Agent Based on Propositional Logic

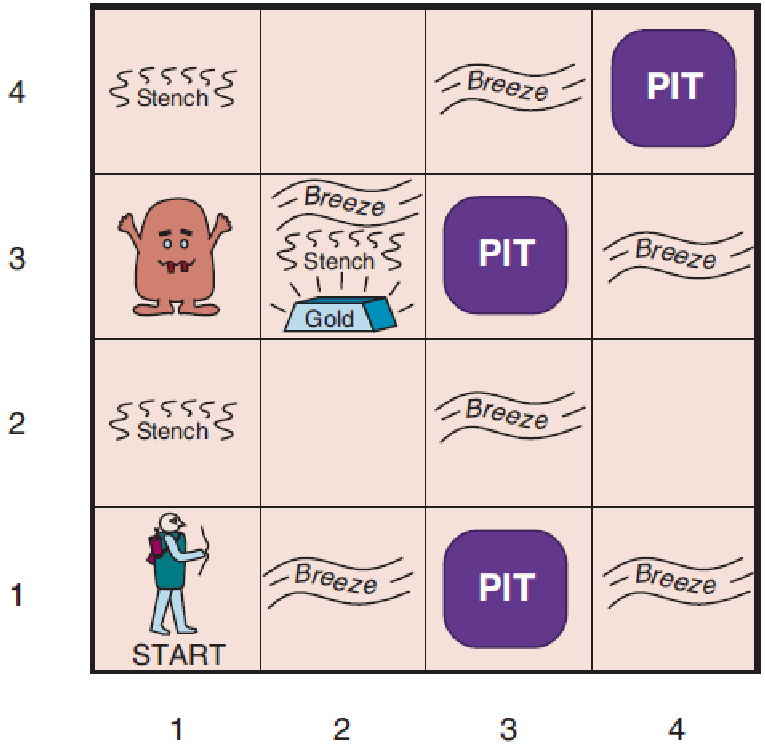
- Write down a complete logical model of the effects of action.
- How logical inference can be used by an agent?.
- How to keep track of the world without resorting to inference history?
- How to use logical inference to construct plans based on the KB?

Knowledge base (KB):

- ♣ general knowledge about how the world works
- ♣ percept sentences obtained in a particular world

* Figures are from the textbook site unless a source is specifically cited.

Current State in the Wumpus World



Axioms (given without being derived):

$\neg P_{1,1}$ $\neg W_{1,1}$

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ // 16 rules of this type

$S_{1,1} \Leftrightarrow (W_{1,2} \vee W_{2,1})$ // 16

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- $P_{x,y}$ = *true* if there is a pit in $[x, y]$.
- $W_{x,y}$ = *true* if there is a wumpus in $[x, y]$, dead or alive.
- $B_{x,y}$ = *true* if the agent perceives a breeze in $[x, y]$.
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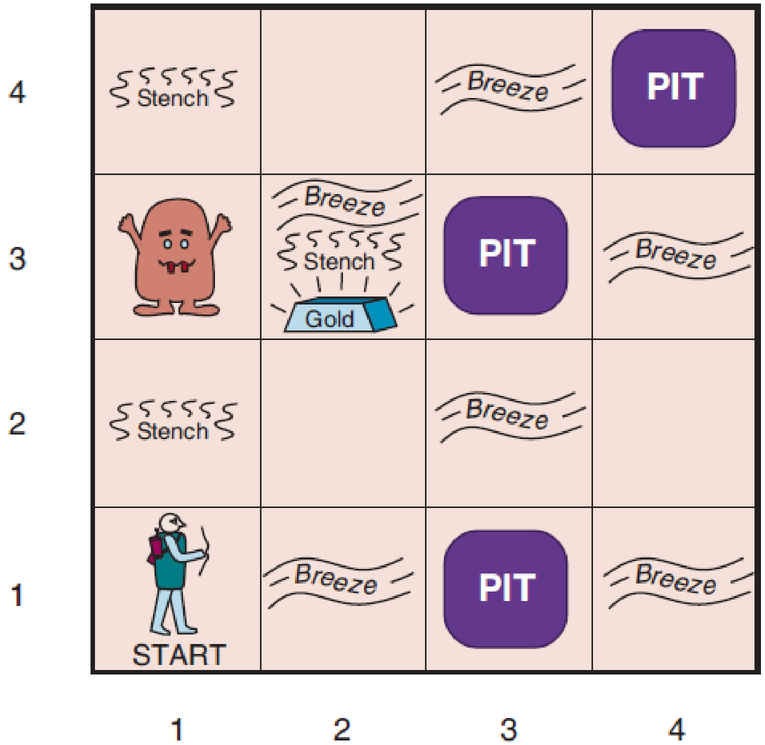

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- Exactly one wumpus

$W_{1,1} \vee W_{1,2} \vee \cdots \vee W_{4,3} \vee W_{4,4}$ // ≥ 1 wumpus

$\neg W_{i,j} \vee \neg W_{k,l}$ $1 \leq i,j,k,l \leq 4$ and $(i,j) \neq (k,l)$

// ≤ 1 Wumpus;

// $\binom{16}{2} = \frac{16 \times 15}{2} = 120$ rules

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Representing Percepts

A percept asserts something only about the current time.

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$O(mn)$ frame axioms for m actions and n fluents

Axioms for Successor States

Successor-state axiom, one for every fluent F, states that

- either the action at t causes F to be true at $t+1$,
- or F was already true at t and the action does not cause it to be false.

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$$L_{1,1}^{t+1} \Leftrightarrow (L_{1,1}^t \land (\neg \mathit{Forward}^t \lor \mathit{Bump}^{t+1})) \lor (L_{1,2}^t \land (\mathit{FacingSouth}^t \lor \mathit{Forward}^t)) \lor (L_{2,1}^t \land (\mathit{FacingWest}^t \lor \mathit{Forward}^t))$$

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Square-OK axiom asserts that a square is free of a pit or live Wumpus.

$$OK_{x,y}^t \Leftrightarrow \neg P_{x,y} \wedge \neg (W_{x,y} \wedge \mathit{WumpusAlive}^t)$$

Initial Percepts and Actions

$\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Scream^0$; $Forward^0$

$\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Scream^1$; $TurnRight^1$

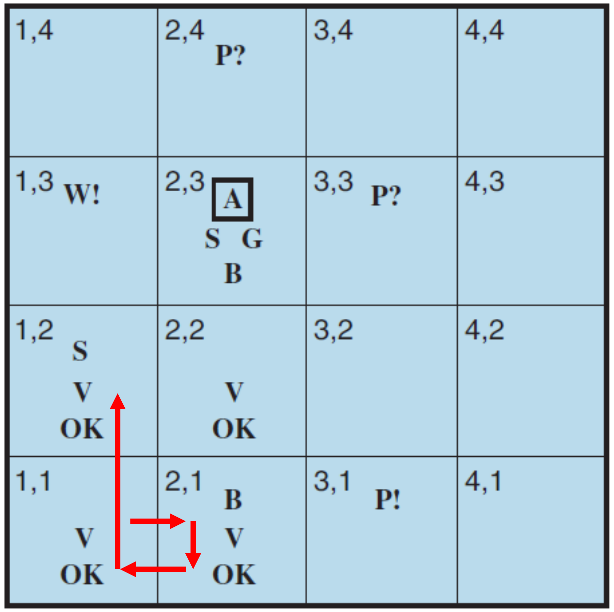
$\neg Stench^2 \wedge Breeze^2 \wedge \neg Glitter^2 \wedge \neg Bump^2 \wedge \neg Scream^2$; $TurnRight^2$

$\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Scream^3$; $Forward^3$

$\neg Stench^4 \wedge \neg Breeze^4 \wedge \neg Glitter^4 \wedge \neg Bump^4 \wedge \neg Scream^4$; $TurnRight^4$

$\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Scream^5$; $Forward^5$

$Stench^6 \wedge \neg Breeze^6 \wedge \neg Glitter^6 \wedge \neg Bump^6 \wedge \neg Scream^6$



Query the knowledge base:

$\text{ASK}(KB, L_{1,2}^6) = true$

$\text{ASK}(KB, W_{1,3}) = true$

$\text{ASK}(KB, P_{3,1}) = true$

$\text{ASK}(KB, OK_{2,2}^6) = true$

// the square [2,2] is OK to move into.