First-Order Logic

Outline

I. Syntax of FOL

II. Quantifiers

III. Model for FOL

IV. Assertions & queries in FOL

* Figures are from the textbook site unless a source is specifically cited.

I. Propositional Logic: Strength and Weakness

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$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

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- ♠ Propositional logic assumes the world contains *facts only*.

Combining Formal and Natural Languages

First-order logic

- built around objects and relations
  - Objects: people, houses, cars, trees, colors, days, …
  - Relations:
    - unary properties such as `big`, `windy`, …
    - n-ary properties such as `bigger than`, `parent of`, `on`, `owns`, …
  - Functions: `square of`, `best friend`, `age`, …
- capable of expressing facts about some or all objects

Formal Languages

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Alphabet of First-Order Logic

- Logical symbols
  - connectives: $\wedge, \vee, \Rightarrow, \Leftrightarrow, \neg$
  - parenthesis: (,) and punctuation ,
  - equality: $=$
  - quantifiers: $\forall$ (universal quantification), $\exists$ (existential quantification)
  - variables: $x, y, z, \ldots$; $x_1, x_2, \ldots$

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    Father(x, y) // x is father of y

    Female(x) // x is female

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    Father(x, y) // x is father of y
    Female(x) // x is female
  - function symbols:
    gcd(x, y) // greatest common divisor of x and y
    FatherOf(x) // father of x

Terms and Atomic Sentences

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function symbol terms

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  - predicates: *true*, *false*
    Mother(Aphrodite, Harmonia)
    Male(John)

- Greek mythology related examples are taken/extended from ones used in the book *Logic for Problem Solving* by Robert Kowalski, Elsevier Science Publishing Co., Inc., 1979.

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function symbol terms

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  - predicates: *true*, *false*

    Mother(Aphrodite, Harmonia)

    Male(John)
  - term equalities

    FatherOf(Apollo) = Zeus

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Complex Sentences

- made of atomic sentences using logical connectives

$$\mathit{Father}(x, y) \Rightarrow \mathit{Male}(x)$$

$$\mathit{Female}(x) \vee \neg \mathit{Mother}(x, y)$$

$$\mathit{Likes}(\text{Mary, John}) \Leftrightarrow \mathit{Likes}(\text{John, Mary})$$

$$(\mathit{Parent}(x, y) \wedge \mathit{Parent}(y, z)) \Rightarrow \mathit{GrandParent}(x, z)$$

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- universal quantification

$\forall x\ \mathit{Circle}(x) \Rightarrow \mathit{Ellipse}(x)$ // Every circle is an ellipse.

$\neg \forall x\ \mathit{Likes}(x, \text{sushi})$ // Not everyone likes sushi.

$\forall x\ \mathit{Integer}(x) \Rightarrow (\mathit{Even}(x) \vee \mathit{Odd}(x))$ // Every integer is either even or odd.

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- existential quantification

$\exists x\ \mathit{Star}(x) \wedge \neg\ (x = \text{Sun})$ // There are stars other than the sun.

$\exists x\ \mathit{Whale}(x) \wedge (\mathit{Age}(x) = 200)$ // Some whales live to 200 years.

Syntax of First-Order Logic

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} & \rightarrow & (\ \textit{Sentence}\ ) \\
& \mid & \neg\ \textit{Sentence} \\
& \mid & \textit{Sentence} \wedge \textit{Sentence} \\
& \mid & \textit{Sentence} \vee \textit{Sentence} \\
& \mid & \textit{Sentence} \Rightarrow \textit{Sentence} \\
& \mid & \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
& \mid & \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\\
\textit{Term} & \rightarrow & \textit{Function}(\textit{Term}, \ldots) \\
& \mid & \textit{Constant} \\
& \mid & \textit{Variable} \\
\\
\textit{Quantifier} & \rightarrow & \forall \mid \exists \\
\textit{Constant} & \rightarrow & A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} & \rightarrow & a \mid x \mid s \mid \cdots \\
\textit{Predicate} & \rightarrow & \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} & \rightarrow & \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\\
\text{OPERATOR PRECEDENCE} & : & \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

II. Scope of a Quantifier

- Quantifiers ∀ and ∃ have the lowest precedence.

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$$\cdots\ (\cdots\ (\cdots\ (\cdots \forall x \quad \cdots \quad)\cdots)\cdots)\cdots$$

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Free and Bound Variables

A variable occurrence is *bound* in a formula if it is quantified.

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$P(x)\ \Rightarrow \exists x\ Q(x)$

free bound

different bound variable

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same free variable

Nested Quantifiers

$\forall x \exists y$ *Student*$(x) \wedge$ *Course*(y) $\wedge$ *Enrolled*(x, y)

$\forall x \exists y$ *Brother*$(x, y) \Rightarrow$ *Sibiling*(x, y)

Order matters for quantifiers of different types:

$\forall x \exists y$ *Loves*(x, y) // Everybody (x) loves somebody (y)

$\exists x \forall y$ *Loves*(y, x) // There is someone (x) whom everyone (y) loves.

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but not for those of the same type and appearing next to each other:

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$\forall x \forall y\ (Brother(x, y) \Rightarrow Sibiling(x, y))$

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Connections Between ∀ and ∃ Through ¬

$$\forall x \ \neg Likes(x, Parsnips) \equiv \neg \exists x \ Likes(x, Parsnips)$$

$$\forall x \ Likes(x, Icecream) \equiv \neg \exists x \ \neg Likes(x, Icecream)$$

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De Morgan's rules still apply:

$$\neg \forall x \ P(x) \equiv \exists x \ \neg P(x)$$

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De Morgan's rules still apply:

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// ($x \equiv$ Poseidon and $y \equiv$ Hades, or $x \equiv$ Hades and $y \equiv$ Poseidon)

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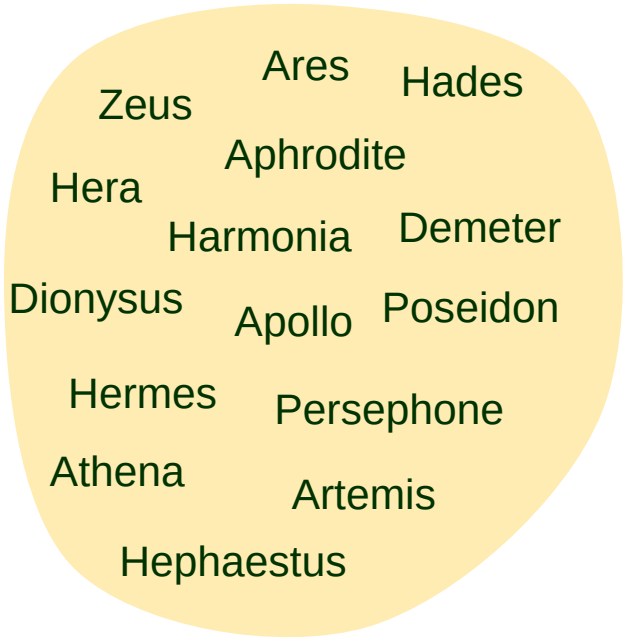
Model Example

Model for the family relationships of the Greek gods (incomplete).



domain D

Father(Zeus, Hermes)

Mother(Hera, Ares)

Mother(Aphrodite, Harmonia)

Father(Zeus, Athena)

⋮

predicates

Weapon(Zeus) // ≡ Thunderbolt

Weapon(Apollo) // ≡ BowAndArrows

Carry(Hermes) // ≡ Flute

Carry(Aphrodite) // ≡ Apple

⋮

functions

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Quantifiers allow us to express properties of a collection of objects instead of enumerating them by name.

∀ (universal): "for all"

∃ (existential): "there exists"

Truths with Quantifications

- $\forall x\ P(x)$ is true in a model M iff $P(x)$ is true with x assuming every object in the model

$\forall x\ Father(x, y) \Rightarrow Male(x)$ — *true* (in every model)

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$\exists x\ \mathit{Mother}(x, \text{Ares}) \wedge \mathit{Mother}(x, \text{Harmonia})$ — *false* (in the model of Greek mythology)

IV. Knowledge Engineering

A field of AI dedicated to representing information about the world in a form that can be utilized by a computer to solve complex tasks such as:

- medical diagnosis
- dialog in a natural language
- etc.

♦ Knowledge representation (logical rules, semantic nets, frames, etc.)

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A *domain* is some part of the world about which we wish to express some knowledge.

Assertions and Queries in FOL

- Add sentences, called *assertions*, to a KB using TELL.

  TELL(*KB*, *Likes*(John, *Icecream*))

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Any query is entailed by the KB should be answered affirmatively.

Substitution

Suppose another KB has the following predicates:

$Bird(\text{Swan})$, $Bird(\text{Crane})$, $Bird(\text{Parrot})$,

- Quantified query

  $\text{ASK}(KB, \exists x\ Bird(x))$

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  substitution or *binding list*

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Kinship relations are represented by binary predicates.

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These are *definitions* in the form of $\forall x, y\ P(x, y) \Leftrightarrow \ldots$ and built upon a basic set of predicates *Child*, *Male*, *Female*, etc.

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$\forall x\ Person(x) \Longrightarrow \cdots$ // partial specification of

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