Using First-Order Logic (FOL)

Outline

I. The wumpus world in FOL

II. Knowledge engineering process

III. Instantiation and Skolemization

* Figures are from the textbook site unless a source is specifically cited.

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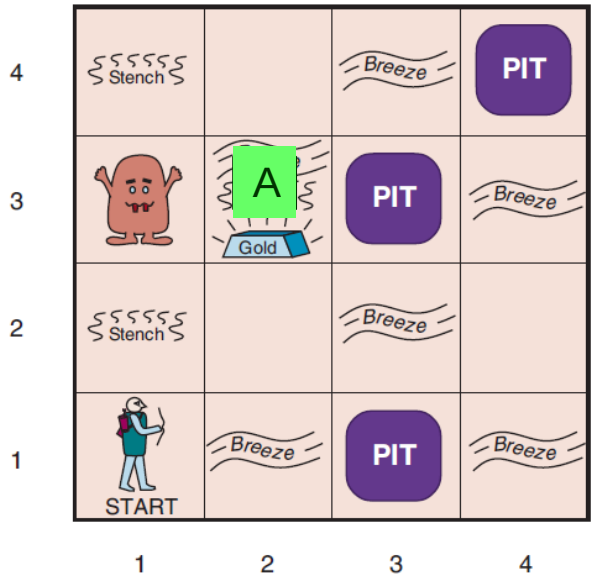

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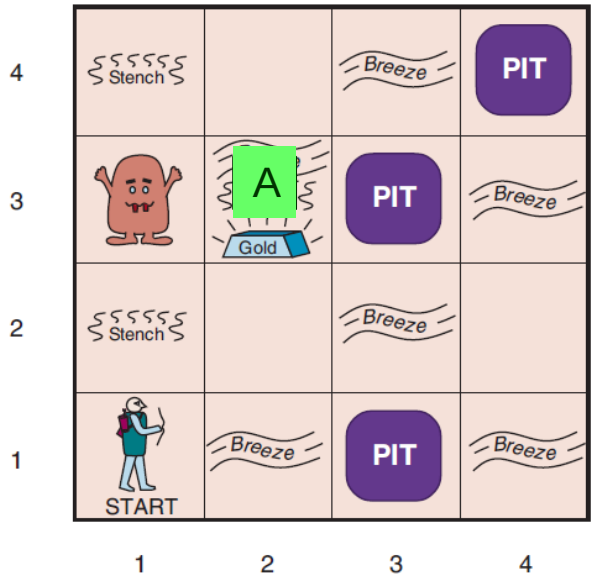

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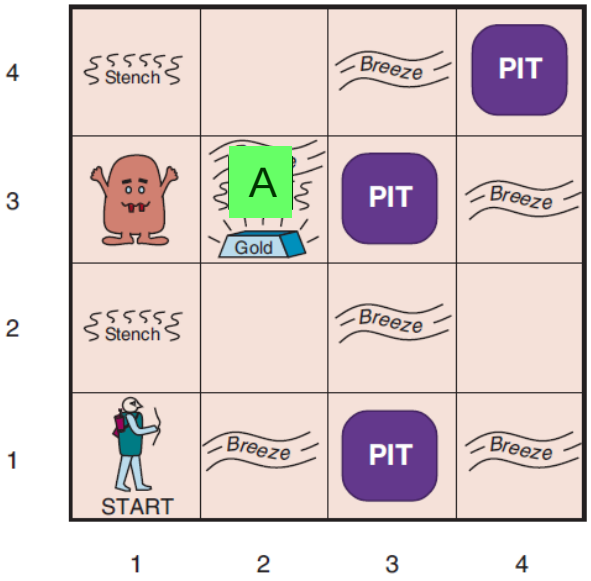

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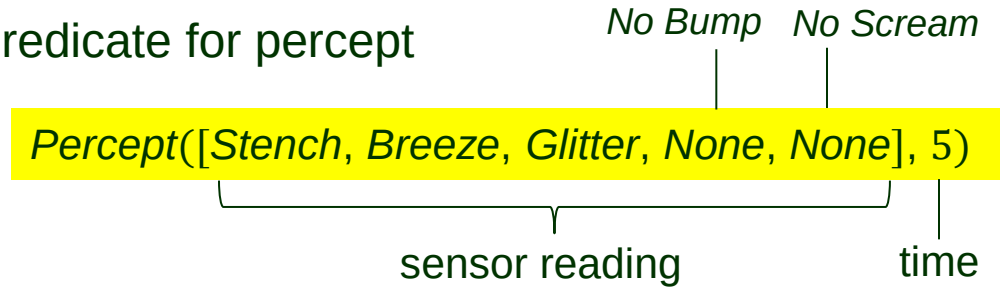

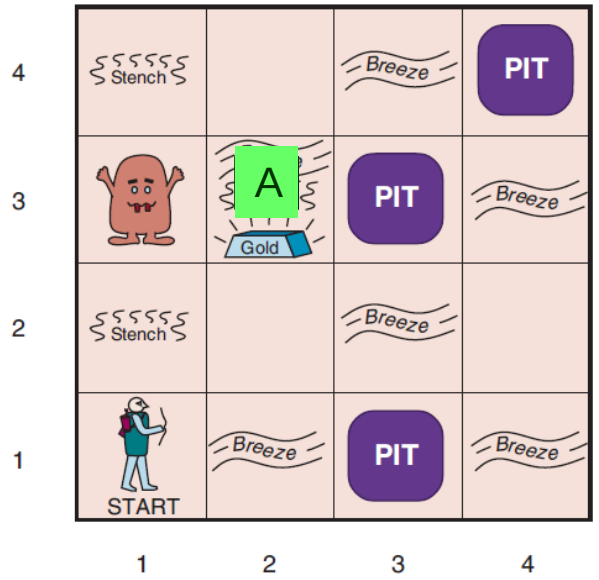

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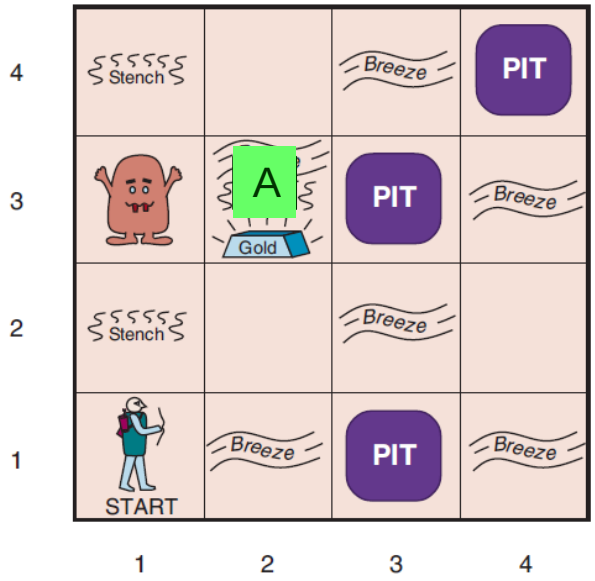

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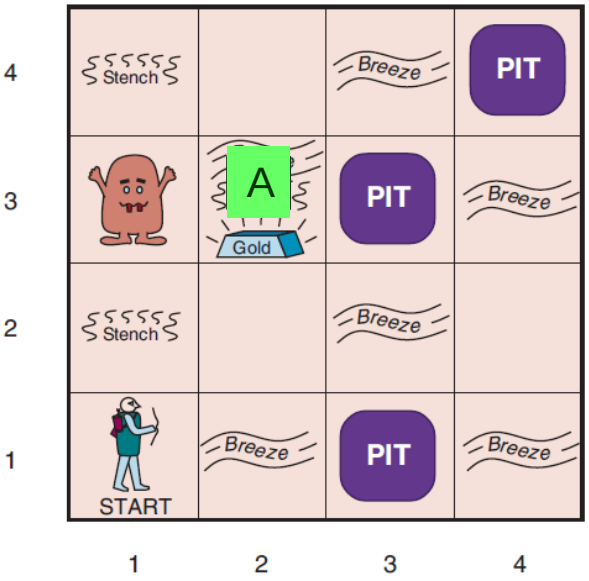
Turn(*Right*), *Turn*(Left), *Forward*, *Shoot*, *Grab*, *Climb*

- Querying the KB for best action

ASKVARS(*KB*, *BestAction*(a, 5))

A binding list, e.g., {a/*Grab*}, is returned.



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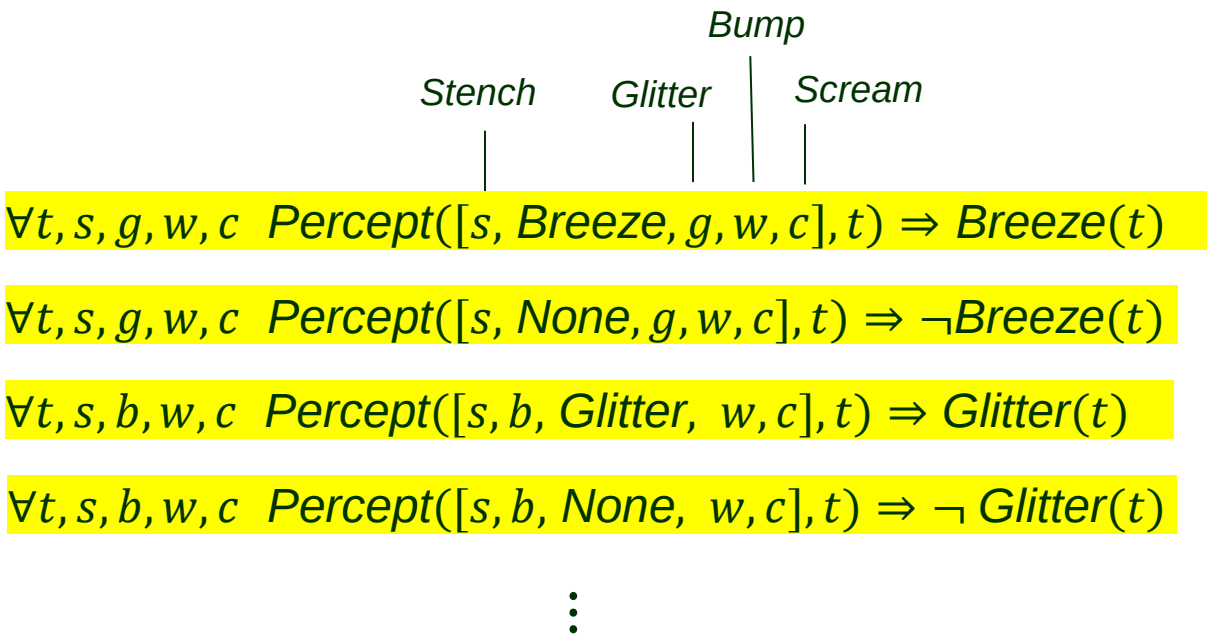

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⋮

$\forall t \;\; Glitter(t) \Rightarrow BestAction(Grab, t)$ // simple “reflex” behavior

Rules for the Environment

- Adjacency of two squares

square at row x and column y

$$\forall x, y, a, b \;\; \textit{Adjacent}([x, y], [a, b]) \Leftrightarrow (x = a \wedge (y = b - 1 \vee y = b + 1)) \vee (y = b \wedge (x = a - 1 \vee x = a + 1))$$

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The first-order logic formulation is no less concise than the English description.

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- Debugging and evaluation of the KB.

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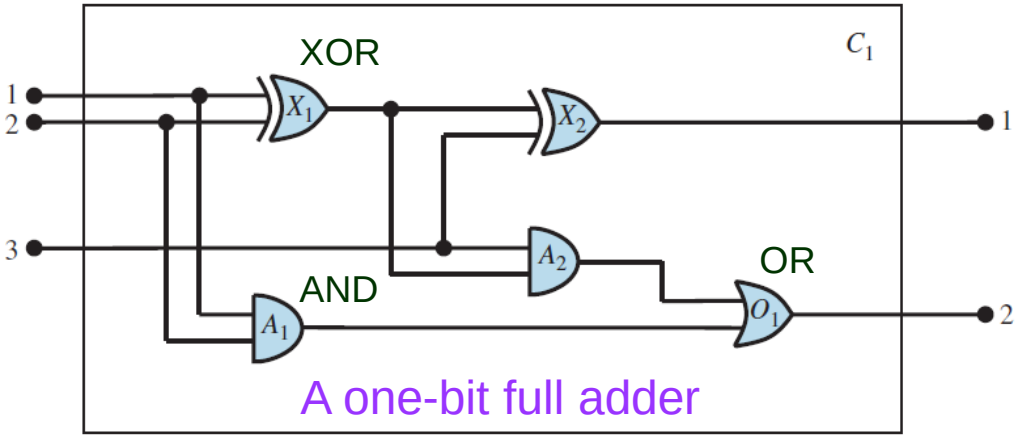

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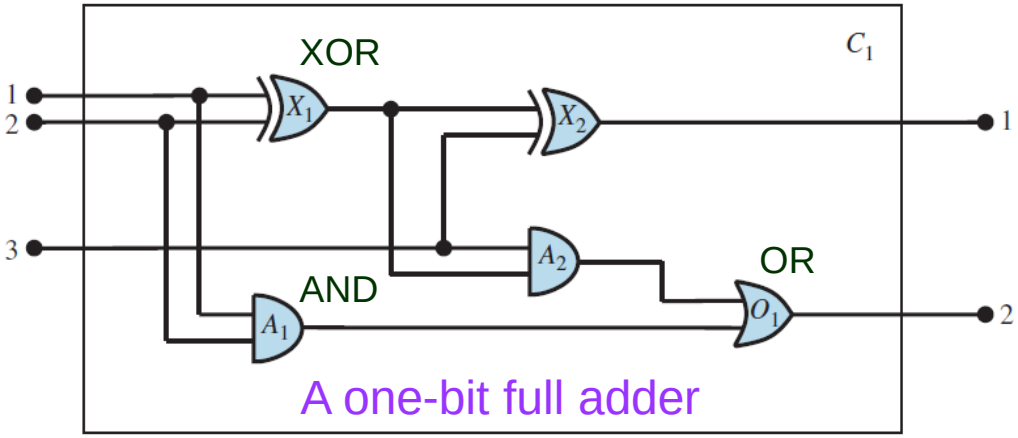

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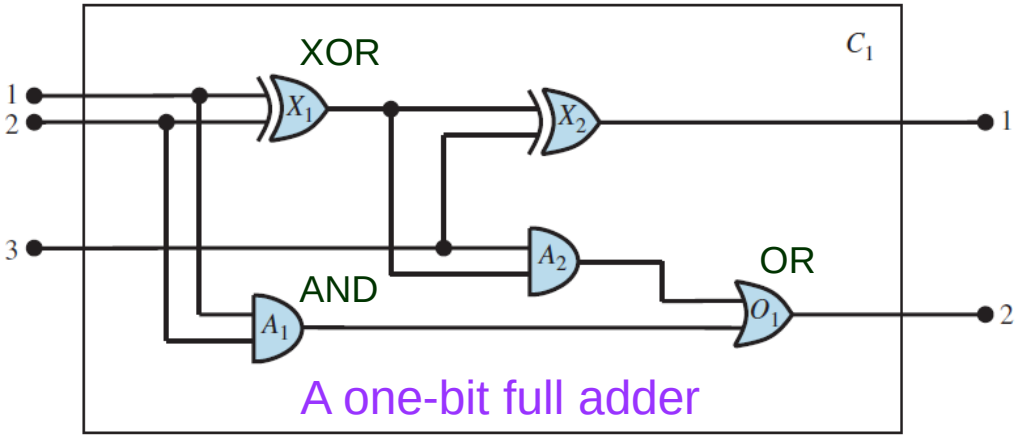

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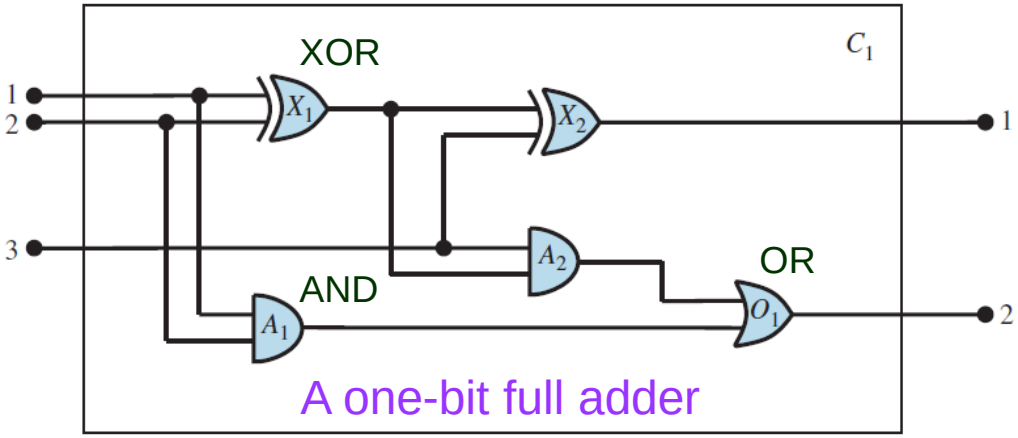

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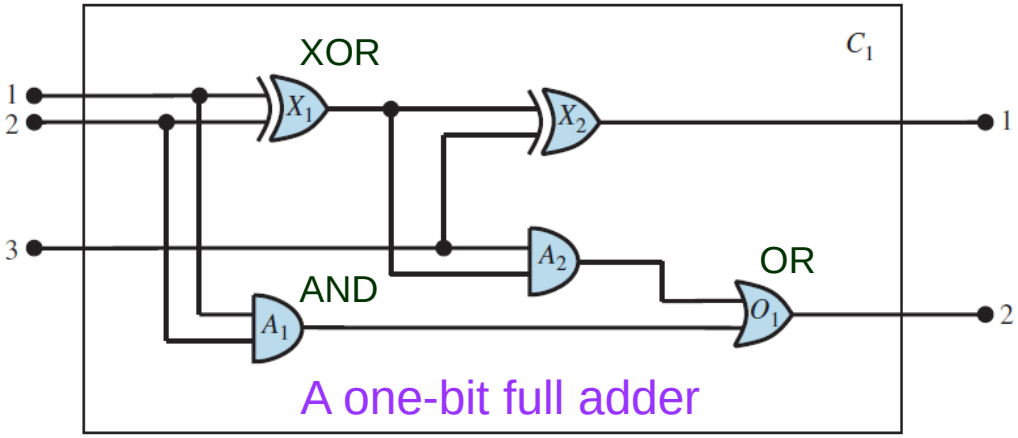

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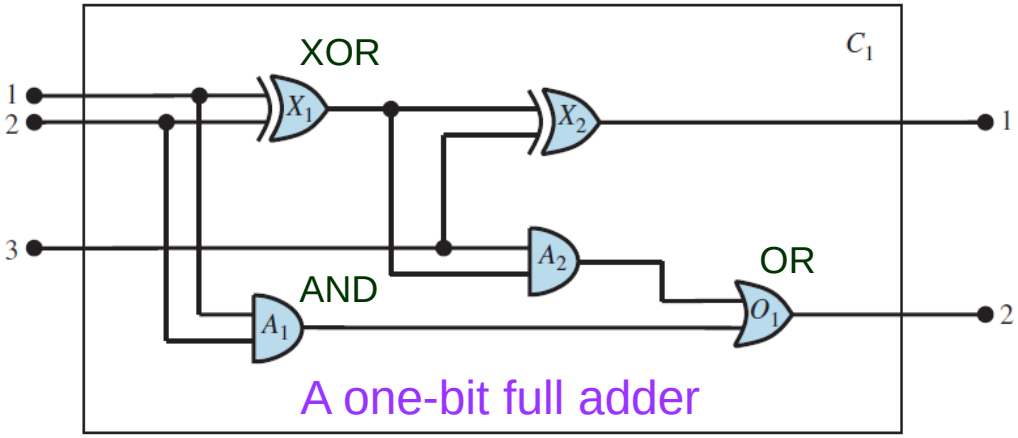

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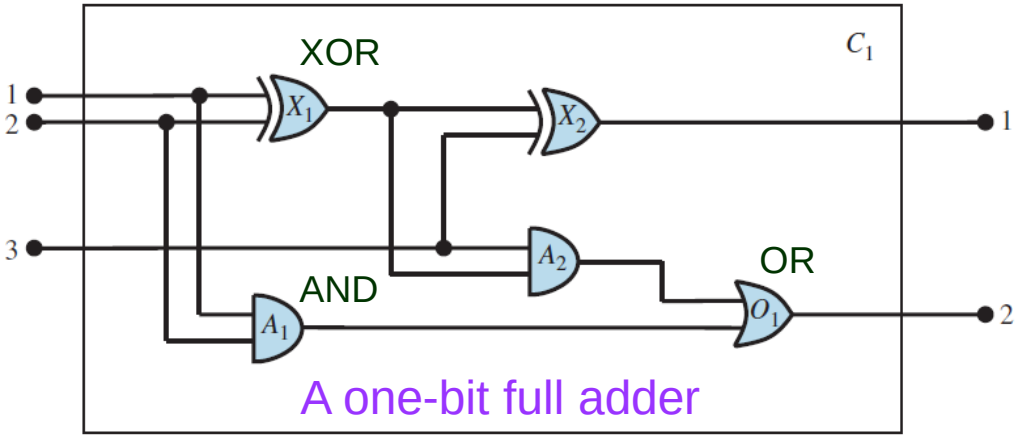

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- Signal function: *Signal*(t) has value 1 or 0 at time t.

Encoding General Domain Knowledge

// Two connected terminals have the same signal.

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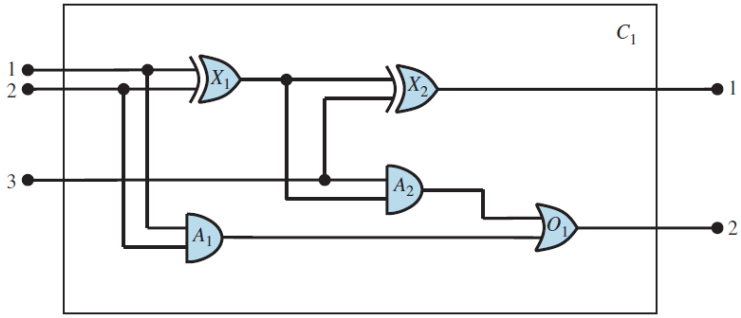

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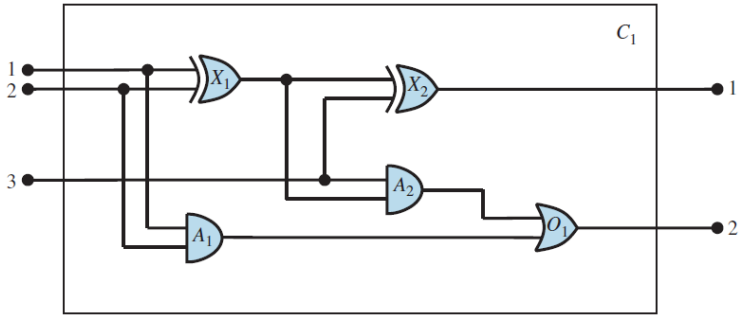

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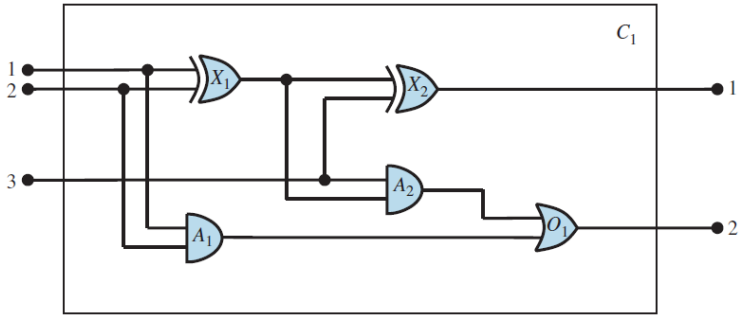

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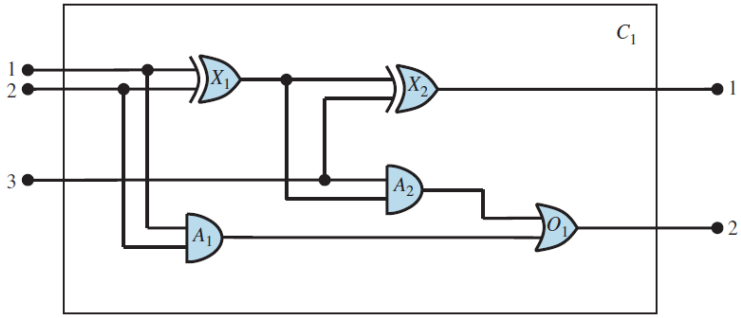

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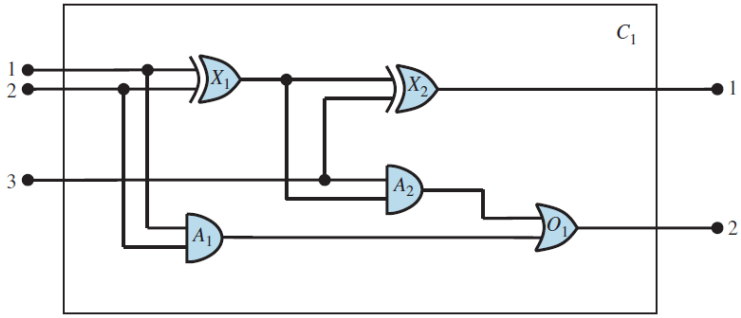

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// An AND gate outputs 0 if and only if any of its input is 0.

$\forall g \quad Gate(g) \wedge Type(g) = AND \Rightarrow (Signal(Out(1, g)) = 0 \Leftrightarrow \exists n \; Signal(In(n, g)) = 0)$

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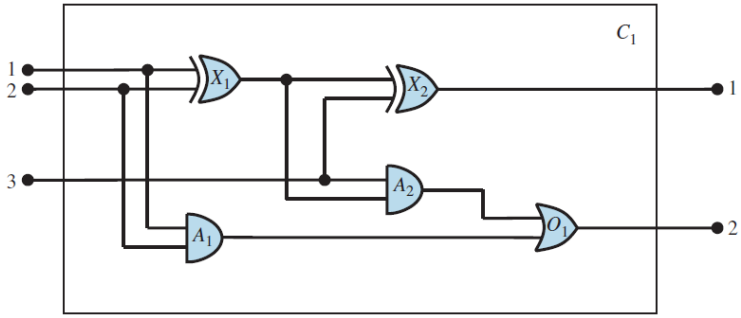
$\forall t \quad Terminal(t) \Rightarrow Signal(t) = 1 \vee Signal(t) = 0$



// Connectivity is commutative.

$\forall t_1, t_2 \quad Connected(t_1, t_2) \Leftrightarrow Connected(t_2, t_1)$

// Four types of gates.

$\forall g, k \quad Gate(g) \wedge k = Type(g) \Rightarrow k = AND \vee k = OR \vee k = XOR \vee k = NOT$

// An AND gate outputs 0 if and only if any of its input is 0.

$\forall g \quad Gate(g) \wedge Type(g) = AND \Rightarrow (Signal(Out(1, g)) = 0 \Leftrightarrow \exists n \; Signal(In(n, g)) = 0)$

// An OR gate outputs 1 if and only if any of its input is 1.

$\forall g \quad Gate(g) \wedge Type(g) = OR \Rightarrow (Signal(Out(1, g)) = 1 \Leftrightarrow \exists n \; Signal(In(n, g)) = 1)$

cont'd

// An XOR gate outputs 1 if and only if its inputs are different.

$$\forall g \;\; Gate(g) \wedge Type(g) = XOR \Rightarrow (Signal(Out(1,g)) = 1 \Leftrightarrow Signal(In(1,g)) \neq Signal(In(2,g)))$$

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$\forall g \;\; Gate(g) \wedge Type(g) = NOT \Rightarrow Arity(g,1,1)$

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$$\forall c,i,j \;\; Circuit(c) \wedge Arity(c,i,j) \Rightarrow$$
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function not predicate

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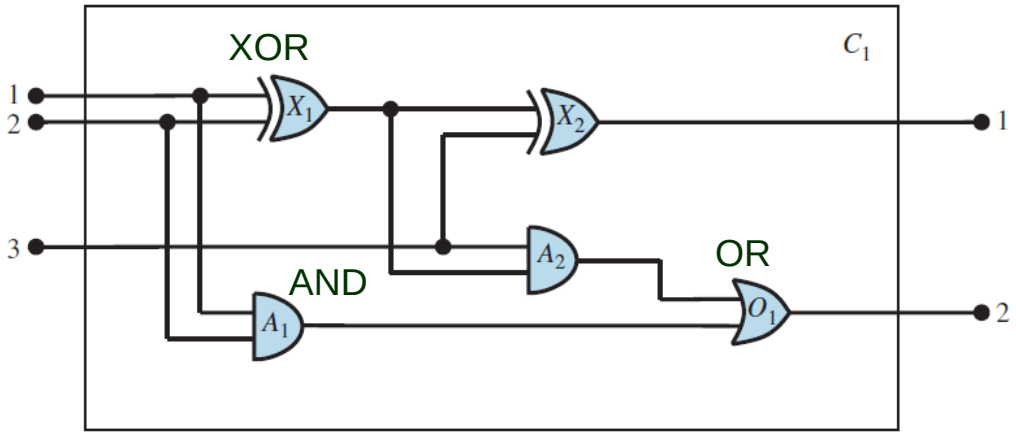
// Gates are circuits.

$\forall g \;\; Gate(g) \Rightarrow Circuit(g)$

(errors/typos in the text)

Encoding a Problem Instance



- Circuit and component gates:

$Circuit(C_1) \wedge Arity(C_1, 3, 2)$

$Gate(X_1) \wedge Type(X_1) = XOR$

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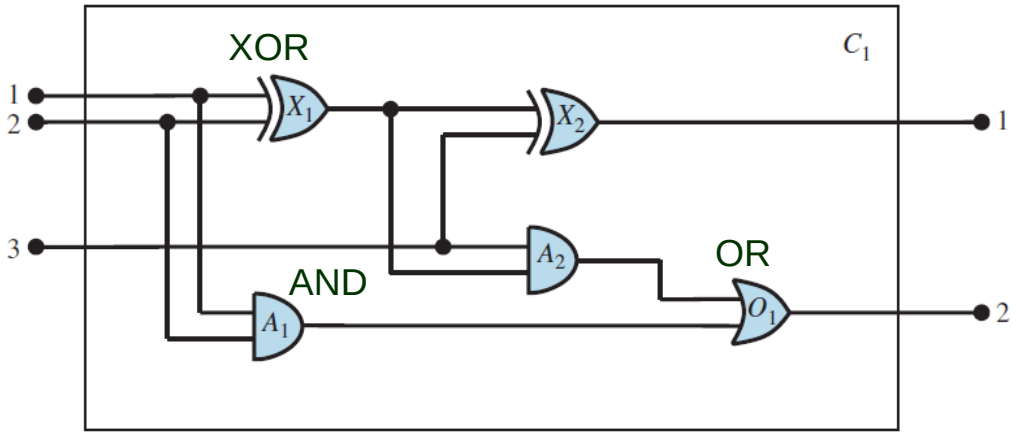

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- Connections between the circuit and component gates:

$Connected(Out(1, X_1), In(1, X_2))$

$Connected(Out(1, X_1), In(2, A_2))$

$Connected(Out(1, A_2), In(1, O_1))$

$Connected(Out(1, A_1), In(2, O_1))$

$Connected(Out(1, X_2), Out(1, C_1))$

$Connected(Out(1, O_1), Out(2, C_1))$

$Connected(In(1, C_1), In(1, X_1))$

$Connected(In(1, C_1), In(1, A_1))$

$Connected(In(2, C_1), In(2, X_1))$

$Connected(In(2, C_1), In(2, A_1))$

$Connected(In(3, C_1), In(2, X_2))$

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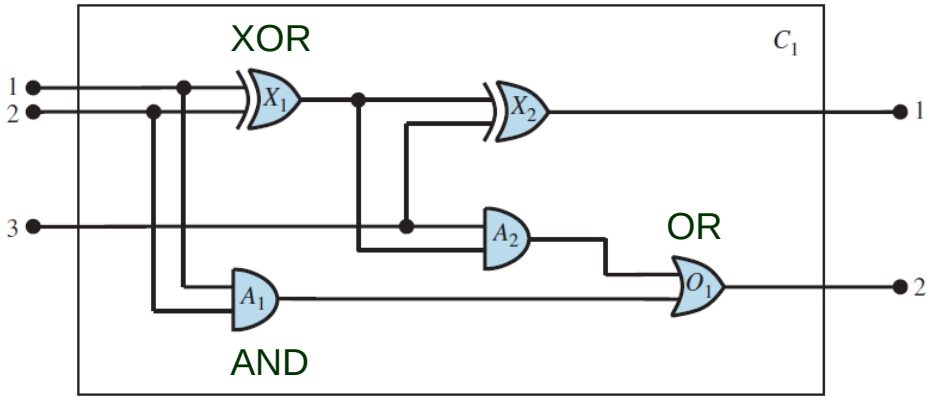
Queries



Q. What combinations of inputs would cause the first output of C_1 to be 0 and its second output to be 1?

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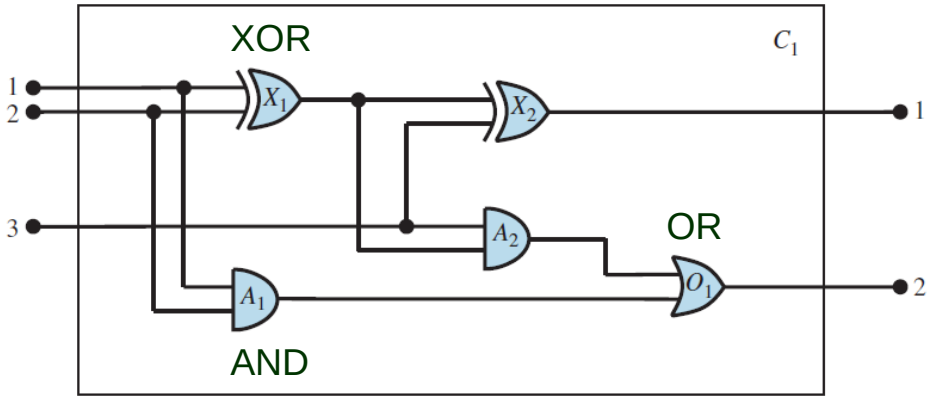

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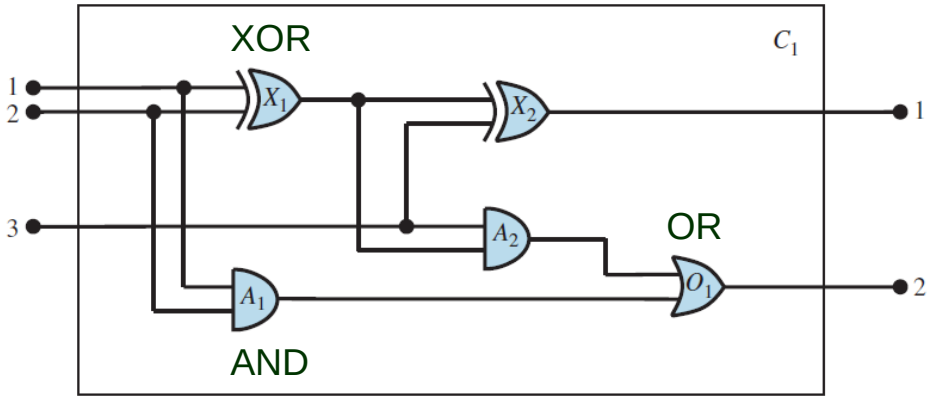

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ASKVARS will give three substitutions as answers.

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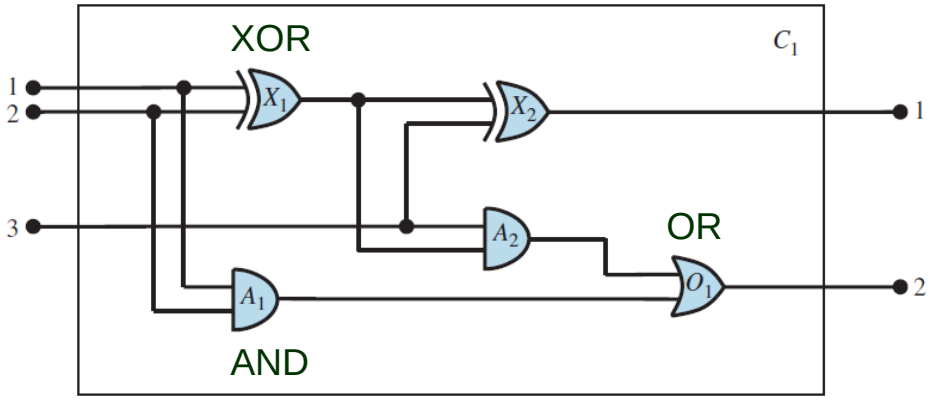

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Debugging: We can also perturb the KB to see what erroneous behaviors would emerge, and then identify missing rules for instance.

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$\forall x \ \mathit{Bird}(x) \Rightarrow \mathit{WarmBlooded}(x) \wedge \mathit{HaveWings}(x)$

We can infer (if $\mathit{Bird}(\mathit{Ostrich})$ and $\mathit{Bird}(\mathit{Peacock})$ are in the KB):

$\mathit{WarmBlooded}(\mathit{Ostrich}) \wedge \mathit{HaveWings}(\mathit{Ostrich})$

$\mathit{WarmBlooded}(\mathit{Peacock}) \wedge \mathit{HaveWings}(\mathit{Peacock})$

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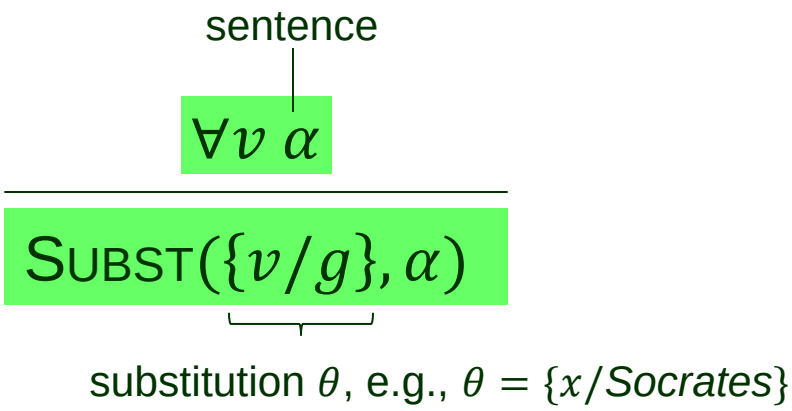

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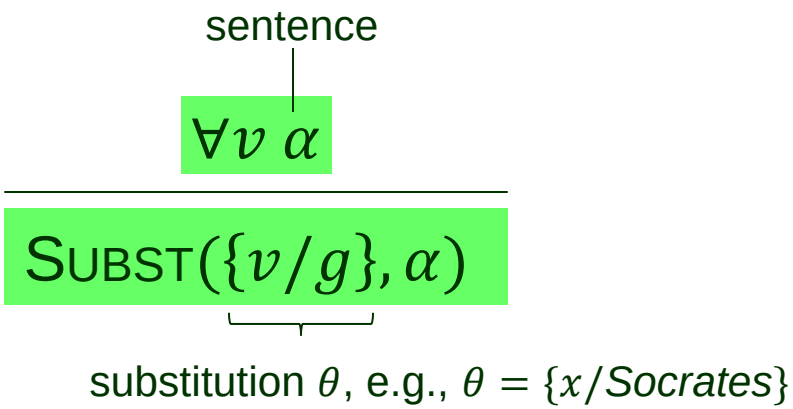

E.g., $\theta = \{x/Ostrich\}$

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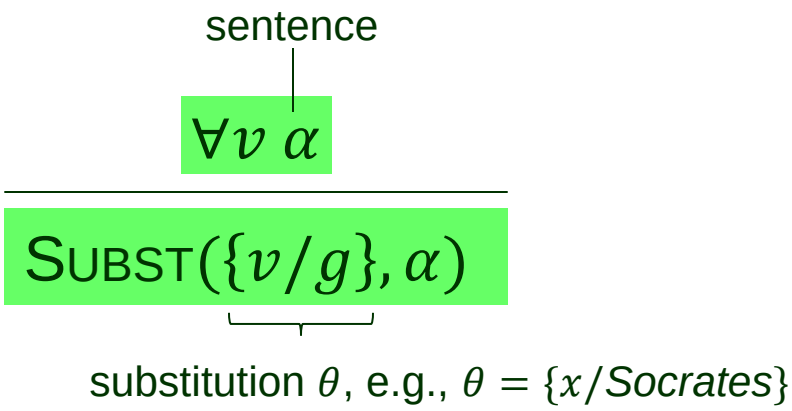

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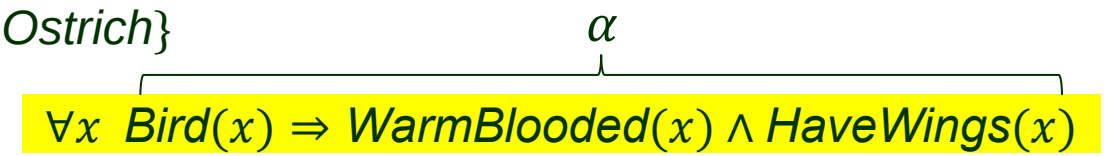

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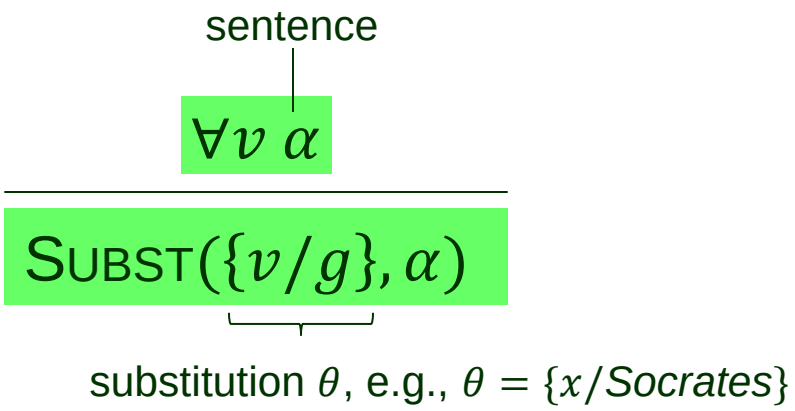

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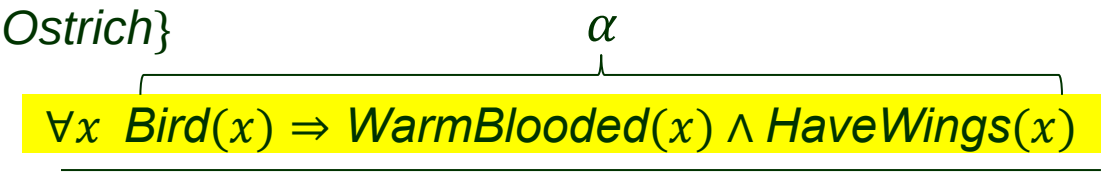

SUBST(θ, α) $\equiv$ $Bird(Ostrich) \Rightarrow WarmBlooded(Ostrich) \wedge HaveWings(Ostrich)$

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Skolem constant

Propositionalization (Non-Standard Way)

$\forall x \forall y \quad Ancestor(x, y) \Rightarrow Parent(x, y) \vee \exists z (Ancestor(x, z) \wedge Ancestor(z, y))$

KB:

$Ancestor(John, David)$

$Parent(John, David)$

$Parent(David, Lisa)$

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$Ancestor(John, David)$
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$Ancestor(John, Lisa)$
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$\exists y \;\; \textit{Mother}(y, \textit{Sophia})$

⟹ new unary function *mom*()

$\textit{Mother}(\textit{mom}(\textit{Liam}), \textit{Liam})$

$\textit{Mother}(\textit{mom}(\textit{Sophia}), \textit{Sophia})$

Skolemization (Standard Way)

A more standard way to eliminate an existential quantifier is to introduce a new function symbol, which is, however, not applicable in generating a PL sentence.

$\exists y \;\; P(y, x_1, \ldots, x_n)$ // y depends on $x_1, \ldots, x_n$

⇩ eliminate y by introducing function f

$P(f(x_1, \ldots, x_n), x_1, \ldots, x_n)$

- That $P(y, x_1, \ldots, x_n) = \textit{true}$ implicitly defines y as a function of $x_1, \ldots, x_n$ (analogous to the implicit function theorem in multivariate calculus).

$\exists y \;\; \textit{Mother}(y, \textit{Liam})$

$\exists y \;\; \textit{Mother}(y, \textit{Sophia})$

new unary function *mom*() ⟹

$\textit{Mother}(\textit{mom}(\textit{Liam}), \textit{Liam})$

$\textit{Mother}(\textit{mom}(\textit{Sophia}), \textit{Sophia})$

- Advantage: one function instead of two new constants to denote the moms of Liam and Sophia.