Monte Carlo Tree Search (MCTS)

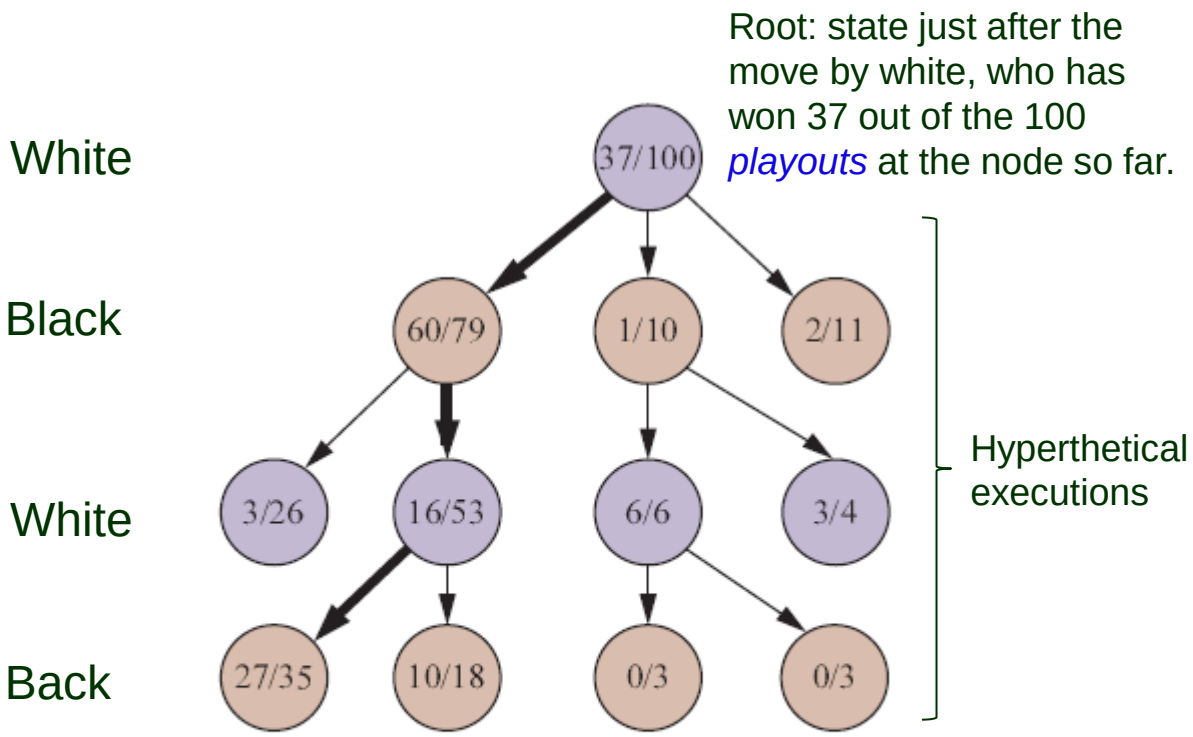
Casino de Monte-Carlo, Monaco

- Online planning algorithm.
- Intelligent tree search that balances exploration and exploitation.
- Random sampling in the form of simulations (*playouts*).
- Storage of the statistics of actions to improve choices in the future.
- Combined with neural nets, it is behind many major successes of AI applications such as games (e.g., AlphaGo), scheduling, planning, and optimization.

* Figures/images are from the textbook site (or by the instructor) unless the sources are given otherwise.

Step 1: Selection

We need to decide a move for black (at the root) to make.



(a) Selection

- Unlike in the minimax game tree, a directed edge represents a move by the player at the edge's destination.

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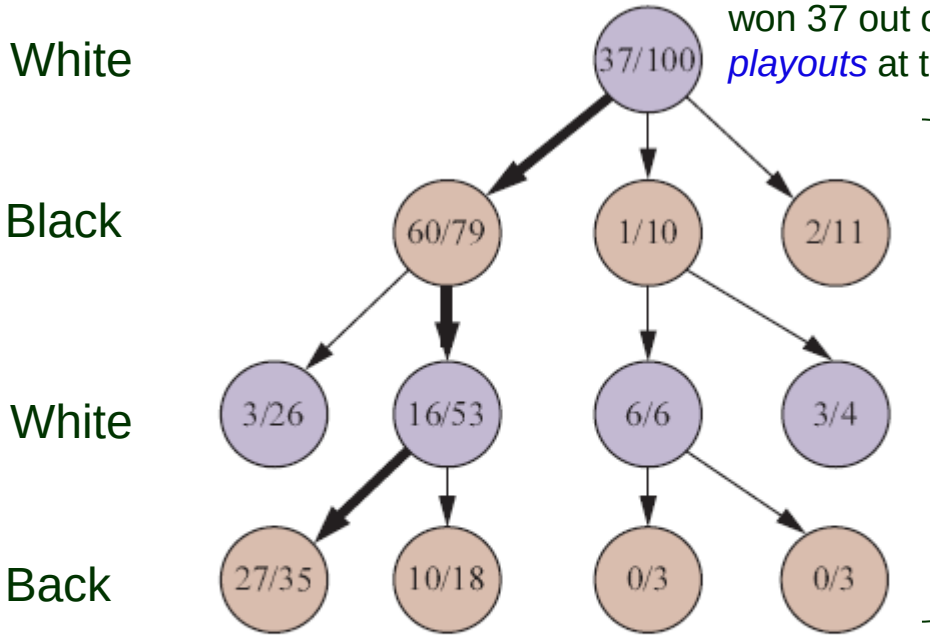
We need to decide a move for black (at the root) to make.

Root: state just after the move by white, who has won 37 out of the 100 *playouts* at the node so far.



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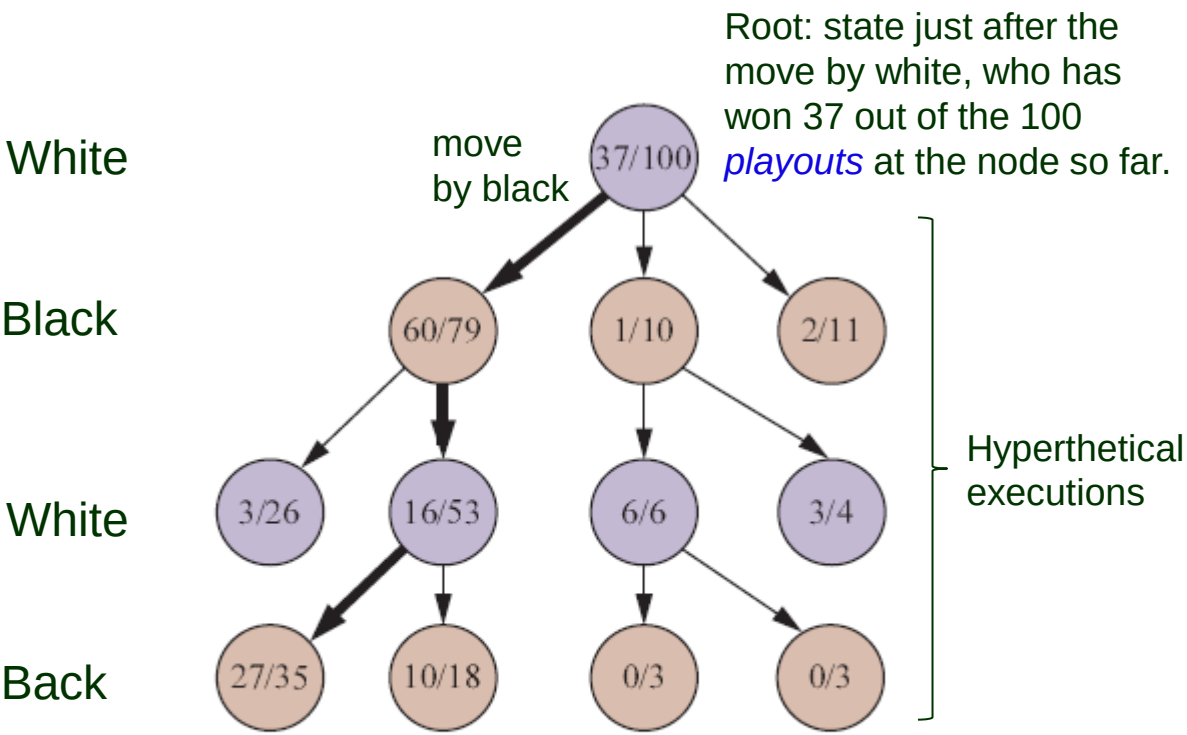

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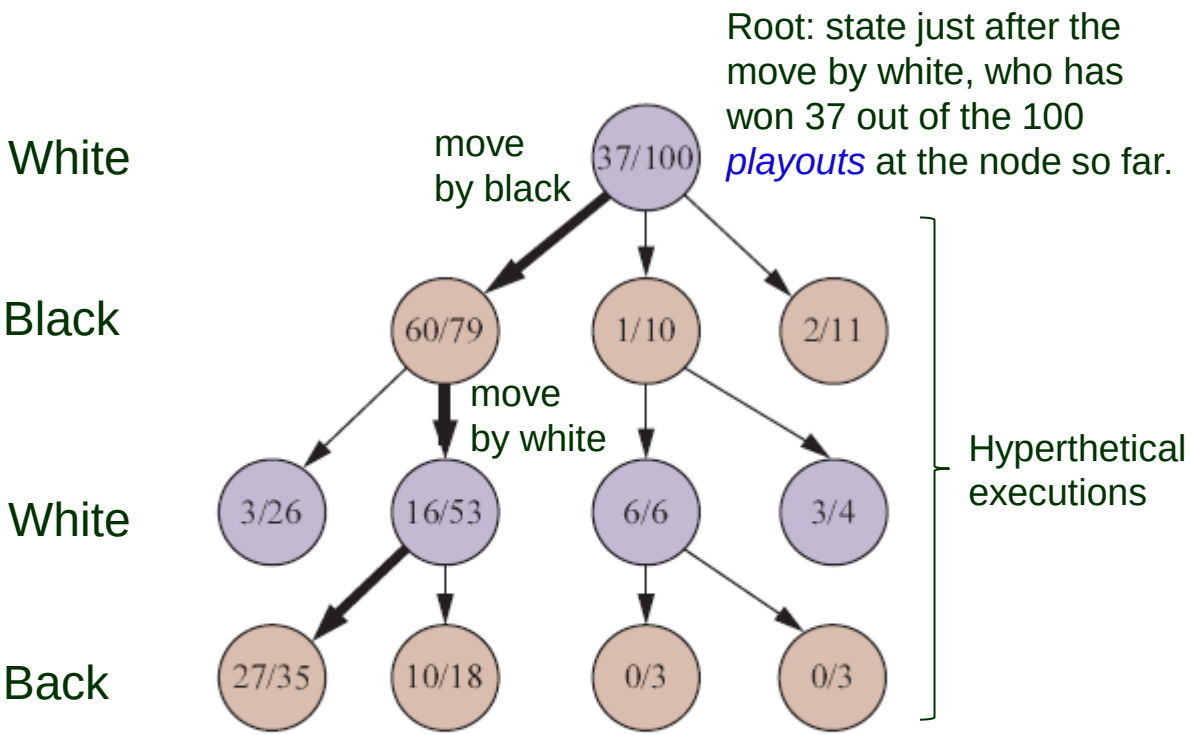

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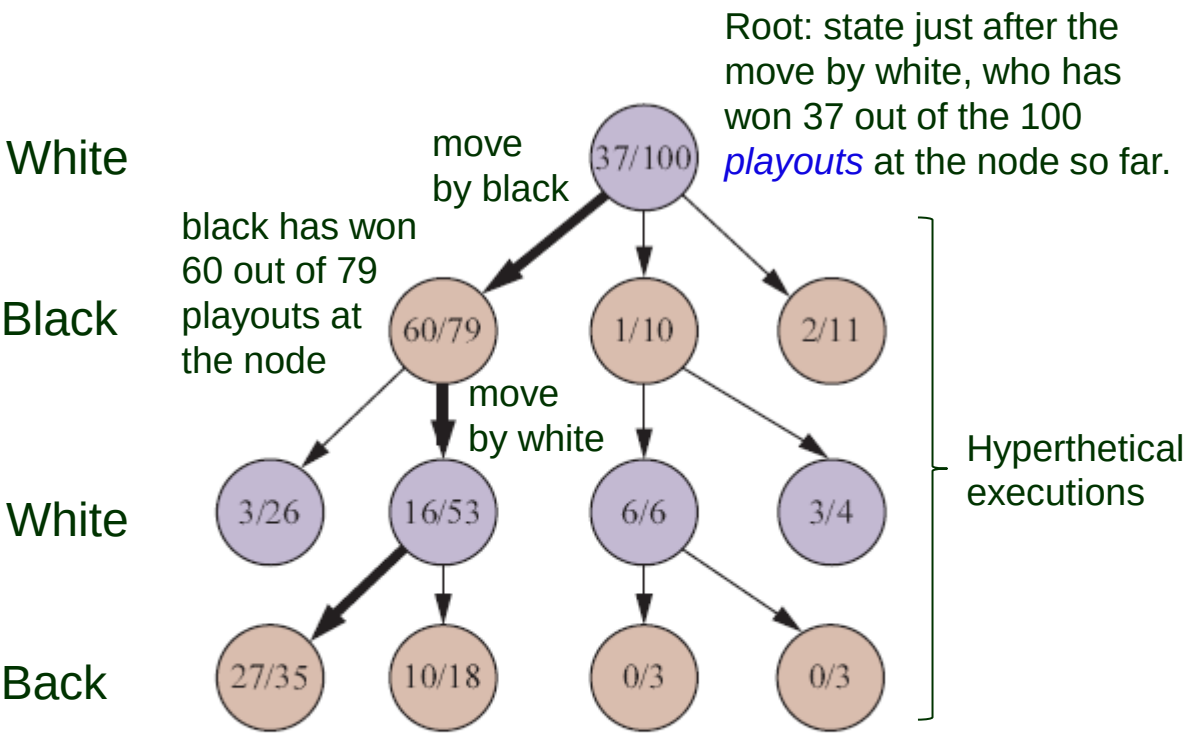

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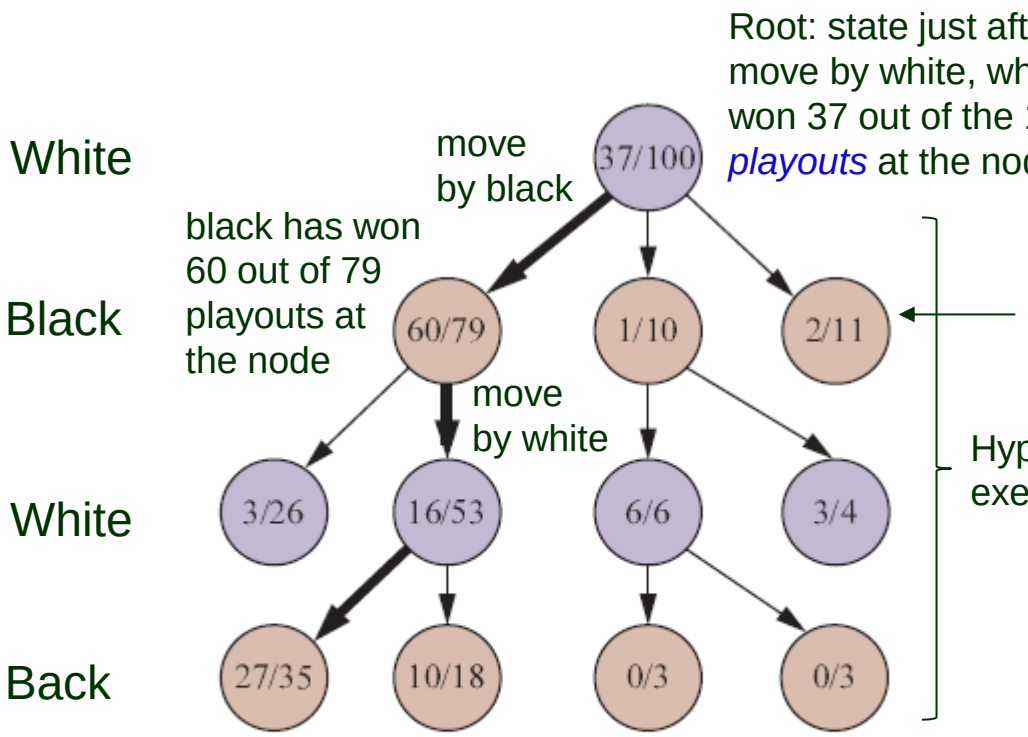

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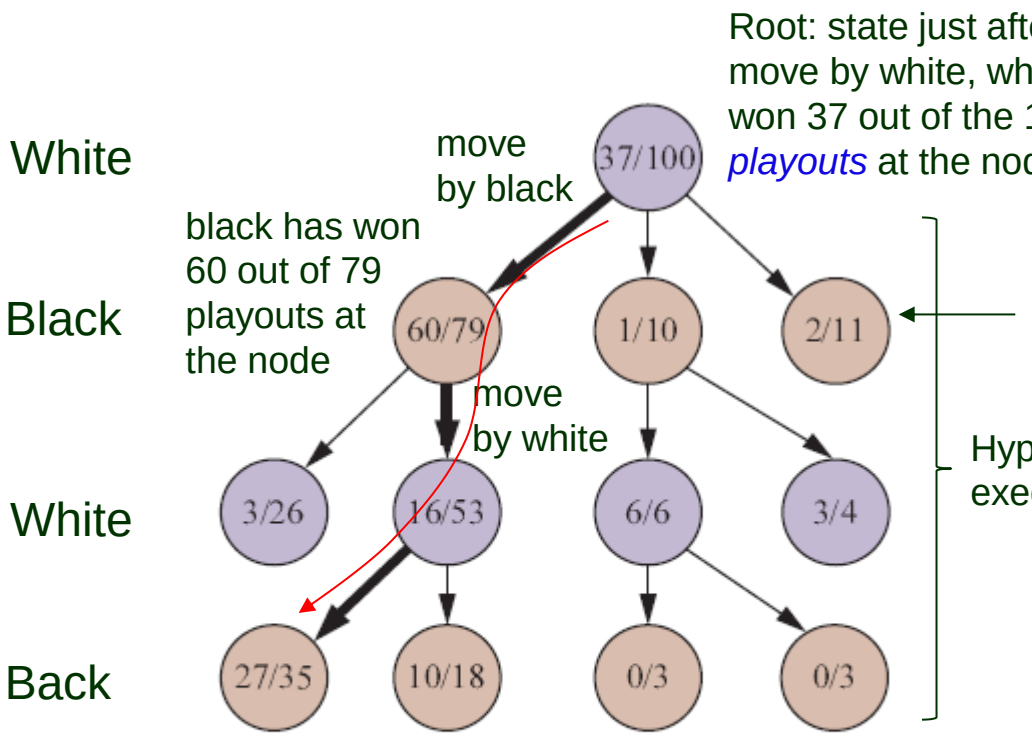

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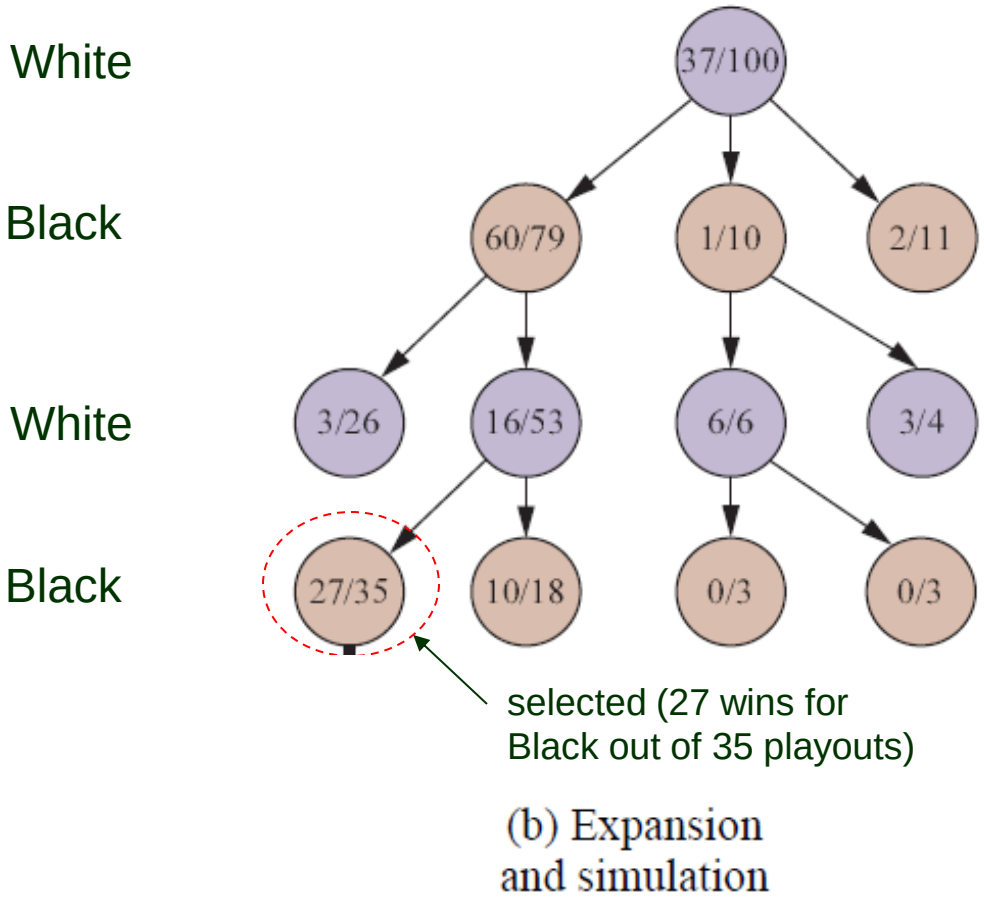

(b) Expansion and simulation

- Create *one or more* child nodes of the selected node.
  - ♦ You may generate a child node by executing every feasible action.
  - ♦ Or you may generate child nodes by executing a subset of feasible actions.
  - ♦ But you shouldn't *always* generate just one child node when there is more than one feasible action. Otherwise, the search tree within each iteration will have just one leaf – which will then be selected – and the tree will degenerate into a chain.

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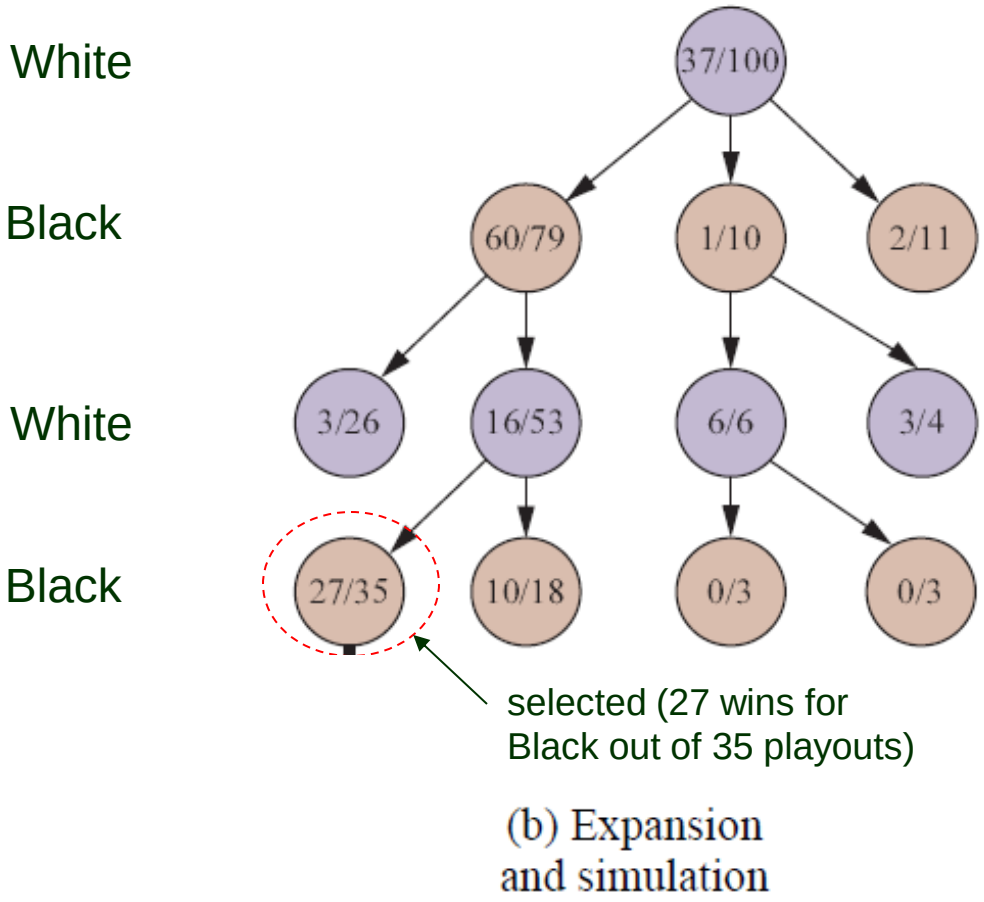

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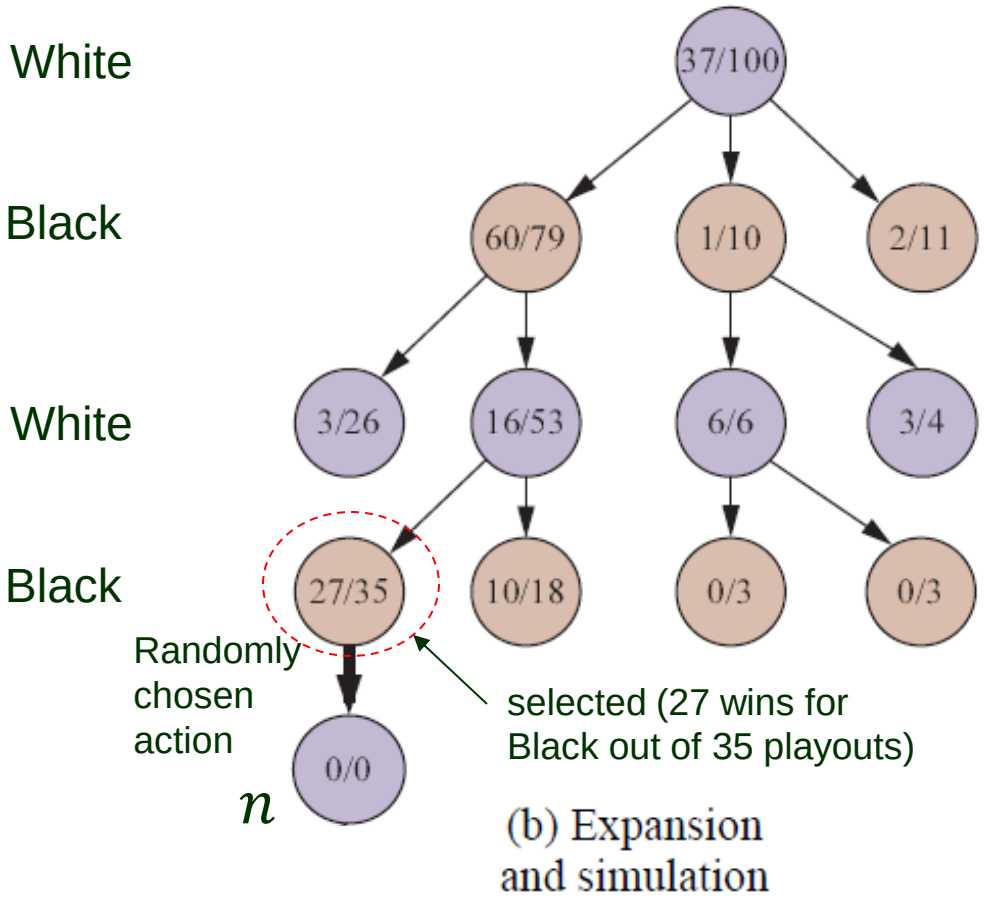

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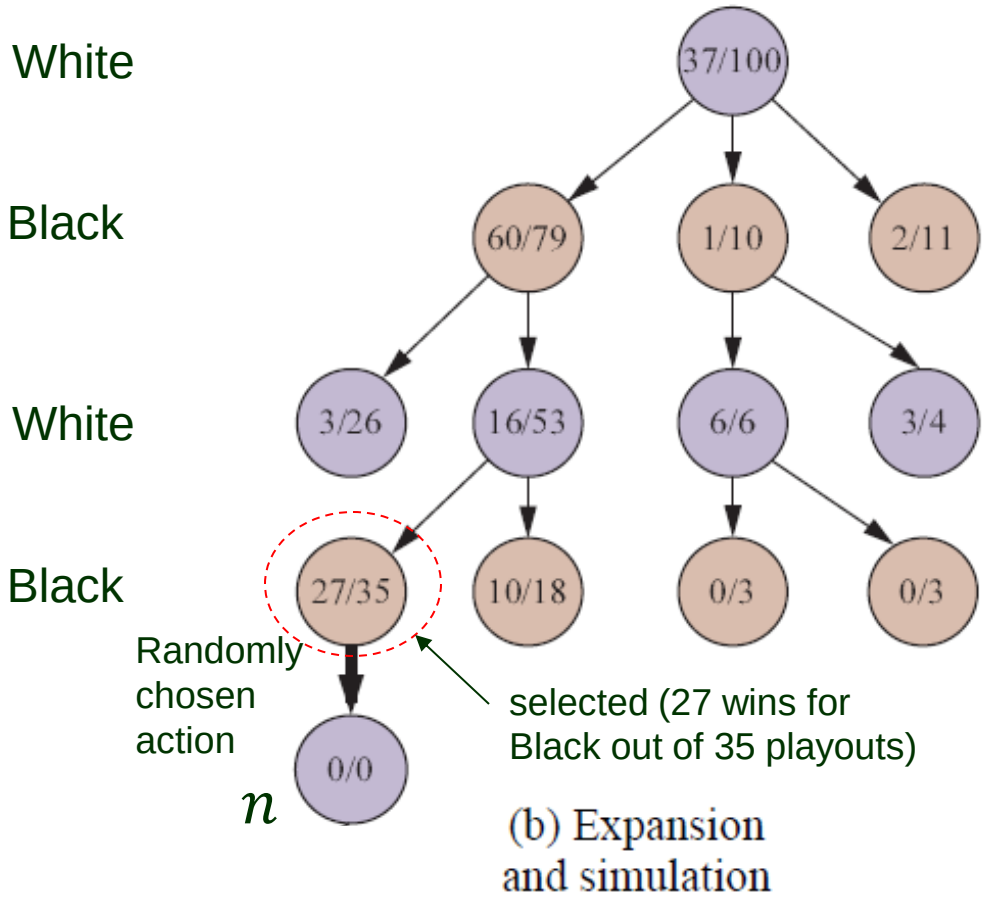

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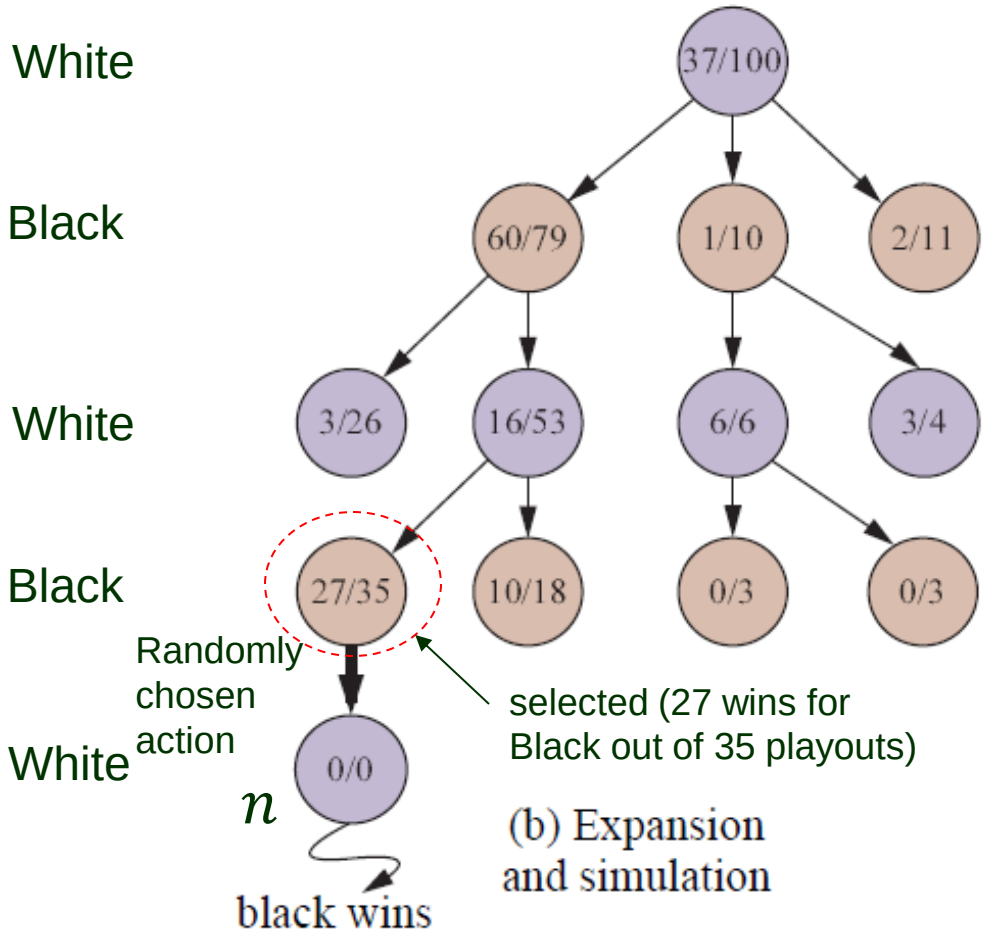

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Step 4: Back Propagation

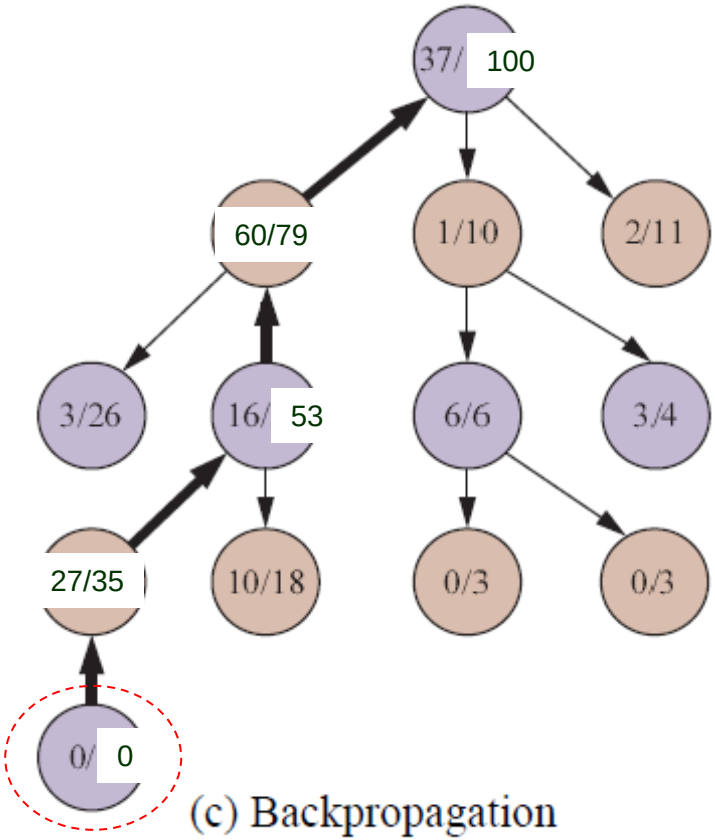
White

Black

White

Black

White



(c) Backpropagation

- Update all the nodes upward along the path until the root.

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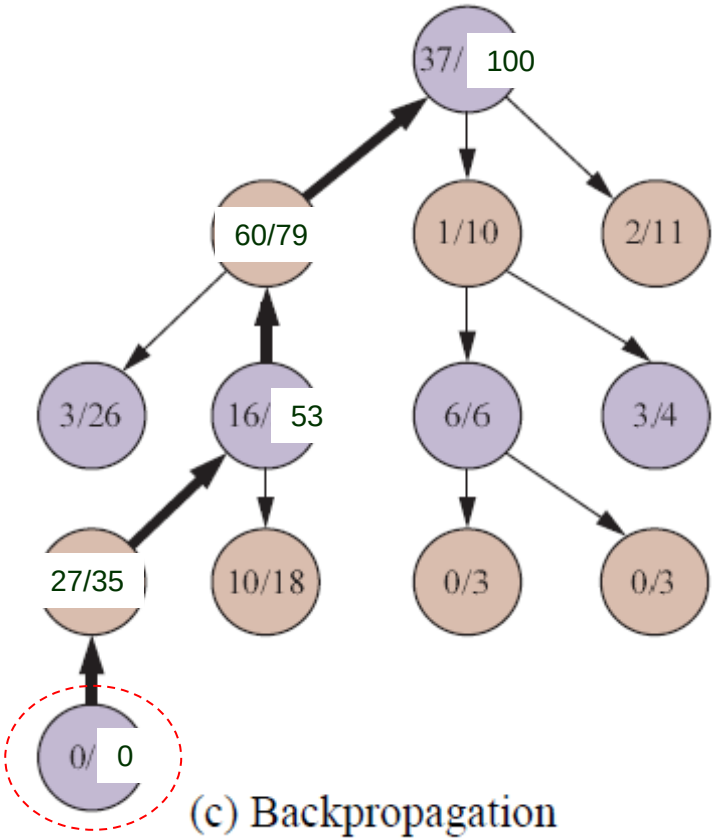

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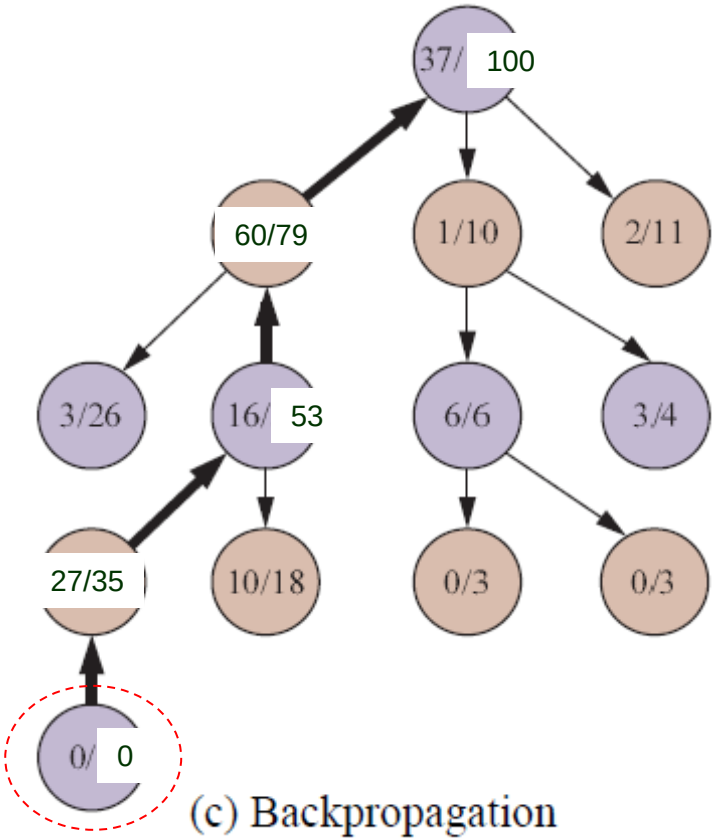

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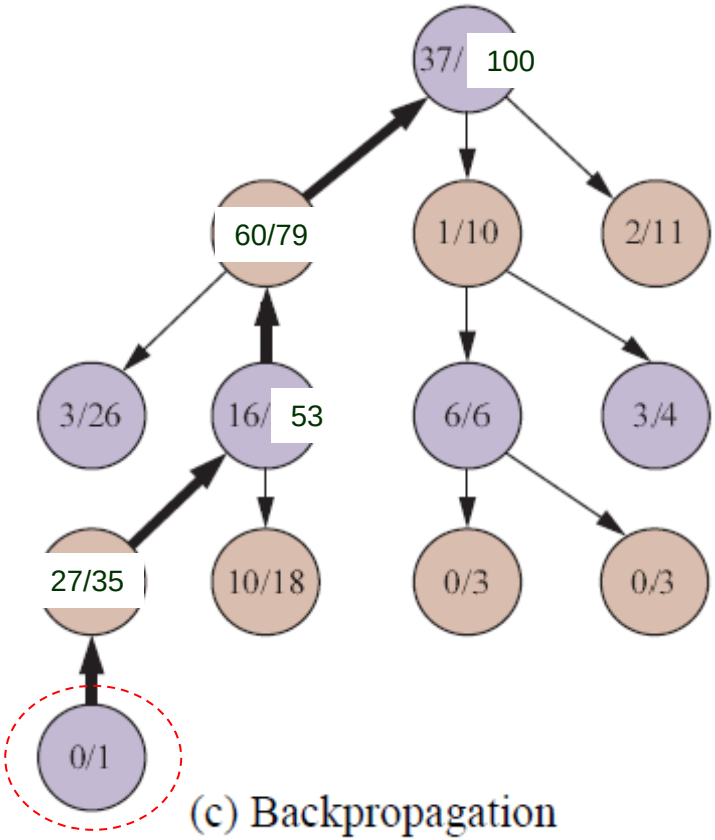

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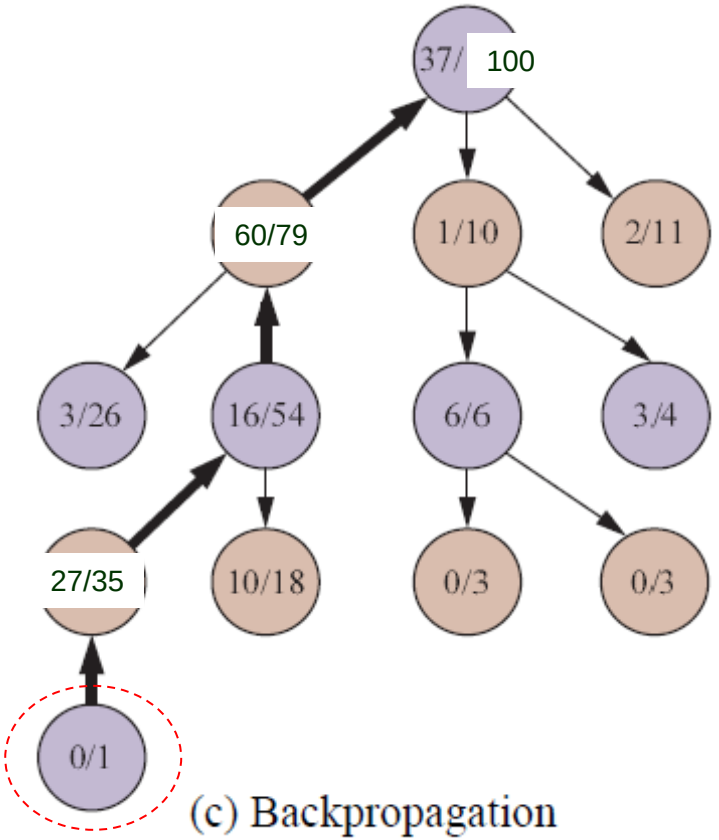

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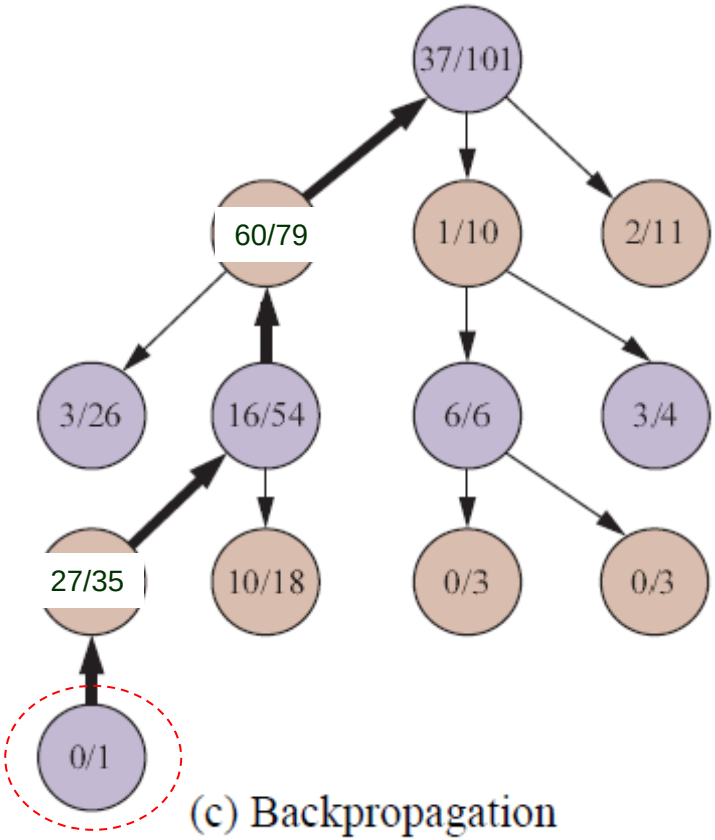

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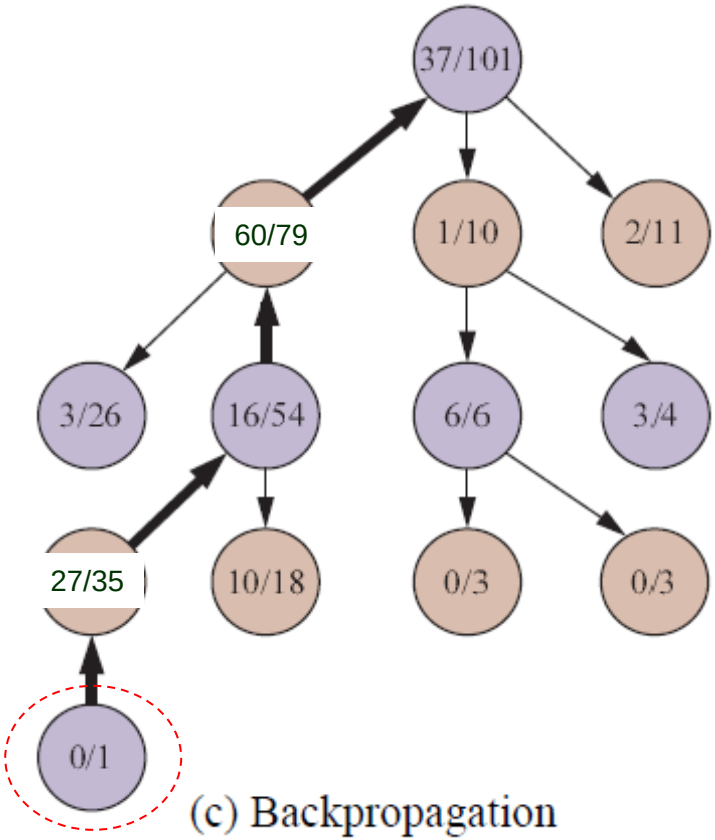

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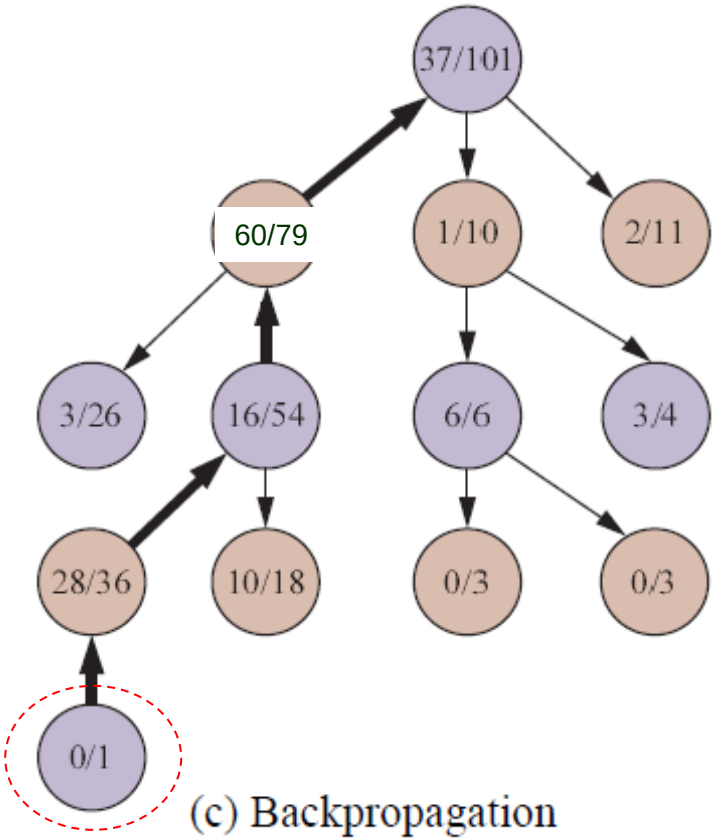

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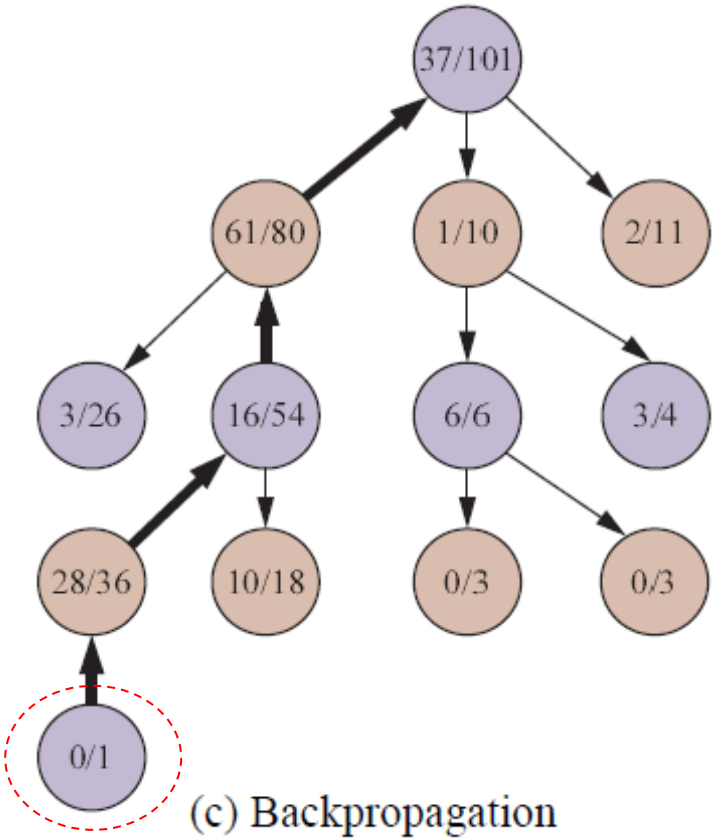

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- MCTS repeats the four steps (selection, expansion, simulation, back-propagation) in order until
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- A node with 65/100 wins is better than one with 2/3 wins (which has a lot of uncertainty).

Monte Carlo Tree Search Algorithm

function MONTE-CARLO-TREE-SEARCH(*state*) **returns** *an action* // decide a move at *state.*
  tree ← NODE(*state*) // initialize the tree with *state* at the root
  while IS-TIME-REMAINING() **do** // N iterations, each expanding the tree by one node.
    leaf ← SELECT(*tree*) // the node to be expanded must be a leaf.
    child ← EXPAND(*leaf*) // tree is expanded to a child of *leaf.*
    result ← SIMULATE(*child*) // playout: moves are not recorded in the tree.
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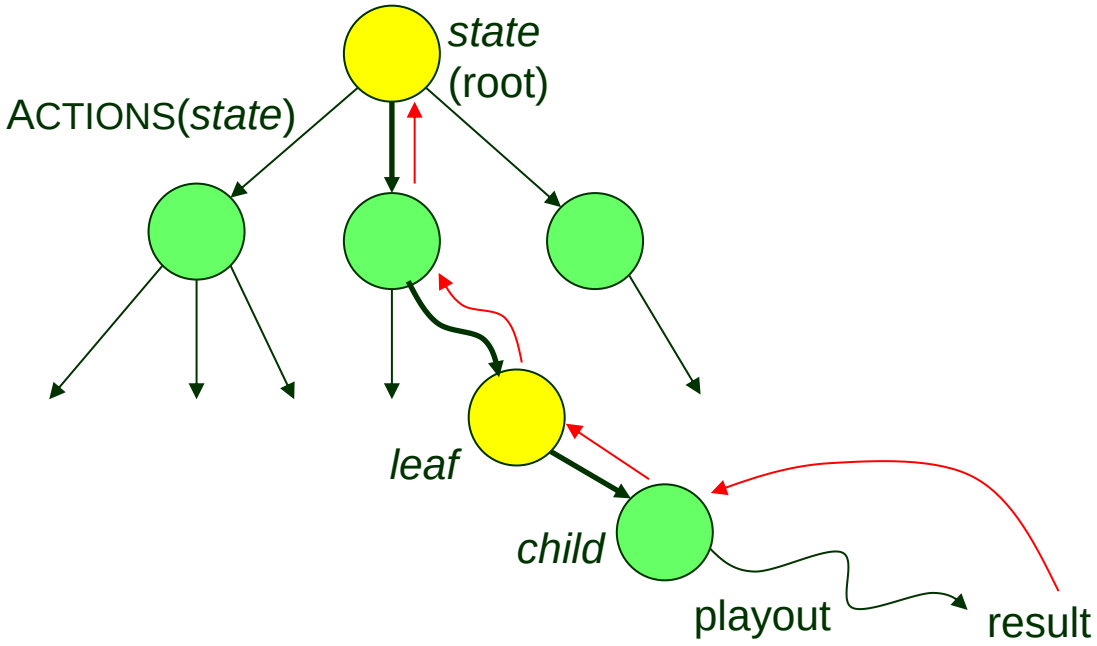
  return the move in ACTIONS(*state*) whose node has highest number of playouts



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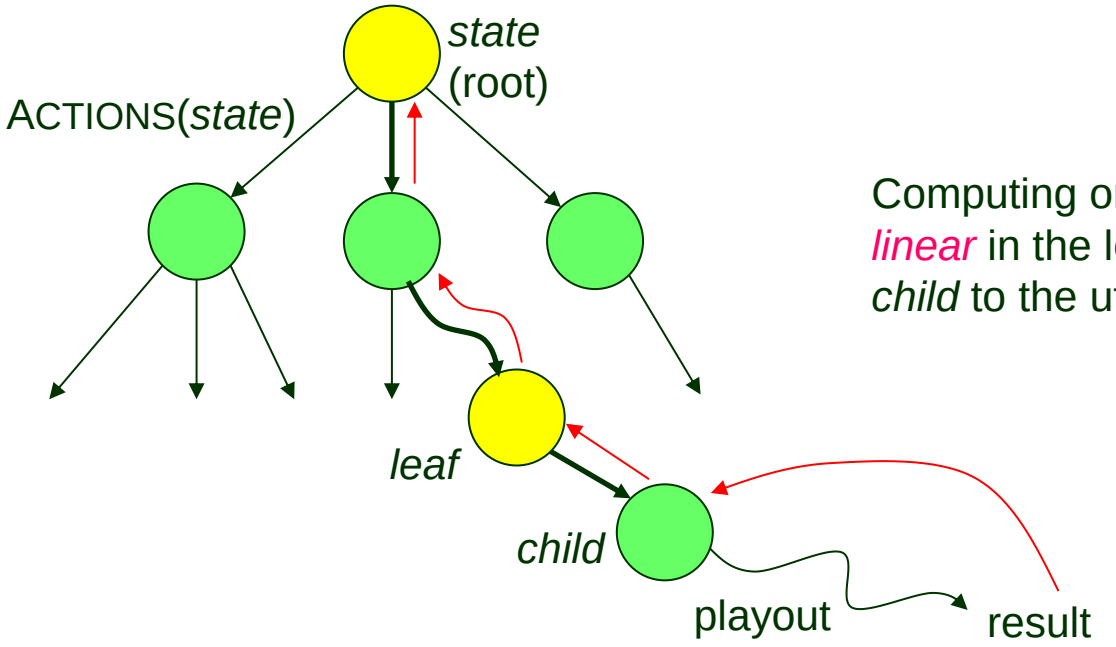
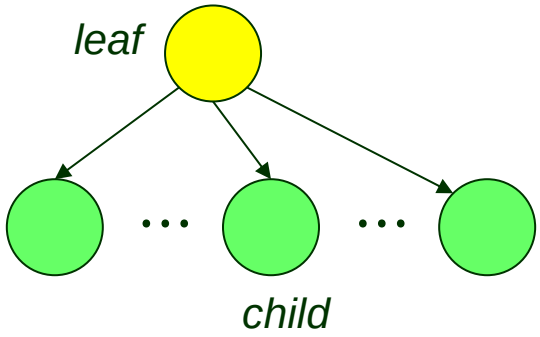


Computing one playout takes time *linear* in the length of the path from *child* to the utility node (result).

Three Issues

- *child* ← EXPAND(*leaf*)



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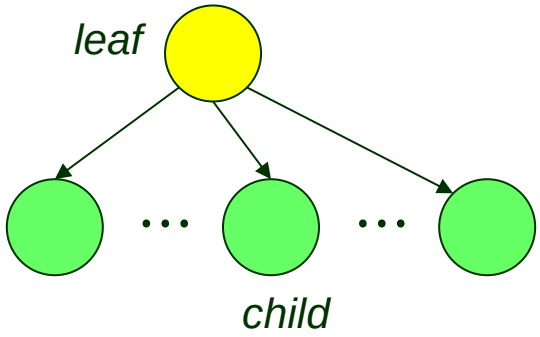

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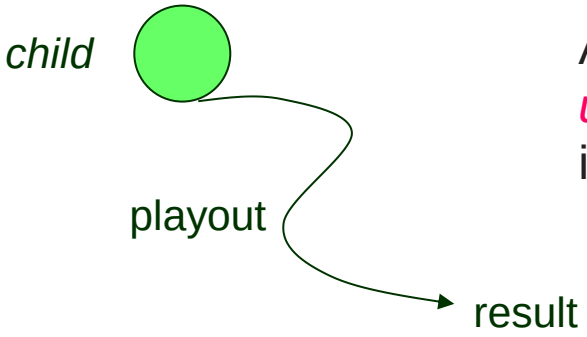

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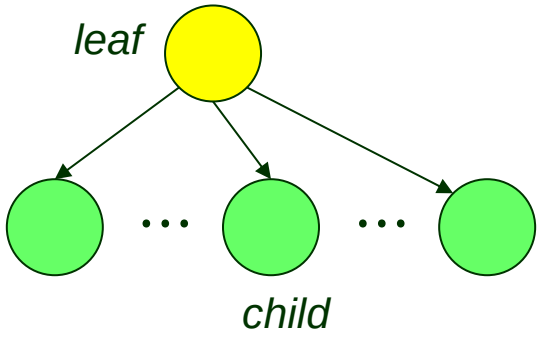

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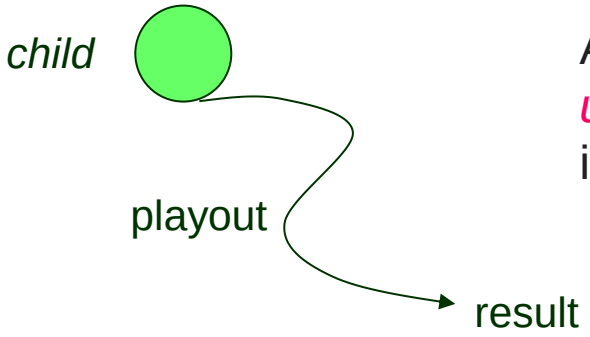

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- IS-TIME-REMAINING()

Pure Monte Carlo search does *N* simulations instead.

Ranking of Possible Moves

At a node n in the expanding MCTS tree rooted at the real *state*.

Upper confidence bound formula:

$$\text{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}}$$

Ranking of Possible Moves

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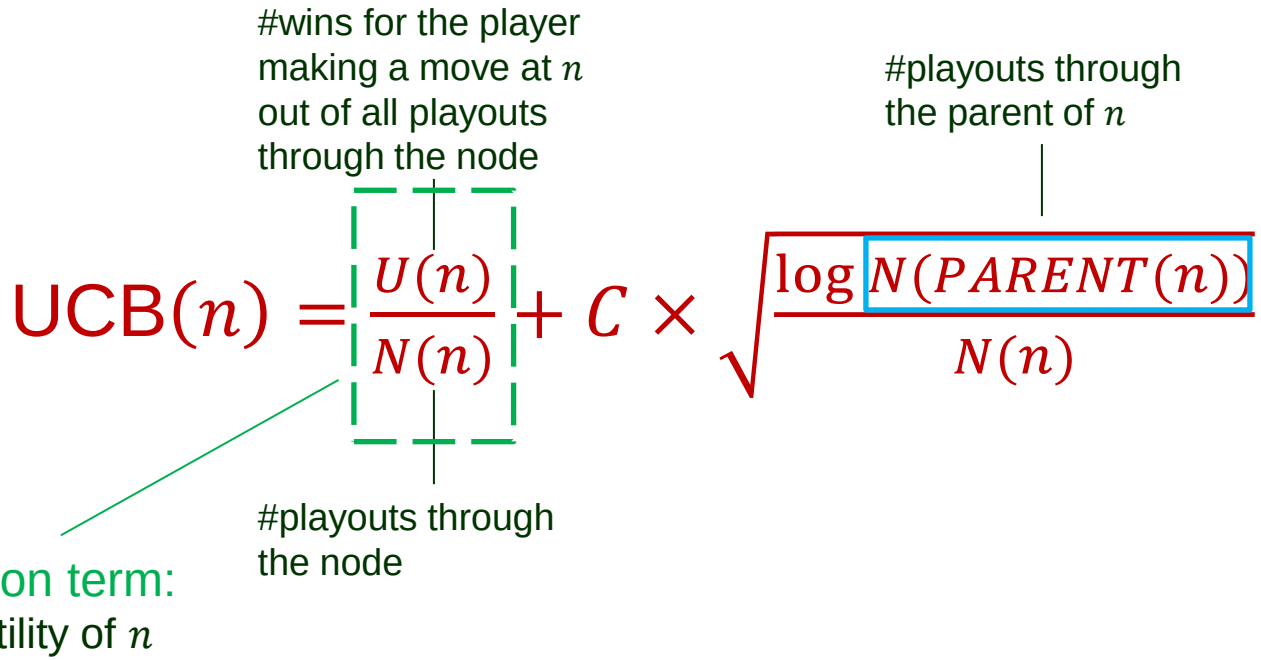

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Exploration term: high value if n has been explored a few times.

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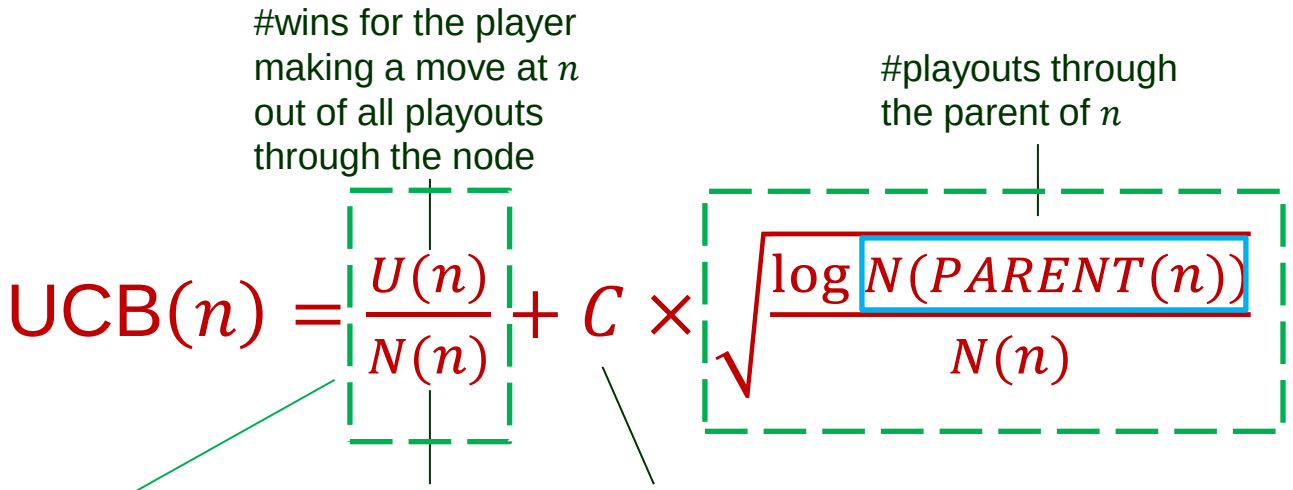

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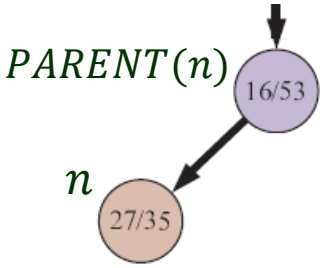

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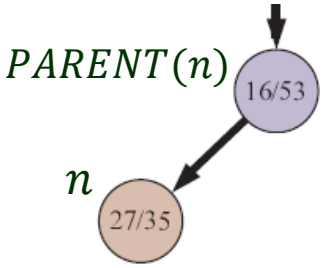
Exploitation term: average utility of n

Exploration term: high value if n has been explored a few times.



$$U(n) = 27$$
$$N(n) = 35$$
$$N(PARENT(n)) = 53$$
$$C = \sqrt{2}$$

(choice by a theoretical argument)

Ranking of Possible Moves

At a node n in the expanding MCTS tree rooted at the real *state*.

Upper confidence bound formula:

#wins for the player making a move at n out of all playouts through the node

#playouts through the parent of n

$$\text{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}}$$

#playouts through the node

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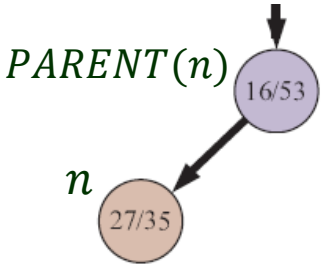

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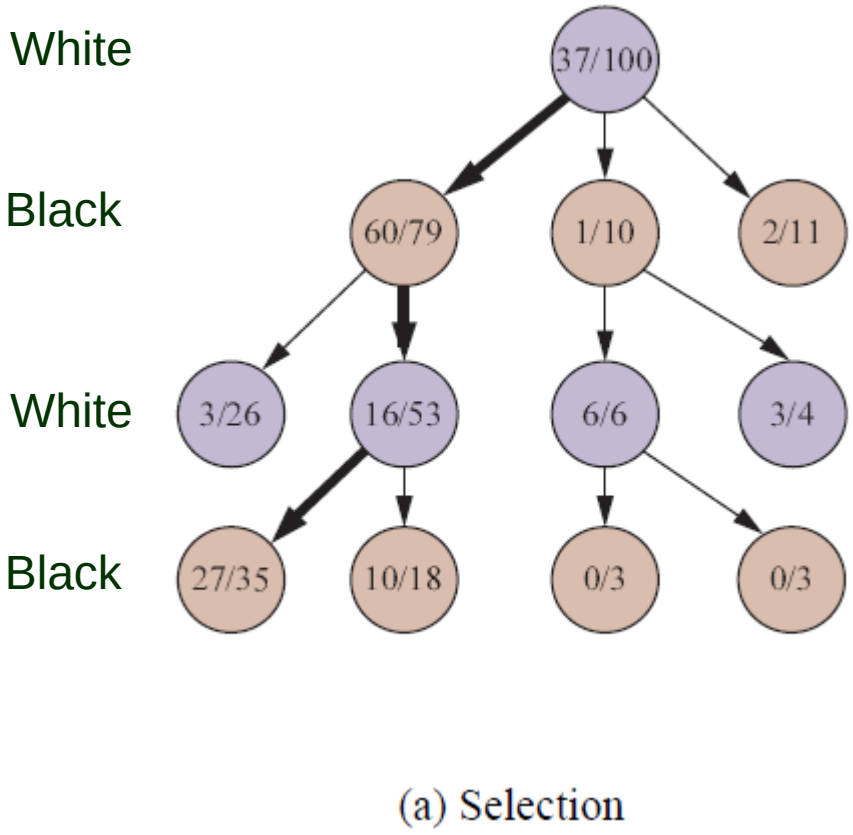
$$\text{UCB}(n) = \frac{27}{35} + \sqrt{2} \cdot \sqrt{\frac{\log 53}{35}}$$

Constant C



(a) Selection

- Balances exploitation and exploration.
- Multiple values are tried and the one that performs the best is chosen.
  - $C = 1.4$

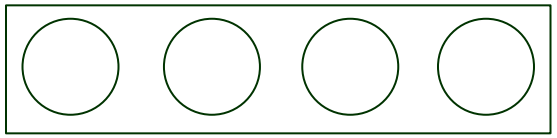
    The 60/79 node has the highest score.
  - $C = 1.5$

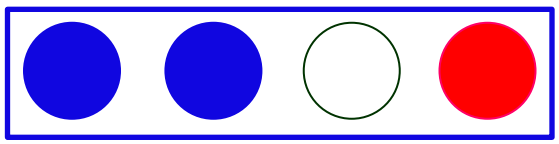
    The 2/11 node has the highest score.

Example: The Pair Game

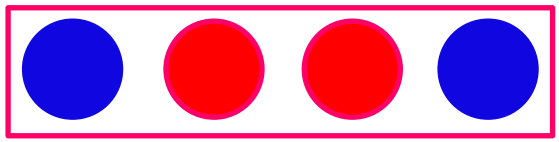
From *Monte Carlo Tree Search: A Guide*

- Four circular holes in the board.
- Two players (Blue and Red) take turns to place disks in their colors to cover the holes.
- Blue goes first.
- Whoever first covers two adjacent holes wins the game.

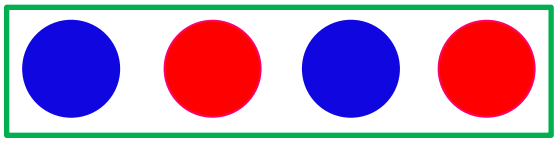




Blue wins

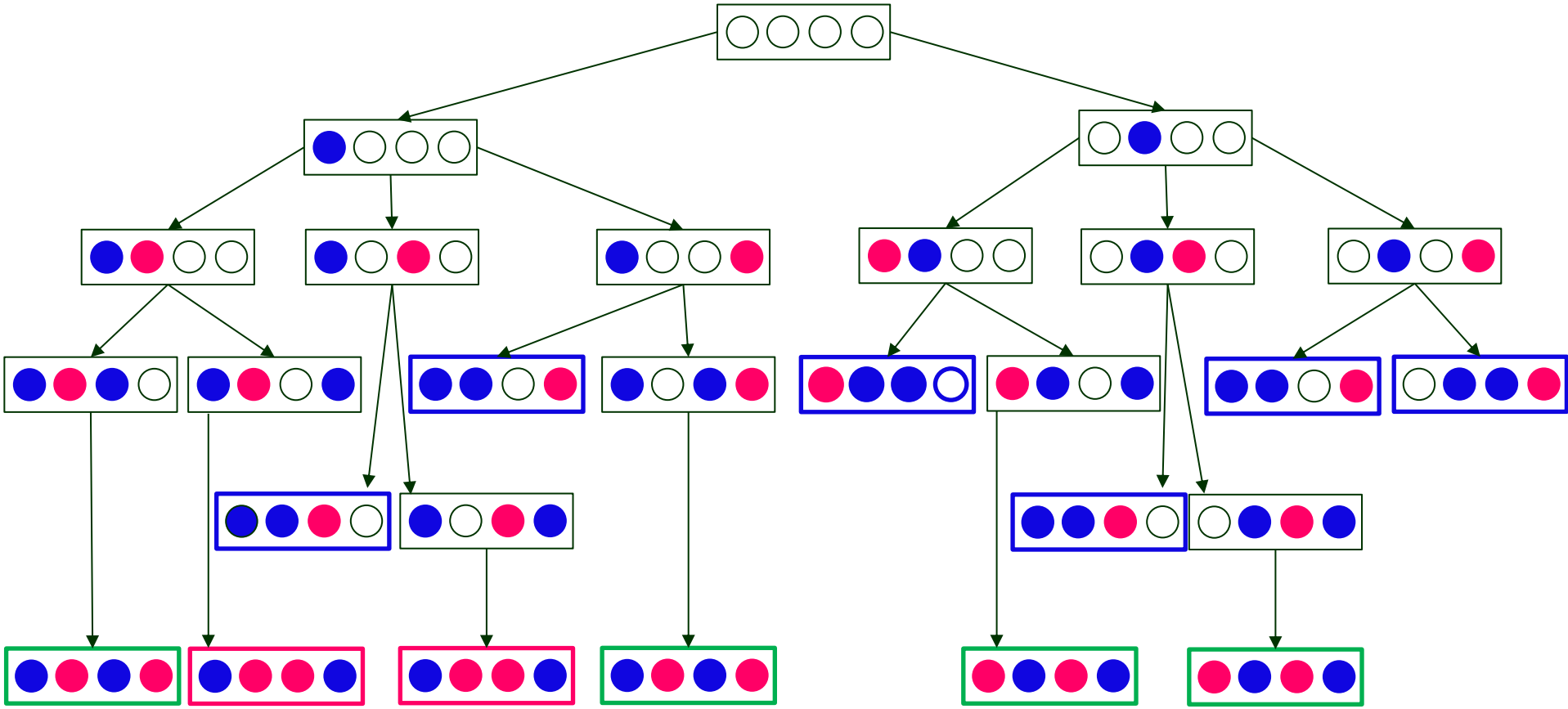


Red wins

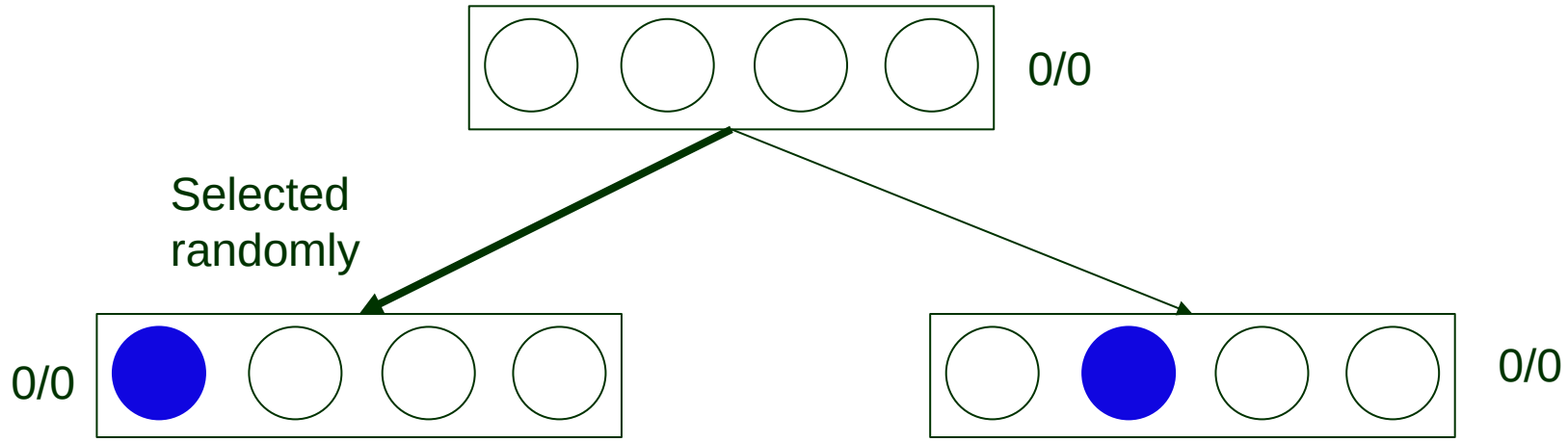


Tie

Complete Game Tree (up to Symmetry)



Iteration 1: Selection & Expansion



Leaf nodes

UCB ← ∞

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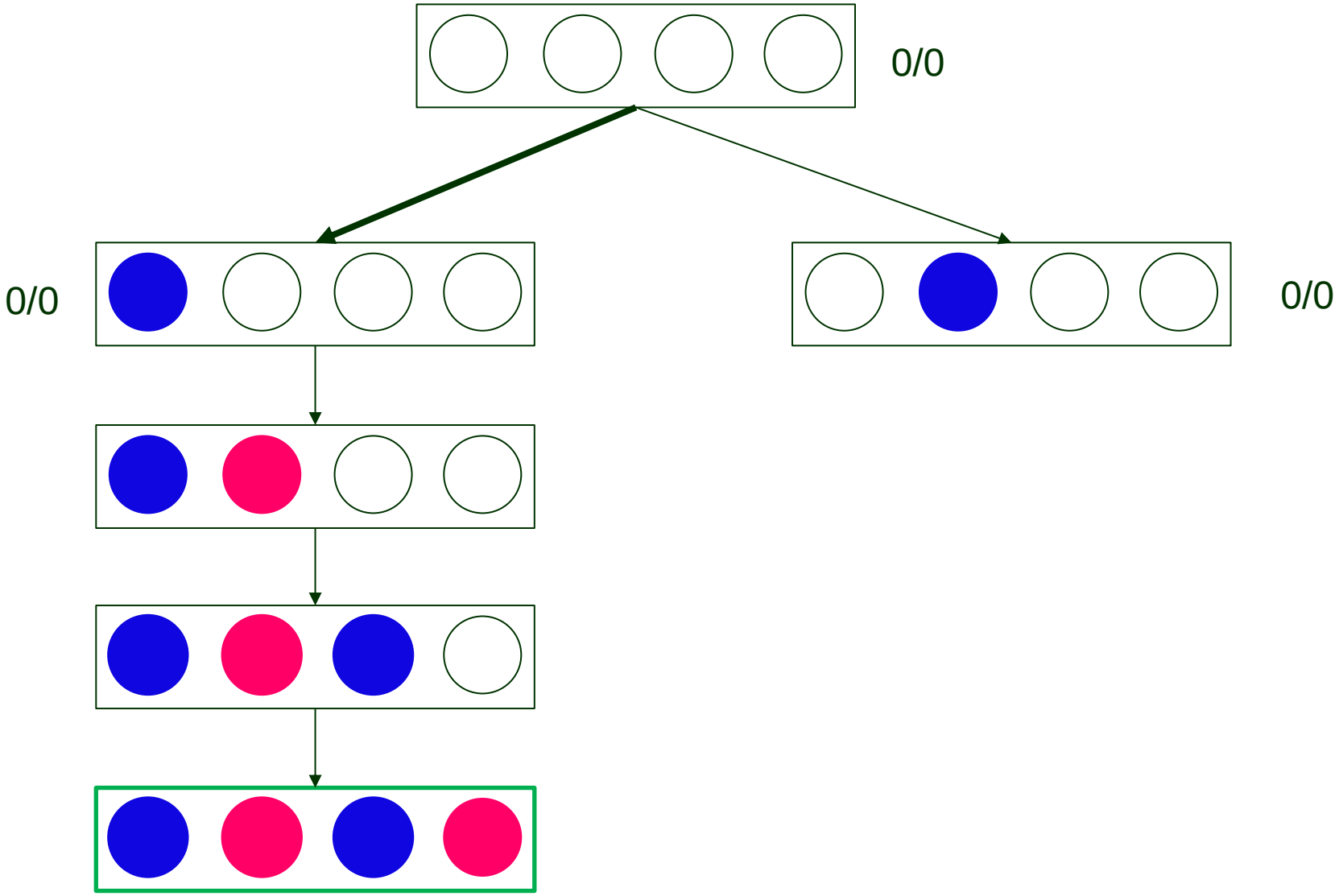
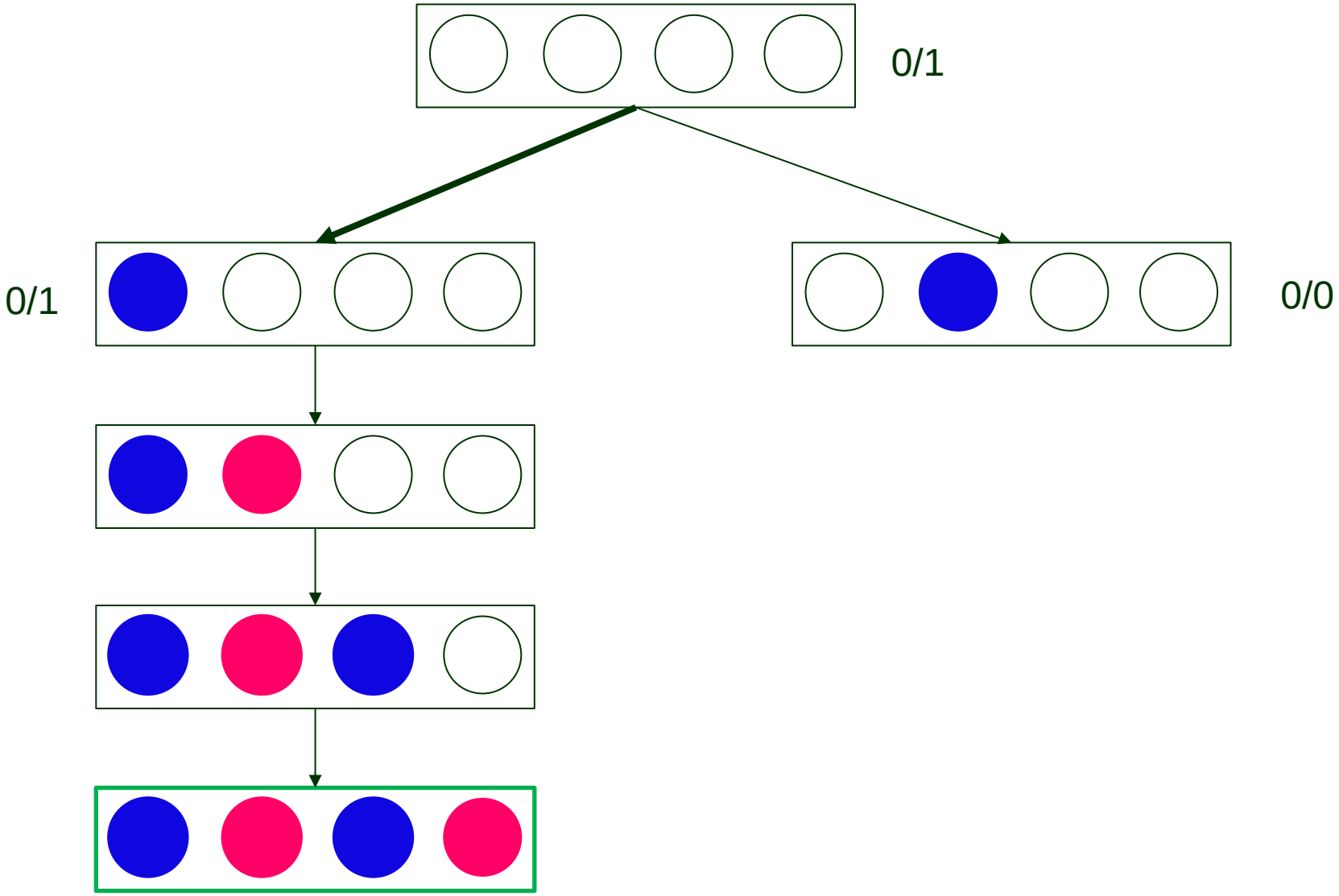
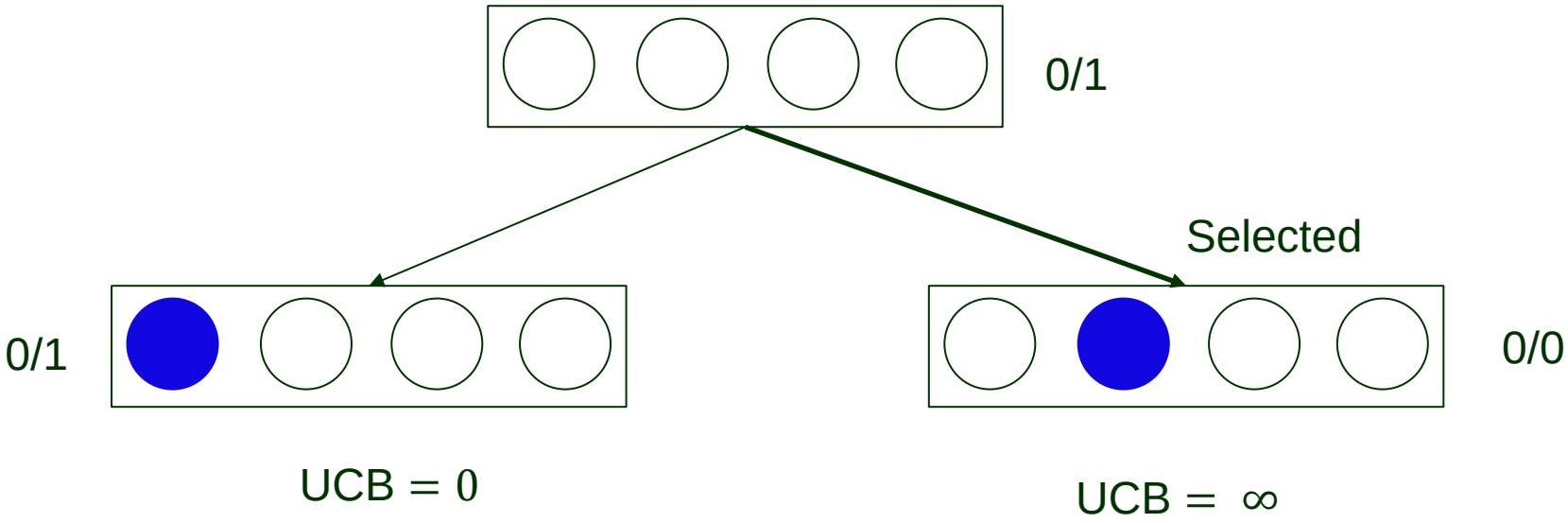
Simulation



Backpropagation



Iteration 2: Selection



$$U(n) = 0$$
$$N(n) = 1$$

$$\text{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}}$$

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♠ MCTS is less desired than alpha-beta on a game like chess with low b and good evaluation function.