Propositional Theorem Proving

Outline

I. The resolution algorithm

II. Horn clauses

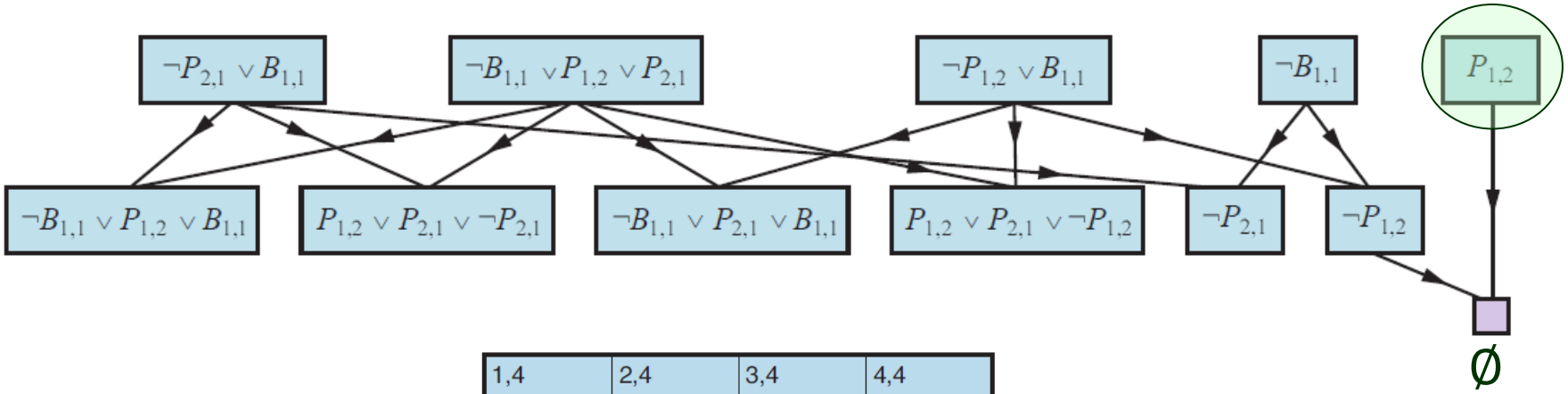
III. Forward and backward chaining

IV. Effective propositional model checking

* Figures are from the textbook site unless a source is specifically cited.

I. Proving $\neg P_{1,2}$ in the Wumpus World

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ transformed into CNF: $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$



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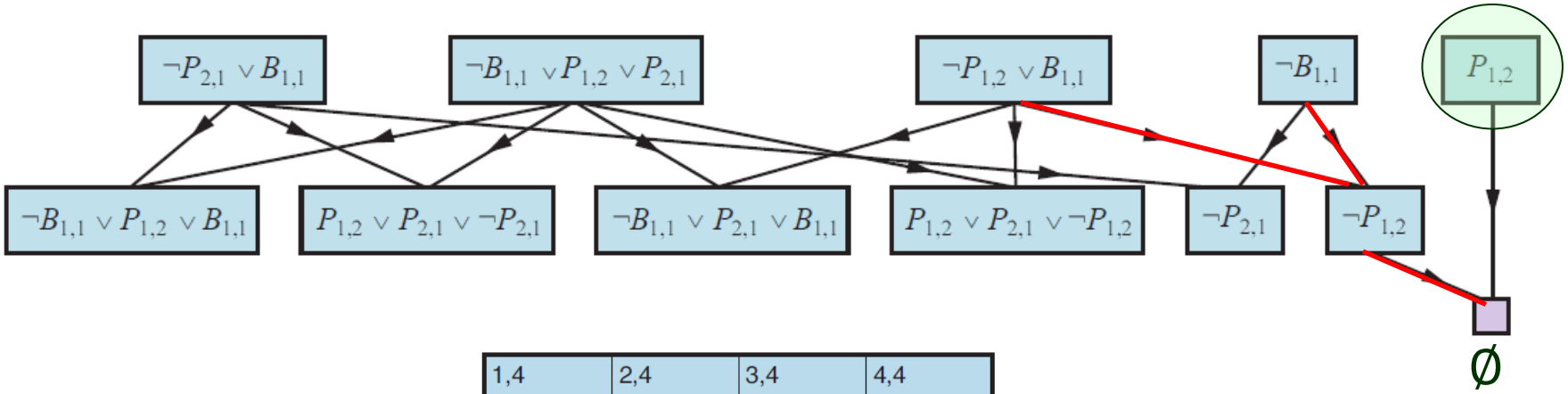

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Resolution Algorithm

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\,\}$
  while *true* **do**
    for each pair of clauses C_i, C_j **in** $clauses$ **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if $resolvents$ contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false* // no new clauses can be added.
    $clauses \leftarrow clauses \cup new$

The process ends in one of two situations below:

- No new clauses can be added, in which case *KB* does not entail α;
- Two clauses resolve to yield the empty clause, in which case *KB* entails α.

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We can construct a proof by explicitly generating an assignment for S if $\emptyset \notin RC(S)$.

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// We are trying to prove $P_1 \wedge P_2 \wedge \cdots \wedge P_k$.

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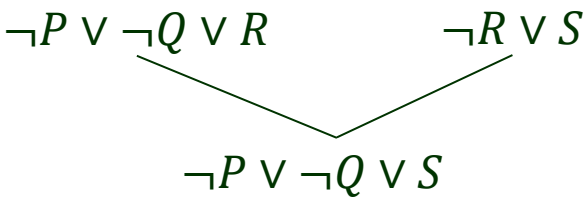

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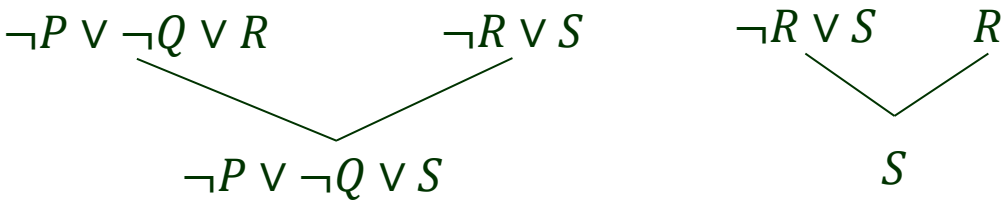

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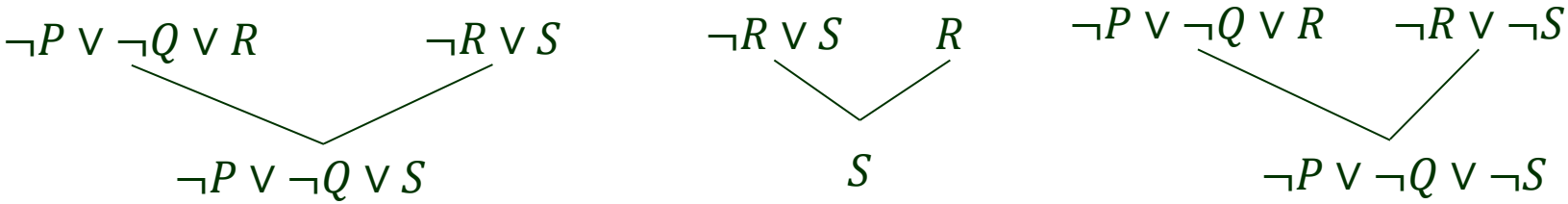

(cont'd)

- Inferences with Horn clauses are through forward- and backward-chaining algorithms.

Logic programming

(natural inference steps easy for humans to follow)

- Low computational complexity: deciding entailment with Horn clauses takes $O(n)$ time.

size of the KB = sum of sizes of clauses in the KB

III. Forward Chaining

Question $KB \models q$?

single proposition symbol

- Begins from positive literals (facts).
- If all the premises of an implications are known, then add its conclusion to KB (as a new fact).
- Continues until q is added or no further inferences can be made.

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- Continues until q is added or no further inferences can be made.

function PL-FC-ENTAILS?(KB, q) **returns** *true* or *false*
  inputs: KB, the knowledge base, a set of propositional definite clauses
    q, the query, a proposition symbol
  count ← a table, where *count*[c] is initially the number of symbols in clause c's premise
  inferred ← a table, where *inferred*[s] is initially *false* for all symbols
  queue ← a queue of symbols, initially symbols known to be true in KB

  while *queue* is not empty **do**
    p ← POP(*queue*)
    if $p = q$ **then return** *true*
    if *inferred*[p] = *false* **then**
      inferred[p] ← *true*
      for each clause c in KB where p is in c.PREMISE **do**
        decrement *count*[c]
        if *count*[c] = 0 **then** add c.CONCLUSION to *queue*
  return *false*

Example of Forward Chaining

KB:

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

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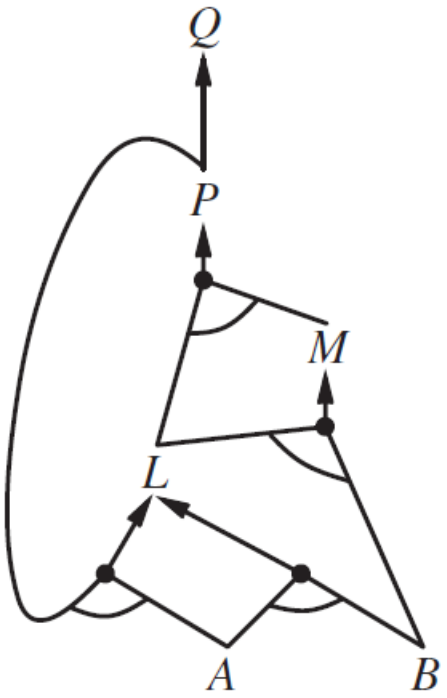
AND-OR graph representation



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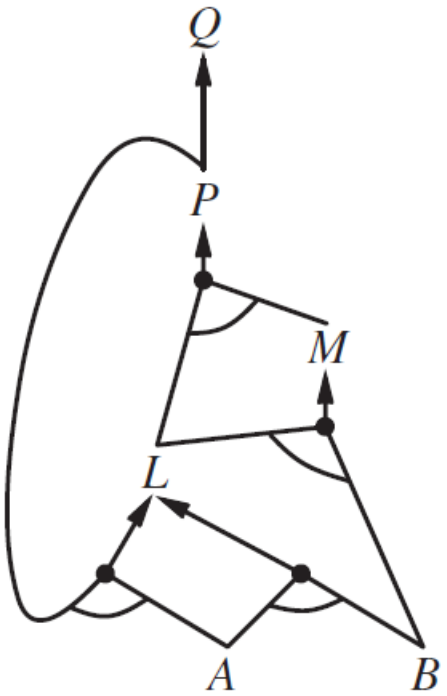

Q. *KB* $\models Q$?

Execution

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

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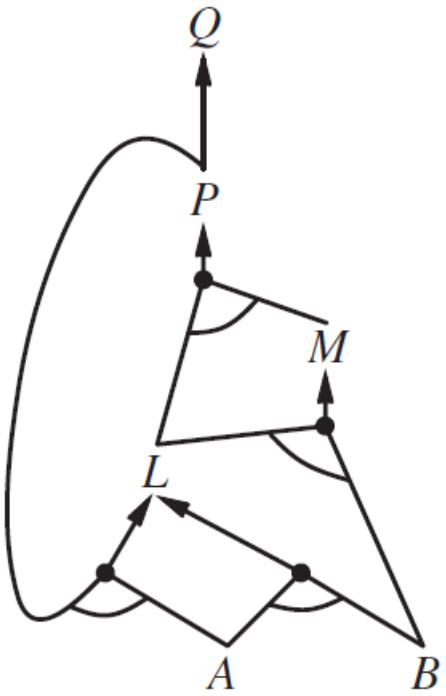
A

B



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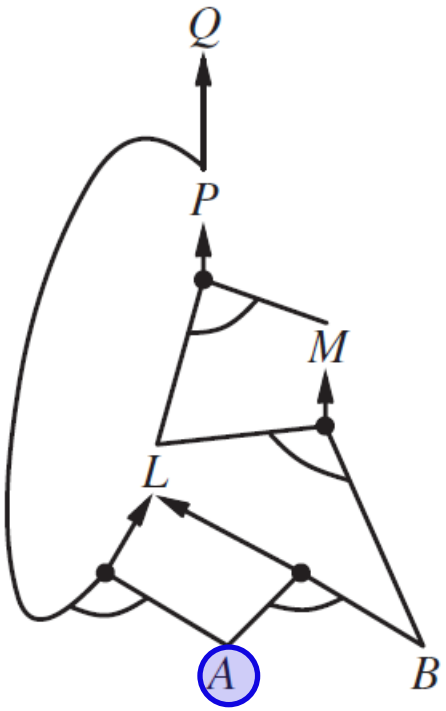

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A

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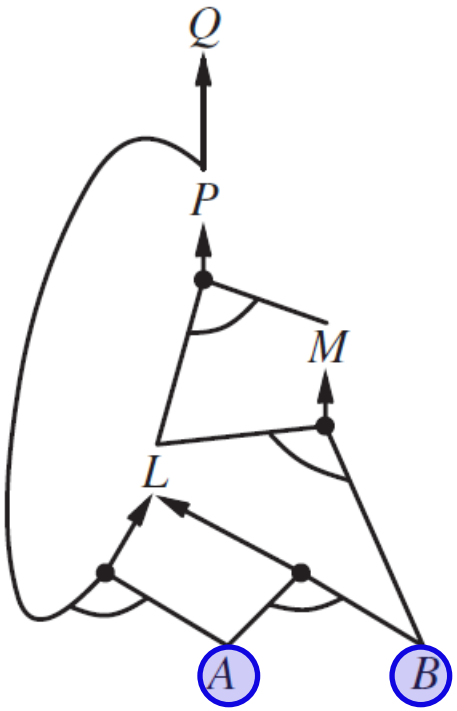


Execution

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

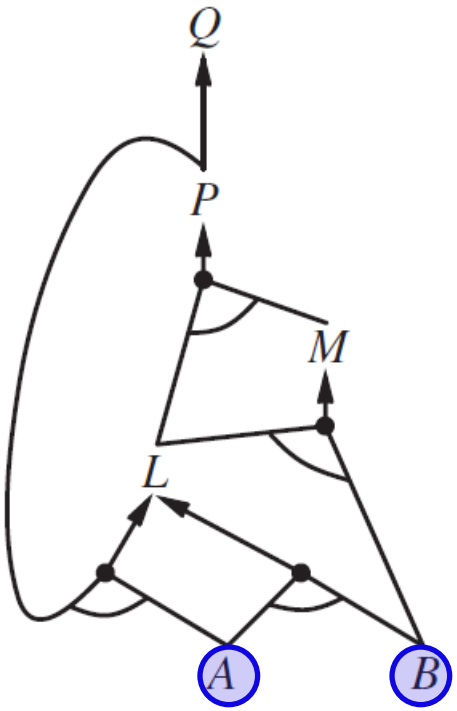
$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$\boxed{A \wedge B \Rightarrow L}$$

$$A$$

$$B$$



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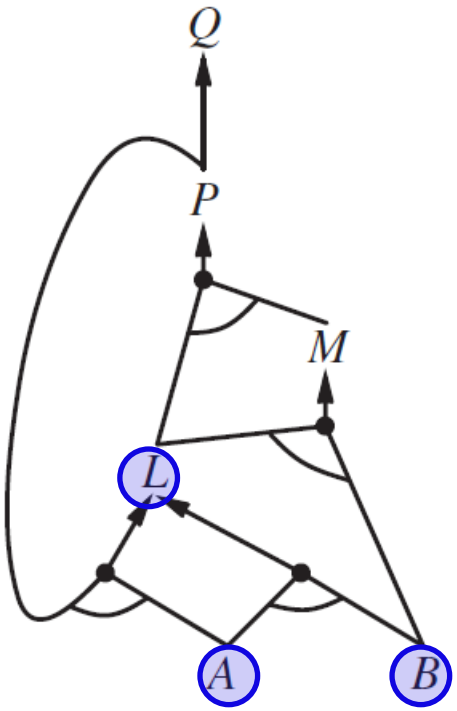

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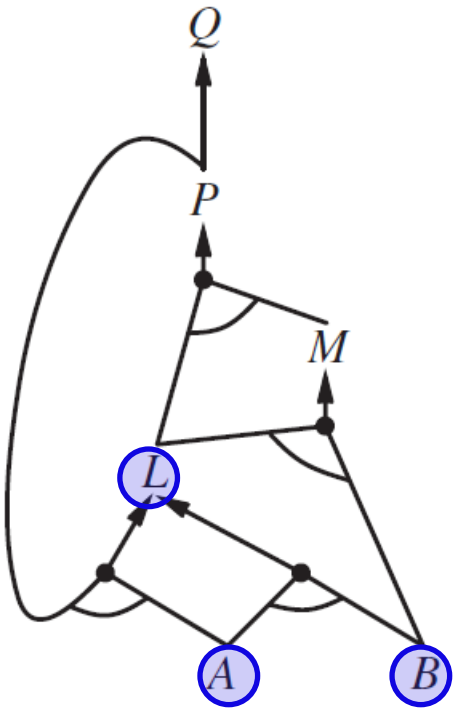

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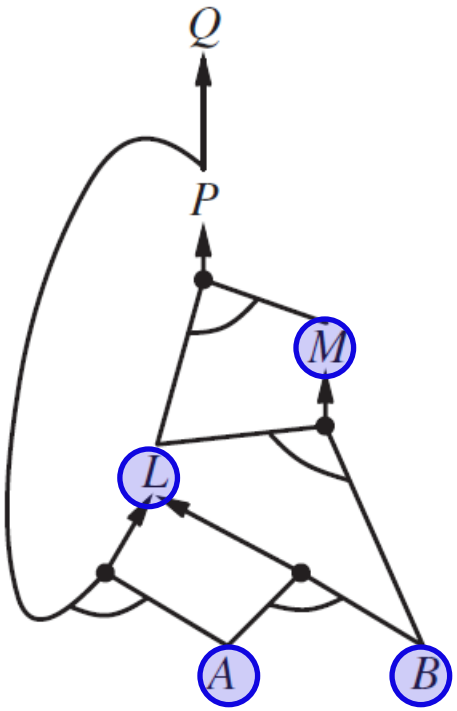

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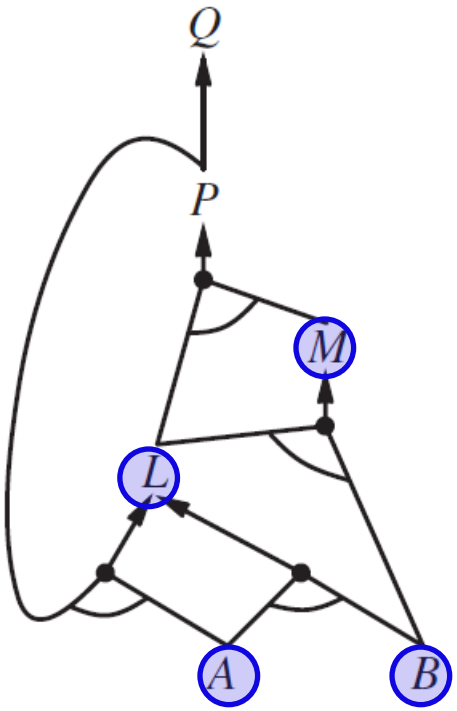

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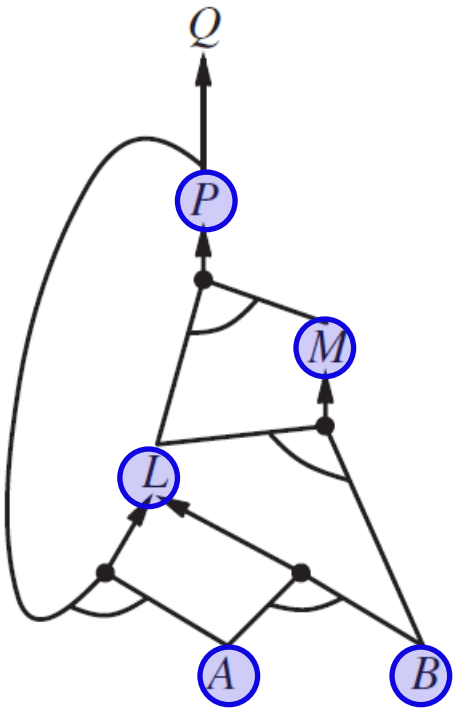

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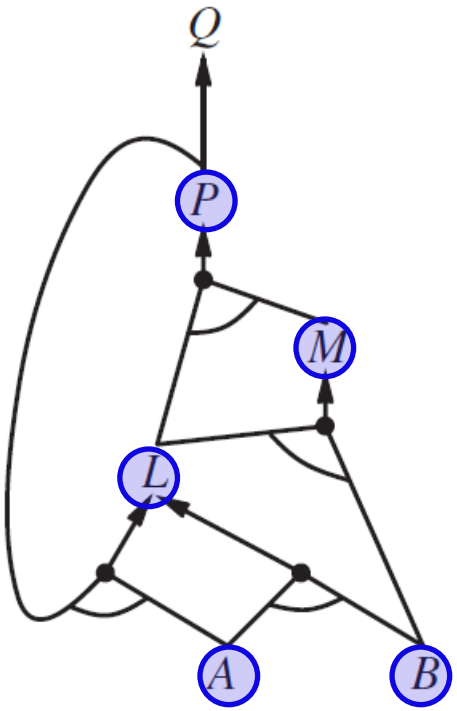

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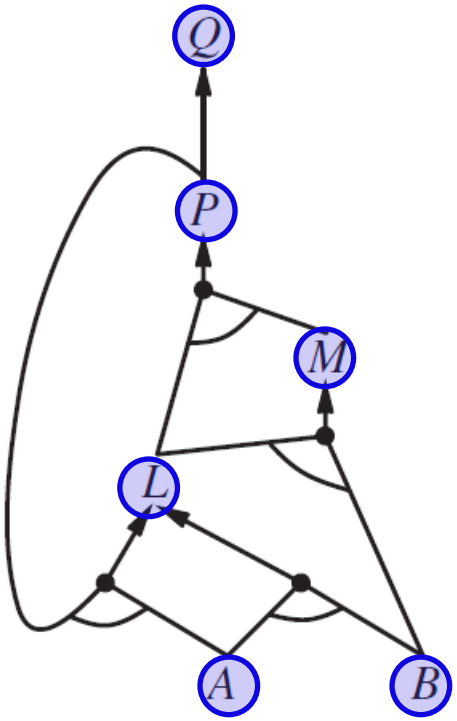

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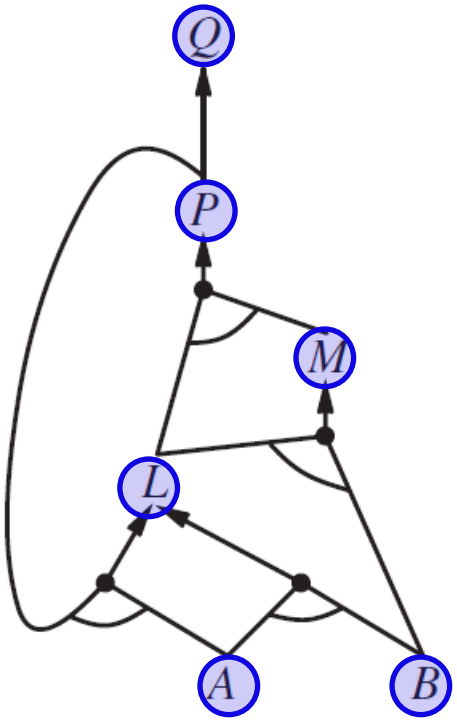

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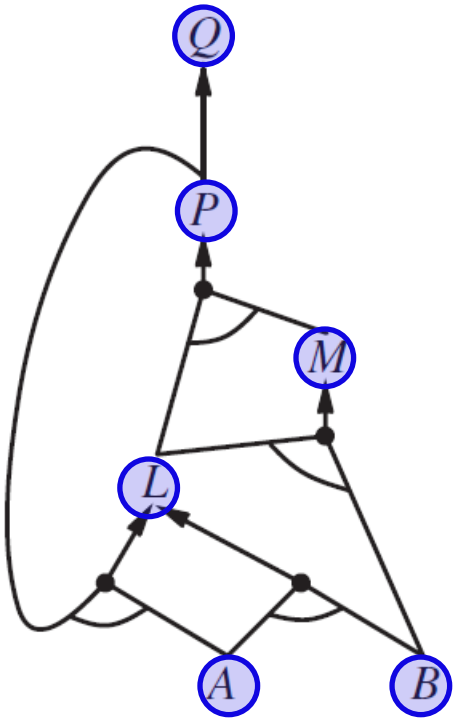

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Completeness: every entailed atomic sentences will be derived.

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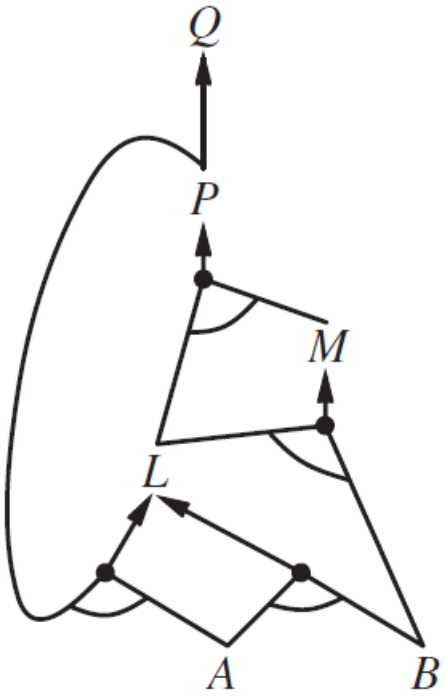

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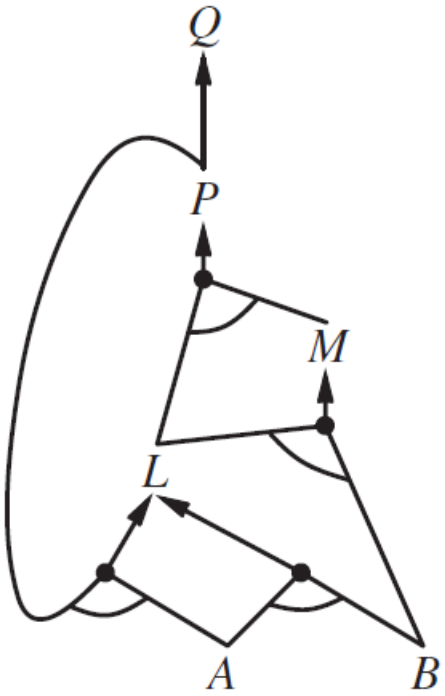

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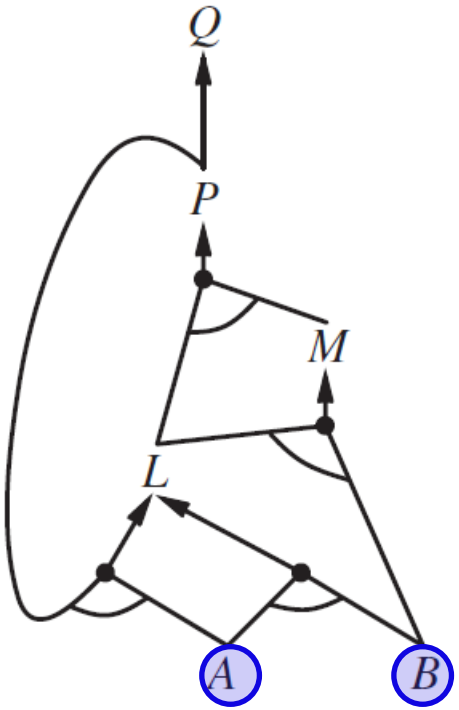

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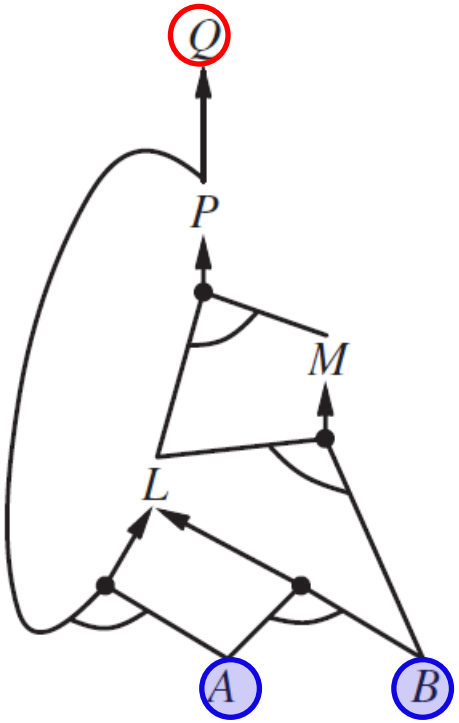

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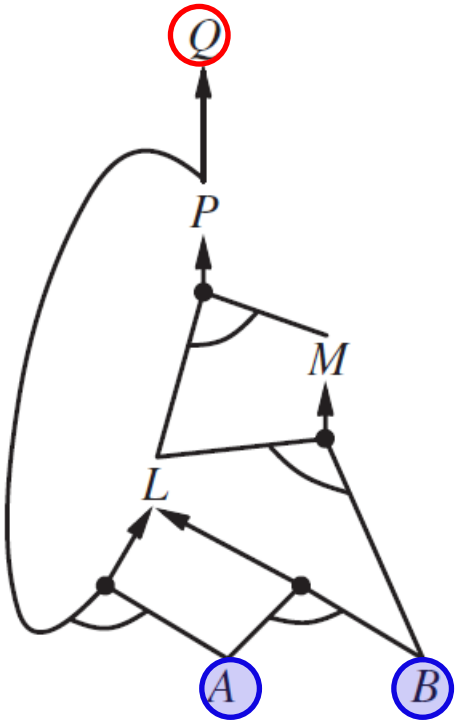

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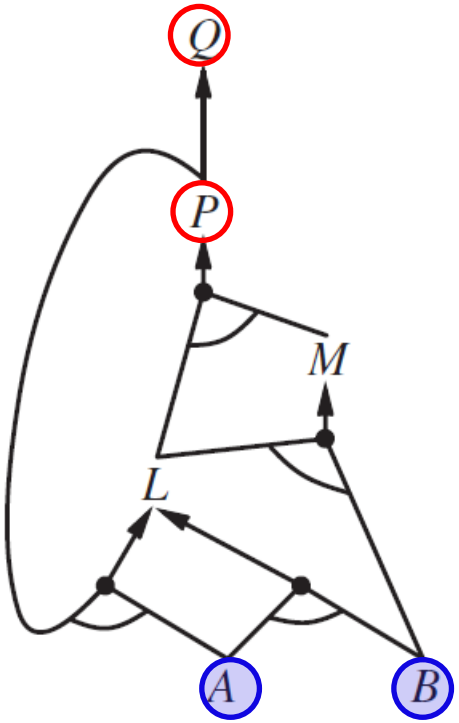

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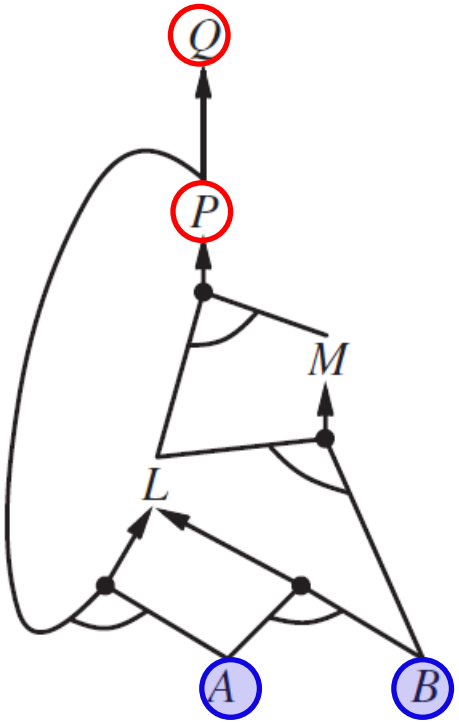

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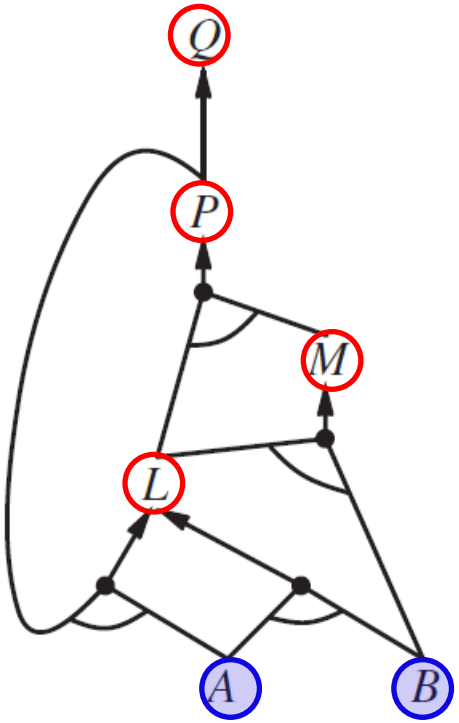

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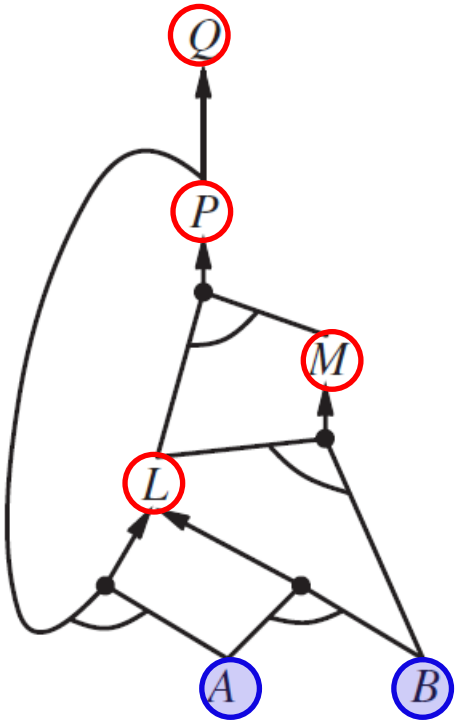

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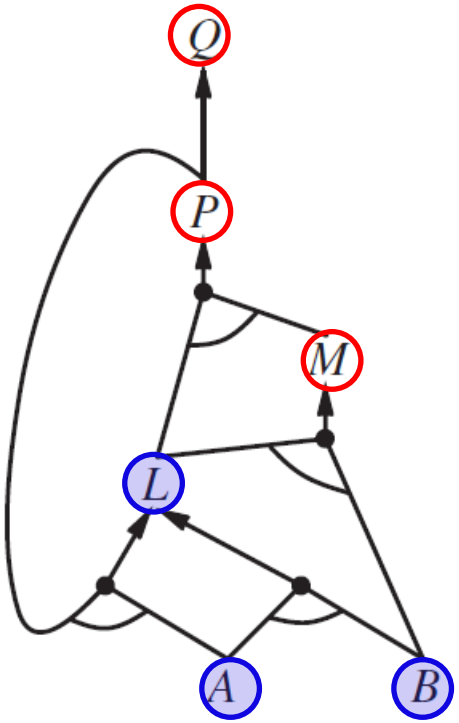

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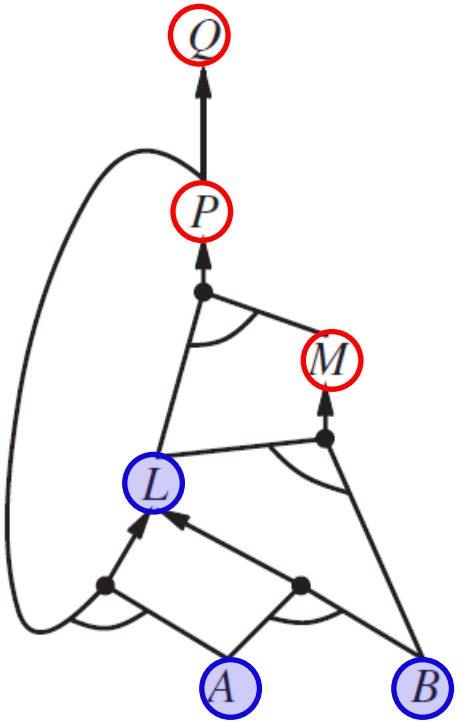

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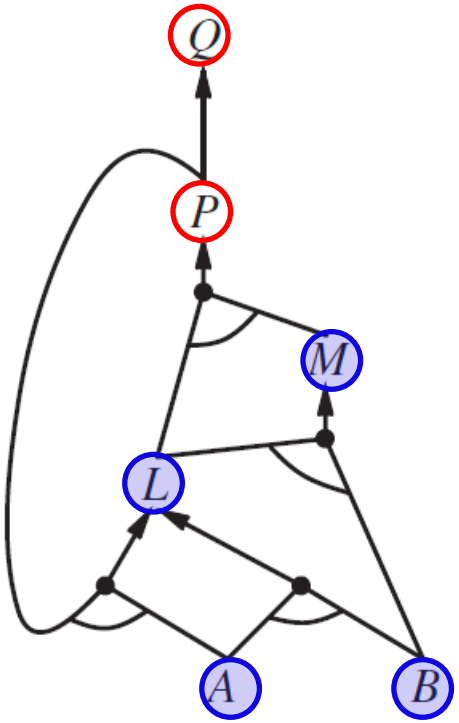

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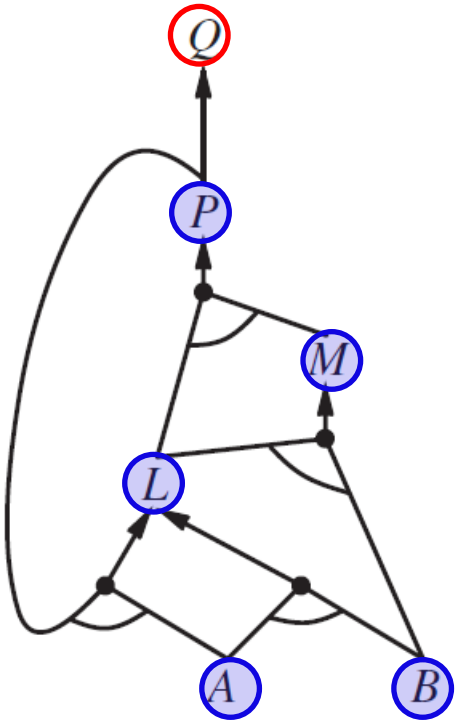

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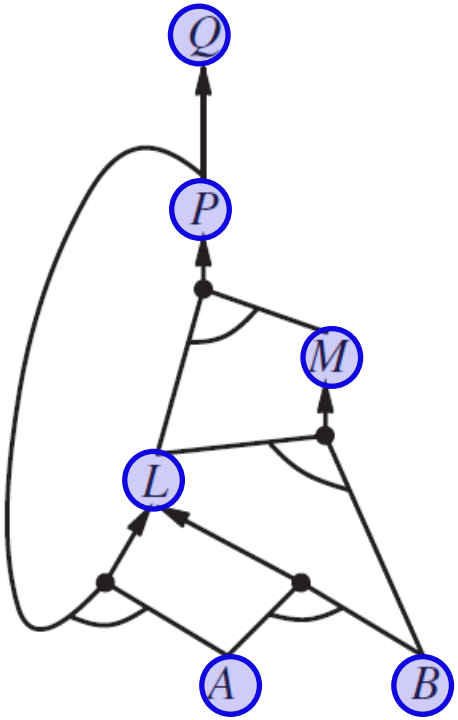

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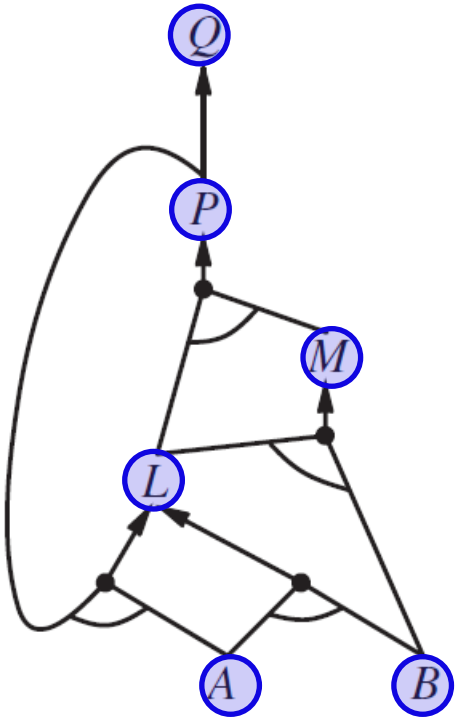

AND-OR graph search!

Forward vs. Backward Chaing

- Applicable range
  - Prove the entailment of a single proposition symbol
  - KB consists of definite clauses only.

Either P or $P_1 \wedge P_2 \wedge \cdots P_k \Rightarrow Q$

- Forward chaining is *data-driven*, automatic, unconscious processing.
- It may perform a lot of work that is irrelevant to the goal.
- Backward chaining is *goal-driven*, and appropriate for problem solving.
- It may run in time sublinear in the size of KB, since it touches only relevant facts.

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- Complete backtracking search (DPLL algorithm)

- Incomplete local search (WALKSAT algorithm)

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With enhancements, modern solvers can handle a problem with a multiple of 10^7 variables.

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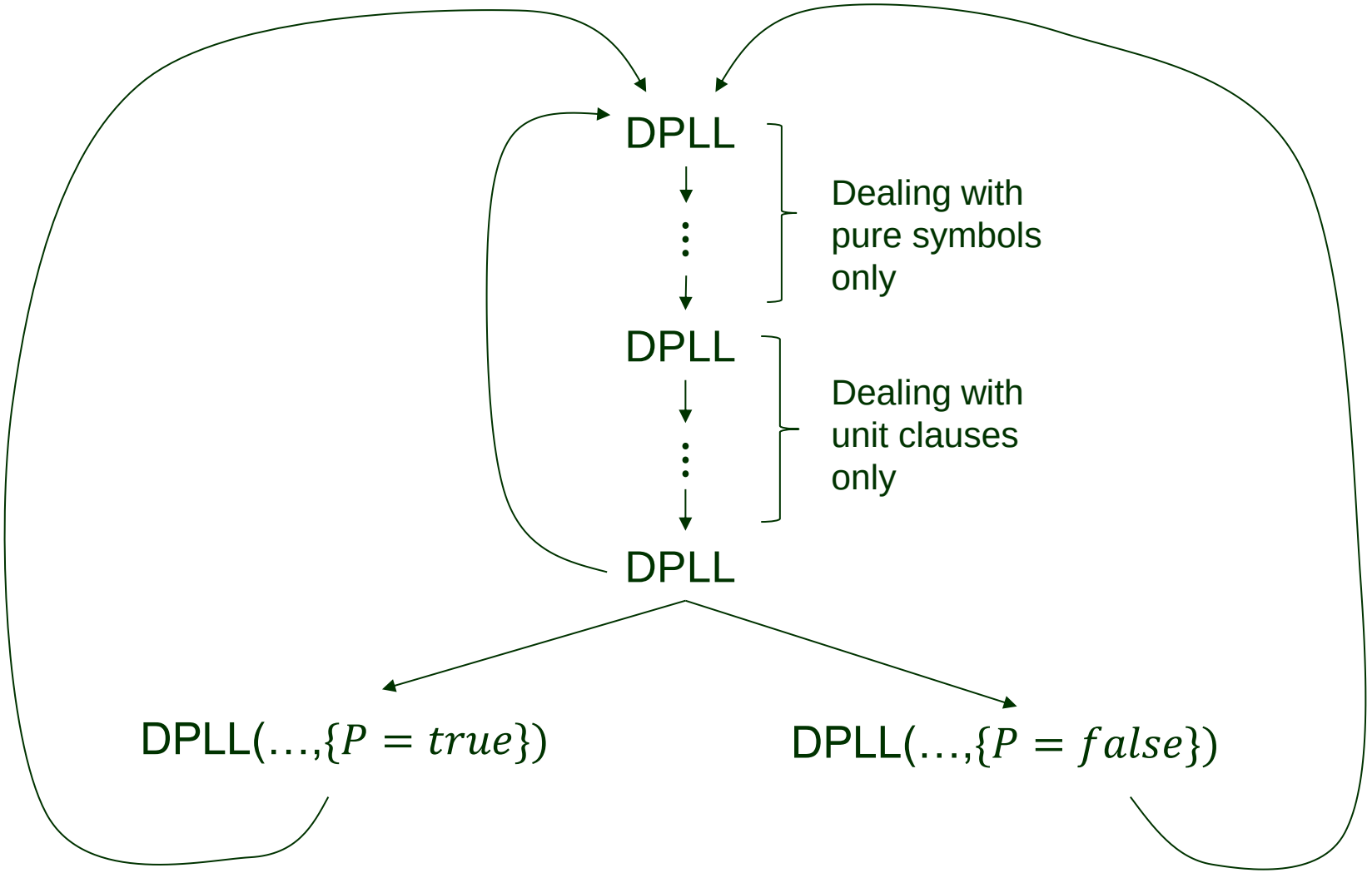
Unit clause propagation on a clause in which all literals but one are assigned *false*. E.g., $\neg B \vee \neg C$ simplifies to the unit clause $\neg C$ if B = *true*.

Recursion Tree



Example (Exercise 7.25)

S_1: $A \Leftrightarrow (B \vee E)$

Convertion into clauses

S_2: $E \Rightarrow D$

S_3: $C \wedge F \Rightarrow \neg B$

S_4: $E \Rightarrow B$

S_5: $B \Rightarrow F$

S_6: $B \Rightarrow C$

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$\big(A \Rightarrow (B \vee E)\big) \wedge \big((B \vee E) \Rightarrow A\big)$

$\Downarrow$

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$\Downarrow$

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$\big(A \Rightarrow (B \vee E)\big) \wedge \big((B \vee E) \Rightarrow A\big)$

$\Downarrow$

$(\neg A \vee B \vee E) \wedge (\neg(B \vee E) \vee A)$

$\Downarrow$

$(\neg A \vee B \vee E) \wedge ((\neg B \wedge \neg E) \vee A)$

C_1: $\neg A \vee B \vee E$

C_2: $\neg B \vee A$

C_3: $\neg E \vee A$

Convertion into clauses

S_2: $E \Rightarrow D$

S_3: $C \wedge F \Rightarrow \neg B$

S_4: $E \Rightarrow B$

S_5: $B \Rightarrow F$

S_6: $B \Rightarrow C$

Example (Exercise 7.25)

S_1: $A \Leftrightarrow (B \vee E)$

$\Downarrow$

$\big(A \Rightarrow (B \vee E)\big) \wedge \big((B \vee E) \Rightarrow A\big)$

$\Downarrow$

$(\neg A \vee B \vee E) \wedge (\neg(B \vee E) \vee A)$

$\Downarrow$

$(\neg A \vee B \vee E) \wedge ((\neg B \wedge \neg E) \vee A)$

C_1: $\neg A \vee B \vee E$

C_2: $\neg B \vee A$

C_3: $\neg E \vee A$

Convertion into clauses

S_2: $E \Rightarrow D$ $\Rightarrow$ C_4: $\neg E \vee D$

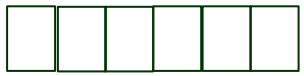
S_3: $C \wedge F \Rightarrow \neg B$ $\Rightarrow$ C_5: $\neg C \vee \neg F \vee \neg B$

S_4: $E \Rightarrow B$ $\Rightarrow$ C_6: $\neg E \vee B$

S_5: $B \Rightarrow F$ $\Rightarrow$ C_7: $\neg B \vee F$

S_6: $B \Rightarrow C$ $\Rightarrow$ C_8: $\neg B \vee C$

A B C D E F

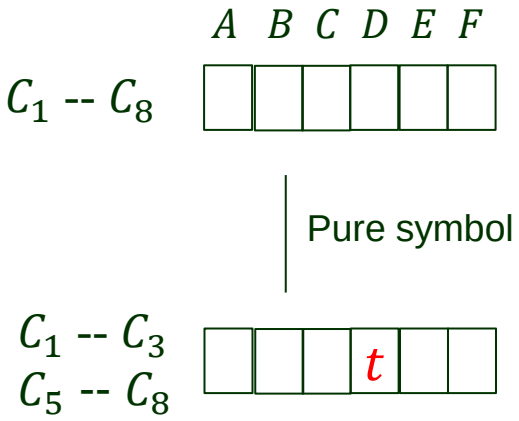
C_1 -- C_8 

C_1: $\neg A \vee B \vee E$ C_2: $\neg B \vee A$

C_3: $\neg E \vee A$ C_4: $\neg E \vee D$

C_5: $\neg C \vee \neg F \vee \neg B$ C_6: $\neg E \vee B$

C_7: $\neg B \vee F$ C_8: $\neg B \vee C$

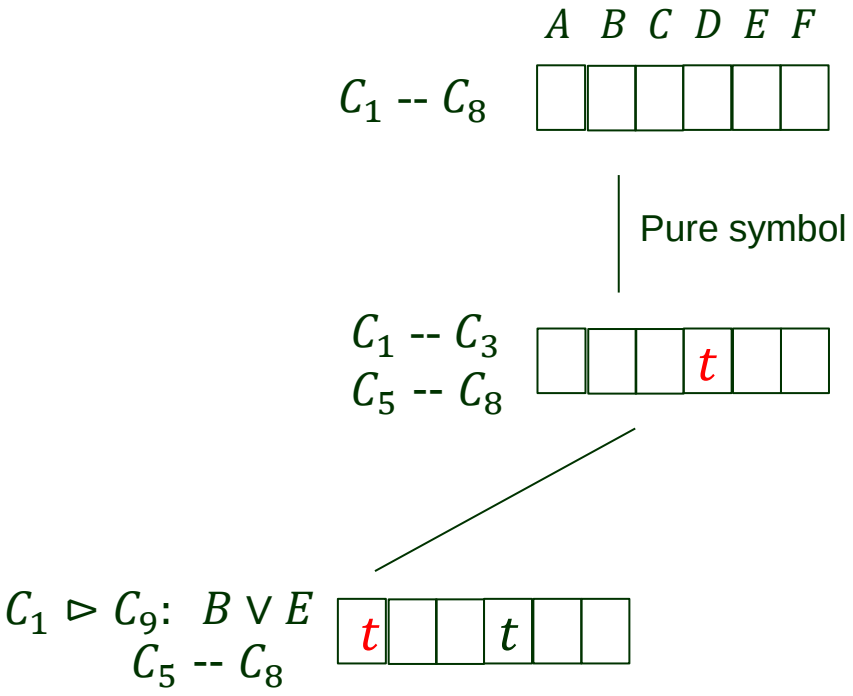


C_1: $\neg A \vee B \vee E$ C_2: $\neg B \vee A$

C_3: $\neg E \vee A$ C_4: $\neg E \vee D$

C_5: $\neg C \vee \neg F \vee \neg B$ C_6: $\neg E \vee B$

C_7: $\neg B \vee F$ C_8: $\neg B \vee C$

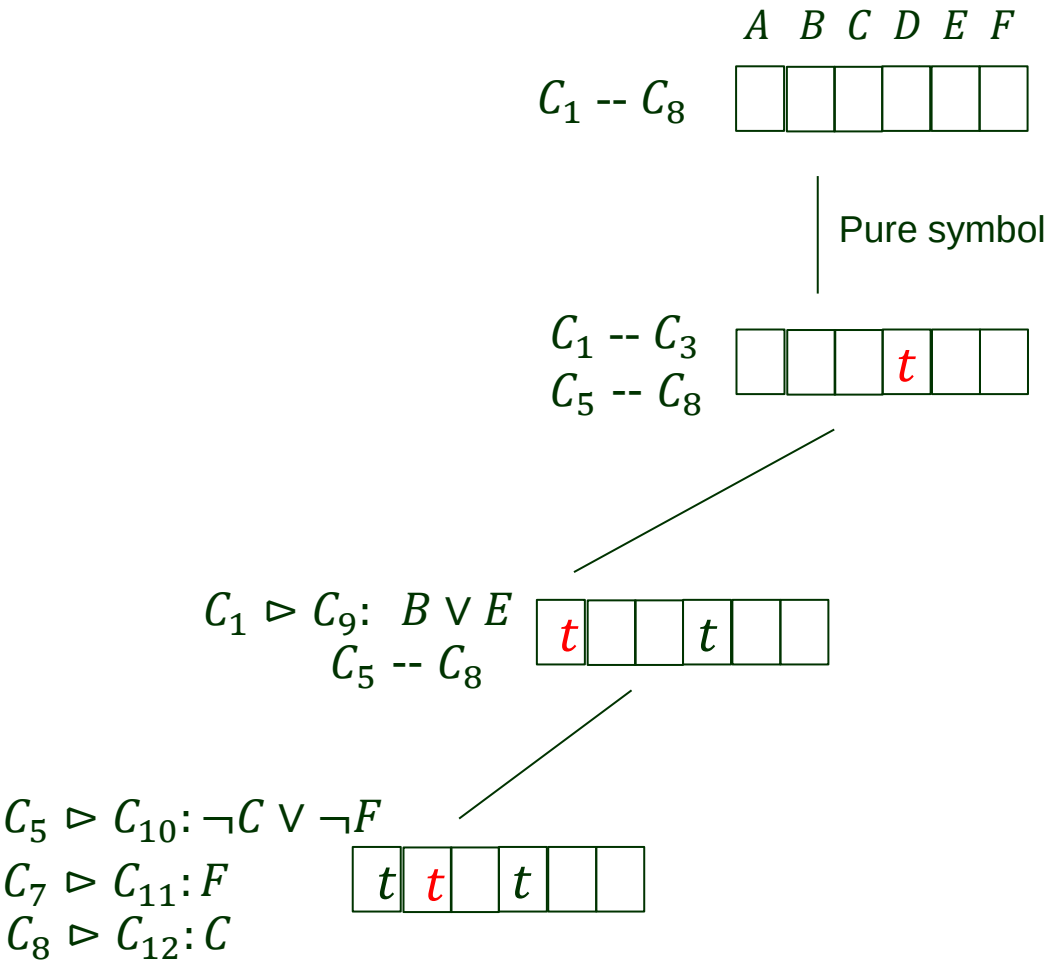


C_1: $\neg A \vee B \vee E$ C_2: $\neg B \vee A$

C_3: $\neg E \vee A$ C_4: $\neg E \vee D$

C_5: $\neg C \vee \neg F \vee \neg B$ C_6: $\neg E \vee B$

C_7: $\neg B \vee F$ C_8: $\neg B \vee C$

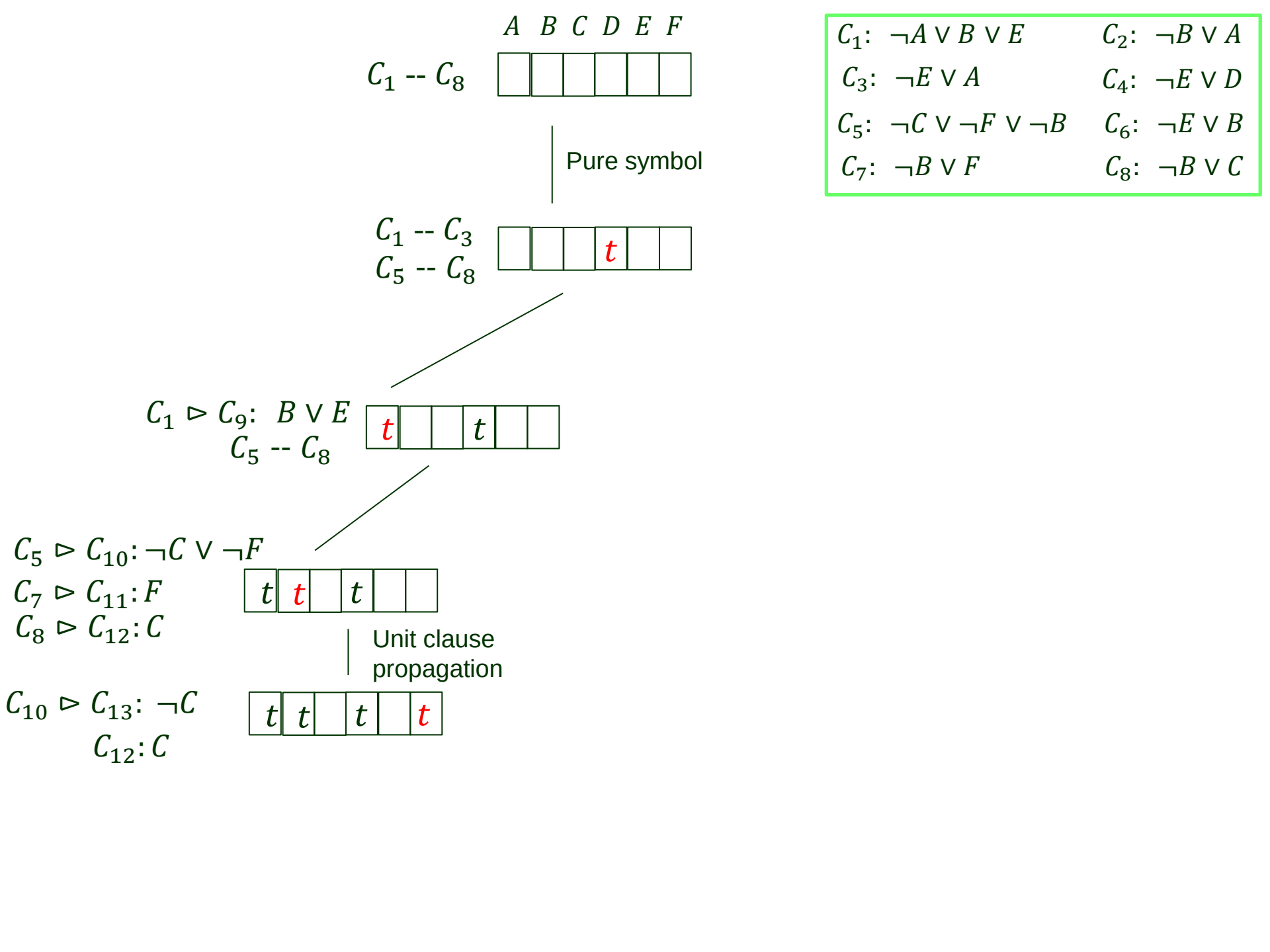


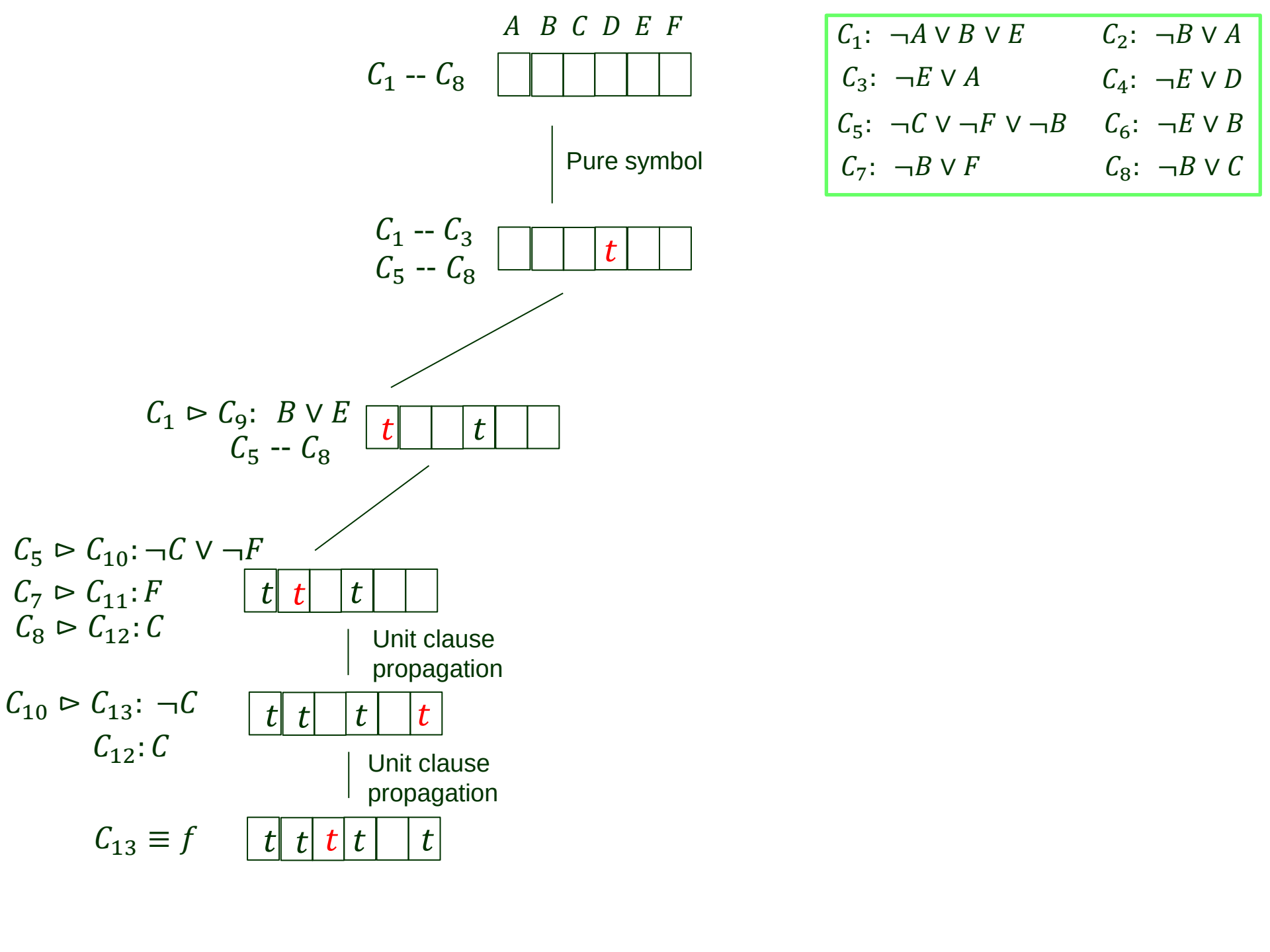
C_1: $\neg A \vee B \vee E$ C_2: $\neg B \vee A$

C_3: $\neg E \vee A$ C_4: $\neg E \vee D$

C_5: $\neg C \vee \neg F \vee \neg B$ C_6: $\neg E \vee B$

C_7: $\neg B \vee F$ C_8: $\neg B \vee C$


A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
$C_1 \triangleright C_9$: $B \vee E$
C_5 -- C_8
t t t
$C_5 \triangleright C_{10}$: $\neg C \vee \neg F$
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t
C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
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C_5 -- C_8
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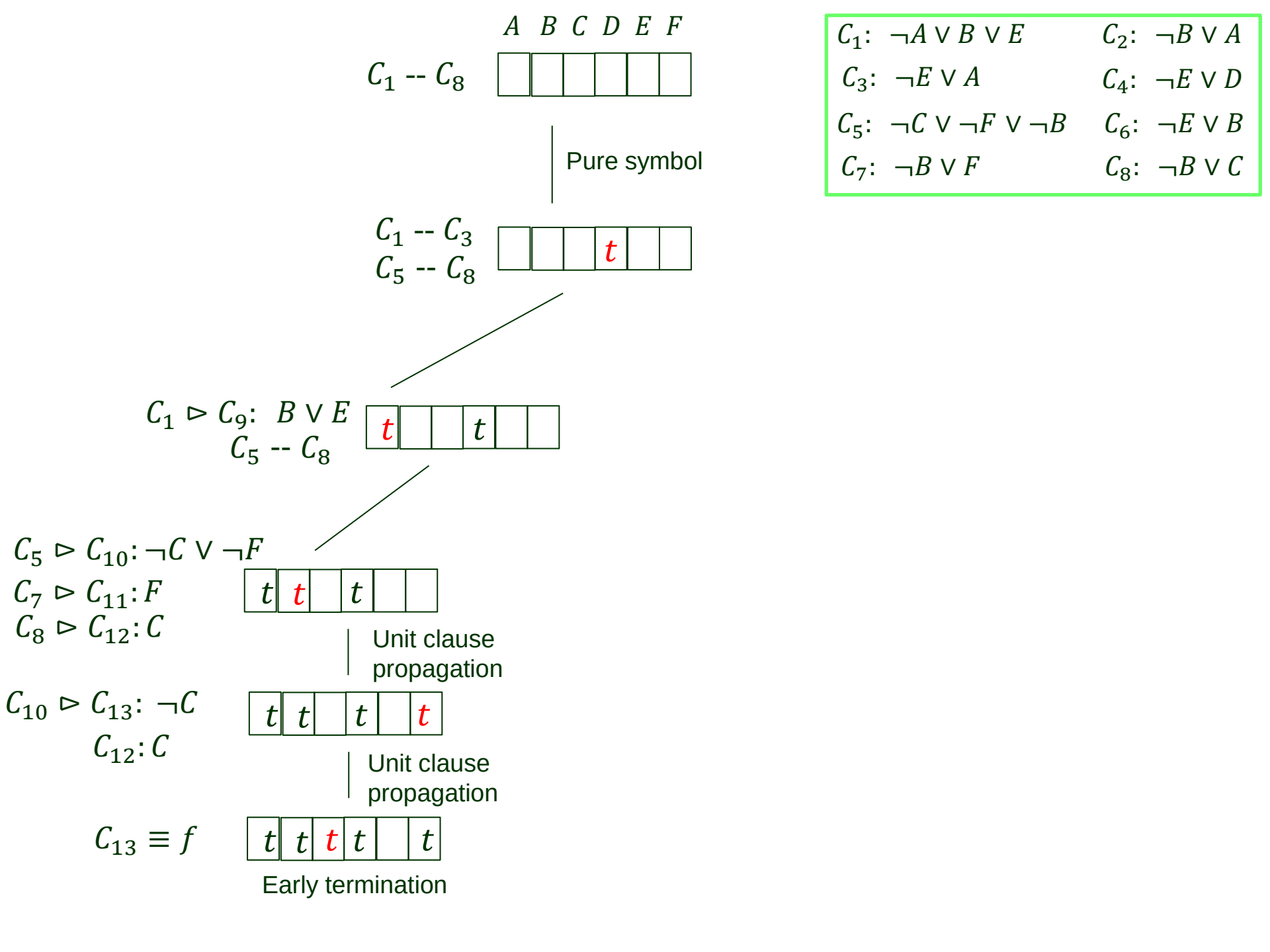
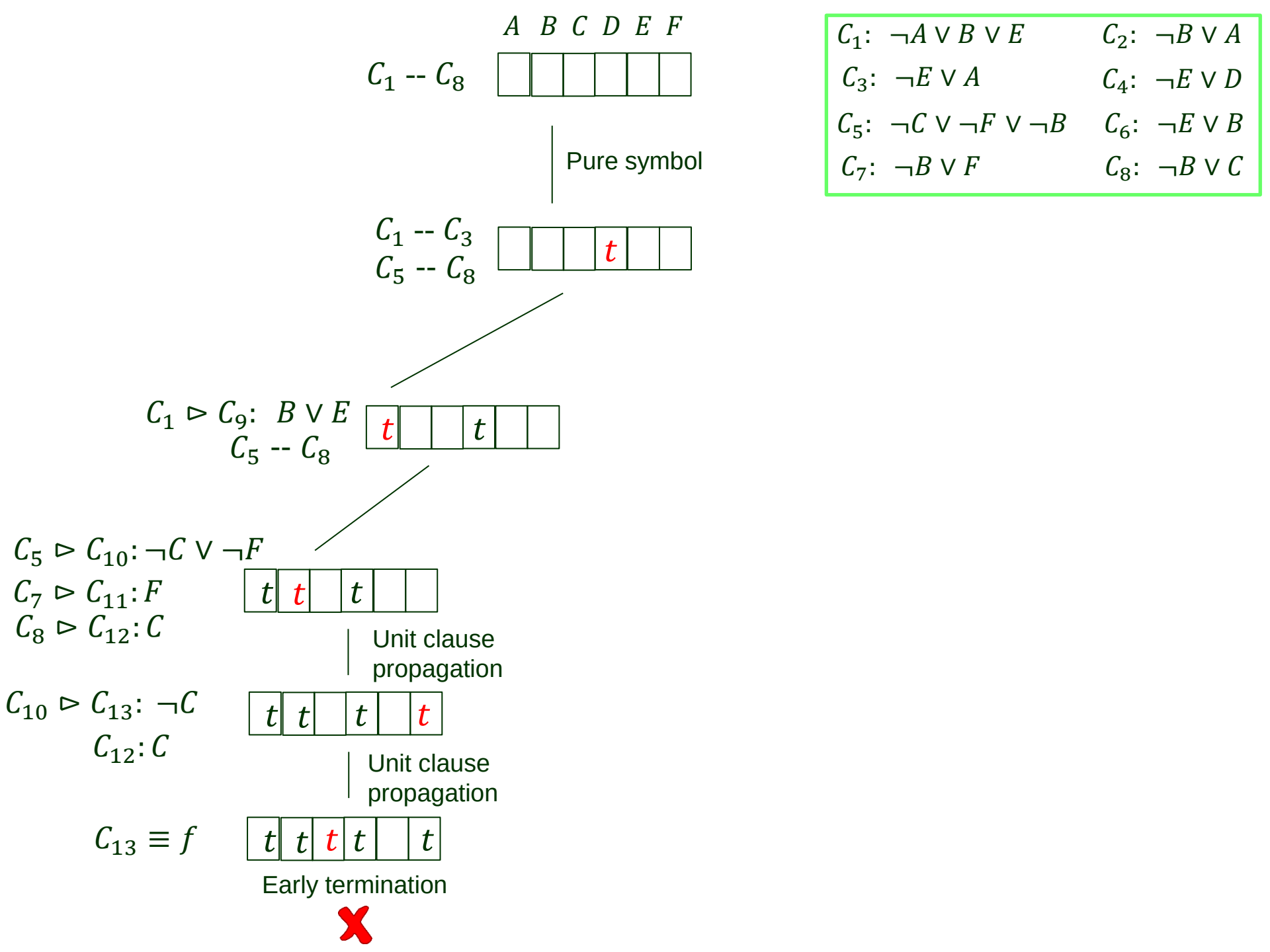
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
$C_1 \triangleright C_9$: $B \vee E$
C_5 -- C_8
t t t
$C_5 \triangleright C_{10}$: $\neg C \vee \neg F$
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
$C_1 \triangleright C_9$: $B \vee E$
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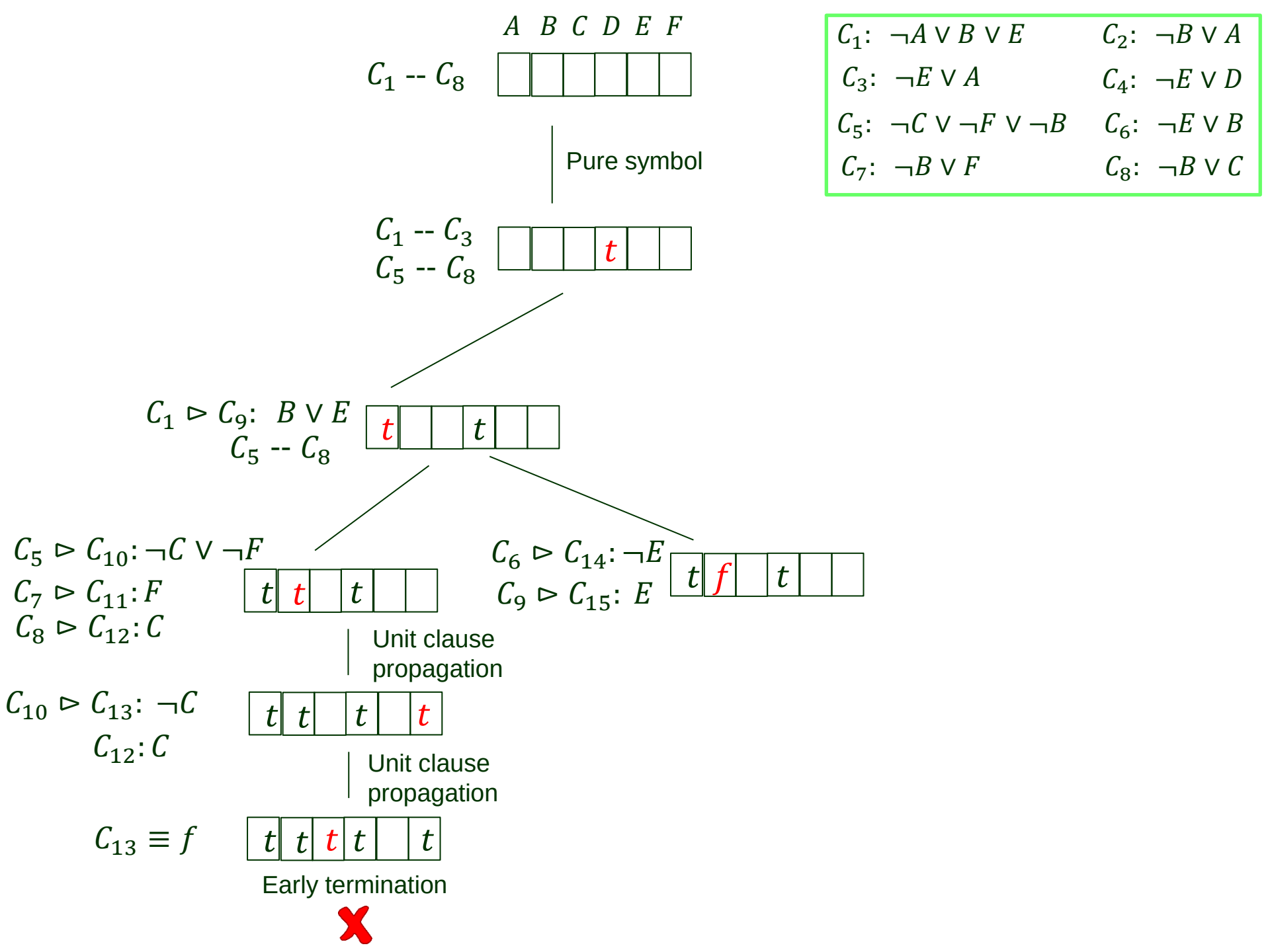
$C_8 \triangleright C_{12}$: C
t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
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C_5 -- C_8
t t t
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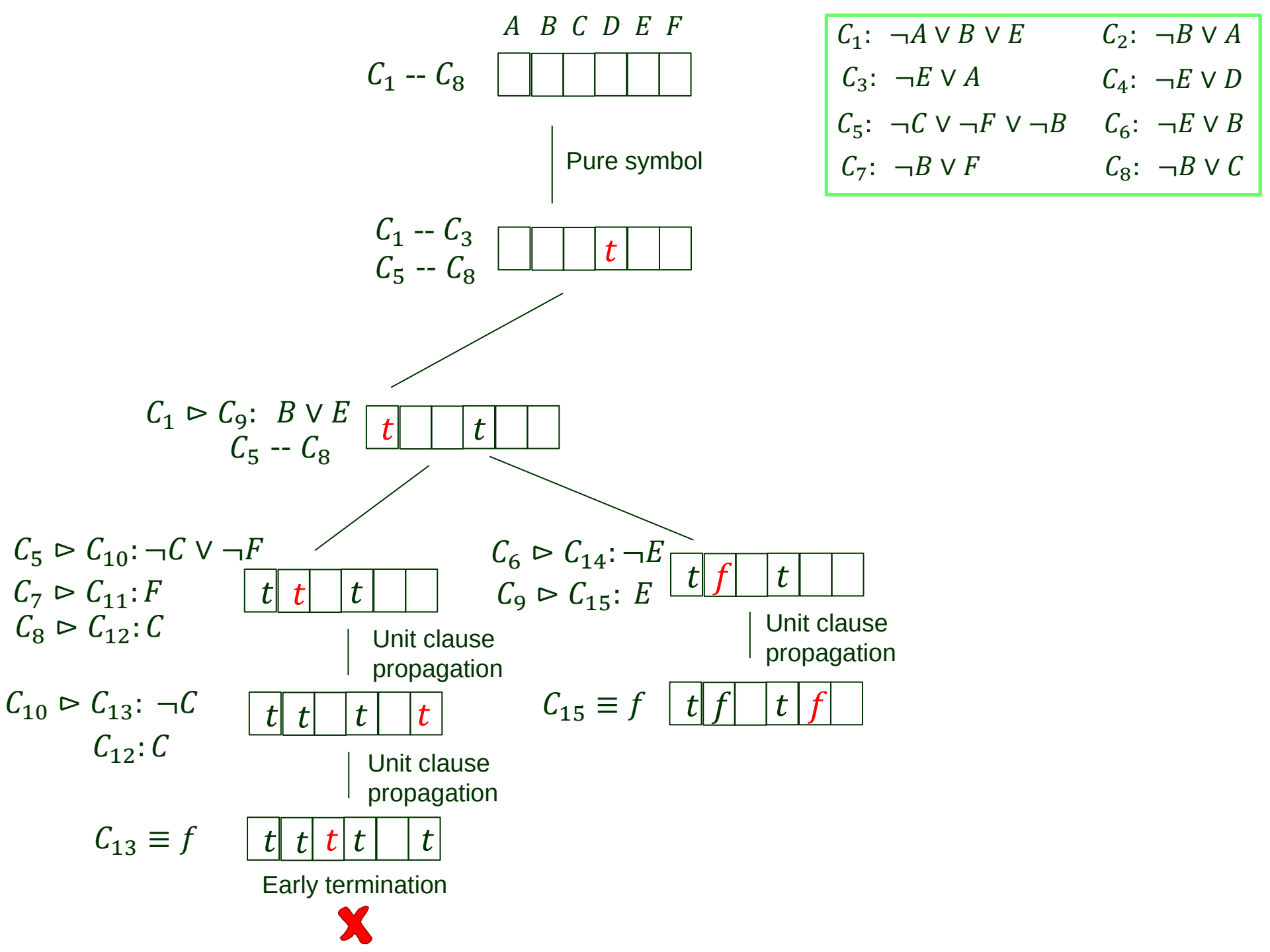
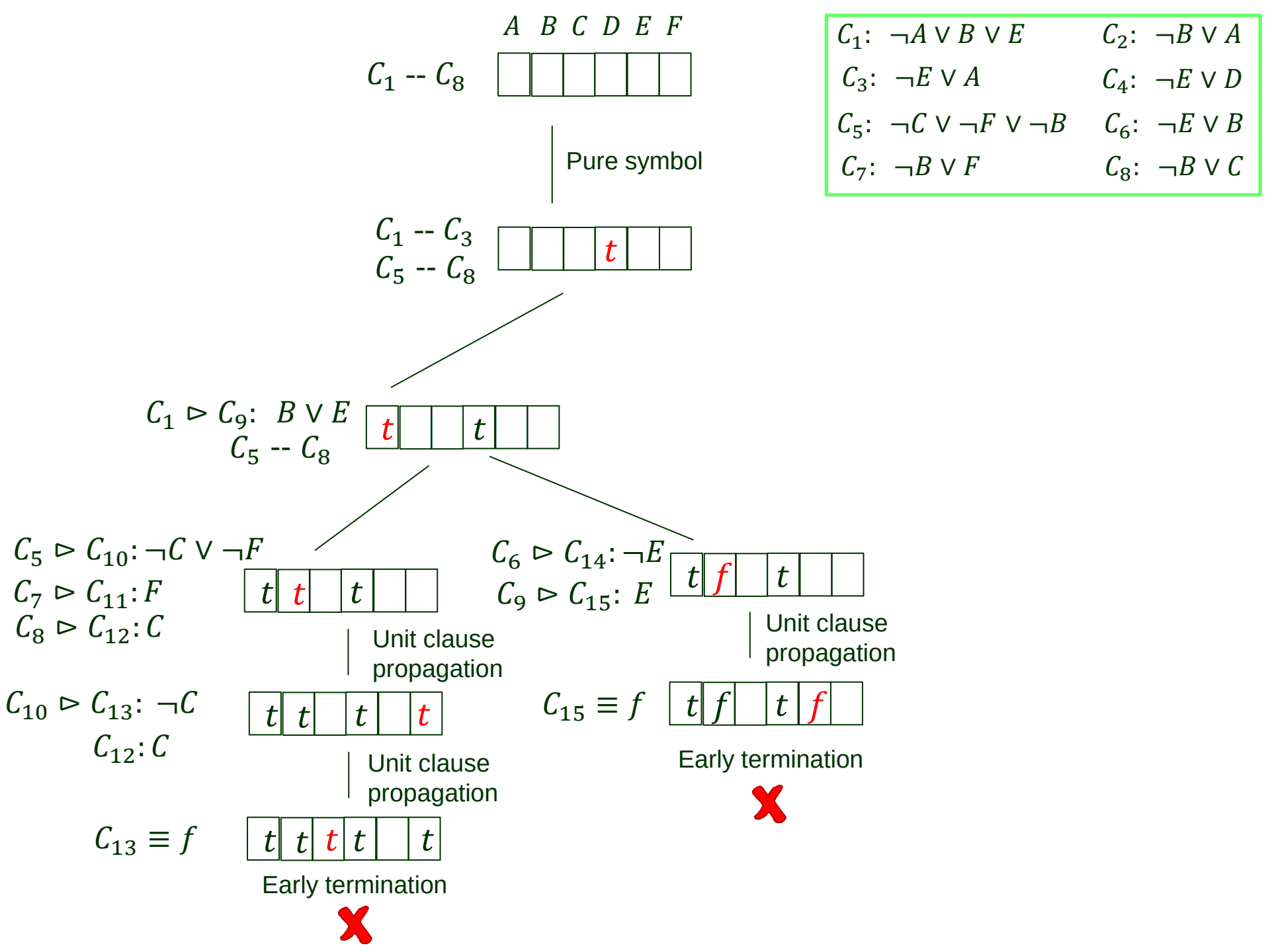
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t
$C_6 \triangleright C_{14}$: $\neg E$
$C_9 \triangleright C_{15}$: E
t f t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
$C_1 \triangleright C_9$: $B \vee E$
C_5 -- C_8
t t t
$C_5 \triangleright C_{10}$: $\neg C \vee \neg F$
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination
$C_6 \triangleright C_{14}$: $\neg E$
$C_9 \triangleright C_{15}$: E
t f t
Unit clause propagation
$C_{15} \equiv f$
t f t f


C_1: $\neg A \vee B \vee E$
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C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
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C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
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Pure symbol
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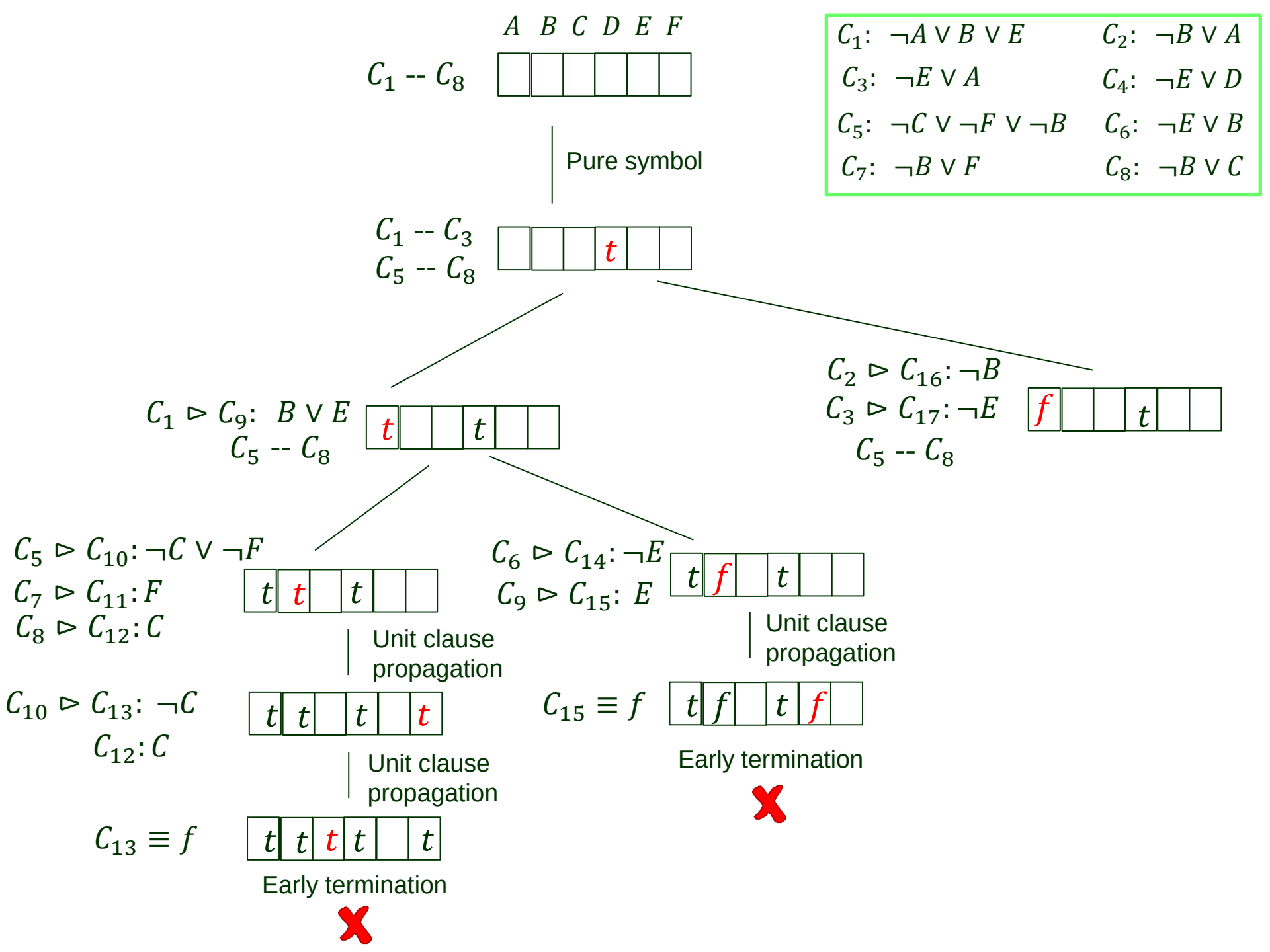
$C_8 \triangleright C_{12}$: C
t t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination
$C_6 \triangleright C_{14}$: $\neg E$
$C_9 \triangleright C_{15}$: E
t f t
Unit clause propagation
$C_{15} \equiv f$
t f t f
Early termination


C_1: $\neg A \vee B \vee E$
C_2: $\neg B \vee A$
C_3: $\neg E \vee A$
C_4: $\neg E \vee D$
C_5: $\neg C \vee \neg F \vee \neg B$
C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
A B C D E F
C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
t
$C_1 \triangleright C_9$: $B \vee E$
C_5 -- C_8
$C_2 \triangleright C_{16}$: $\neg B$
$C_3 \triangleright C_{17}$: $\neg E$
C_5 -- C_8
f
$C_5 \triangleright C_{10}$: $\neg C \vee \neg F$
$C_7 \triangleright C_{11}$: F
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Unit clause propagation
Unit clause propagation
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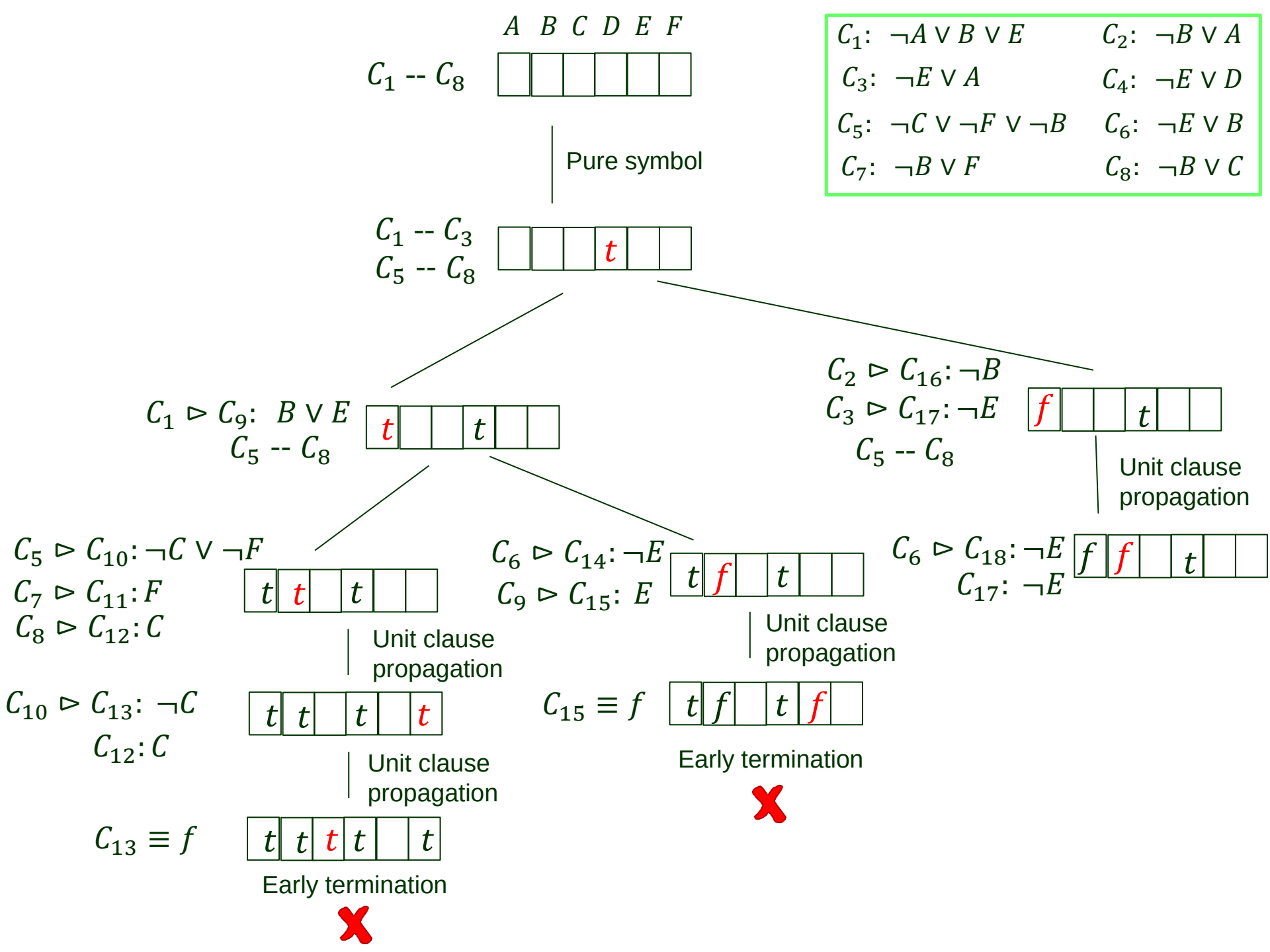
Unit clause propagation
Early termination
$C_{13} \equiv f$
Early termination


C_1: $\neg A \vee B \vee E$
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$C_9 \triangleright C_{15}$: E
$C_6 \triangleright C_{18}$: $\neg E$
C_{17}: $\neg E$
Unit clause propagation
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Early termination
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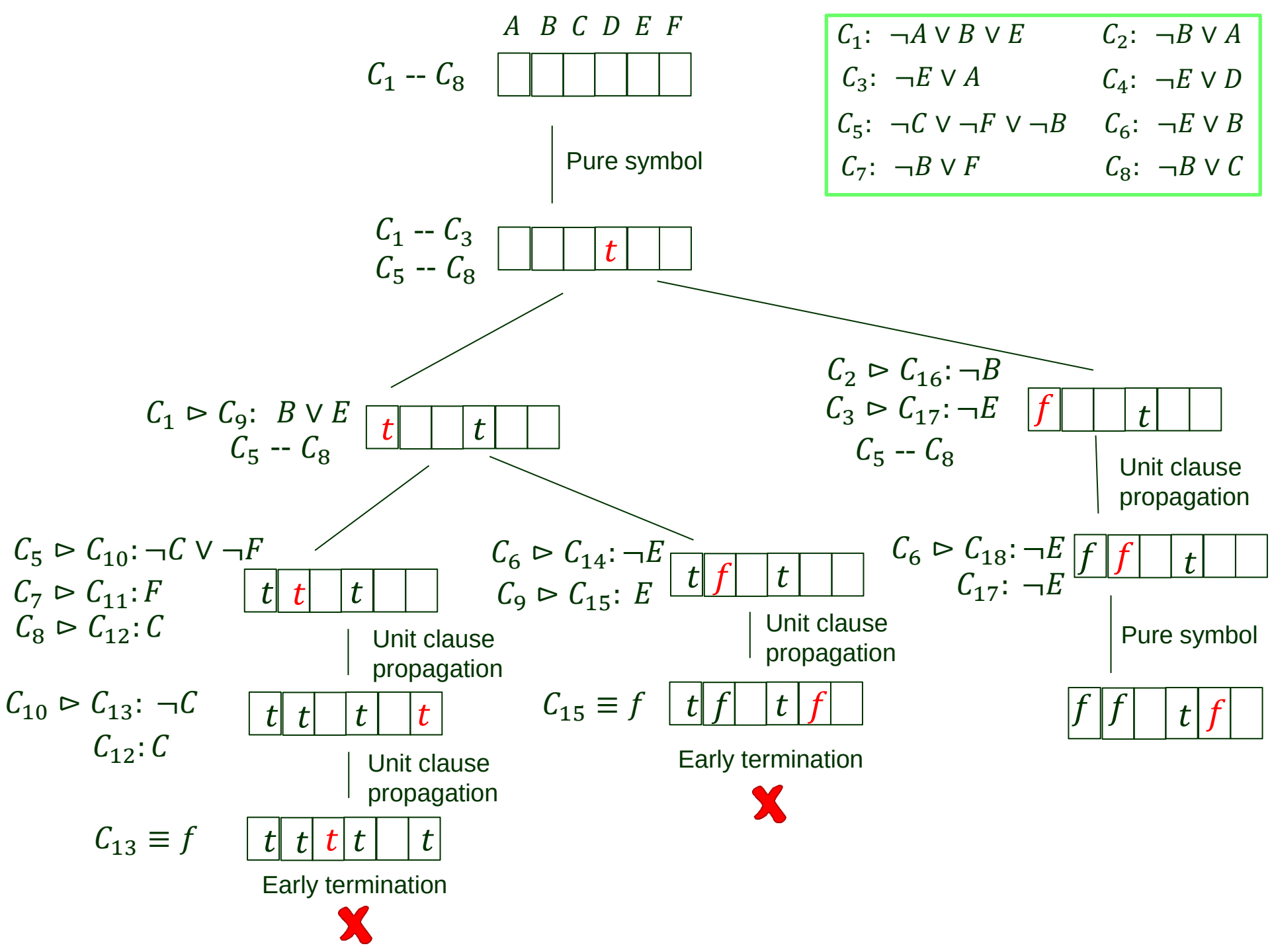
C_1: $\neg A \vee B \vee E$
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C_6: $\neg E \vee B$
C_7: $\neg B \vee F$
C_8: $\neg B \vee C$
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C_1 -- C_8
Pure symbol
C_1 -- C_3
C_5 -- C_8
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Unit clause propagation
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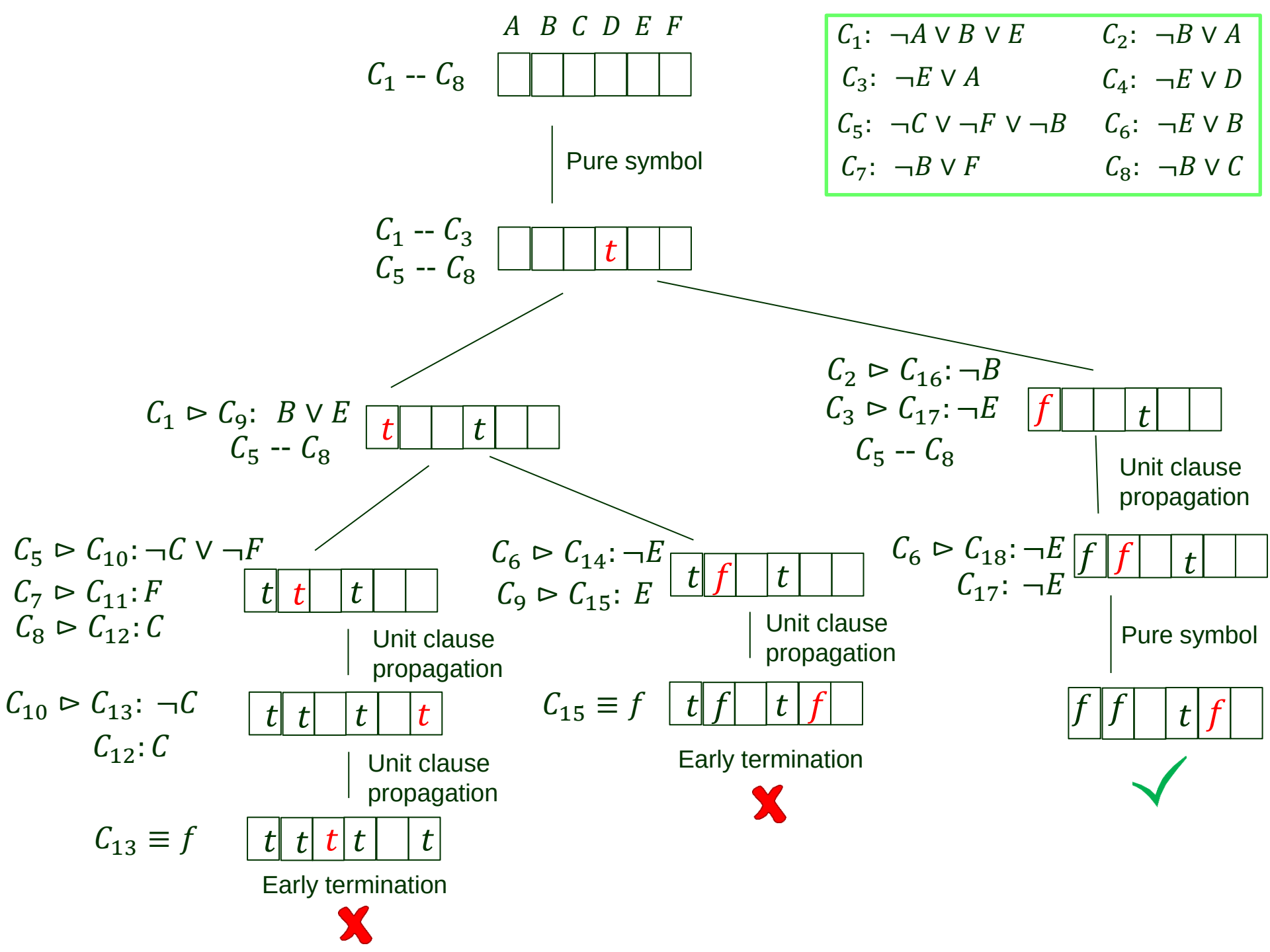
C_1: $\neg A \vee B \vee E$
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C_3: $\neg E \vee A$
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Pure symbol
C_1 -- C_3
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t t t
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$C_3 \triangleright C_{17}$: $\neg E$
C_5 -- C_8
f t t
Unit clause propagation
$C_5 \triangleright C_{10}$: $\neg C \vee \neg F$
$C_7 \triangleright C_{11}$: F
$C_8 \triangleright C_{12}$: C
t t t t
Unit clause propagation
$C_{10} \triangleright C_{13}$: $\neg C$
C_{12}: C
t t t t t
Unit clause propagation
$C_{13} \equiv f$
t t t t t
Early termination
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t f t
Unit clause propagation
$C_{15} \equiv f$
t f t f
Early termination
$C_6 \triangleright C_{18}$: $\neg E$
C_{17}: $\neg E$
f f t
Pure symbol
f f t f

Local Search Algorithms

- Take steps in the space of complete assignments, flipping the truth value of one symbol at a time.
- Use an evaluation that counts the number of unsatisfied clauses.
- Escape local minima using various forms of randomness.
- Find a good balance between greediness and randomness.

The WALKSAT Algorithm

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move, typically around 0.5
    max_flips, number of value flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for each $i = 1$ **to** *max_flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    if RANDOM$(0, 1) \leq p$ **then**
      flip the value in *model* of a randomly selected symbol from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*

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    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*