

# Logical Agents

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## Outline

I. Knowledge-based agents

II. The Wumpus world

III. Logic

# I. Knowledge-Based Agents

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- ♣ Problem solving agents do not know general facts.

An 8-puzzle agent does not know that two tiles cannot occupy the same space.

- ♣ Their atomic representations are very limited.

e.g., a list of all possible concrete states.

# I. Knowledge-Based Agents

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An 8-puzzle agent does not know that two tiles cannot occupy the same space.

- ♣ Their atomic representations are very limited.

e.g., a list of all possible concrete states.

- ◆ Intelligent agents need *knowledge about the world* in order to carry out reasoning for good decision making.

# I. Knowledge-Based Agents

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An 8-puzzle agent does not know that two tiles cannot occupy the same space.

- ♣ Their atomic representations are very limited.

e.g., a list of all possible concrete states.

- ◆ Intelligent agents need *knowledge about the world* in order to carry out reasoning for good decision making.

- Represent states, actions, etc.
- Incorporate new percepts.
- Update internal representation of the world.
- Deduce hidden properties of the world.
- Deduce appropriate actions.

# Knowledge Base

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A *knowledge base (KB)* is a set of sentences that represents some assertion about the world.

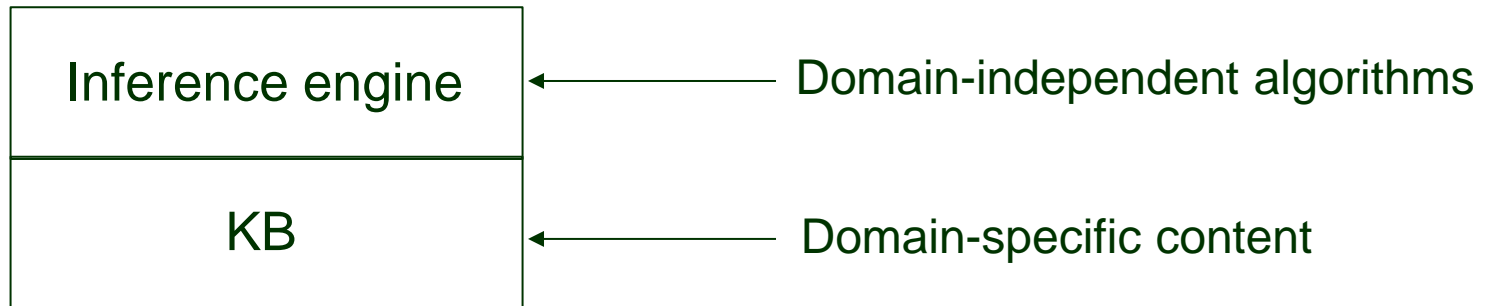
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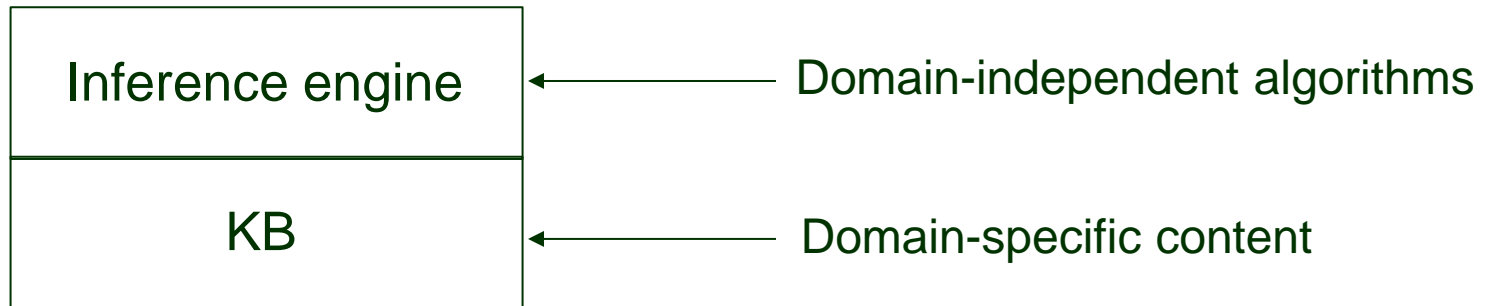


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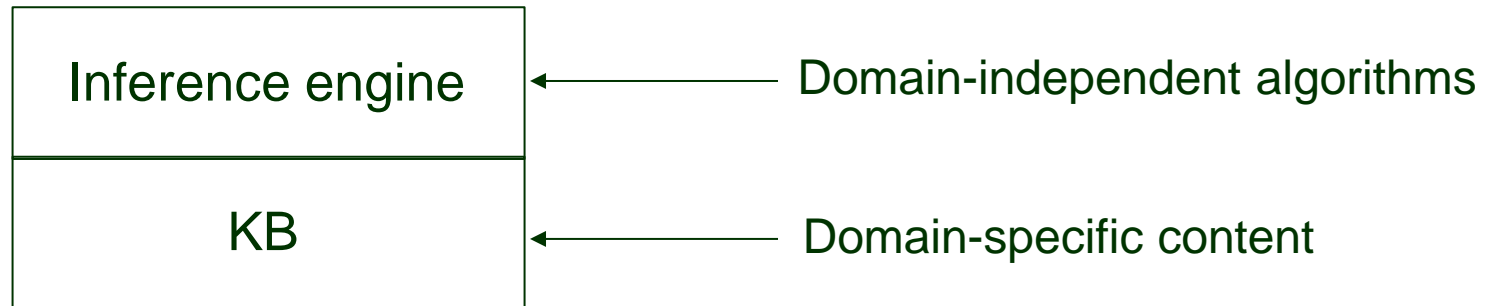
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**TELL:** Add new sentences to the KB.

**ASK:** Query the KB.

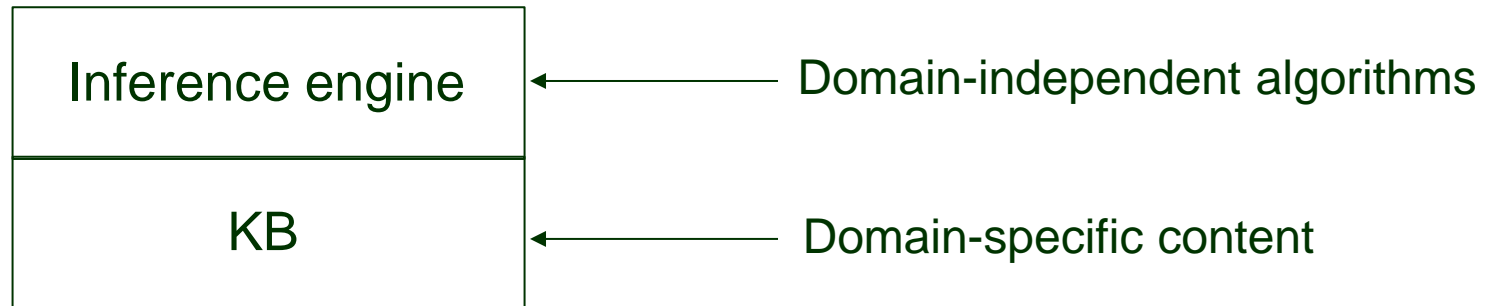


# Knowledge Base

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A *knowledge base (KB)* is a set of sentences that represents some assertion about the world.

An *axiom* is such a sentence that is taken to be true without being derived from other sentences.



**TELL:** Add new sentences to the KB.

**ASK:** Query the KB.

**Inference:** Derive new sentences from old.

# Generic Knowledge-Based Agent

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**function** KB-AGENT(*percept*) **returns** an *action*

**persistent:** *KB*, a knowledge base

*t*, a counter, initially 0, indicating time

TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))

*action* ← ASK(*KB*, MAKE-ACTION-QUERY(*t*)) // asks what action  
// it should perform.

TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*)) // tells what action  
// was chosen.

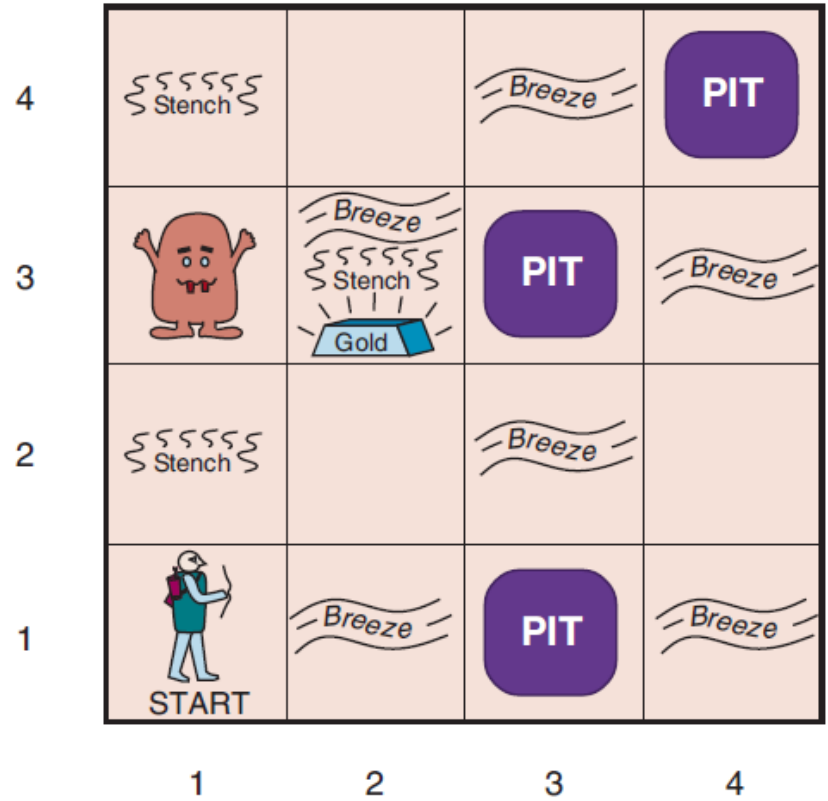
*t* ← *t* + 1

**return** *action*

# II. The Wumpus World

Cave consisting of connected rooms.

- Some rooms contain pits that will trap whoever enters them.
- The wumpus lurks in one room ready to eat whoever enters the room.
- The wumpus can be shot by the agent, who has **only one** arrow.
- A heap of gold is in a different room than where the wumpus lurks.

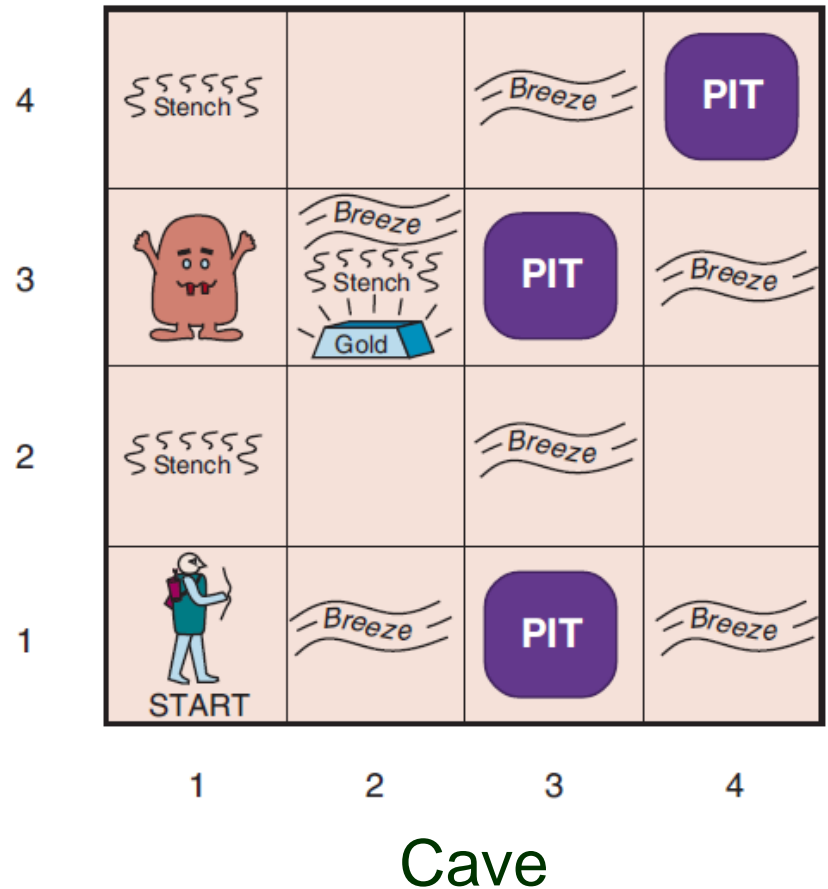


Cave

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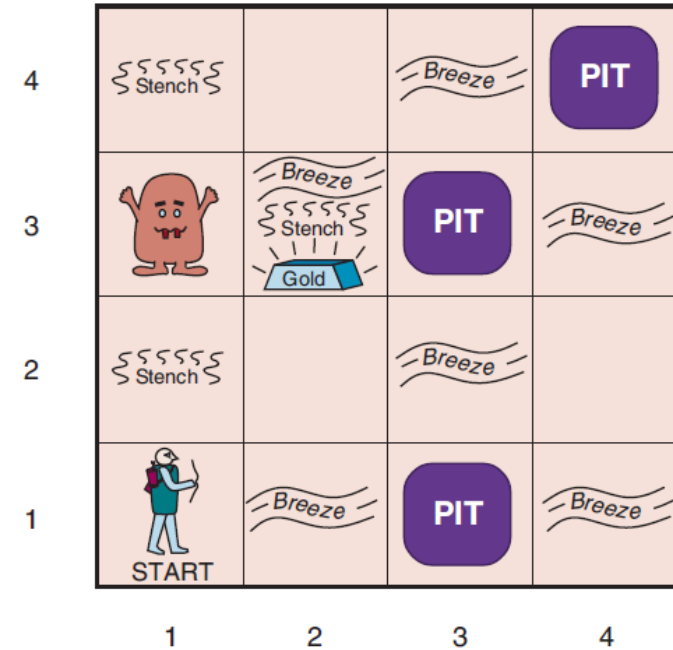


**Goal:** Find the gold and bring it back to the start without getting killed.

# Task Environment

## Performance measure

- +1000 (climbing out of the cave with the gold)
- -1000 (falling into a pit or being eaten by the wumpus)
- -1 (each action taken)
- -10 (using up the arrow)



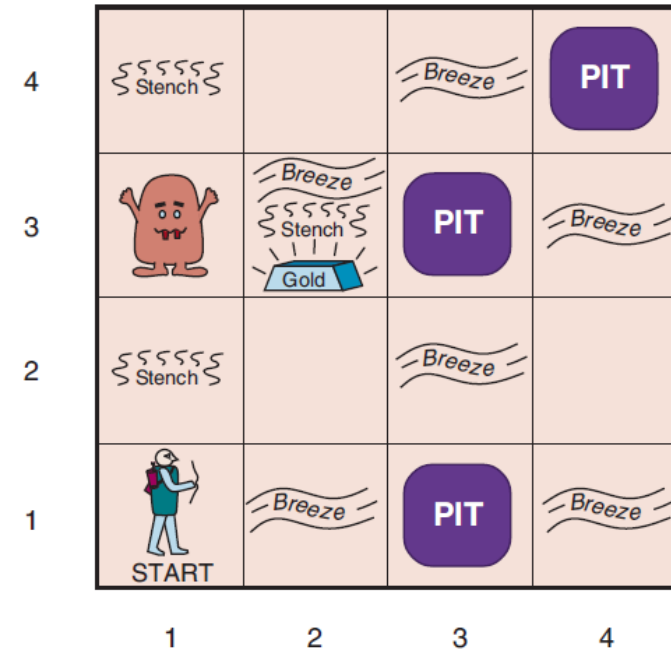
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- -1000 (falling into a pit or being eaten by the wumpus)
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- -10 (using up the arrow)

## Environment

- $4 \times 4$  grid surrounded by walls
- [1,1]: the starting square for the agent, who faces east
- locations of the gold and the wumpus:
  - $\neq [1, 1]$
  - otherwise randomly generated under uniform distribution (two distinct locations)
- 0.2 probability for a square other than [1, 1] and without gold or wumpus to be a pit



# Actuators

## Actuators:

### 1) *Forward*, *TurnLeft* by 90°, *TurnRight* by 90°

- Death of the agent if it enters a square containing a pit or a live wumpus.
- No movement if bumping into a wall.

### 2) *Grab*

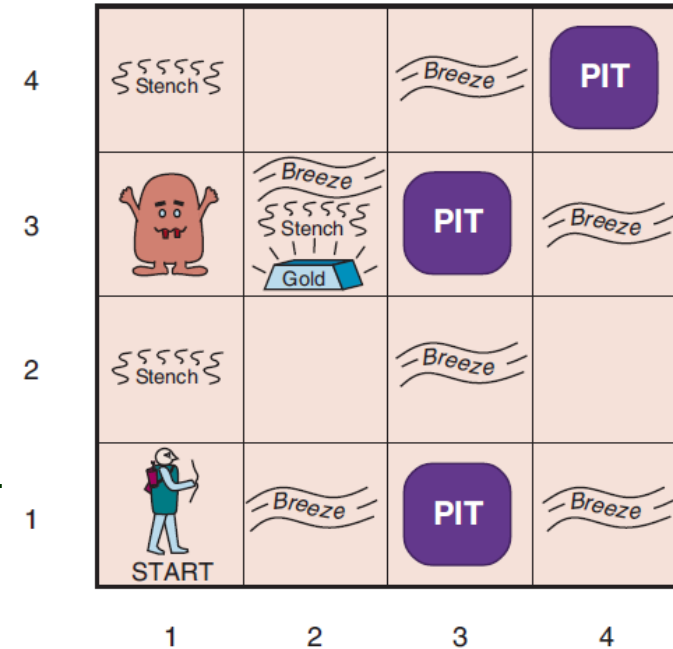
- Picks up the gold if it is in the same square as the agent.

### 3) *Shoot*

- Fire an arrow in the direction the agent is facing.
- The arrow continues until hitting the wumpus (who gets killed consequently) or a wall.

### 4) *Climb*

- Climb out of the cave if at [1, 1].



# Sensors

---

5 Sensors, each providing one bit of information:

1) *Stench*

- in the squares directly (not diagonally) adjacent to the wumpus

2) *Breeze*

- in the squares directly (not diagonally) adjacent to a pit

3) *Glitter*

- in the square where the gold is

4) *Bump*

- when the agent walks into a wall

5) *Scream*

- when the wumpus is killed



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4) *Bump*

- when the agent walks into a wall

5) *Scream*

- when the wumpus is killed

Percepts in the form of a 5-vector:

e.g., [*Stench*, *Breeze*, *None*, *None*, *None*]

# Characteristics of WW

---

- ◆ Deterministic, discrete, static, and single-agent

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Outcome specified.

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The wumpus does not move.

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Outcome specified.

The wumpus does not move.

- ◆ Partially observable

Locations of the pits and the wumpus are unknown.

# Characteristics of WW

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- ◆ Deterministic, discrete, static, and single-agent

Outcome specified.

The wumpus does not move.

- ◆ Partially observable

Locations of the pits and the wumpus are unknown.

**Challenge:** The agent needs to use logical reasoning to overcome its initial lack of knowledge about the environment's configuration.

# Solution by a Knowledge-Based Agent

---

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

- A** = Agent
- B** = Breeze
- G** = Glitter, Gold
- OK** = Safe square
- P** = Pit
- S** = Stench
- V** = Visited
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# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
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[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
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[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

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Forward  


1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
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Forward



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



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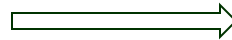
[None, Breeze, None, None, None]

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

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1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

[None, Breeze, None, None, None]



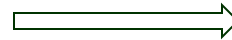
A pit in [1,1], [2,2], or [3, 1].

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
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1,3	2,3	3,3	4,3
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[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

[None, Breeze, None, None, None]



A pit in [1,1], [2,2], or [3, 1].



[1,1] has just been visited.

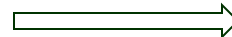
A pit in [2,2] or [3, 1].

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
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1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

[Stench, Breeze, Glitter, Bump, Scream]

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

[None, Breeze, None, None, None]



A pit in [1,1] , [2,2] , or [3, 1].



[1,1] has just been visited.

A pit in [2,2] or [3, 1].

# Next Step

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
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1,3	2,3	3,3	4,3
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1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

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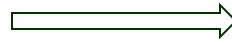
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1,2 A S OK	2,2 OK	3,2	4,2
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[Stench, None, None, None, None]



The wumpus is in [1,1], [2,2], or [1, 3].

Only one unexplored square [1,2] is OK.

Be prudent: Turn around, go back to [1,1] and move onto [1, 2].

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[Stench, Breeze, Glitter, Bump, Scream]

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[Stench, None, None, None, None]



The wumpus is in [1,1], [2,2], or [1, 3].

- [1,1] is OK
- [2,2] is impossible because no stench was detected at [2, 1].

Only one unexplored square [1,2] is OK.

Be prudent: Turn around, go back to [1,1] and move onto [1, 2].

# Next Step

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
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[Stench, None, None, None, None]



The wumpus is in [1,1], [2,2], or [1, 3].



[1,1] is OK  
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The wumpus is in [1, 3].

Only one unexplored square [1,2] is OK.

Be prudent: Turn around, go back to [1,1] and move onto [1, 2].

# Next Step

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

[Stench, Breeze, Glitter, Bump, Scream]

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# More Inference

1,4	2,4 <b>P?</b>	3,4	4,4
1,3 <b>W!</b>	2,3 <b>A</b> S G B	3,3 <b>P?</b>	4,3
1,2 S V OK	2,2 V <b>OK</b>	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 <b>P!</b>	4,1

[Stench, None, None, None, None]



The wumpus is in [1, 3].



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# More Inference

1,4	2,4 <b>P?</b>	3,4	4,4
1,3 <b>W!</b>	2,3 <b>A</b> S G B	3,3 <b>P?</b>	4,3
1,2 S V OK	2,2 V <b>OK</b>	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 <b>P!</b>	4,1

[Stench, None, None, None, None]



The wumpus is in [1, 3].



No breeze in [1,2].

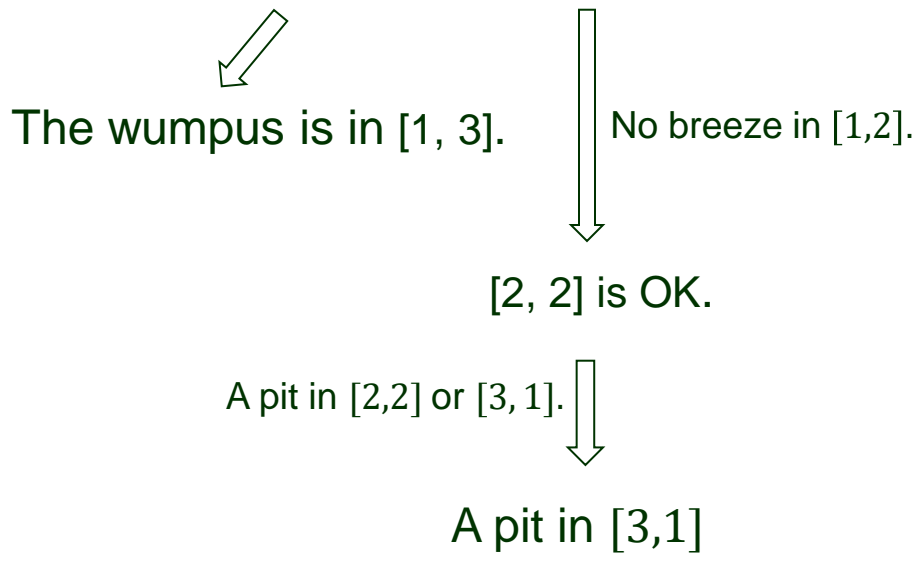
[2, 2] is OK.

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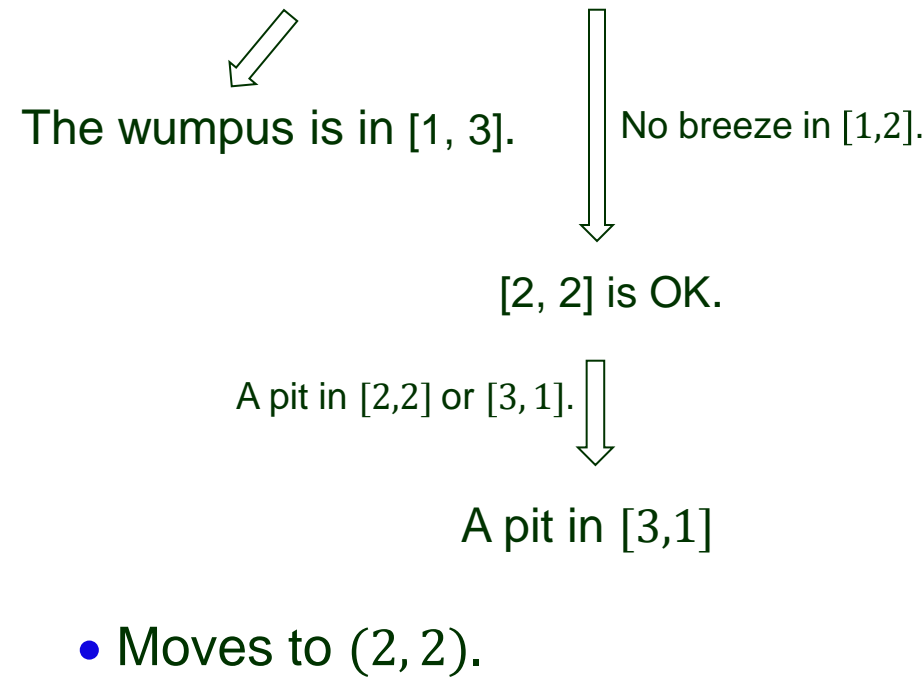


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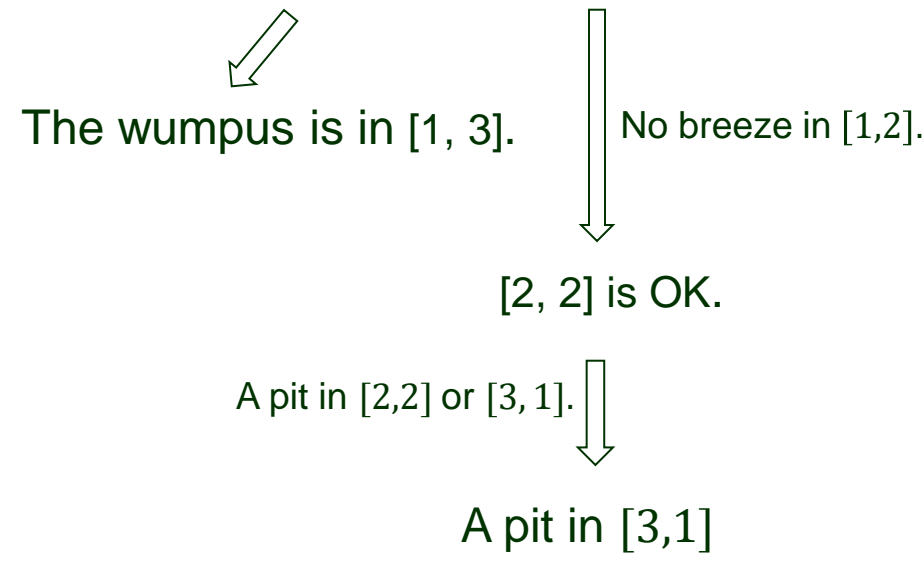


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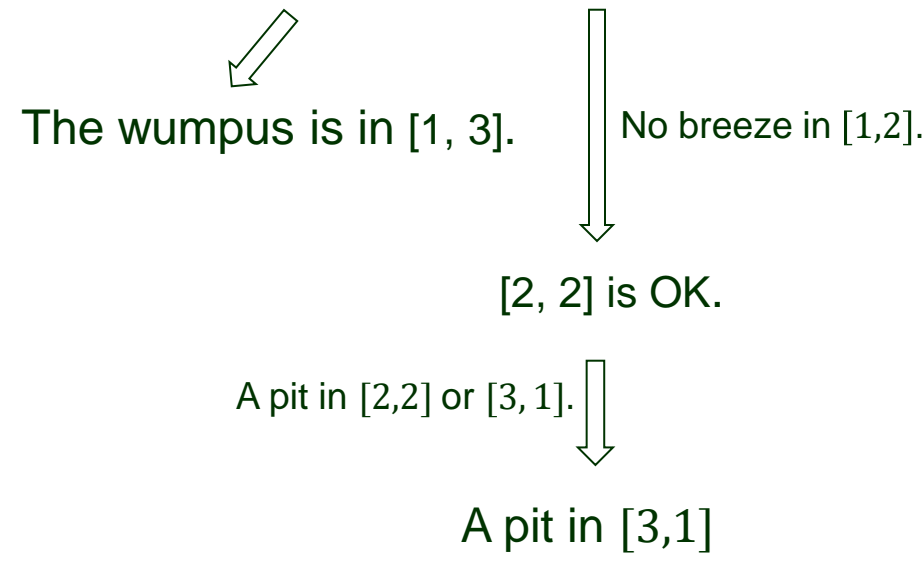
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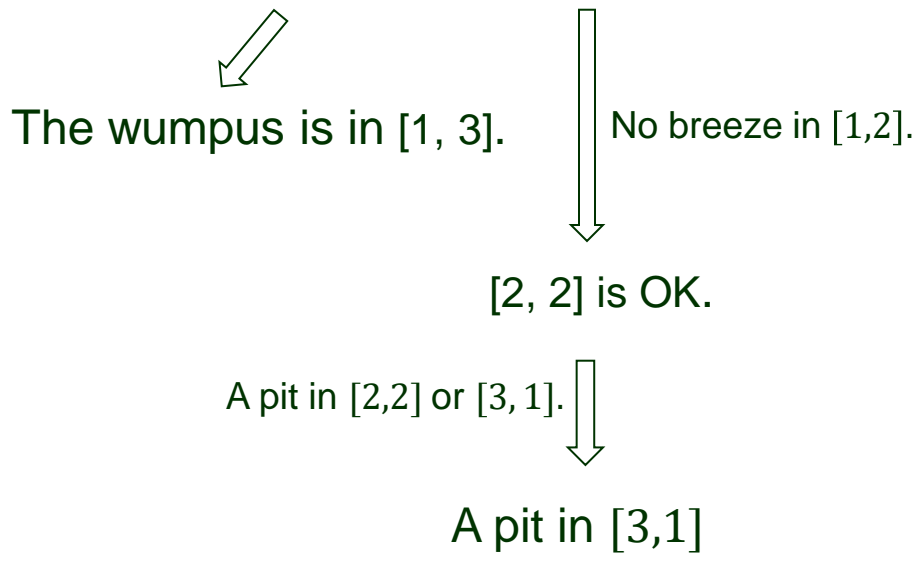
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A conclusion drawn is guaranteed if the available information is correct.

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# III. Logic

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- A formal language for representing information such that conclusions can be drawn.

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Every sentence must be either true or false in each possible world.

# Model and Reasoning

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Model  $m$ : assigns values to variables.

$m$  *satisfies* a sentence  $\alpha$ , or  $m$  is a model of  $\alpha$ , if  $\alpha$  is true in  $m$ .

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**Example**  $x = 0$  entails  $xy = 0$ .

# Back to the Wumpus World




1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P? 	3,2	4,2
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Knowledge base (KB) includes

- All the rules.
- Percepts:
  - [None, None, None, None, None]* in [1,1]
  - [None, Breeze, None, None, None]* in [2,1]

[Stench, Breeze, Glitter, Bump, Scream]

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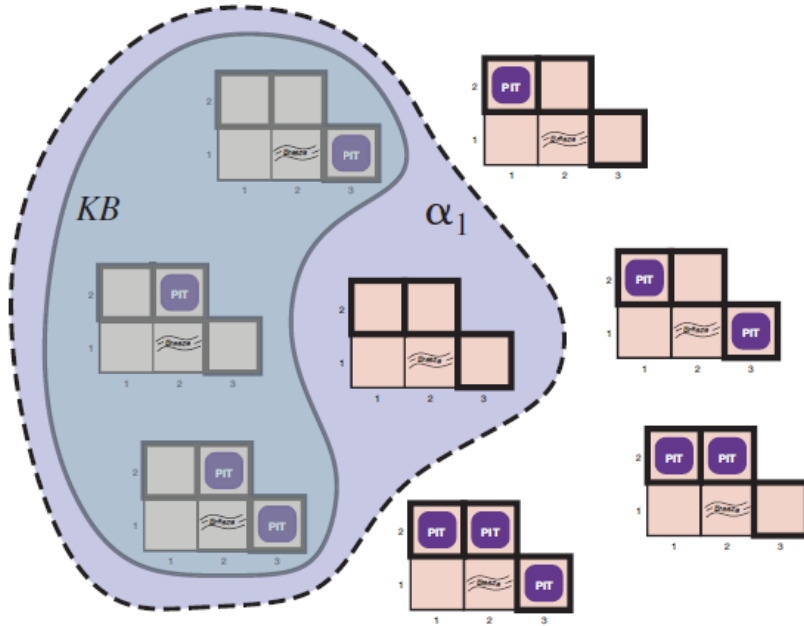
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8 possibilities if ignoring the KB.

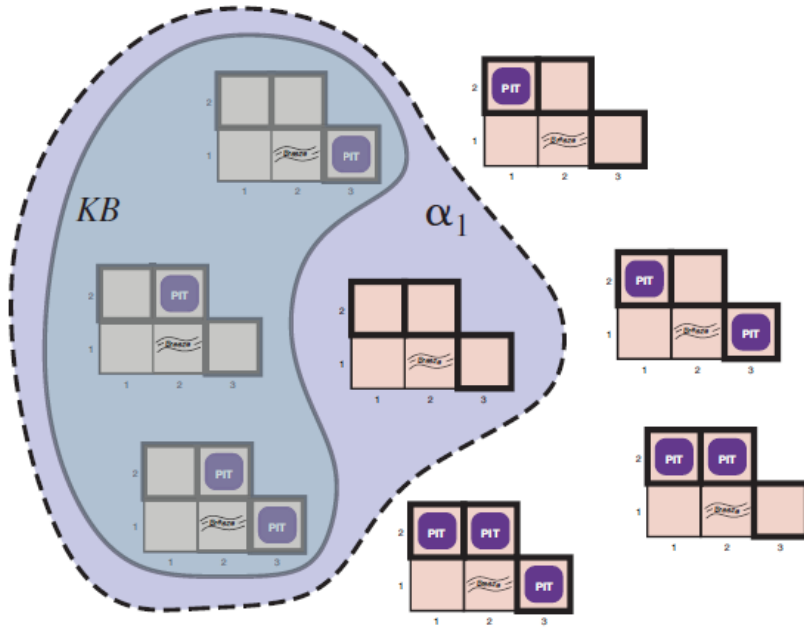
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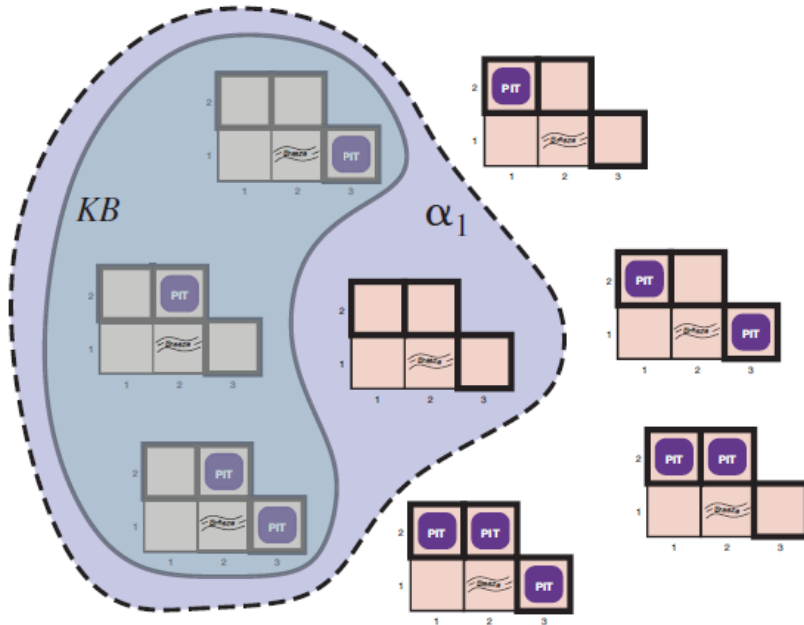
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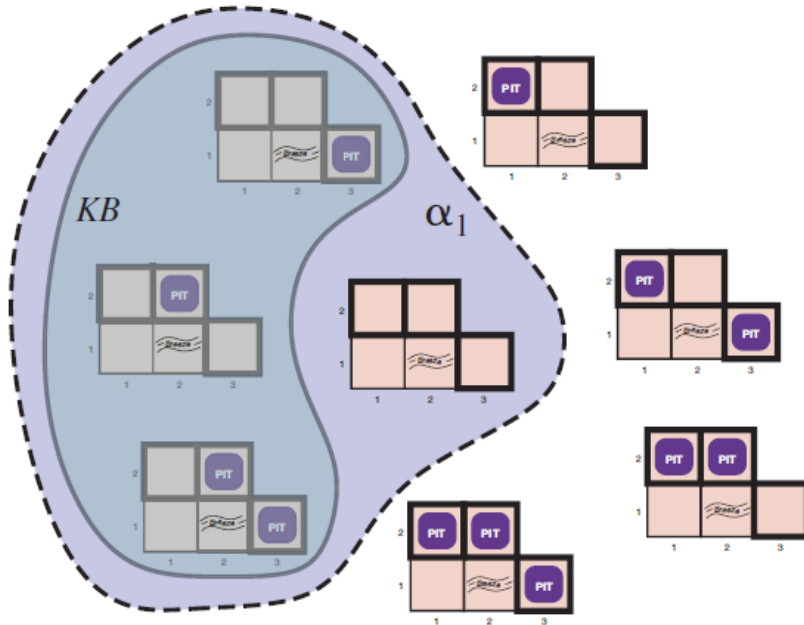


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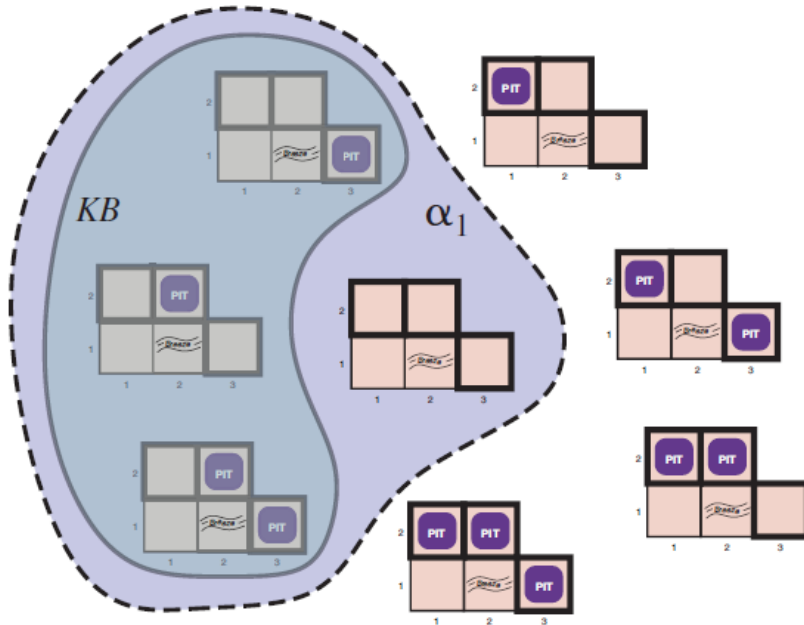
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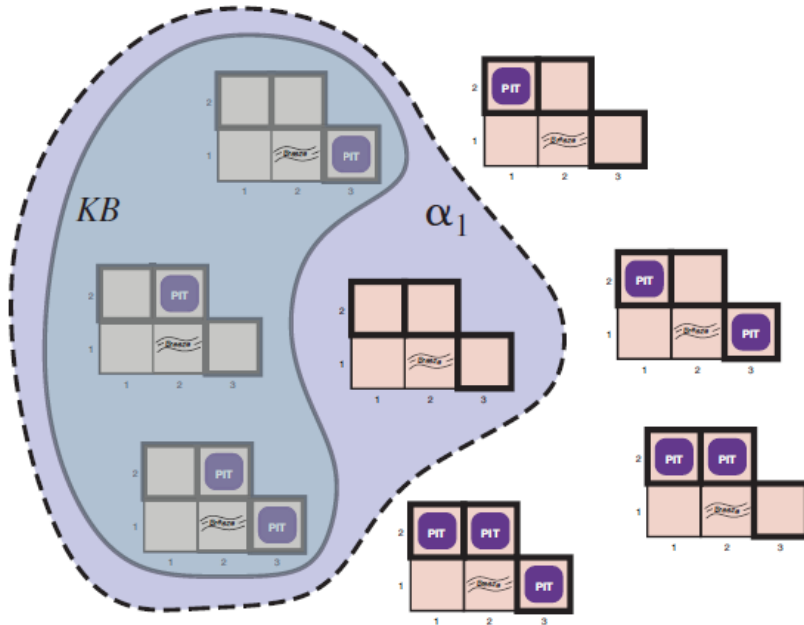
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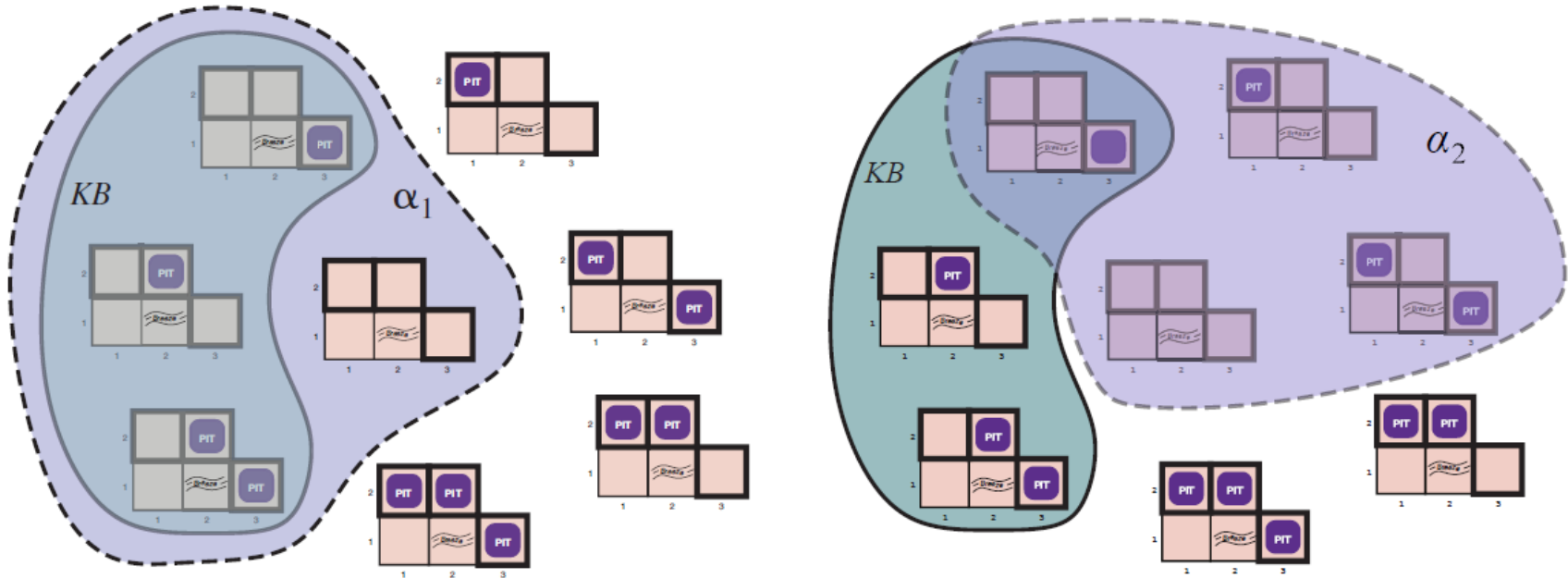
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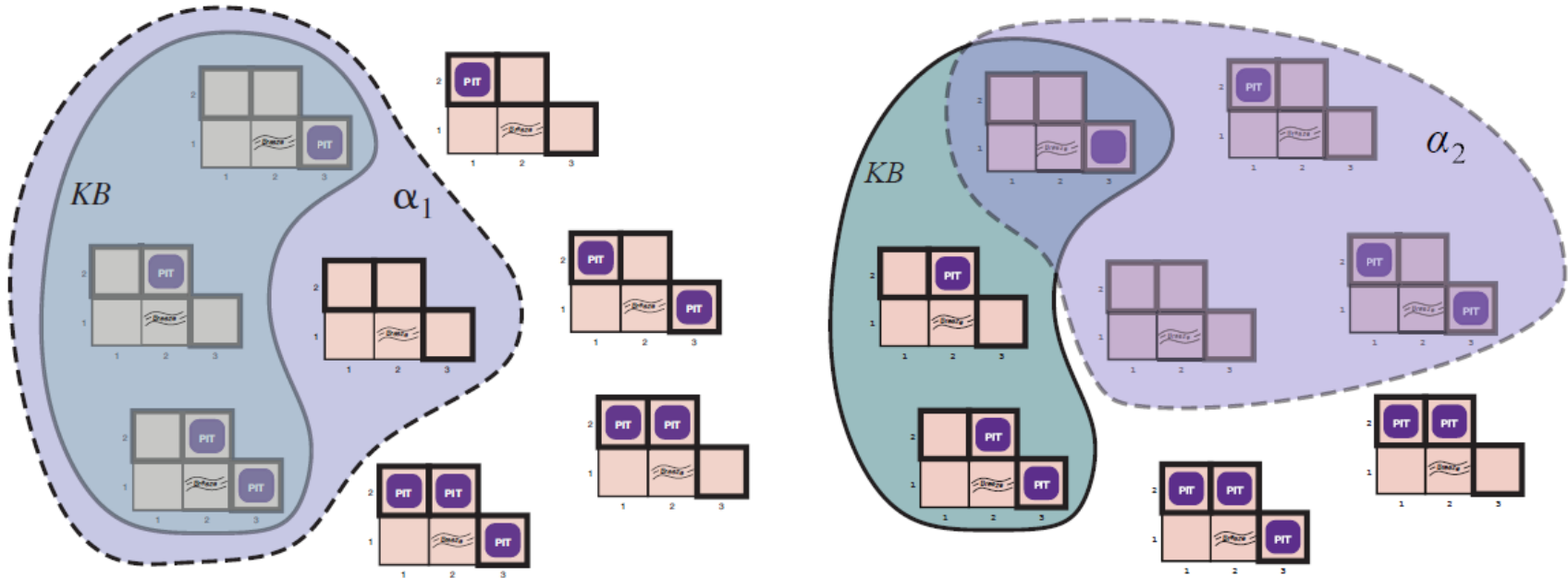
$\alpha_2$  = "There is no pit in  $[2, 2]$ ."

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$KB \not\models \alpha_2$  since  $M(KB) \not\subseteq M(\alpha_2)$

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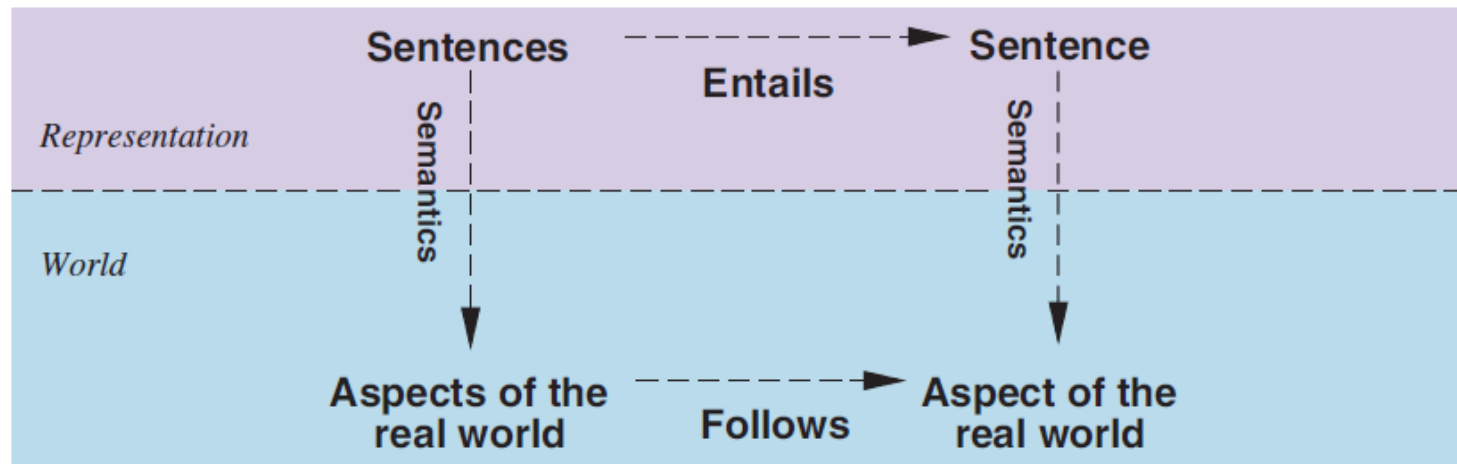
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- ♣ It is *complete* if it can derive any sentence that is entailed, that is,

$KB \vdash_i \alpha$  whenever  $KB \models \alpha$

# Logical Reasoning

A process whose conclusions are guaranteed to be true in any world in which the premises (the *KB* in this case) are true.



Correspondence between world and representation