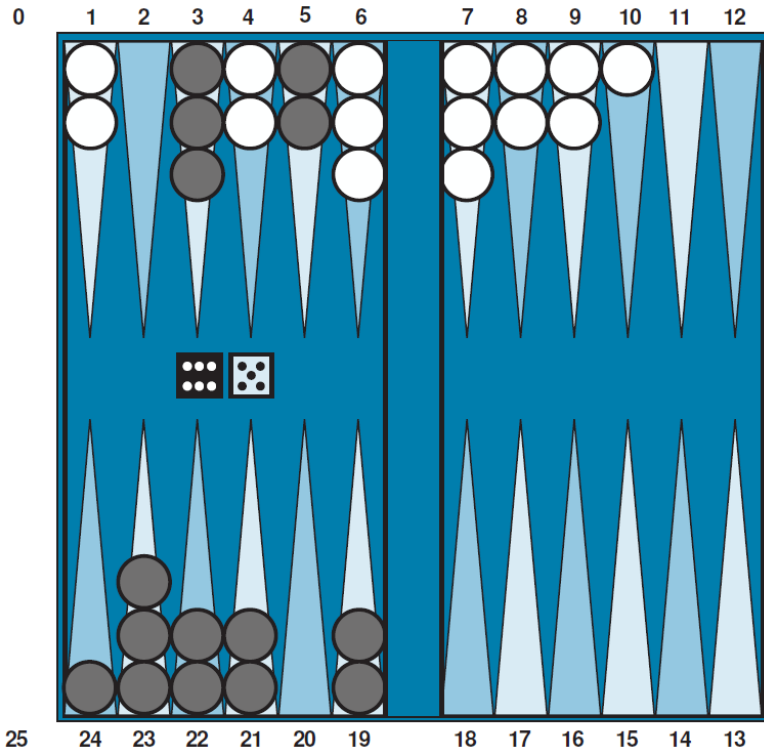


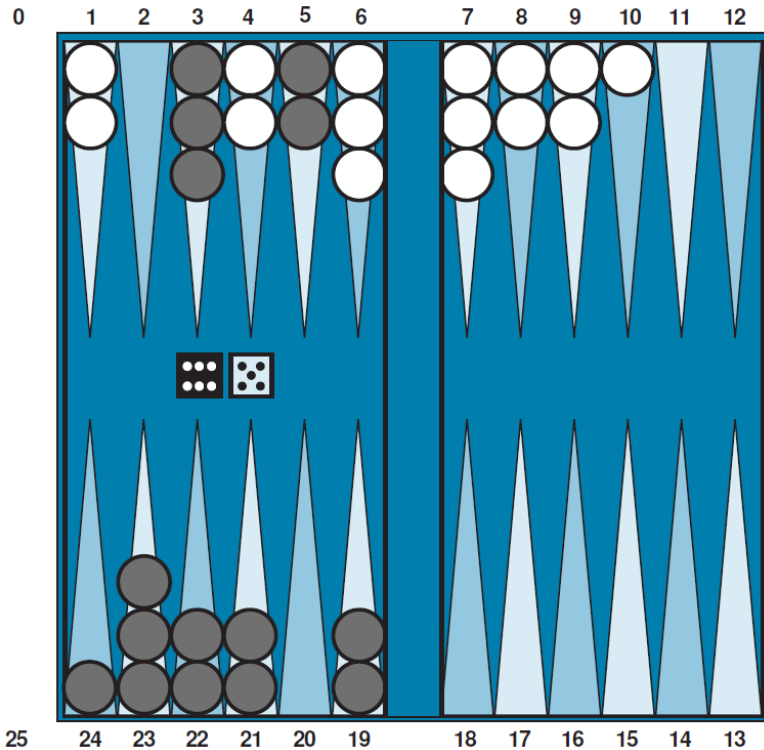
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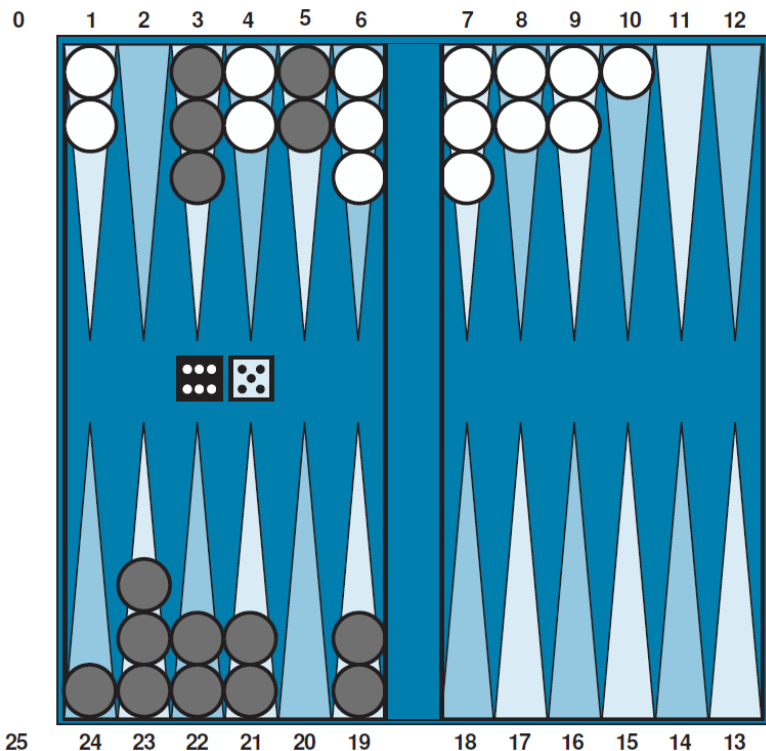
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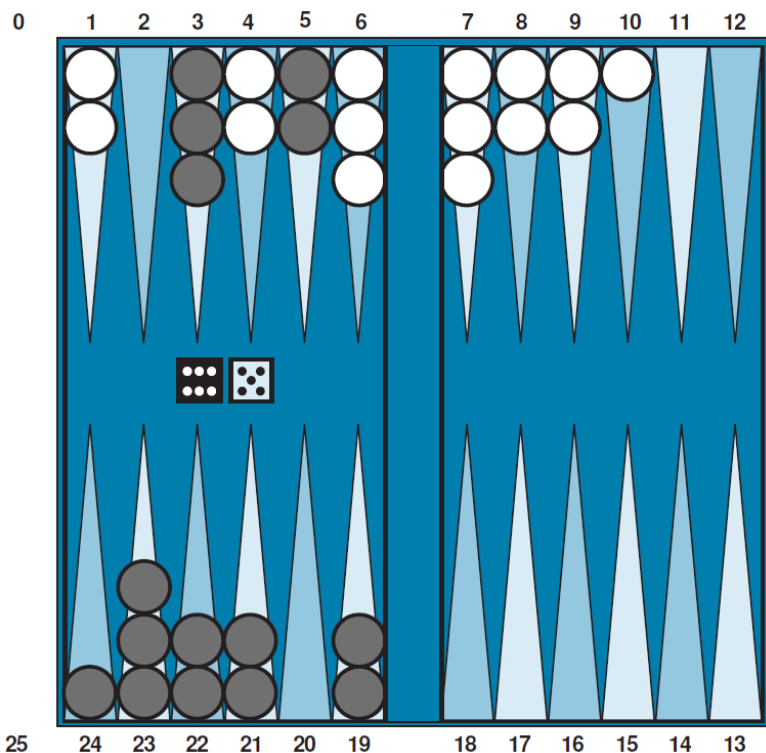


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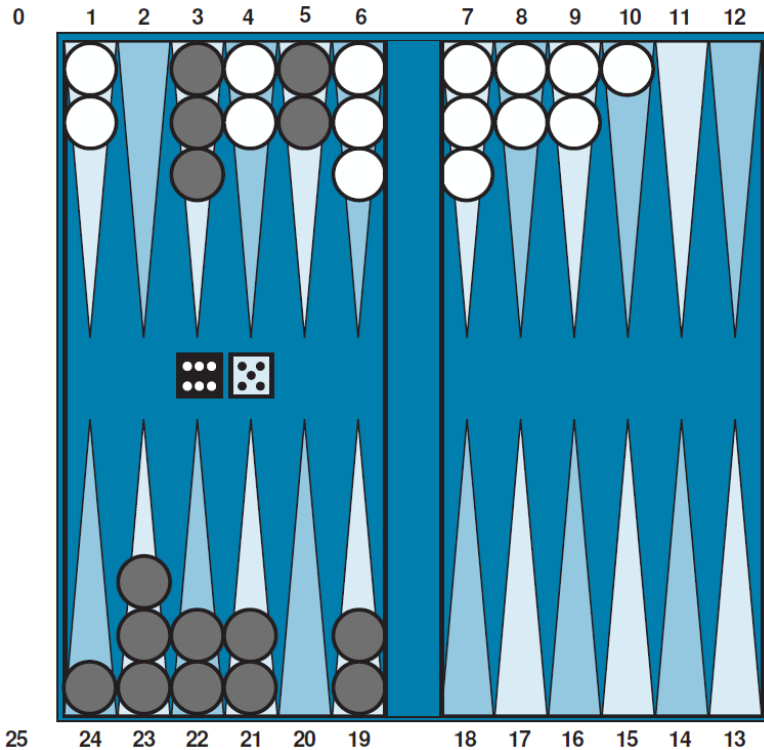
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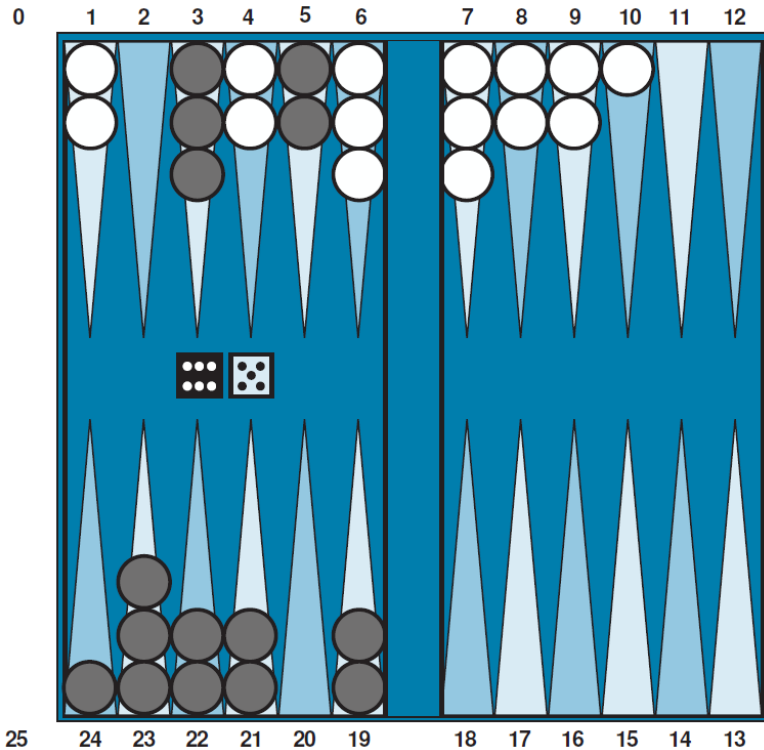
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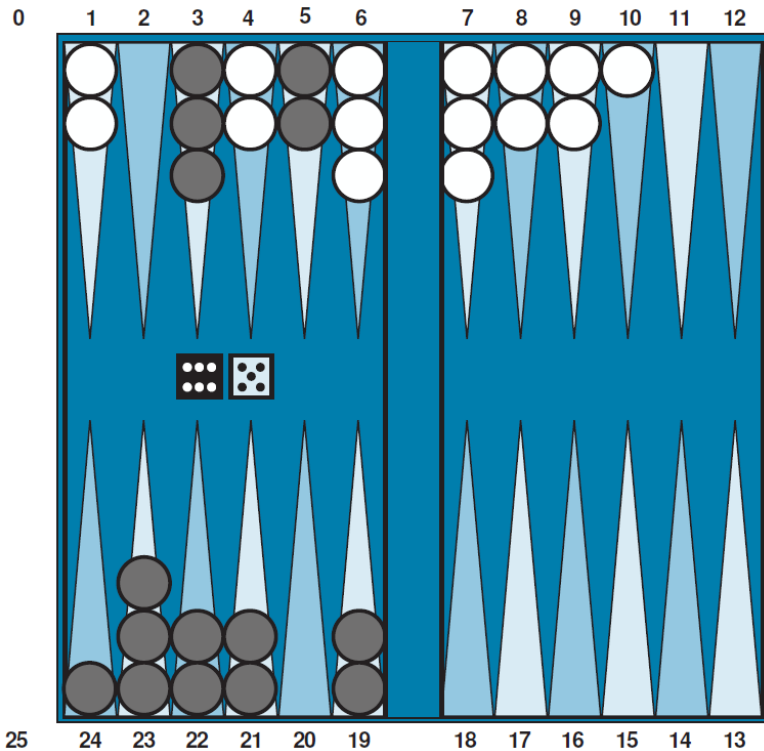
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└──────────┘  
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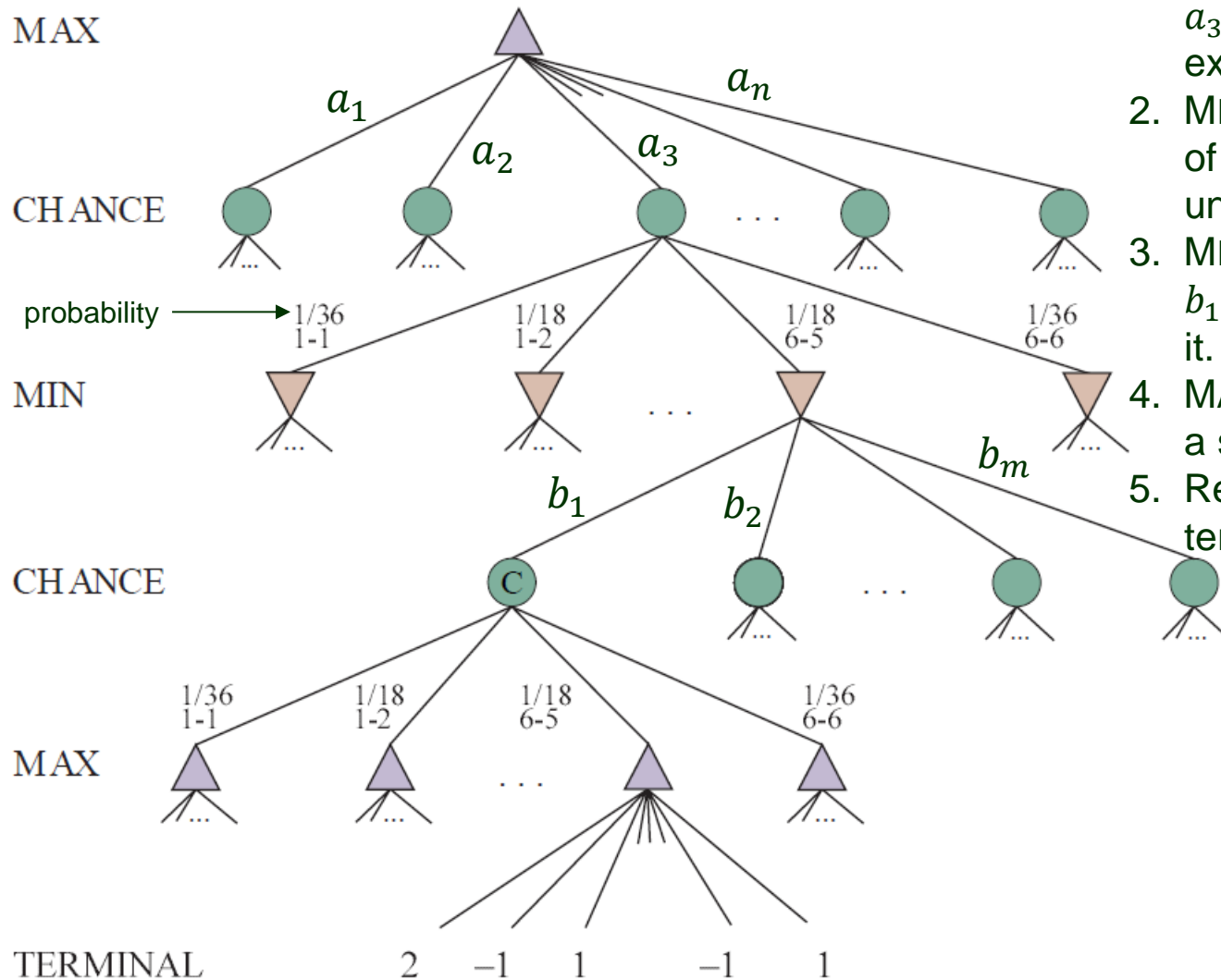
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- ◆ Calculate expected value (called **expectiminimax** value) of a position.

# Game Tree for a Backgammon Position



1. MAX chooses an action, say,  $a_3$  out of  $\{a_1, \dots, a_n\}$  and executes it.
2. MIN rolls a die and gets a set of legal actions  $\{b_1, \dots, b_m\}$  under some CHANCE node.
3. MIN chooses an action, say,  $b_1$ , from the set and executes it.
4. MAX rolls a die and gets a set of legal actions.
5. Repeat steps 1-4 until a terminal state is reached.



# Expectiminimax Value

---

EXPECTIMINIMAX( $s$ ) =

$$\left\{ \begin{array}{l} \text{UTILITY}(s, \text{MAX}) \\ \max_a \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) \\ \min_a \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) \\ \sum_r P(r) \text{EXPECTIMINIMAX}(\text{RESULT}(s, r)) \end{array} \right.$$

one possible dice roll

expected value

if IS-TERMINAL( $s$ )

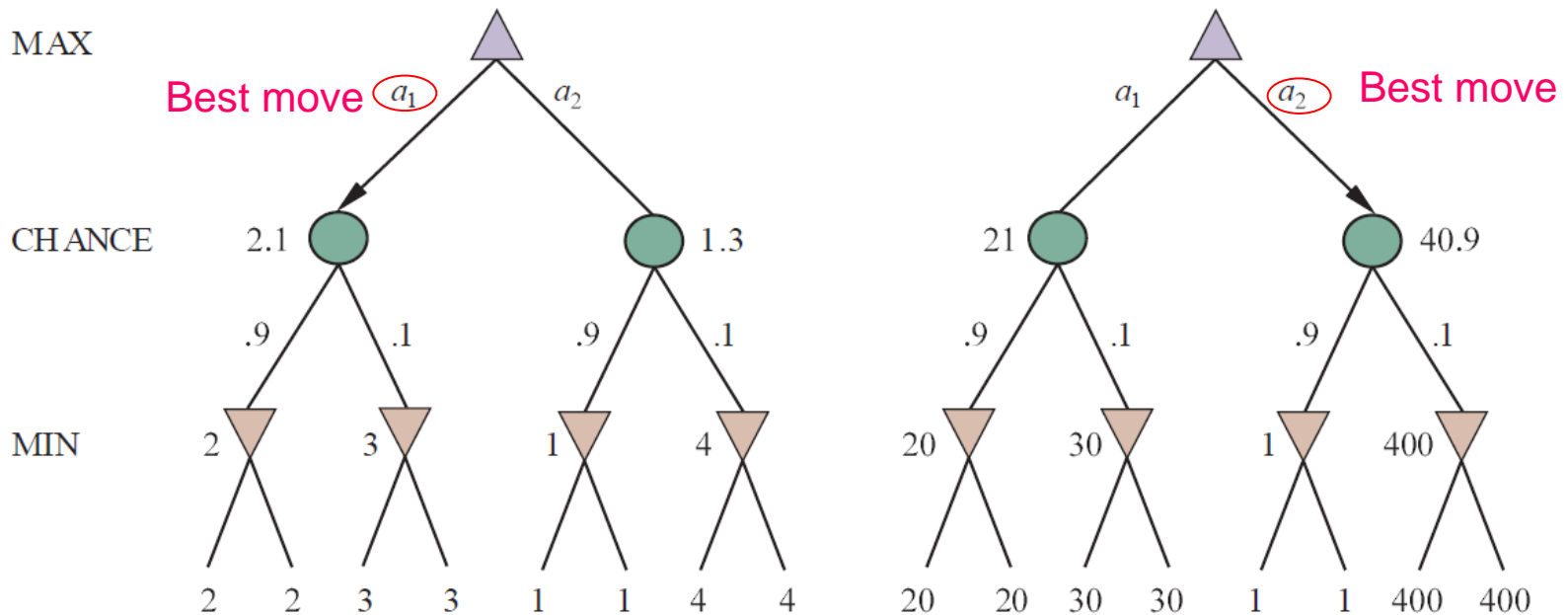
if TO-MOVE( $s$ ) = MAX

if TO-MOVE( $s$ ) = MIN

if TO-MOVE( $s$ ) = CHANCE

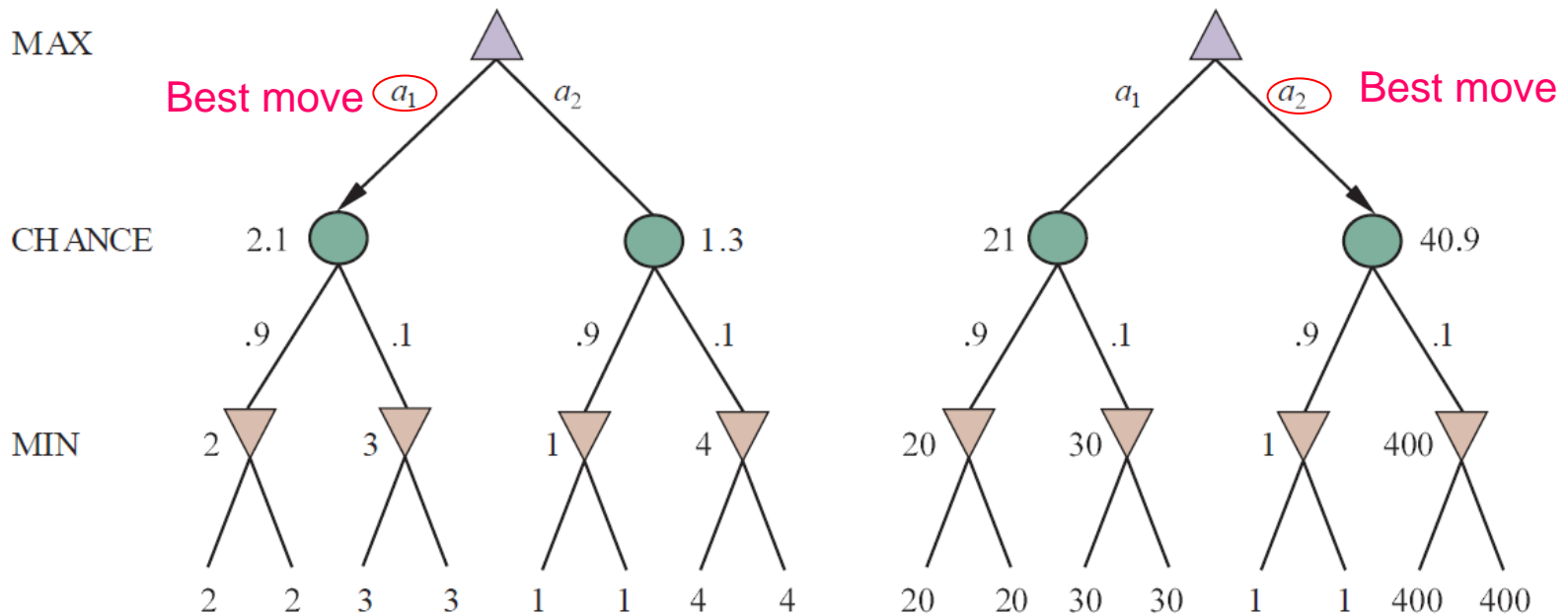
# Evaluation Functions

Evaluation functions with the same order of leaf values can yield different move choices at a state.



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Alpha-beta pruning is still applicable if we can bound values on chance nodes.