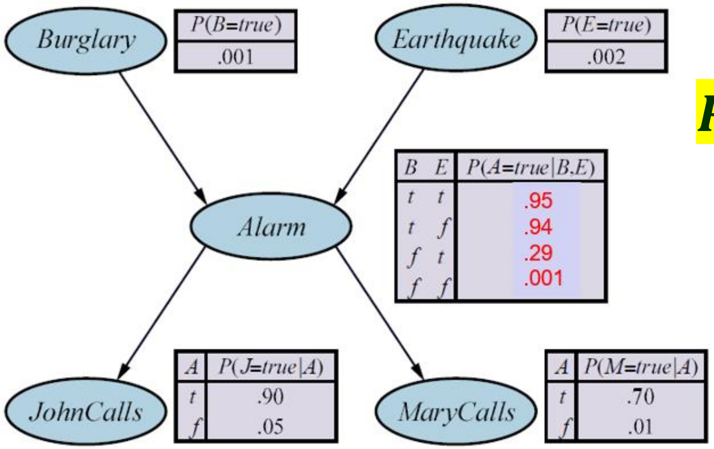
- Answer the query $\mathbf{P}(X \mid \mathbf{e})$ using a BN.

Burglary Example (revisited)



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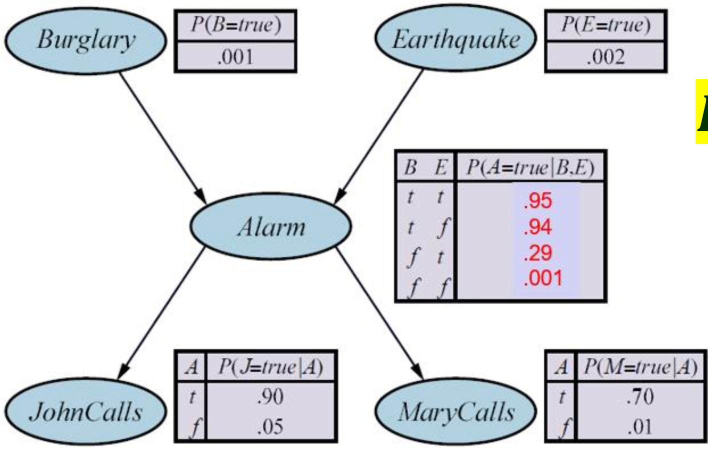

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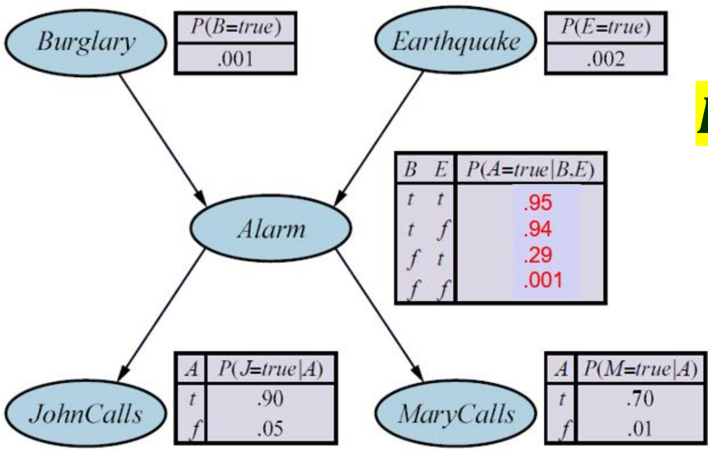

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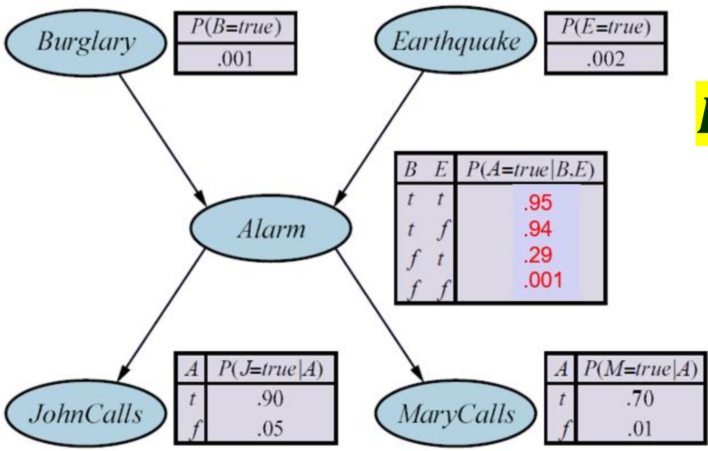

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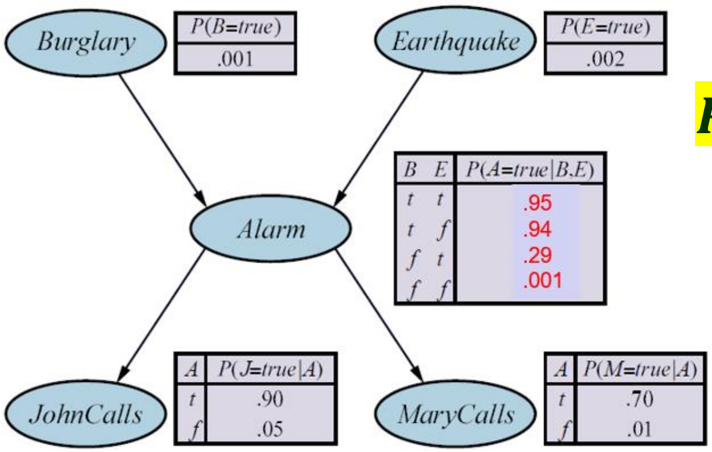

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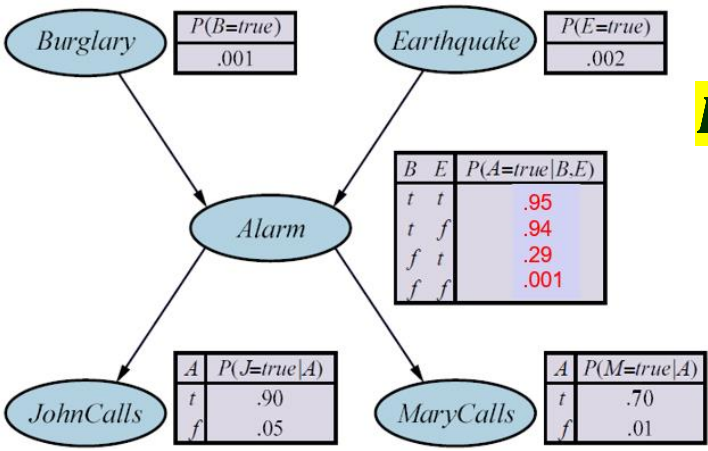

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In the general case with n variables, there are 2^n summands, each as a product requires $O(n)$ computation time.

Expression Tree

Take advantage of the nested structure to move summations inwards as far as possible.

$$P(b \mid j, m) = \alpha \sum_{E} \sum_{A} P(b)P(E)P(A \mid b, E)P(j \mid A)P(m \mid A)$$

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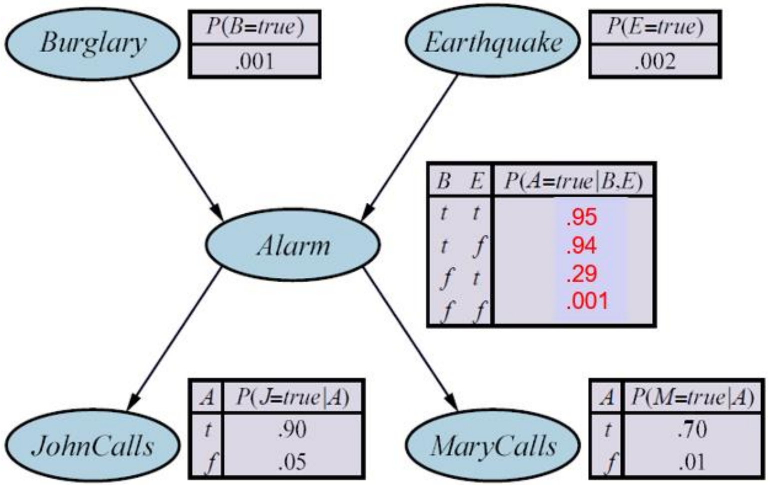

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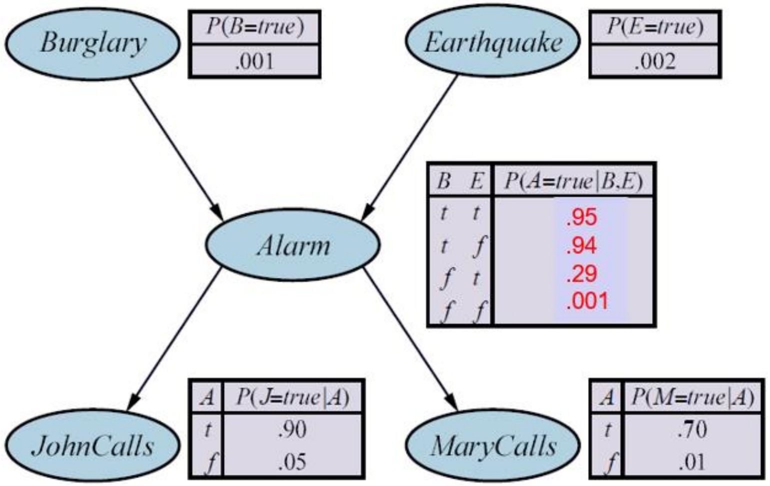

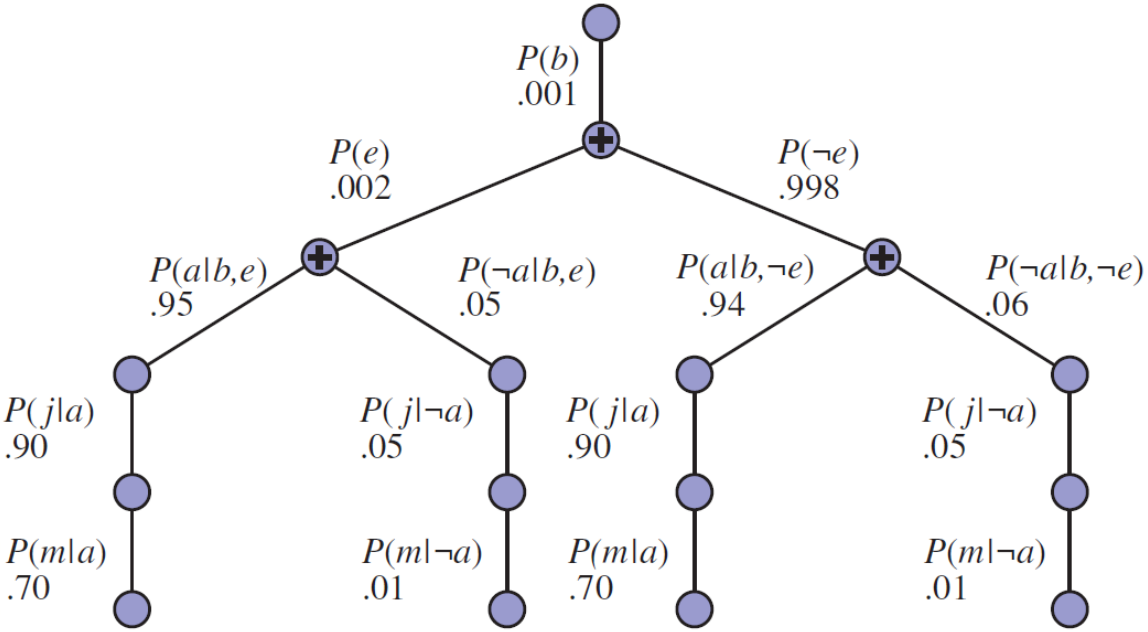

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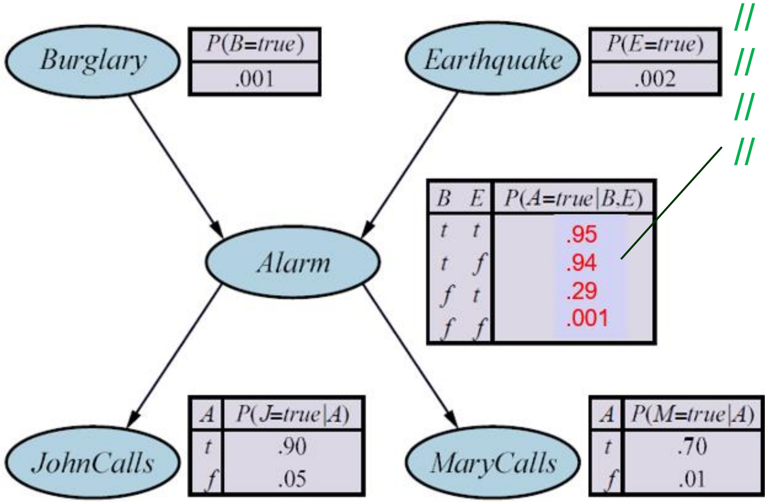

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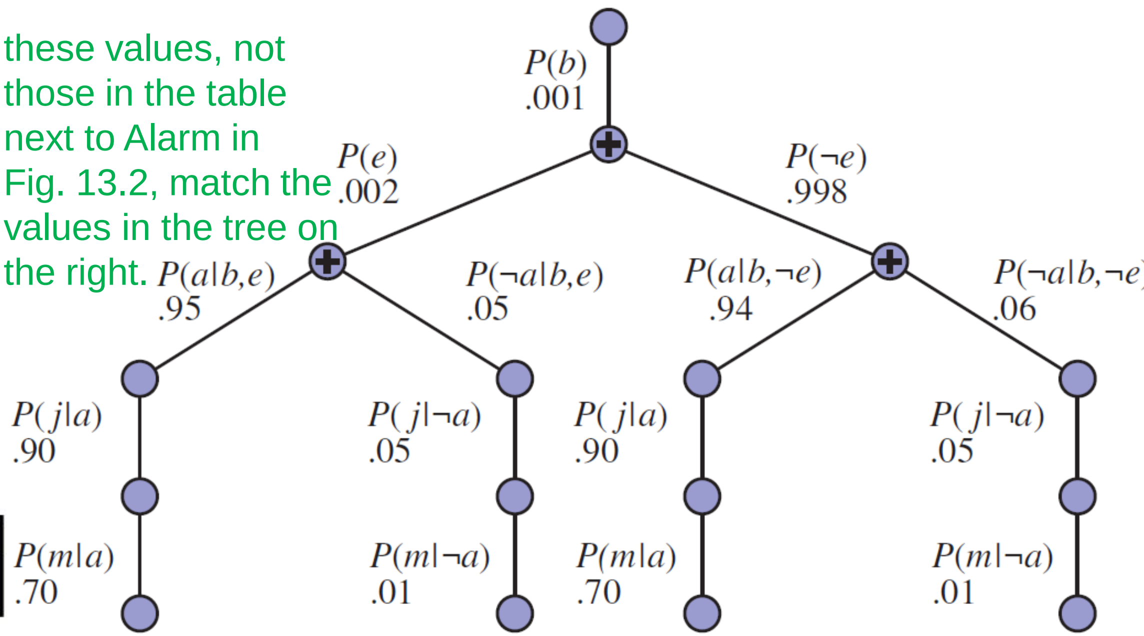

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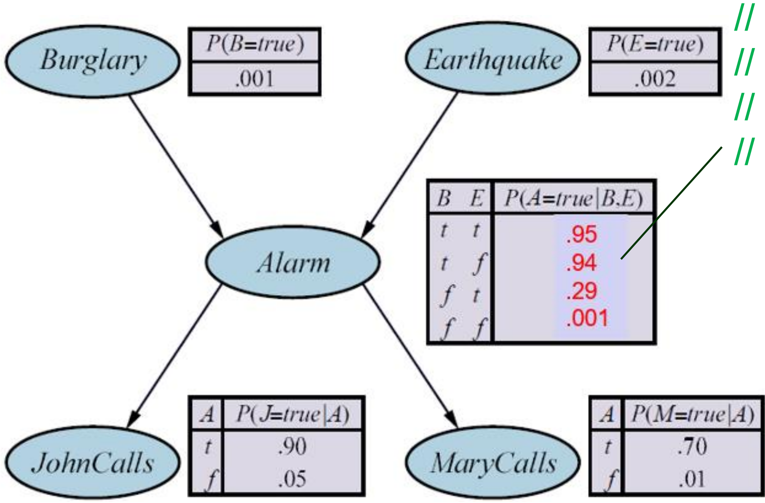

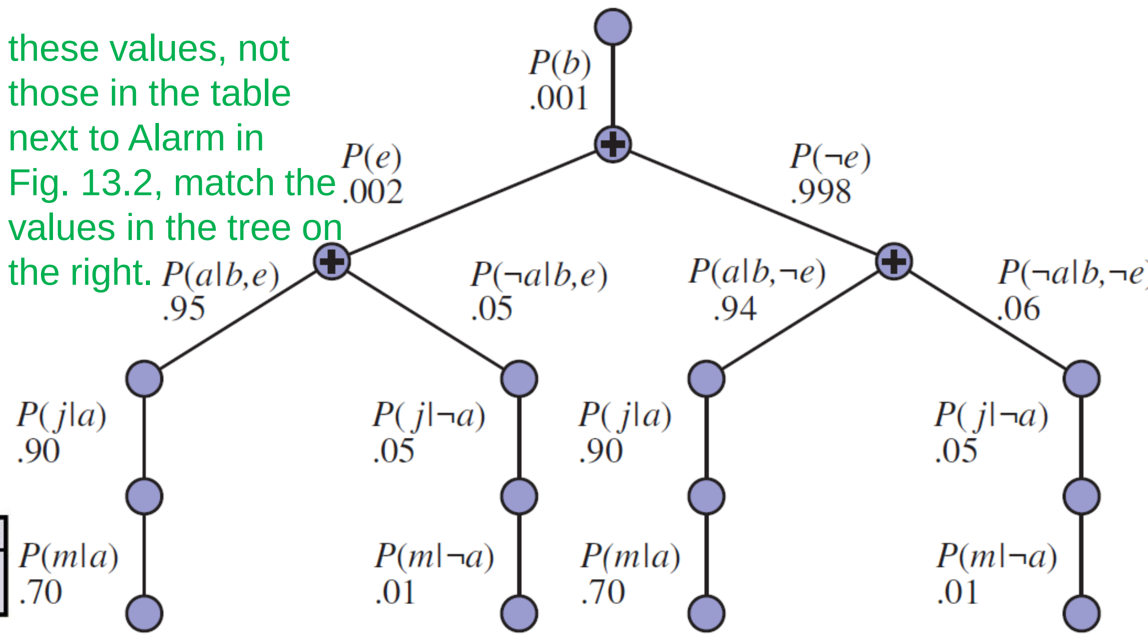

- Multiply values along each path.
- Sum at the “+” nodes.

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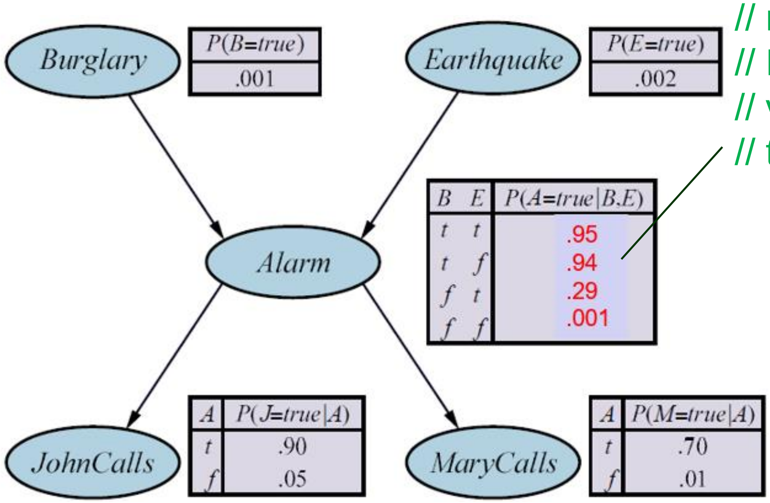

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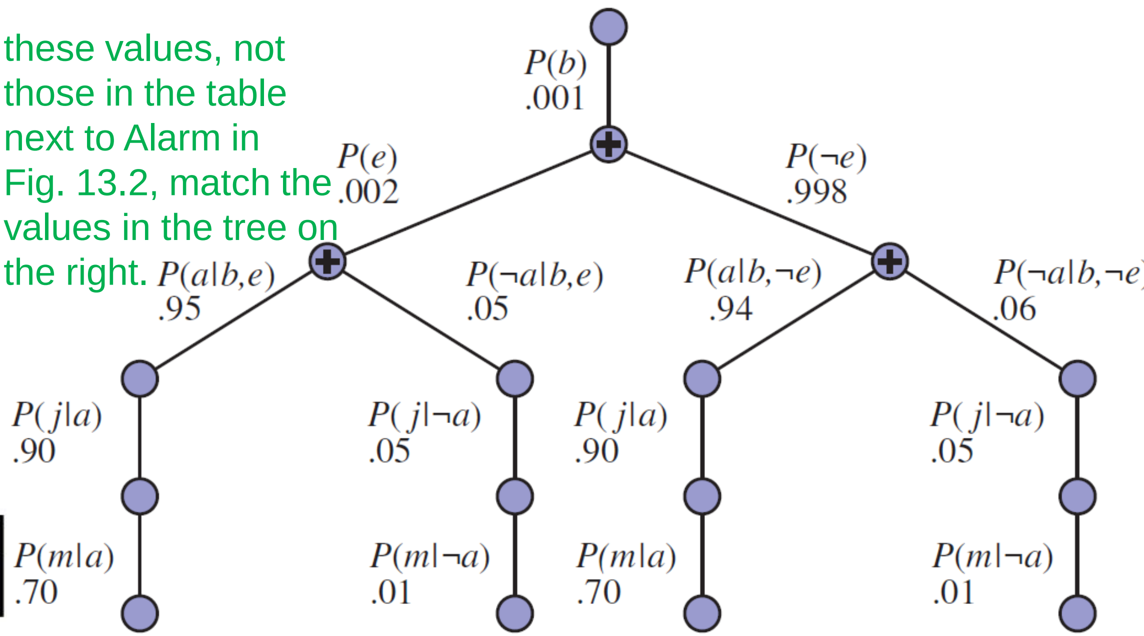

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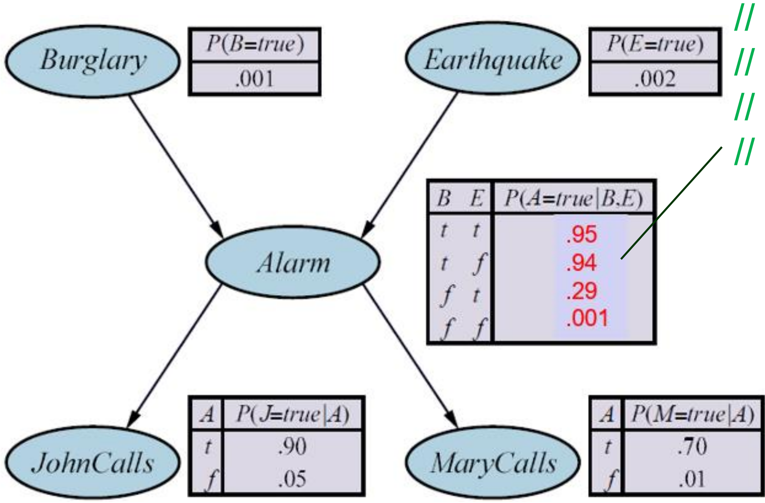

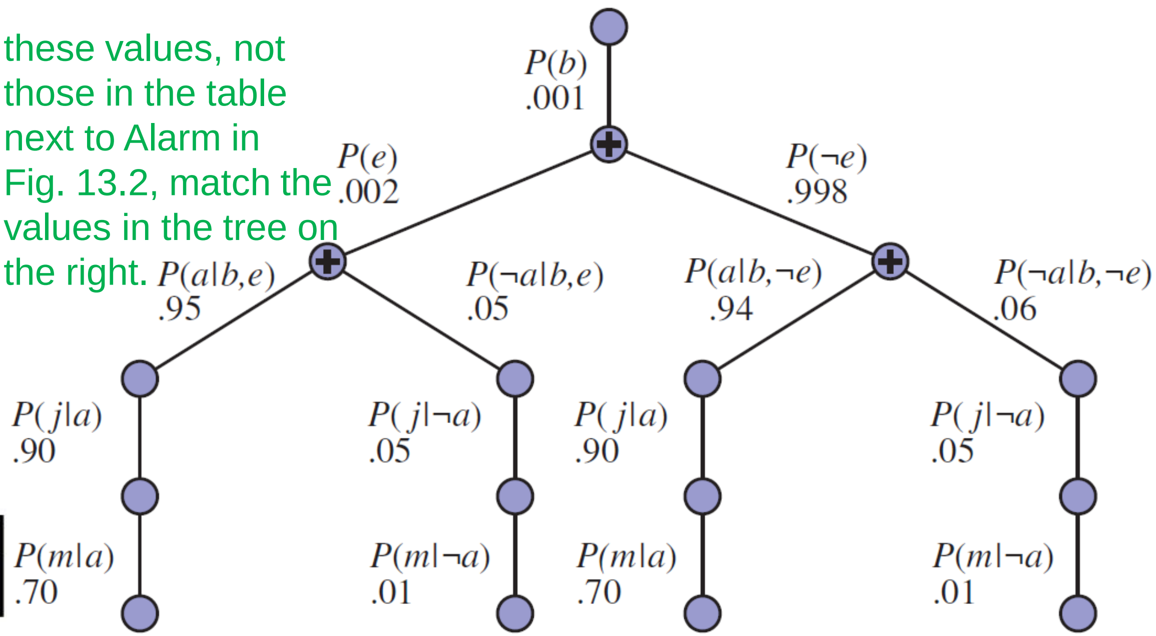

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(computed using a tree of the same structure but with b replaced by $\neg b$ and probabilities changed)

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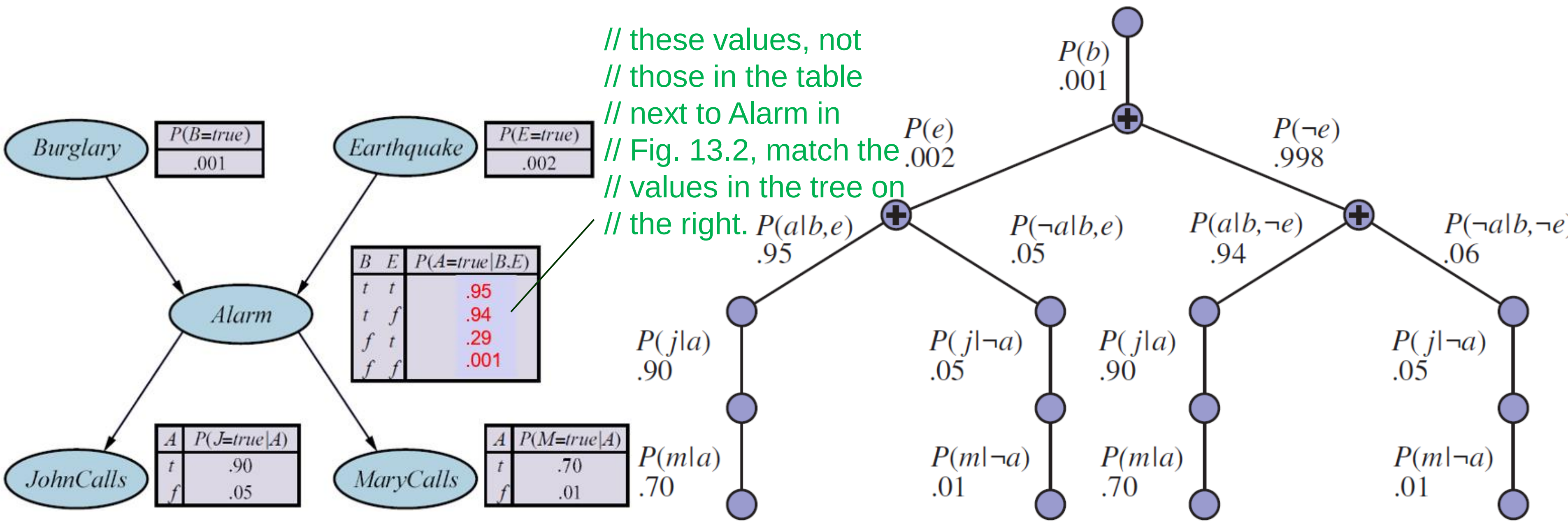

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$\Longrightarrow$ $\mathbf{P}(B \mid j,m) = \alpha\langle 0.00059224, 0.0014919\rangle \approx \langle 0.284, 0.716\rangle$

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Enumeration Algorithm

function ENUMERATION-ASK(X, $\mathbf{e}$, bn) **returns** a distribution over X
  inputs: X, the query variable
    $\mathbf{e}$, observed values for variables $\mathbf{E}$
    bn, a Bayes net with variables $vars$

  $\mathbf{Q}(X) \leftarrow$ a distribution over X, initially empty
  for each value x_i of X **do**
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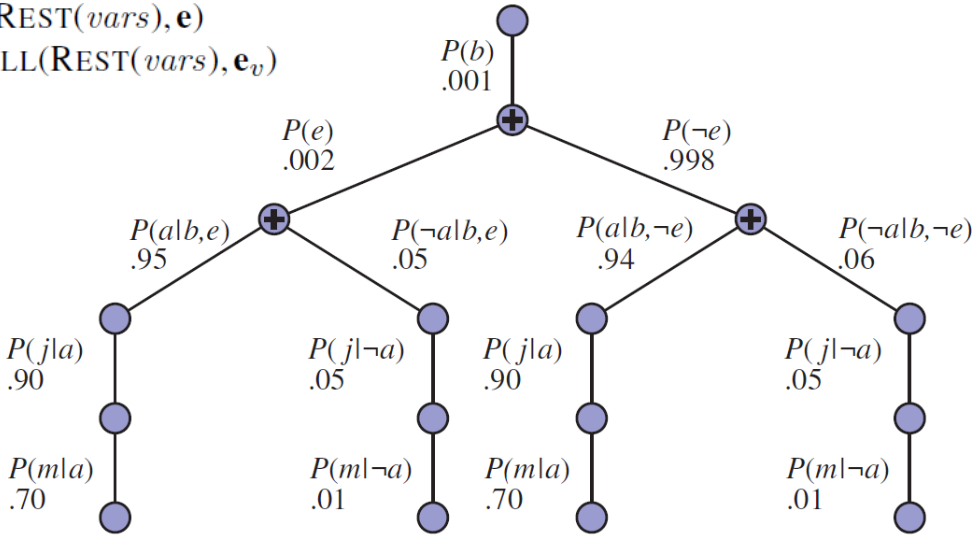

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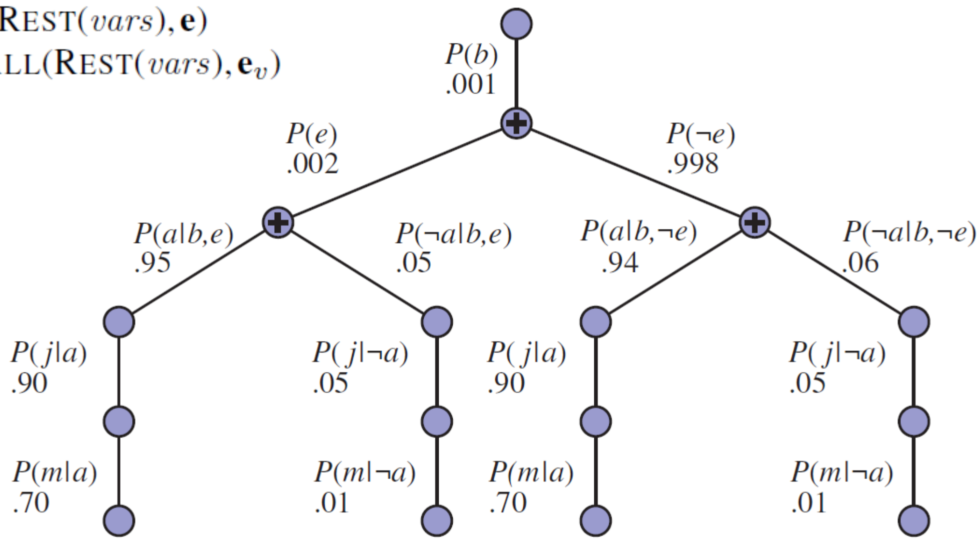

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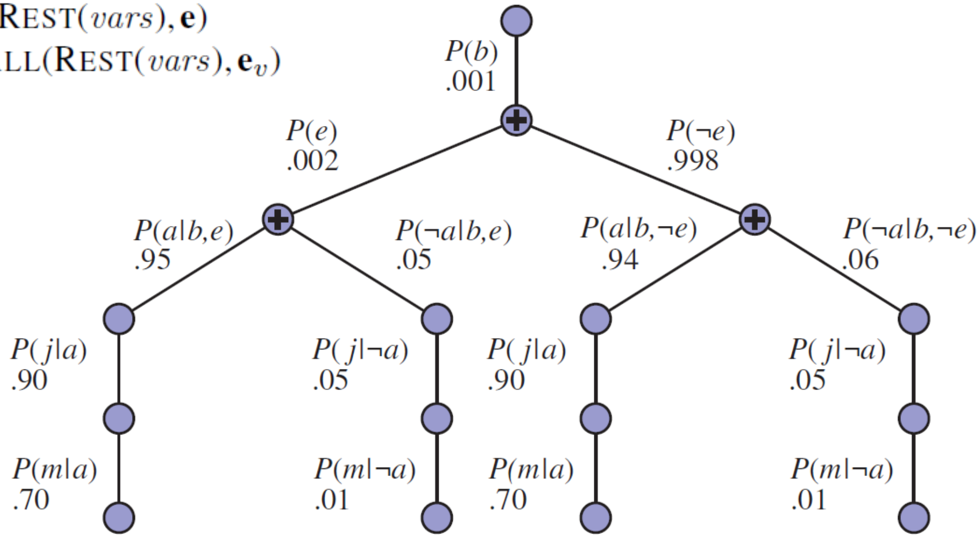

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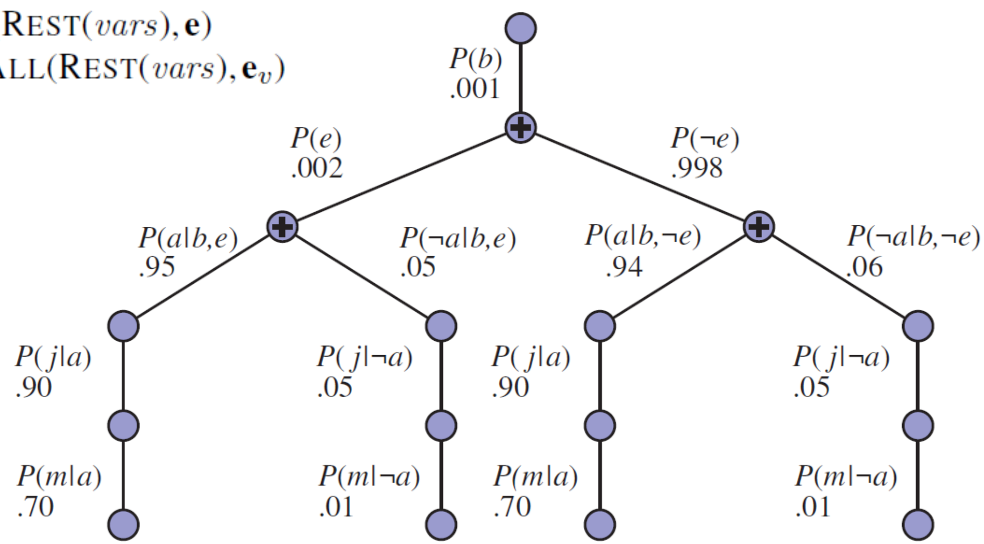

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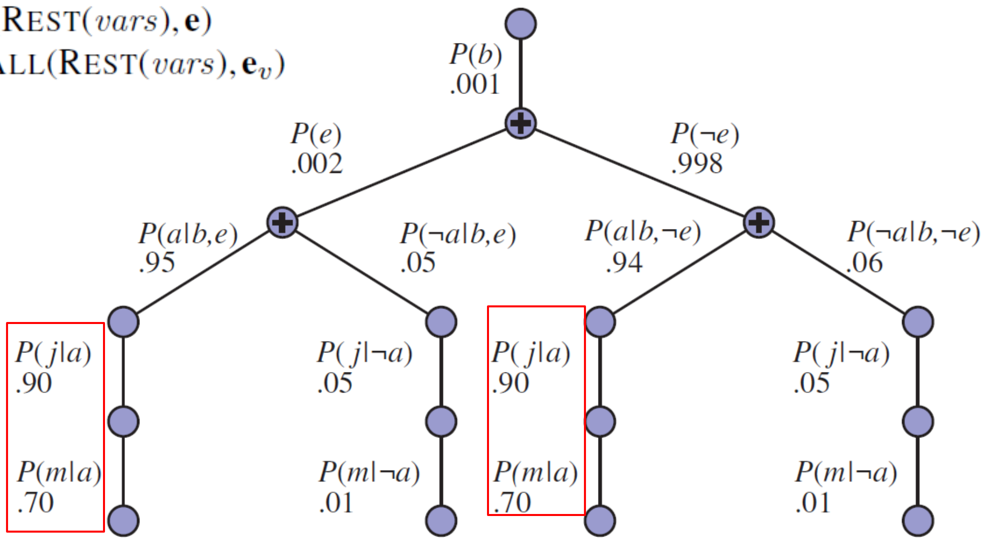

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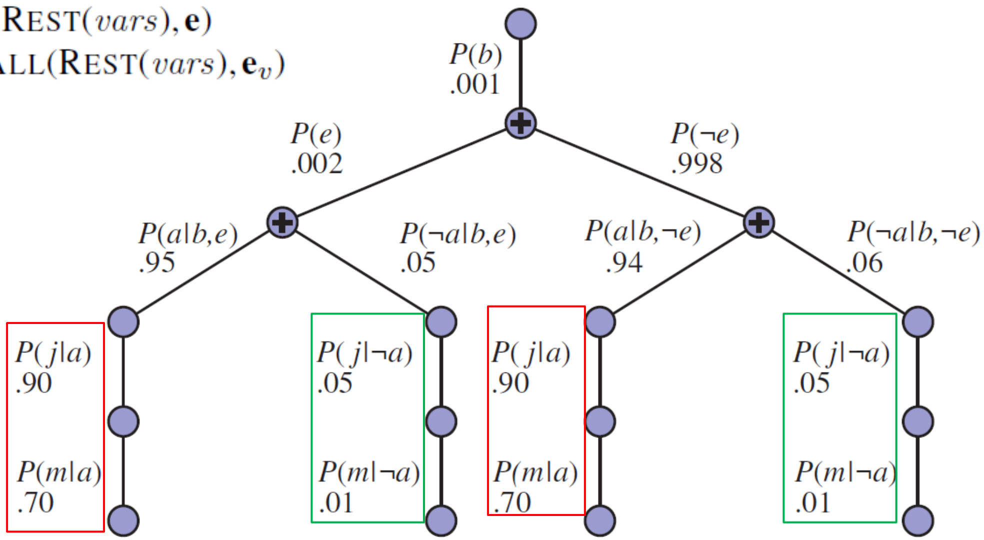

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factors:	$\boldsymbol{f}_1(B)$	$\boldsymbol{f}_2(E)$	$\boldsymbol{f}_3(A, B, E)$	$\boldsymbol{f}_4(A)$	$\boldsymbol{f}_5(A)$
dimensions:	2×1	2×1	$2 \times 2 \times 2$	2×1	2×1

Evaluation Example

$$\boldsymbol{P}(B \mid j, m) = \alpha \boldsymbol{f}_{\mathbf{1}}(B) \times \sum_{E} \boldsymbol{f}_{\mathbf{2}}(E) \times \sum_{A} \boldsymbol{f}_{\mathbf{3}}(A, B, E) \times \boldsymbol{f}_{\mathbf{4}}(A) \times \boldsymbol{f}_{\mathbf{5}}(A)$$

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2×2

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$$= \left(\underset{2 \times 2}{\boldsymbol{f}_3(a, B, E)} \times P(a) \times P(a)\right) + \left(\underset{2 \times 2}{\boldsymbol{f}_3(\neg a, B, E)} \times P(\neg a) \times P(\neg a)\right)$$

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$$\mathbf{P}(B \mid j, m) = \alpha \boldsymbol{f}_{\mathbf{1}}(B) \times \sum_{E} \boldsymbol{f}_2(E) \times \boldsymbol{f}_6(B, E)$$

- Sum out E from the product of $\boldsymbol{f}_2$ and $\boldsymbol{f}_6$.

$$\underset{1\times 2}{\boldsymbol{f}_7(B)} = \sum_{E \in \{e, \neg e\}} \boldsymbol{f}_{\mathbf{2}}(E) \times \boldsymbol{f}_{\mathbf{6}}(B, E) = P(e) \times \underset{1\times 2}{\boldsymbol{f}_6(B, e)} + P(\neg e) \times \underset{1\times 2}{\boldsymbol{f}_6(B, \neg e)}$$

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$$\mathbf{P}(B \mid j, m) = \alpha \boldsymbol{f}_{\mathbf{1}}(B) \times \sum_{E} \boldsymbol{f}_2(E) \times \boldsymbol{f}_6(B, E)$$

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- Finally, carry out the following pointwise product:

$$\mathbf{P}(B \mid j, m) = \alpha \boldsymbol{f}_{\mathbf{1}}(B) \times \boldsymbol{f}_{\mathbf{7}}(B)$$

Pointwise Product of Two Factors

$$\boldsymbol{f}(X_1,\ldots,X_j,Y_1,\ldots,Y_k)\times \boldsymbol{g}(Y_1,\ldots,Y_k,Z_1,\ldots,Z_l)=\boldsymbol{h}(X_1,\ldots,X_j,Y_1,\ldots,Y_k,Z_1,\ldots,Z_l)$$

X	Y	$\mathbf{f}(X,Y)$	Y	Z	$\mathbf{g}(Y,Z)$	X	Y	Z	$\mathbf{h}(X,Y,Z)$
t	t	.3	t	t	.2	t	t	t	$.3 \times .2 = .06$
t	f	.7	t	f	.8	t	t	f	$.3 \times .8 = .24$
f	t	.9	f	t	.6	t	f	t	$.7 \times .6 = .42$
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common variables

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Variable Elimination Algorithm

Move outside the summation any factor independent of the variable to be summed out.

$$\sum_X \boldsymbol{f}(X,Y) \times \boldsymbol{g}(Y,Z) = \boldsymbol{g}(Y,Z) \times \sum_X \boldsymbol{f}(X,Y)$$

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- Use a greedy heuristic: eliminate whichever variable minimizes the size of the next factor to be constructed.

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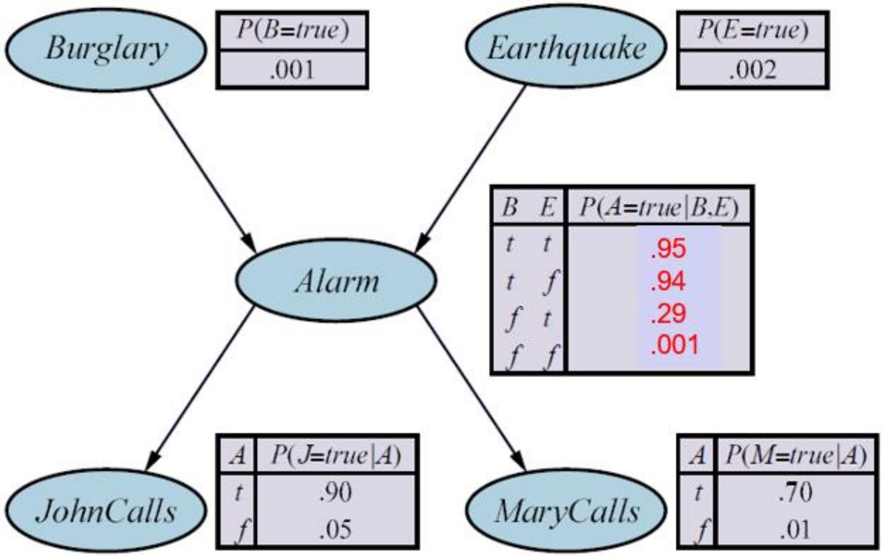

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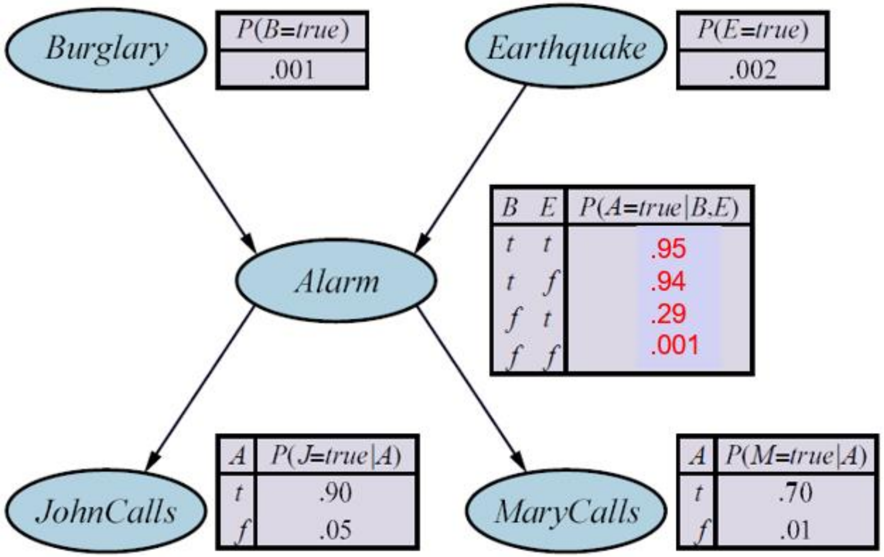
$$\mathbf{P}(J \mid b) = \alpha P(b) \sum_{E} P(E) \sum_{A} P(A \mid b, E)\, \mathbf{P}(J \mid A) \sum_{M} P(M \mid A)$$



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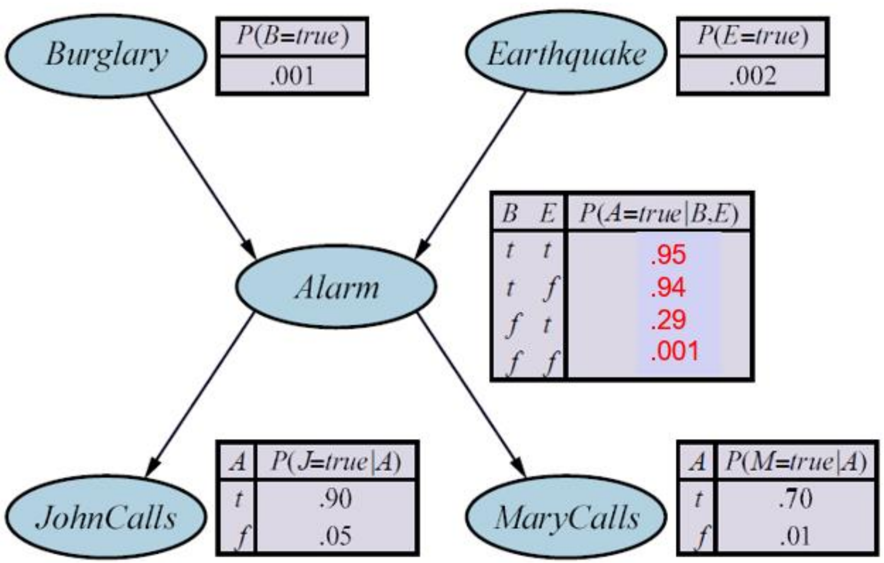

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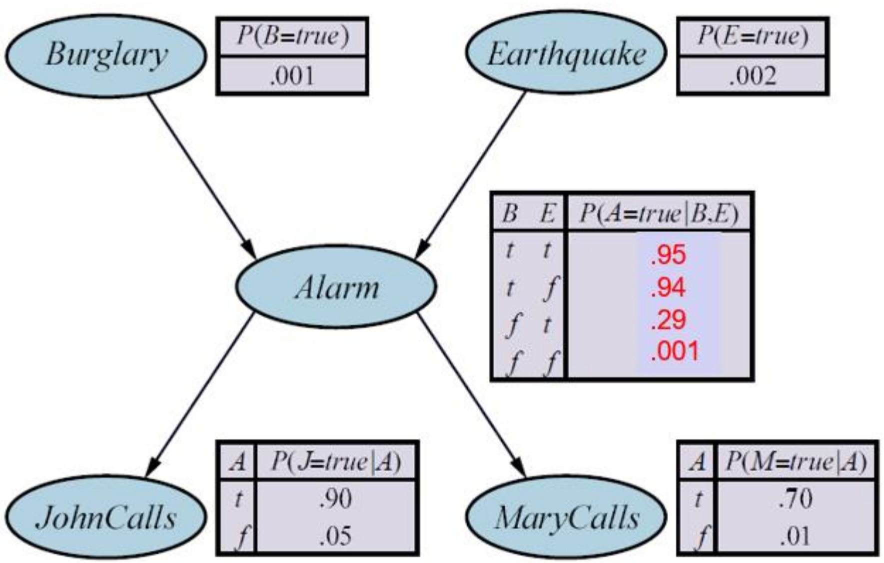

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$\Downarrow$

The variable M is irrelevant to the query.

Inference with Elimination

- We can remove a leaf node (e.g., M) that is neither a query variable nor an evidence variable.

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- ♦ Using *reverse topological order* for variables, exact inference with elimination can be 1,000 times faster than the enumeration algorithm.

- ♦ If we want to compute posterior probabilities for all the variables rather than answer individual queries, we can use clustering algorithms (i.e., join tree algorithms).