Robot Motion Planning

Outline:

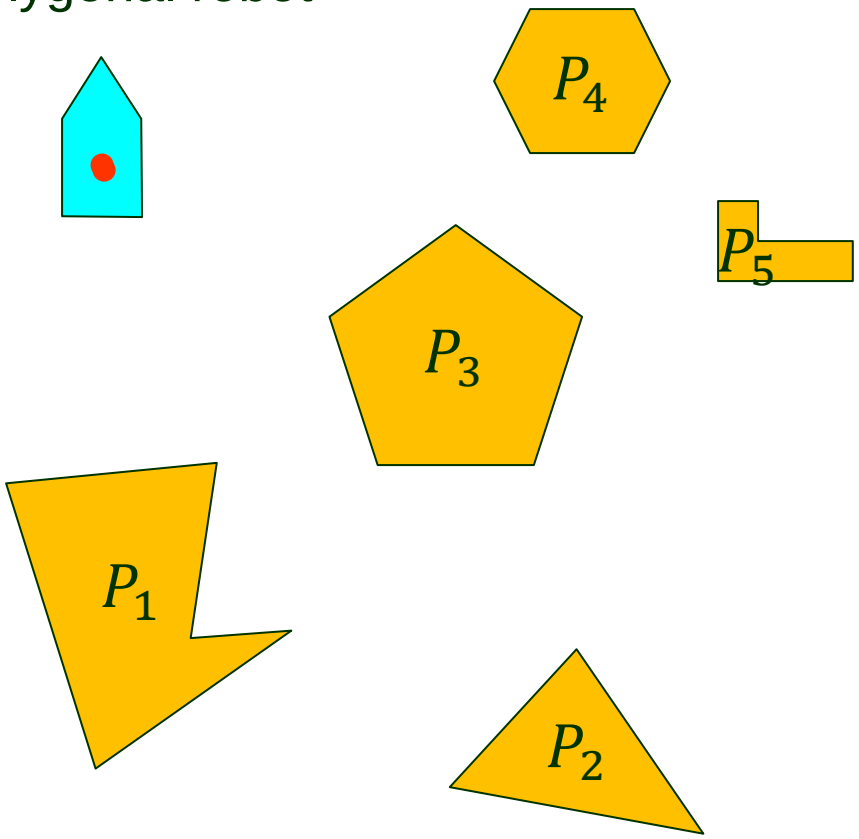
I. Configuration and degrees of freedom

II. Configuration space (C-space)

III. Planning of a collision-free path

I. The Planning Problem

Polygonal robot

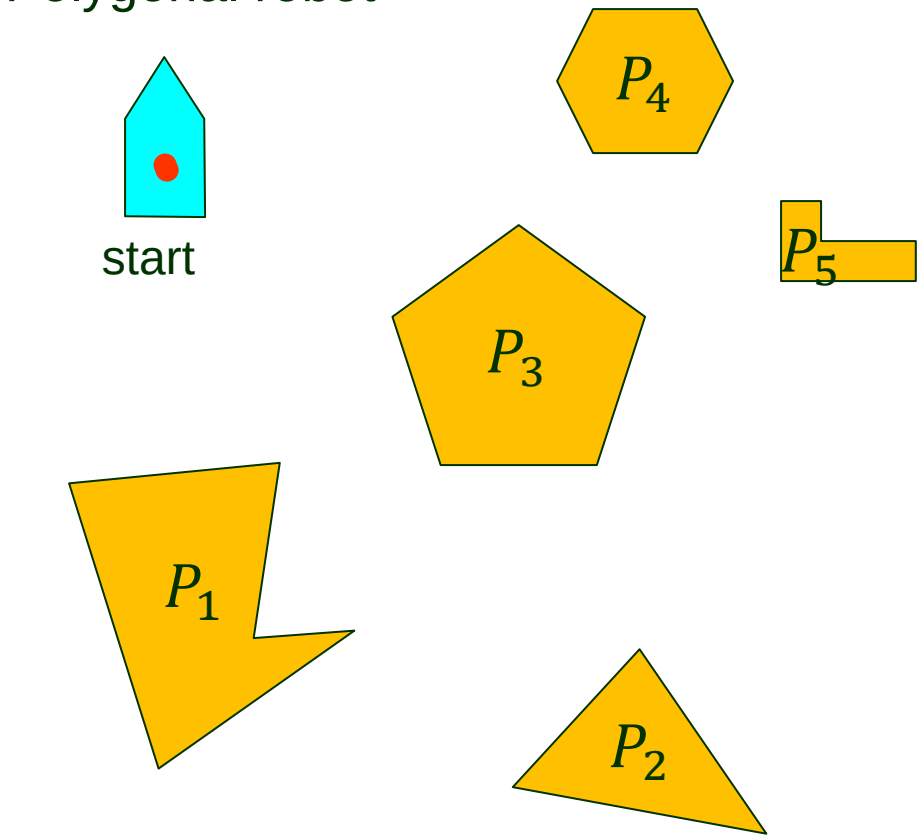


Static environment:

- 2-dimensional (2D)
- with polygonal obstacles

$$S = \{P_1, P_2, \ldots, P_t\}$$

I. The Planning Problem



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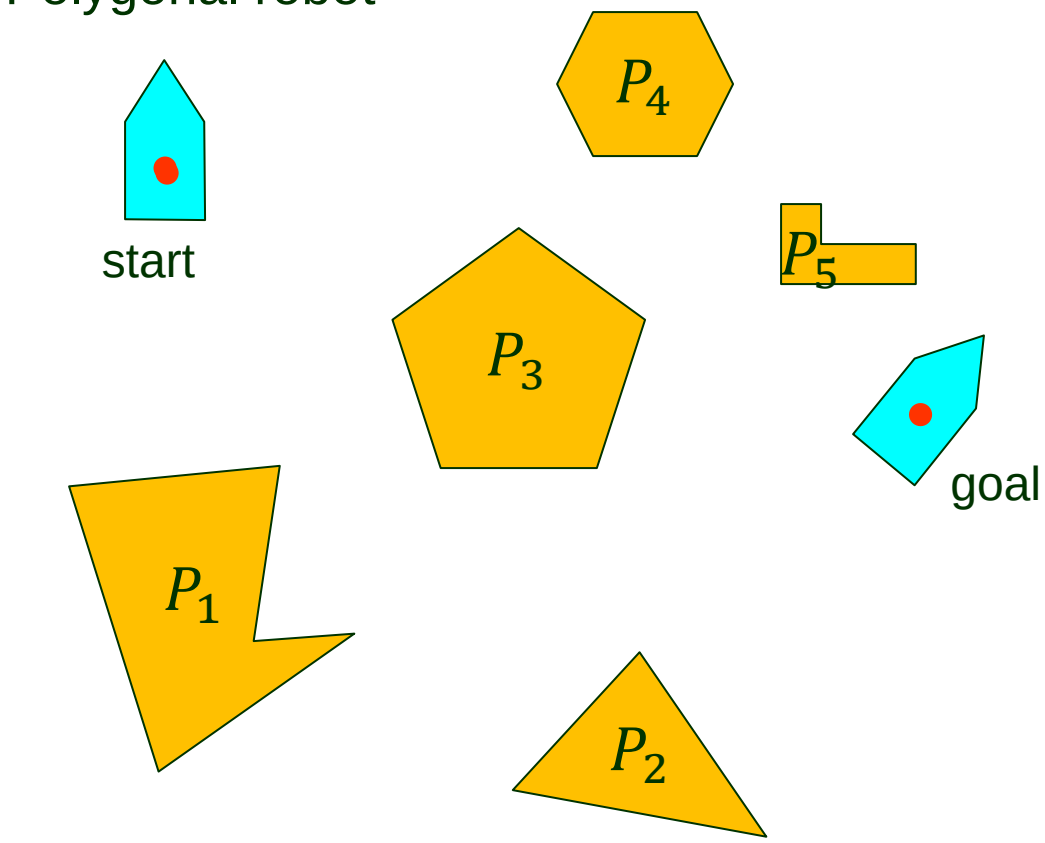

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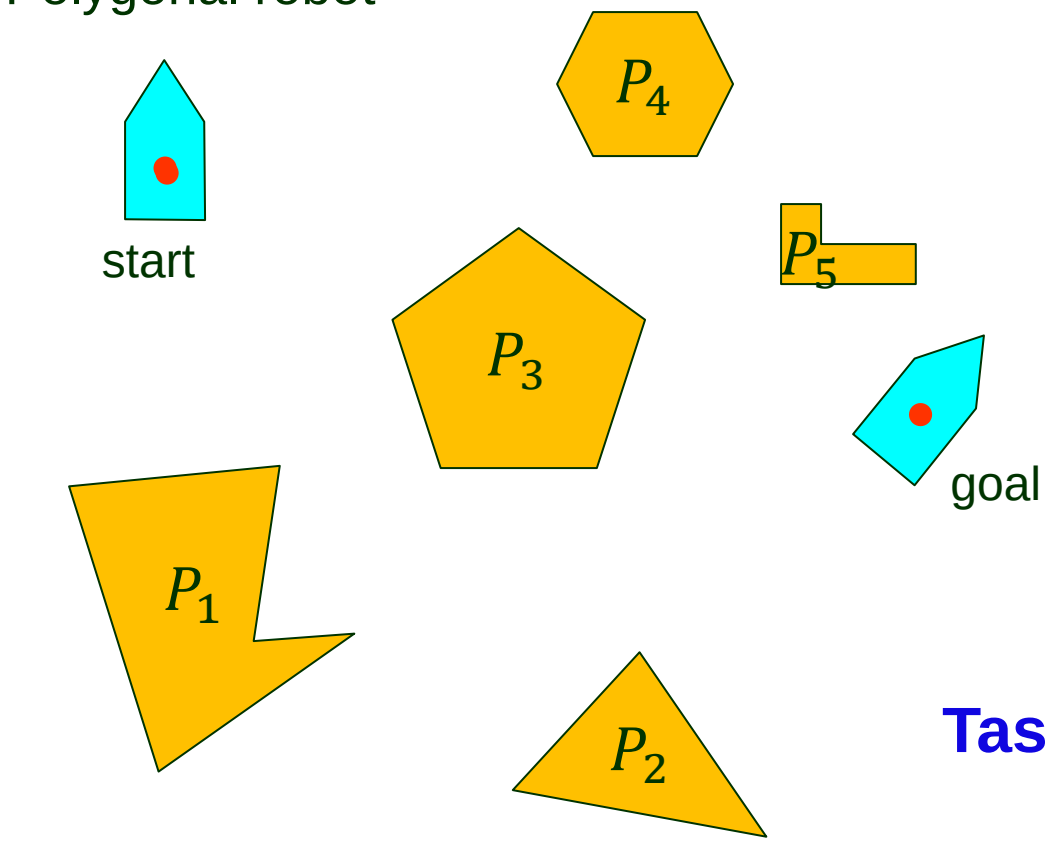

Static environment:

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Task: Move the robot from the start placement to the goal placement *without colliding* with any obstacles.

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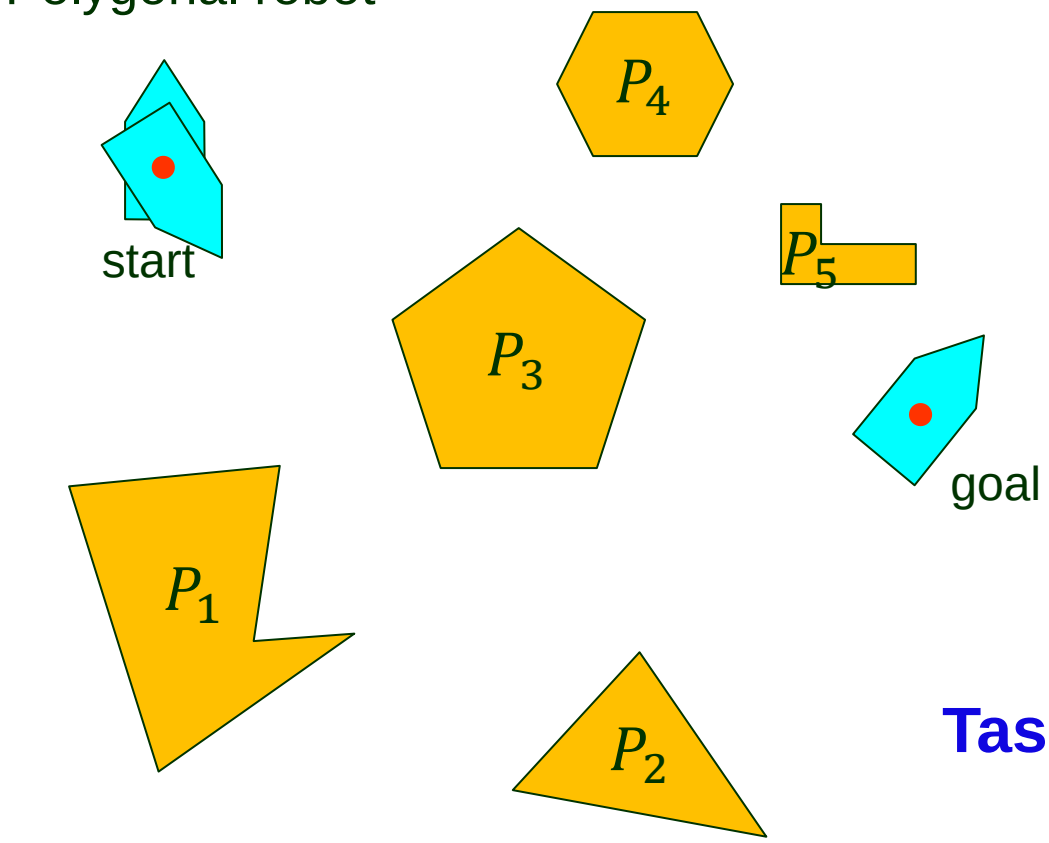

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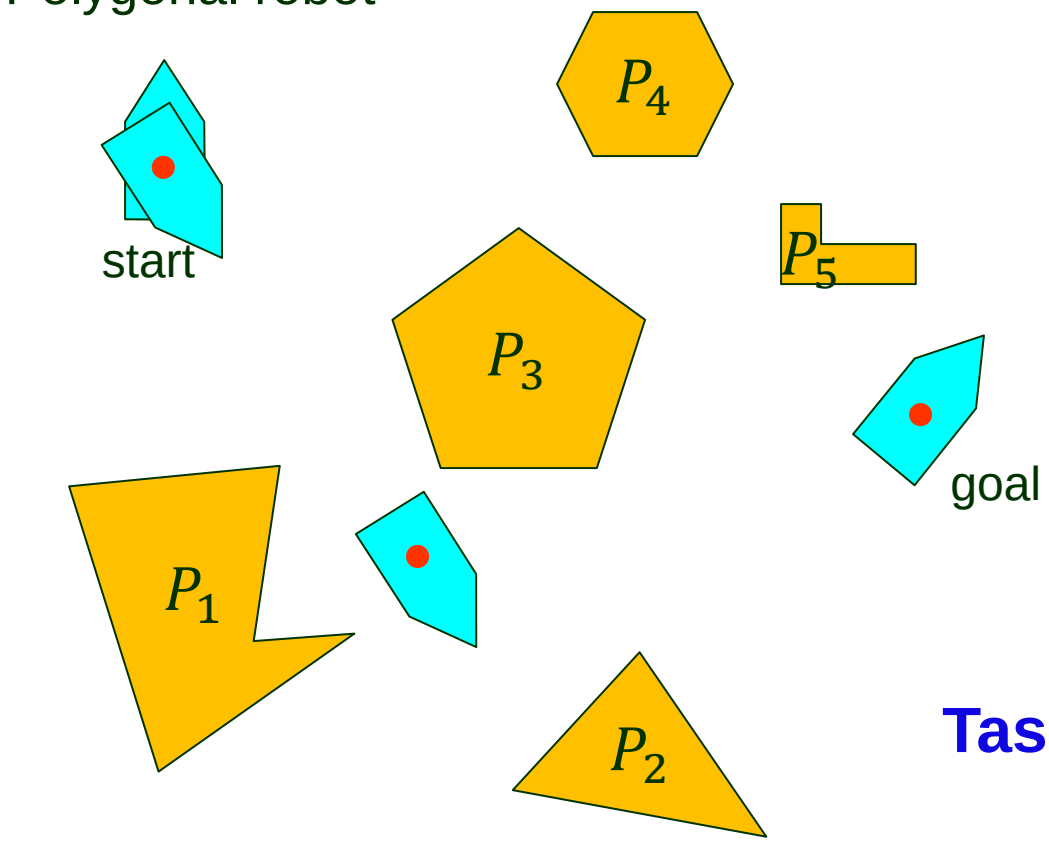

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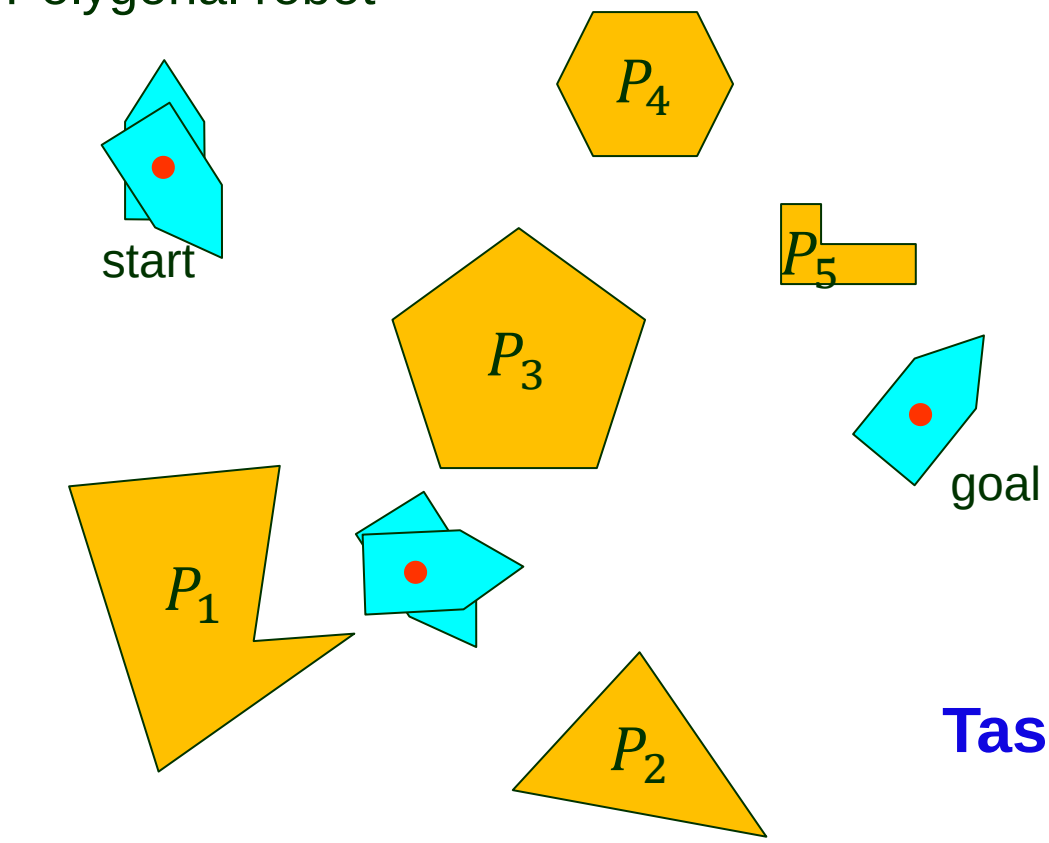

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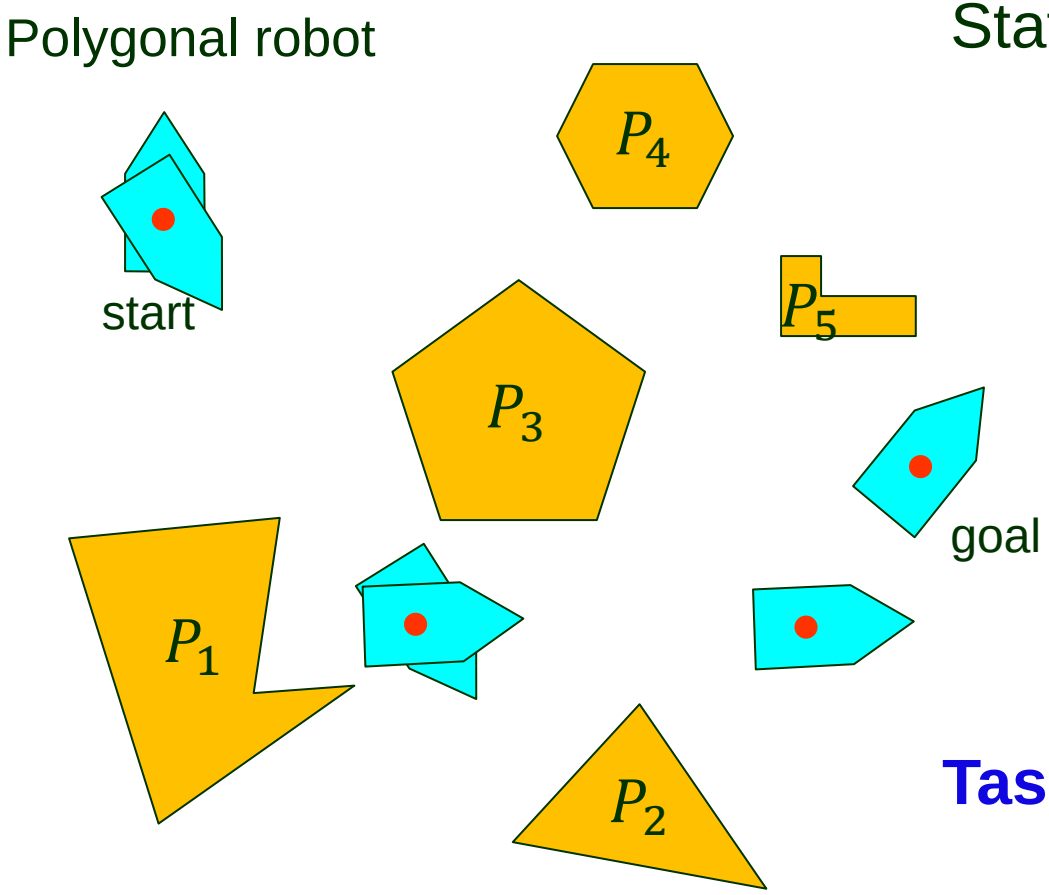

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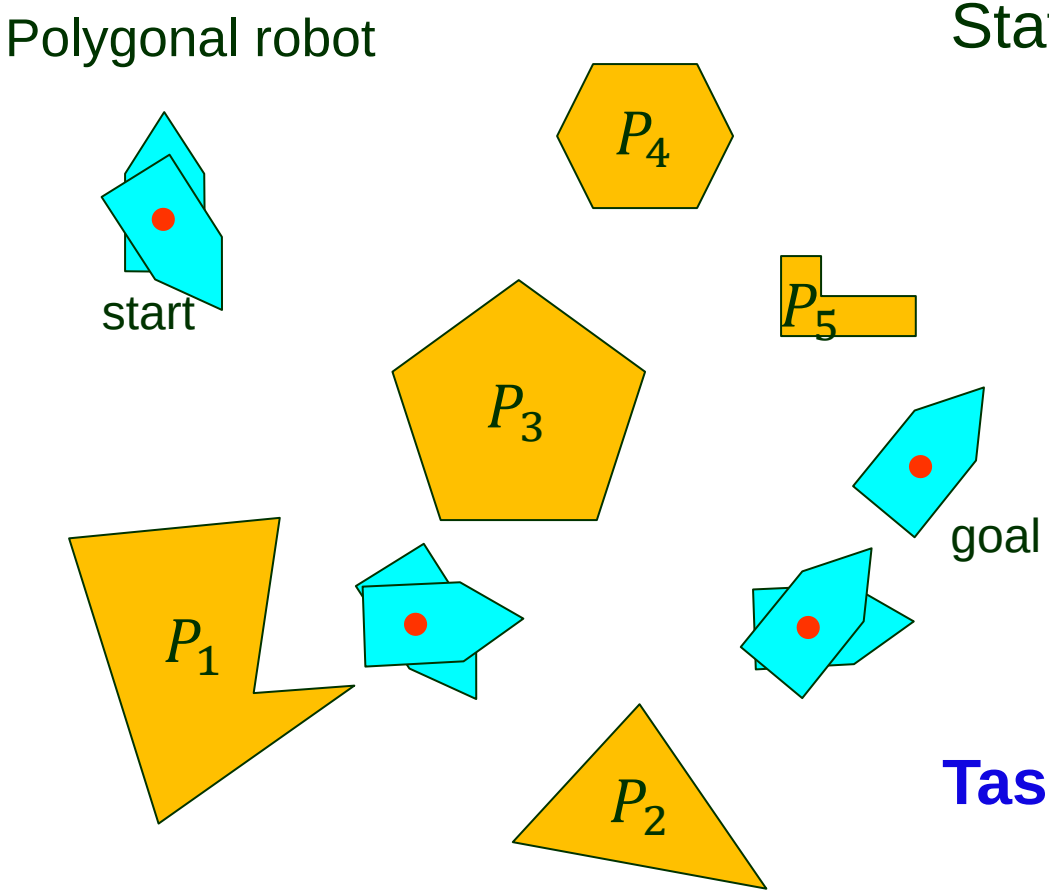

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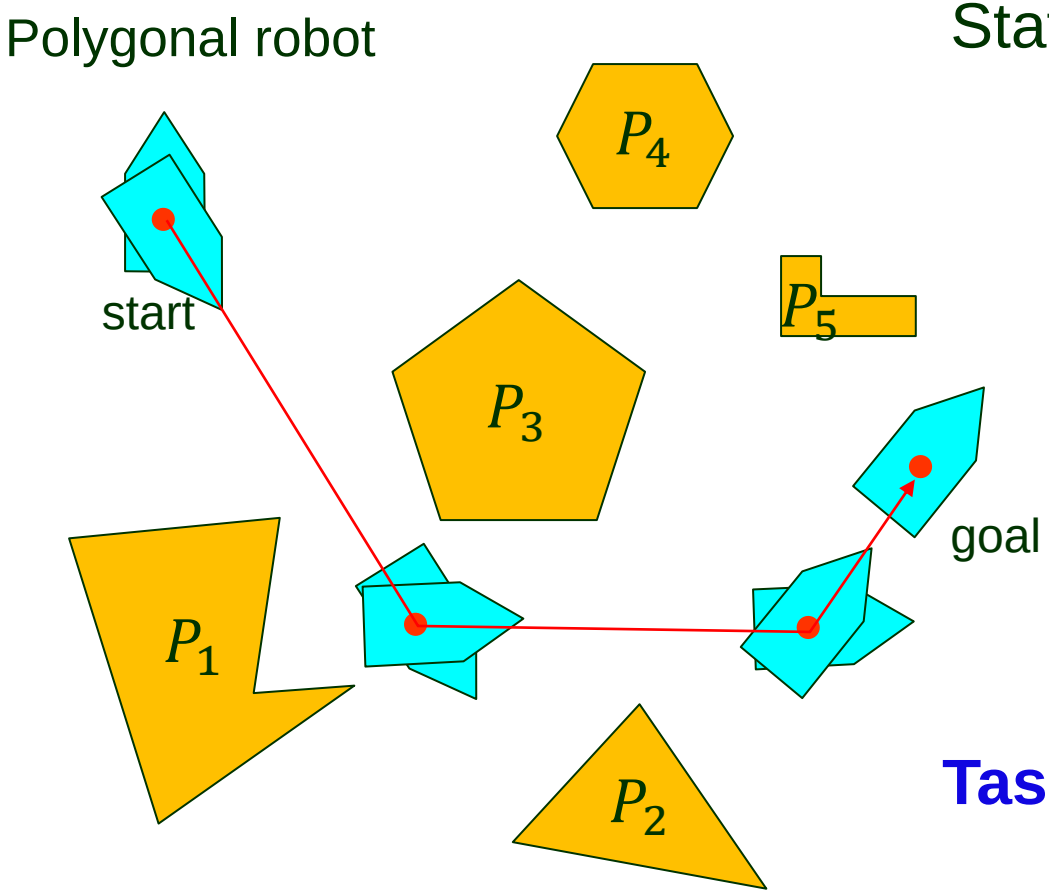

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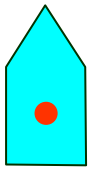
Task: Move the robot from the start placement to the goal placement *without colliding* with any obstacles.

Reference Point

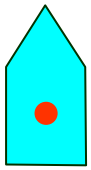
The robot as a polygon R.



Reference Point

The robot as a polygon R.

How to represent its placement, or *configuration*?



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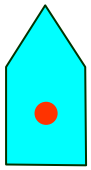

Pick a *reference point* (usually in the interior of R).

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The robot as a polygon R.

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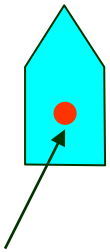

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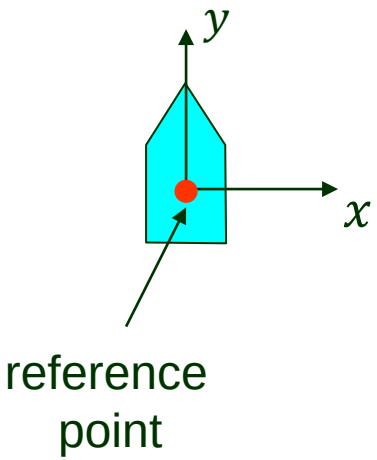

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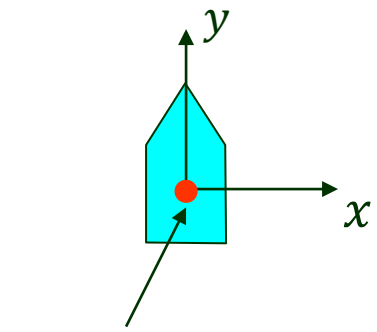

Pick a *reference point* (usually in the interior of R).

- No rotation

Reference Point

The robot as a polygon R.

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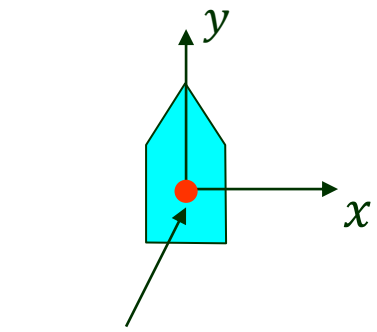


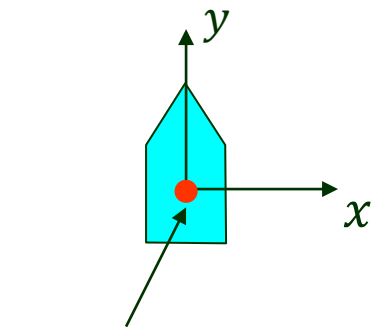
Pick a *reference point* (usually in the interior of R).

- No rotation
  - The robot can only translate.

Reference Point

The robot as a polygon R.

How to represent its placement, or *configuration*?



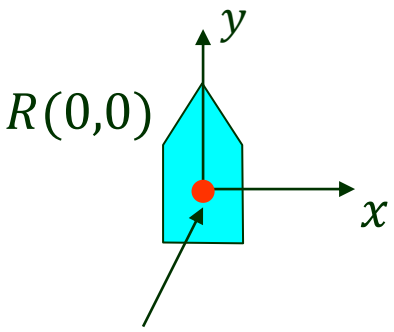
Pick a *reference point* (usually in the interior of R).

- No rotation
  - The robot can only translate.
  - $R(x, y)$: the robot placement with its reference point at (x, y).

Reference Point

The robot as a polygon R.

How to represent its placement, or *configuration*?



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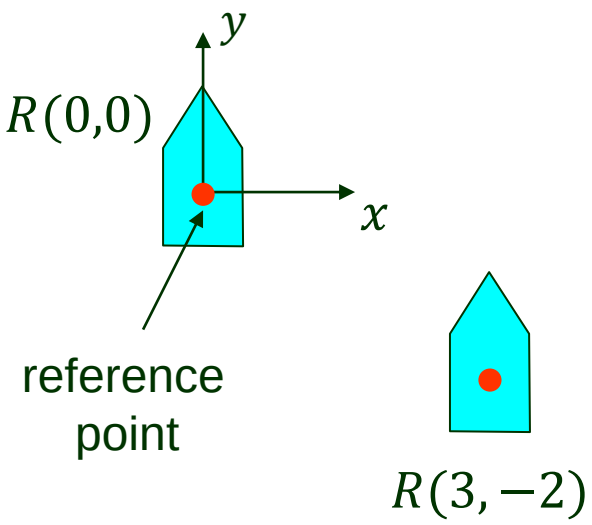

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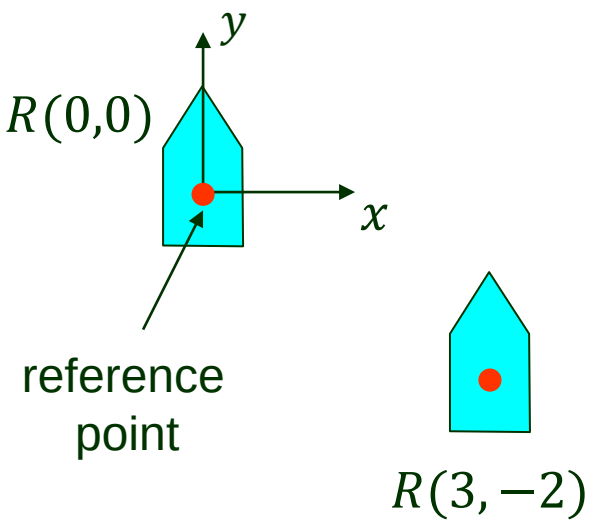
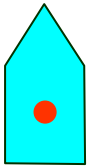


Pick a *reference point* (usually in the interior of R).

- No rotation
  - The robot can only translate.
  - $R(x, y)$: the robot placement with its reference point at (x, y).
- It has only two *degrees of freedom* (2 DOFs): x and y.

Configuration

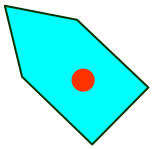
- With rotation
  - The robot can additionally change its orientation around the reference point.

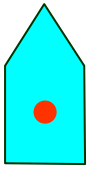


$R(0,0,0)$

Configuration

- With rotation
  - The robot can additionally change its orientation around the reference point.

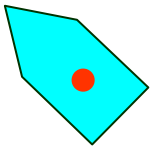


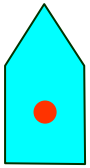


$R(0,0,0)$

Configuration

- With rotation
  - The robot can additionally change its orientation around the reference point.
  - Need to specify the rotation angle θ.





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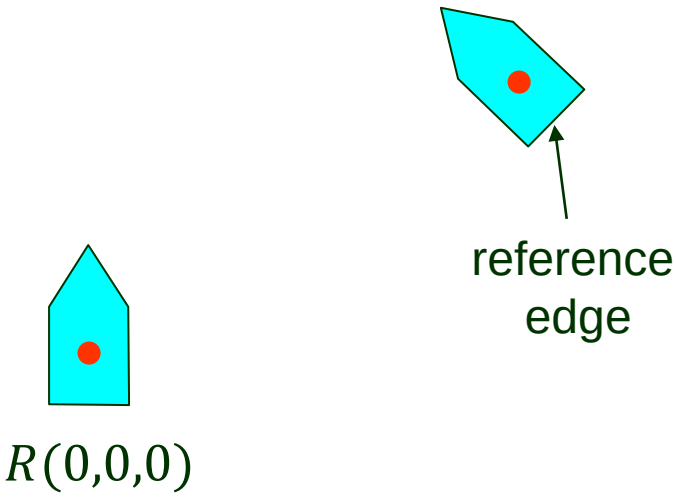

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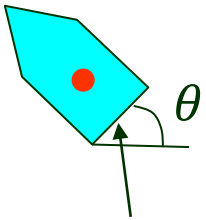
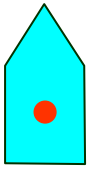


reference edge



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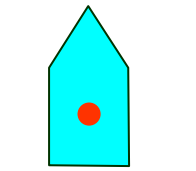

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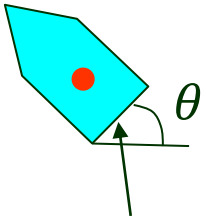
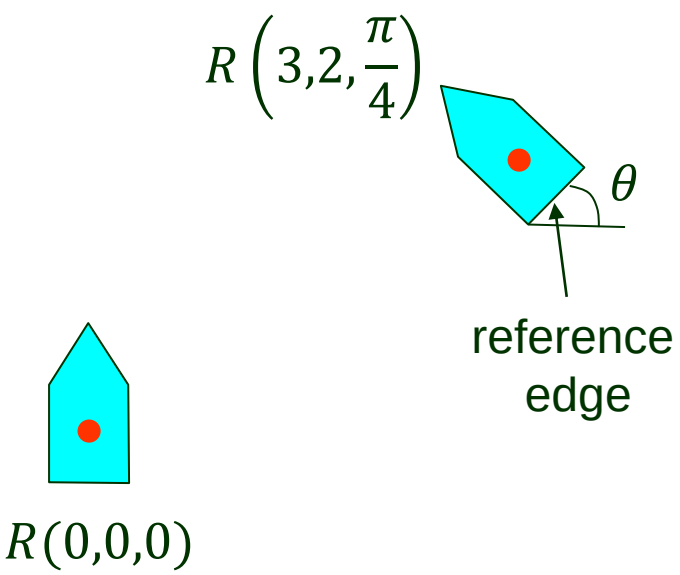


reference edge

- With rotation
  - The robot can additionally change its orientation around the reference point.
  - Need to specify the rotation angle θ.
  - $R(x, y, \theta)$: the robot placed at (x, y) with orientation θ.

Configuration



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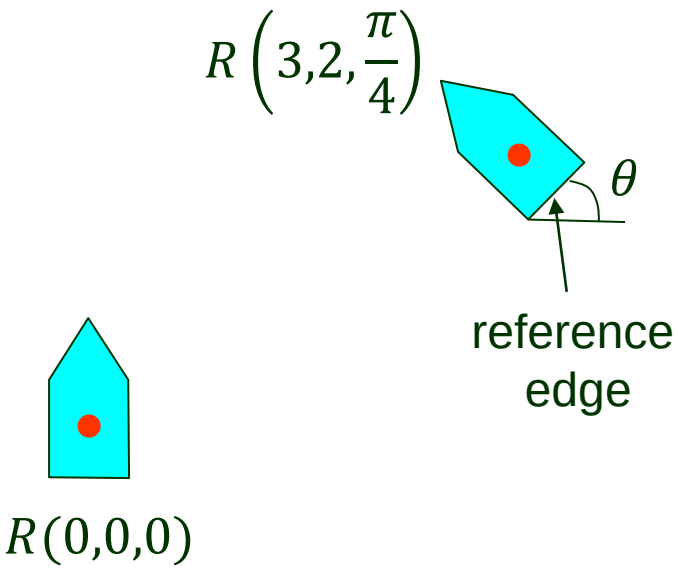

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3 DOFs: x, y, θ

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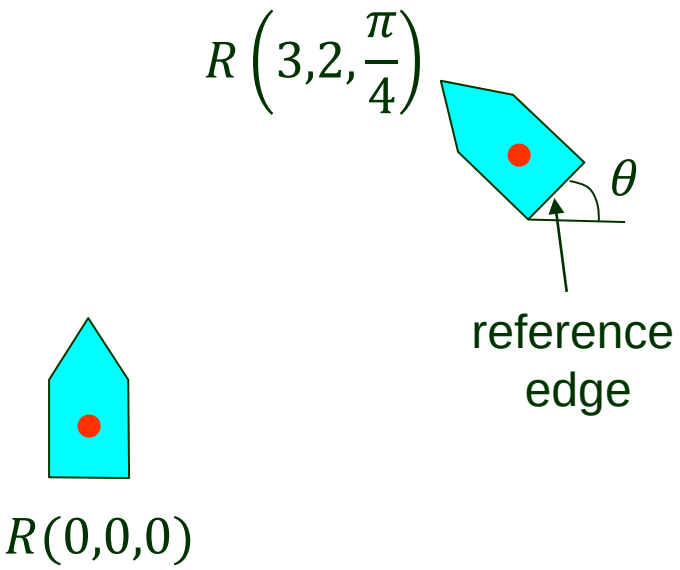

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3 DOFs: x, y, θ

Configuration could also include the robot's velocity and angular velocity.

Pose refers to position and orientation only.

Degrees of Freedom

Number of *independent parameters* that define the configuration of an object.

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point 

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point  2 DOFs: (x, y)

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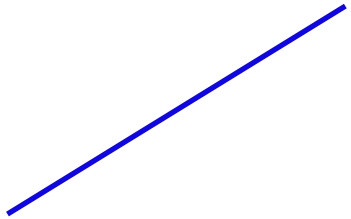
2 DOFs: (x, y)

line



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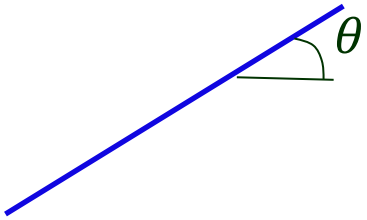

- θ: orientation

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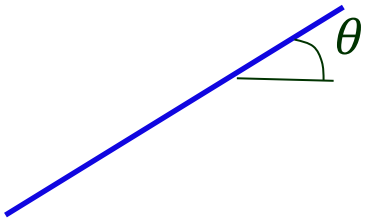

- θ: orientation
- translation in the direction of the line does not change its configuration.

Degrees of Freedom

Number of *independent parameters* that define the configuration of an object.

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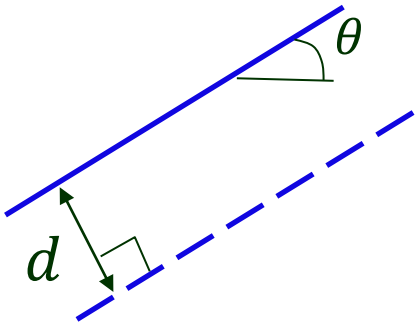
2 DOFs: (x, y)

line



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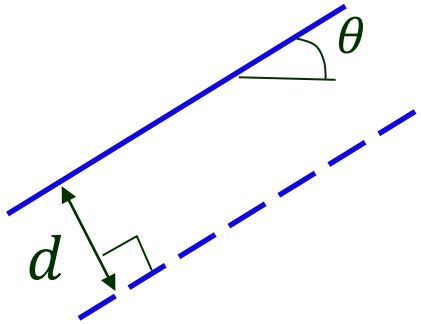


- θ: orientation
- translation in the direction of the line does not change its configuration.
- d: signed distance of translation in the perpendicular direction

Degrees of Freedom

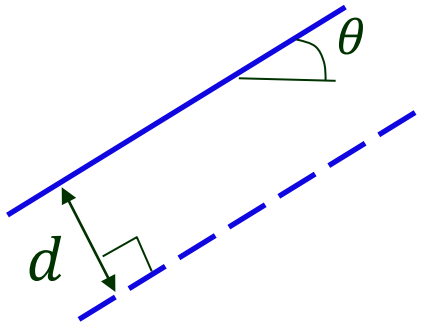
Number of *independent parameters* that define the configuration of an object.

point •

2 DOFs: (x, y)

line

2 DOFs: (θ, d)



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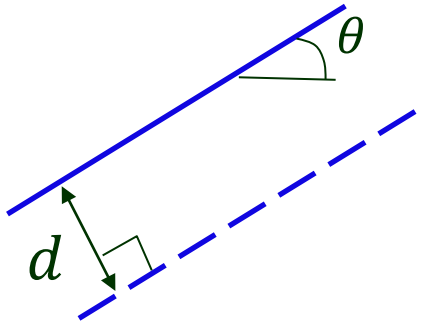


2 DOFs: (θ, d)

- θ: orientation
- translation in the direction of the line does not change its configuration.
- d: signed distance of translation in the perpendicular direction

line segment

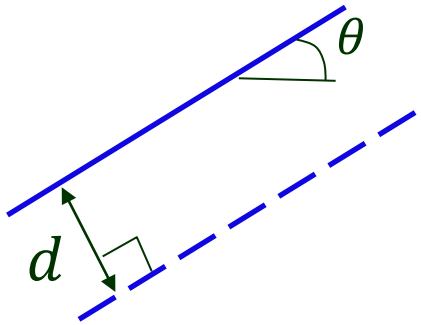
Degrees of Freedom

Number of *independent parameters* that define the configuration of an object.

point

2 DOFs: (x, y)

line



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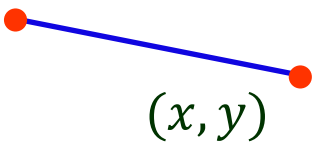

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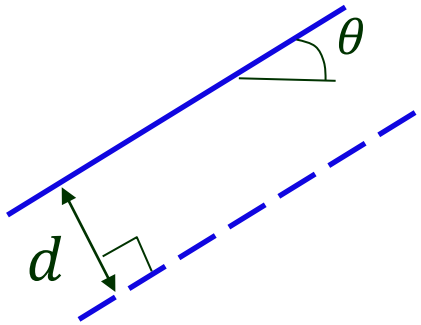
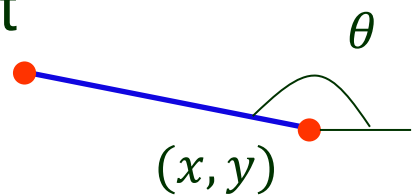
line



2 DOFs: (θ, d)

- θ: orientation
- translation in the direction of the line does not change its configuration.
- d: signed distance of translation in the perpendicular direction

line segment



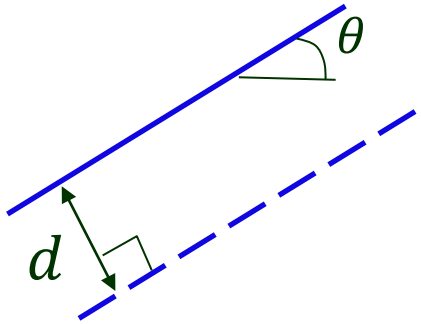
Degrees of Freedom

Number of *independent parameters* that define the configuration of an object.

point

2 DOFs: (x, y)

line



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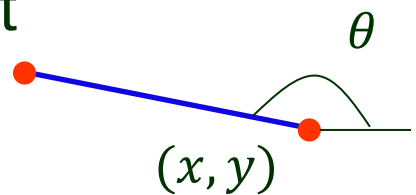


- translation along the segment changes its configuration.

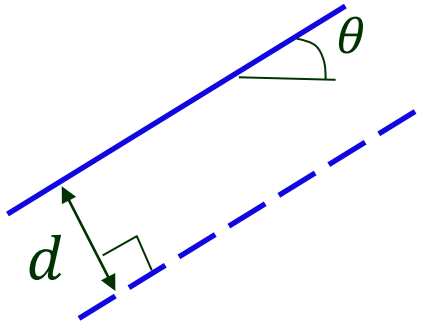
Degrees of Freedom

Number of *independent parameters* that define the configuration of an object.

point

2 DOFs: (x, y)

line



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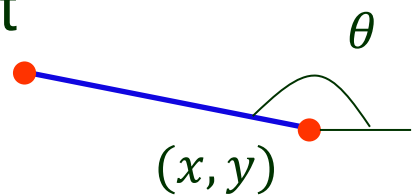

3 DOFs: (x, y, θ)

- translation along the segment changes its configuration.

DOF (cont'd)

DOFs for a robot as one rigid body:

	2D	3D
translation	(x, y)	(x, y, z)
translation & rotation	(x, y, θ)	$(x, y, z, \theta, \phi, \psi)$

DOF (cont'd)

DOFs for a robot as one rigid body:

	2D	3D
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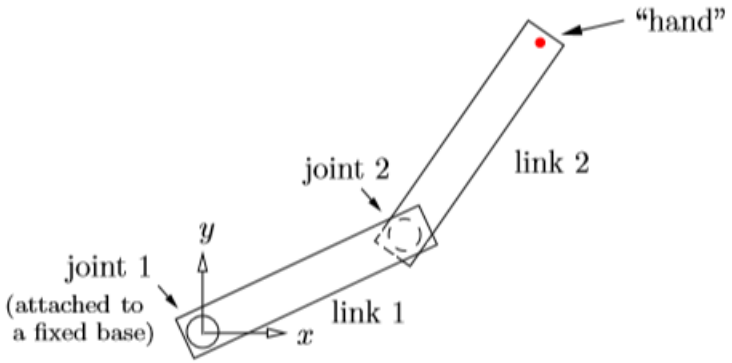
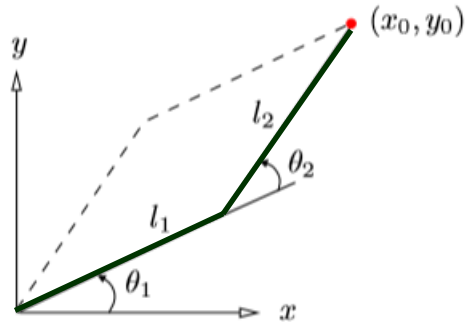
	2D	3D
translation	(x, y)	(x, y, z)
translation & rotation	(x, y, θ)	$(x, y, z, \theta, \phi, \psi)$

rotation angles about stationary x, y, z axes, or three Euler angles: roll, pitch, and yaw

Serial Manipulators

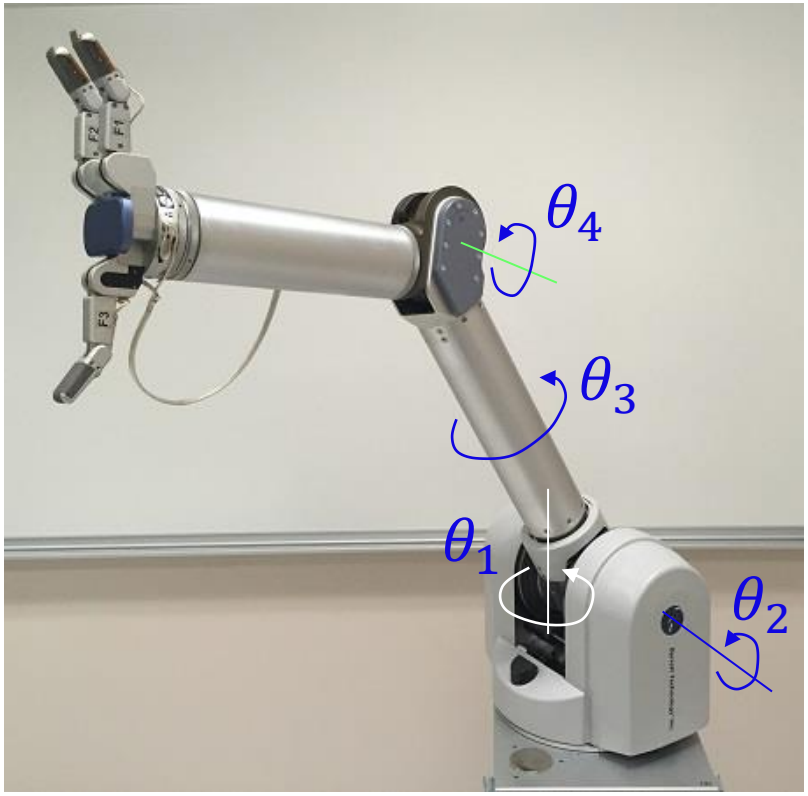
2-DOF manipulator: (θ_1, θ_2)





Serial Manipulators

4-DOF WAM Arm + 3-DOF BarretHand
(both from Barret Technology, Inc.)



Shadow Hand

- Anthropomorphic
- 24 joints.

20 DOFs almost like the human hand!

Image: Shadow Robot Company

II. Configuration Space (C-space)

Work space W: 2D environment

Configuration space $C(R)$: parameter space of the robot R in W

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II. Configuration Space (C-space)

Work space W: 2D environment

Configuration space $C(R)$: parameter space of the robot R in W

translation + rotation $\Rightarrow$ C-space: $\mathbb{R}^2 \times [0,2\pi)$

From the C-space to the workspace:

$$C(R) \rightarrow W$$

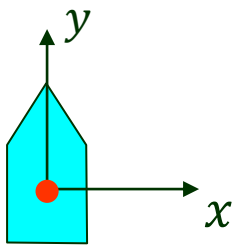
$$(x, y, \theta) \mapsto R(c)$$

placement

Point in the C-space

A polygonal robot is *represented as a point* in the C-space.

$(0, 0, 0)$ $\mapsto$



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A polygonal robot is *represented as a point* in the C-space.

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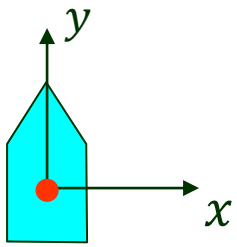

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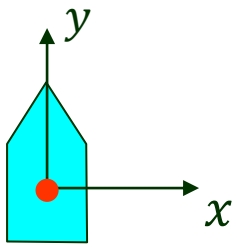

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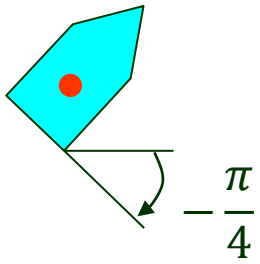
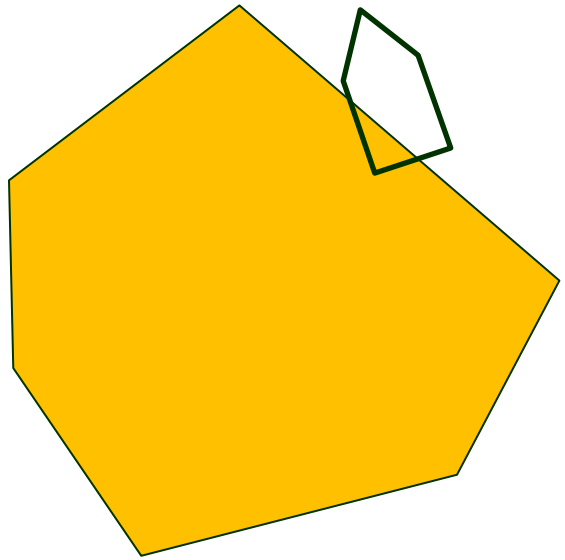


Collision and Forbidden Configuration

A configuration (x, y, θ) is *forbidden* if the placement $R(x, y, \theta)$ collides with an obstacle. Otherwise, it is free.



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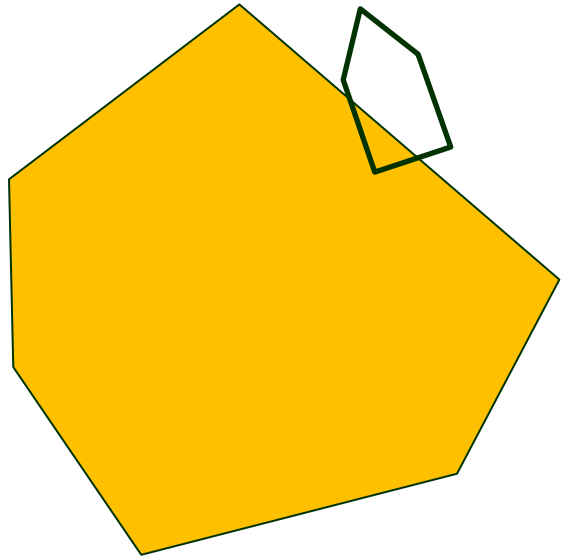

set of obstacles

↓

Forbidden C-space $C_{forb}(R, S)$:
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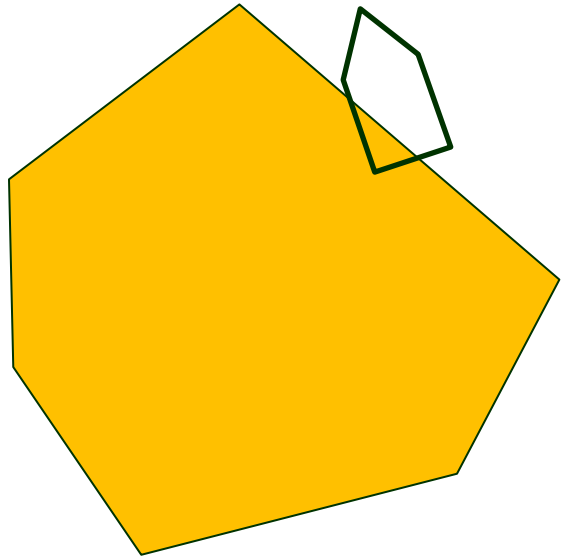

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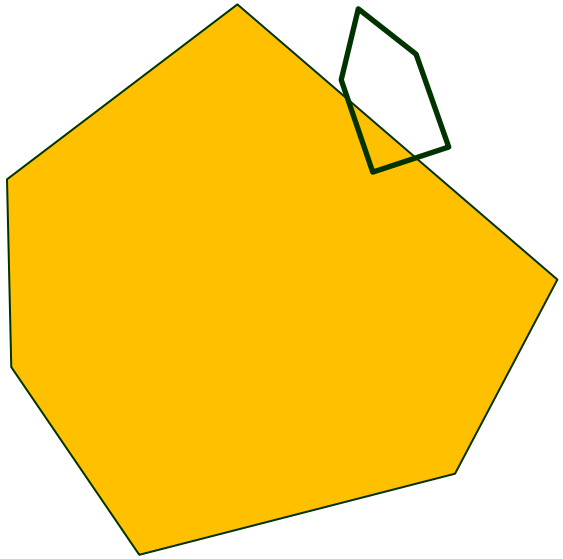

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$$C_{forb}(R, S) \cup C_{free}(R, S) = C(R)$$

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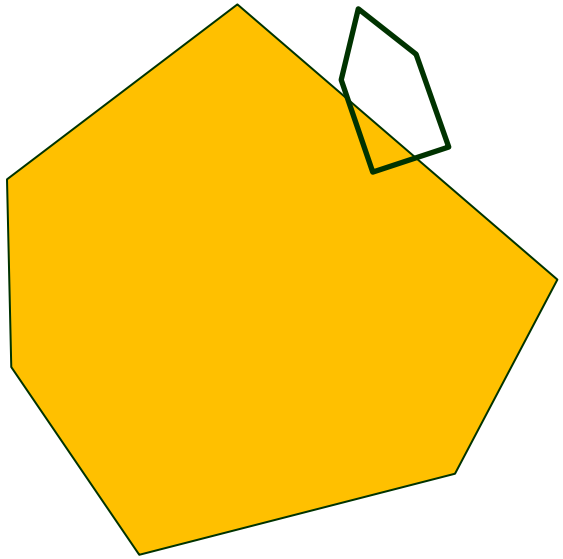

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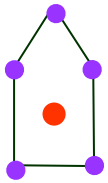
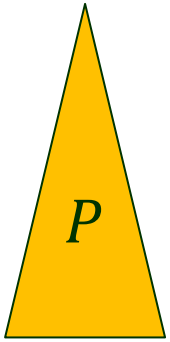
$$C_{forb}(R, S) \cap C_{free}(R, S) = \emptyset$$

C-obstacle for a Translating Robot

Robot Obstacle





$P \mapsto$ *C-obstacle* $O(P)$: $\{ c \mid c \in C(R)$ and $\underbrace{R(c) \cap P \neq \emptyset}_{\text{collision}} \}$

C-obstacle for a Translating Robot

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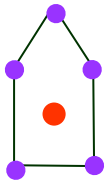

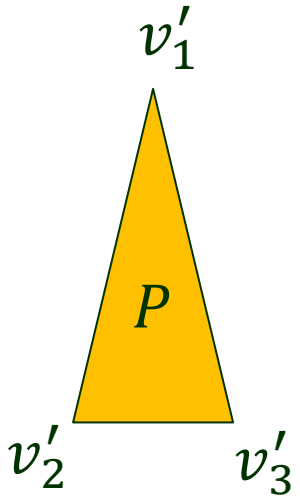

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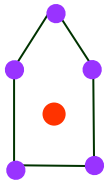

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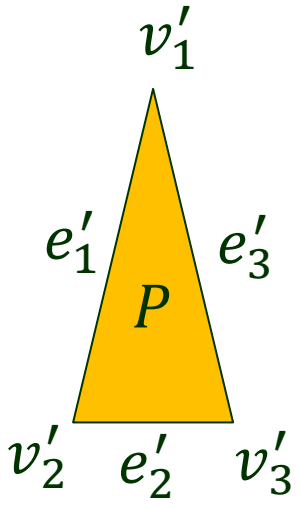

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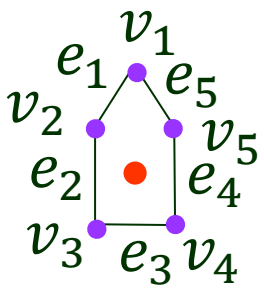

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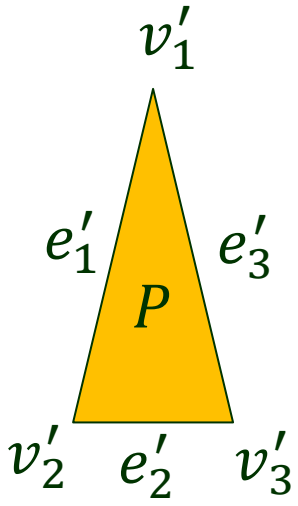

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C-obstacle for a Translating Robot

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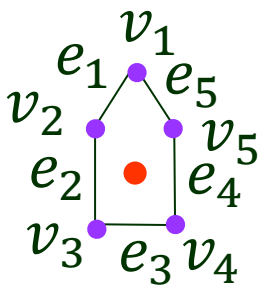

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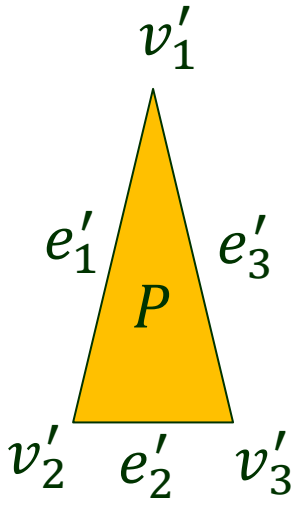

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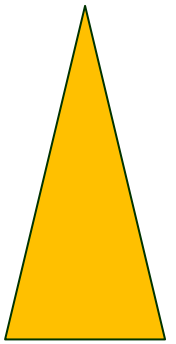

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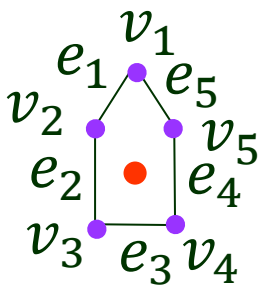
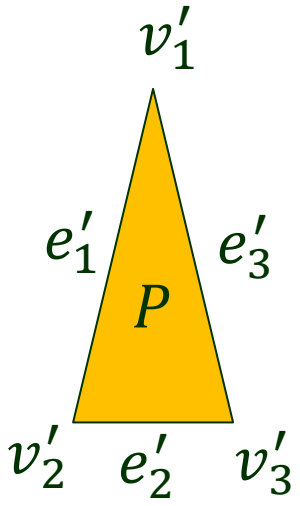
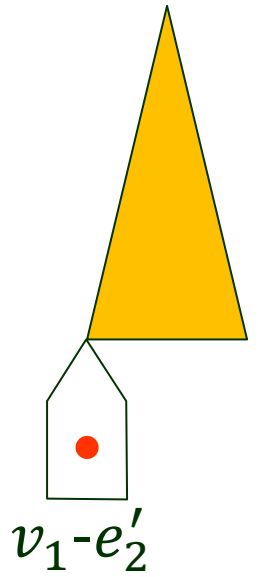
C-obstacle for a Translating Robot

Robot Obstacle C-obstacle







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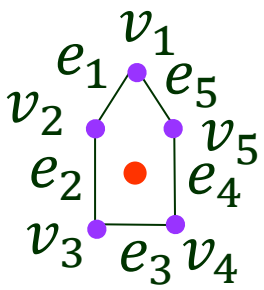
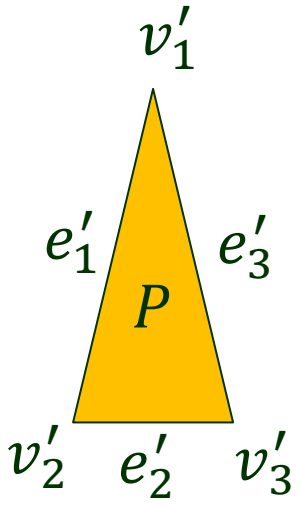
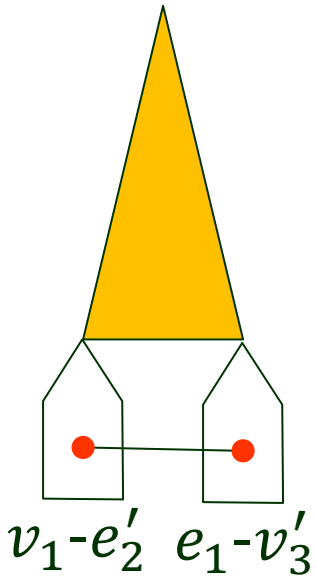
C-obstacle for a Translating Robot

Robot Obstacle C-obstacle







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C-obstacle for a Translating Robot

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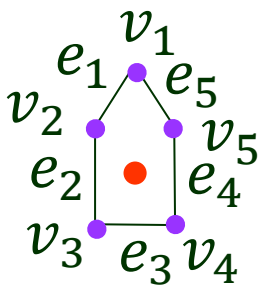

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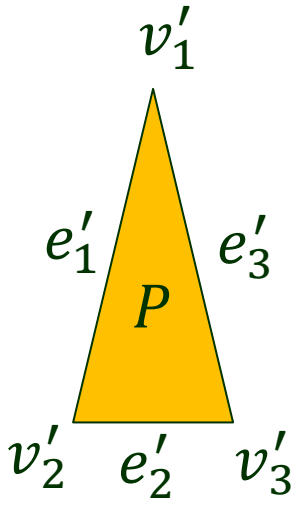

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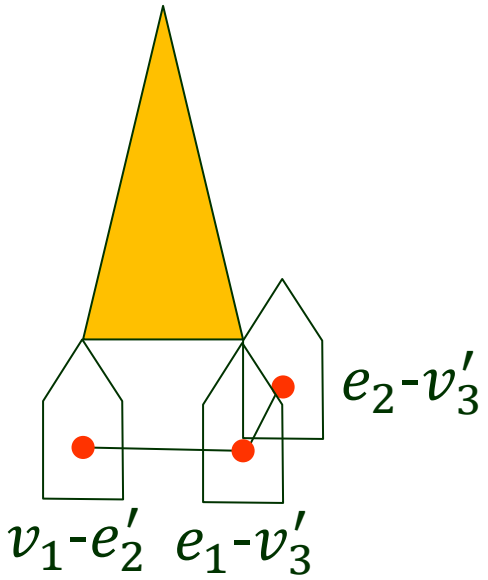

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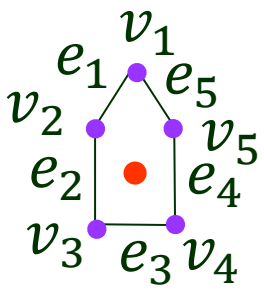

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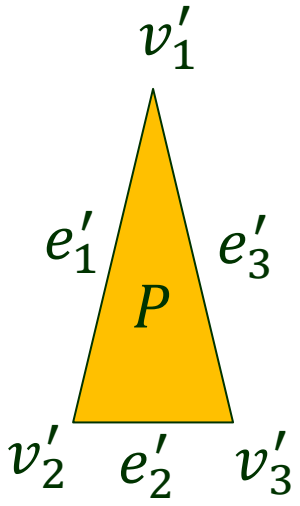

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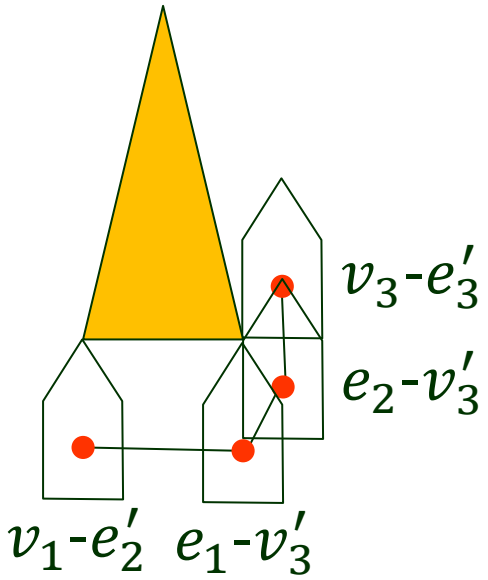

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C-obstacle for a Translating Robot

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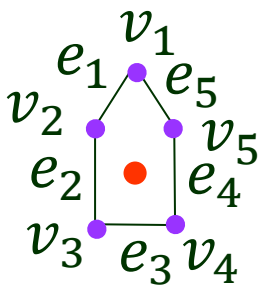

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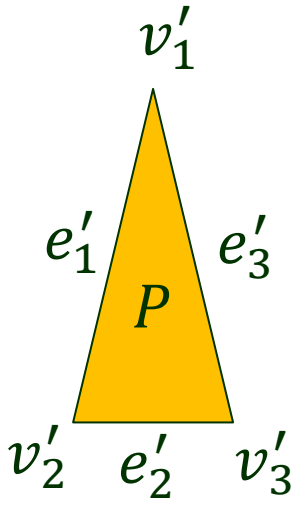

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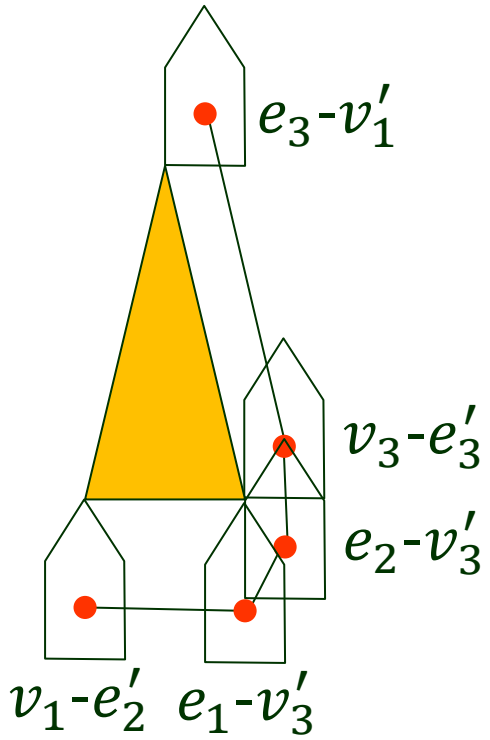

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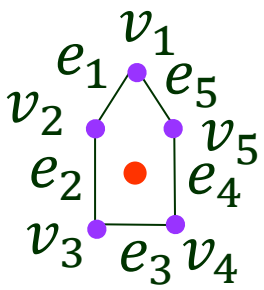
C-obstacle for a Translating Robot

Robot



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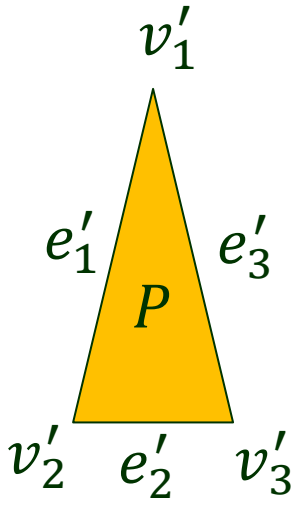

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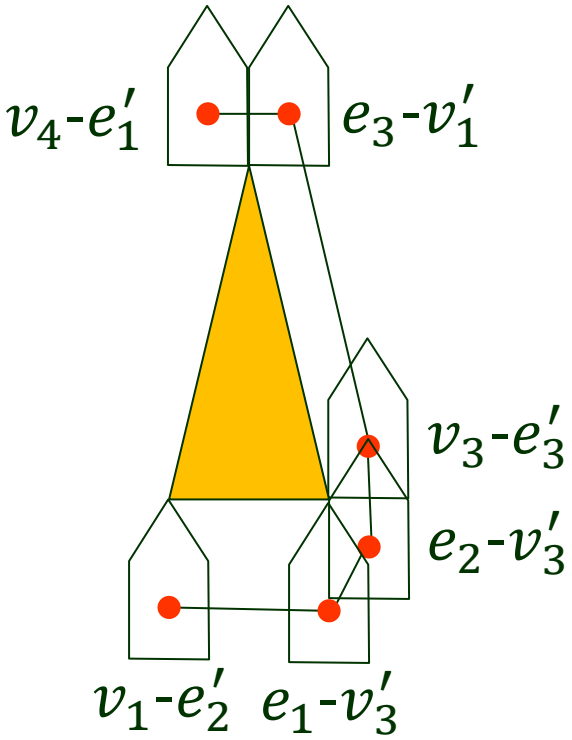

$P \mapsto$ *C-obstacle* $O(P)$: $\{ c \mid c \in C(R)$ and $\underbrace{R(c) \cap P \neq \emptyset}_{\text{collision}} \}$

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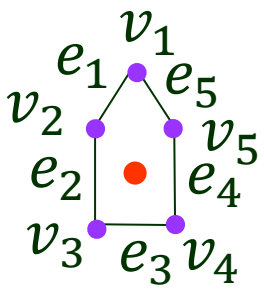

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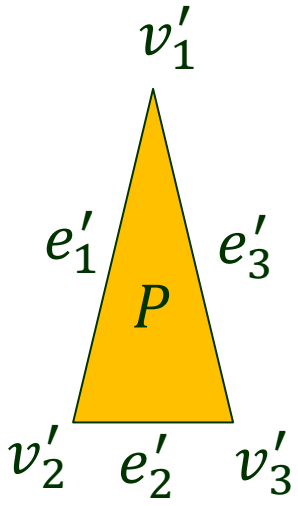

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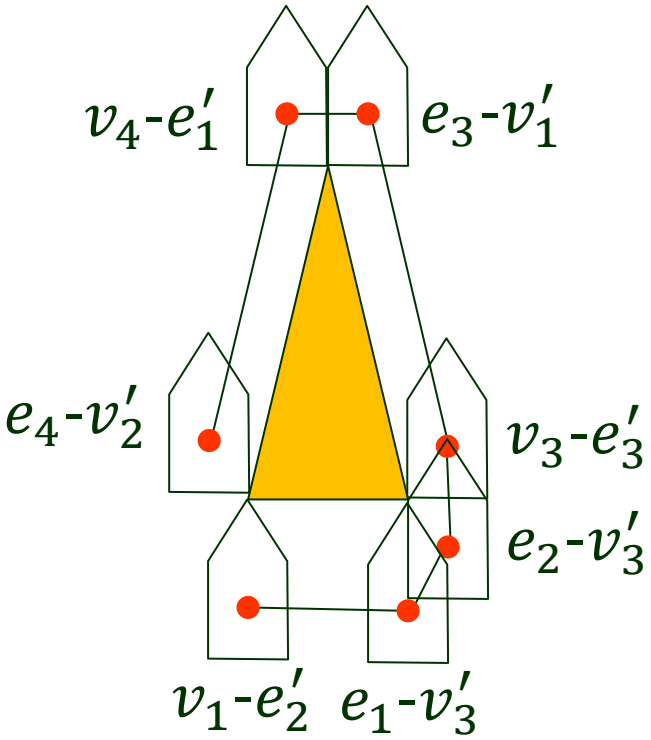

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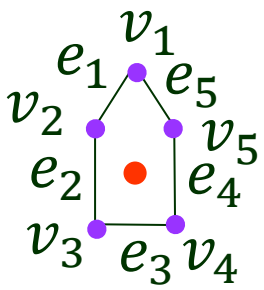

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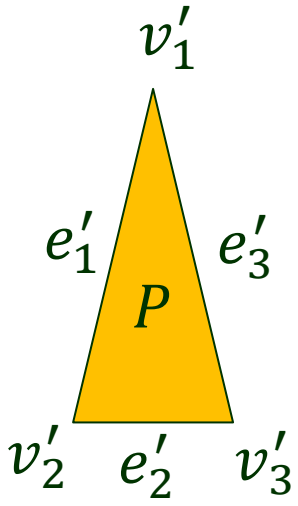

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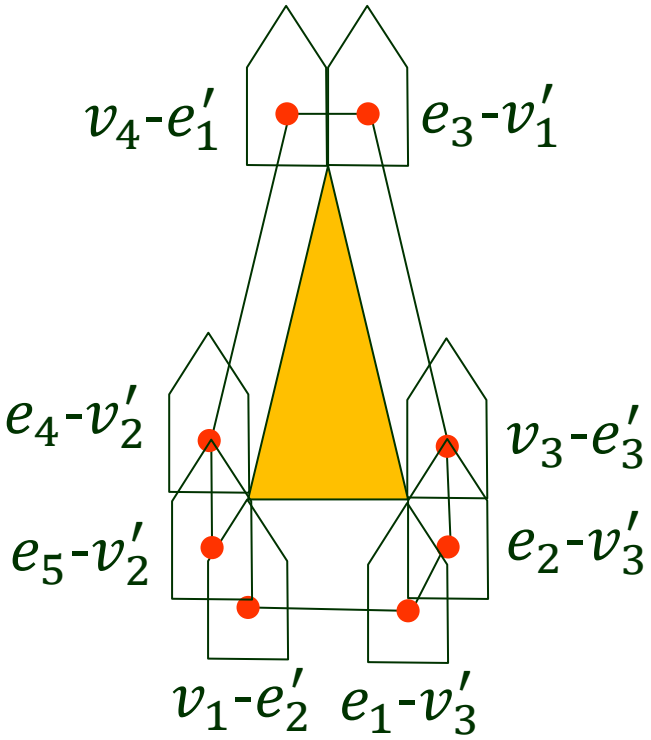

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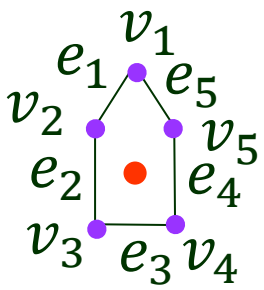
C-obstacle for a Translating Robot

Robot



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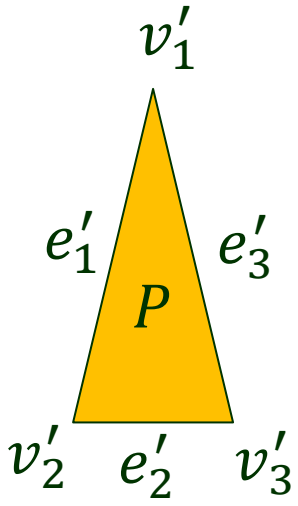

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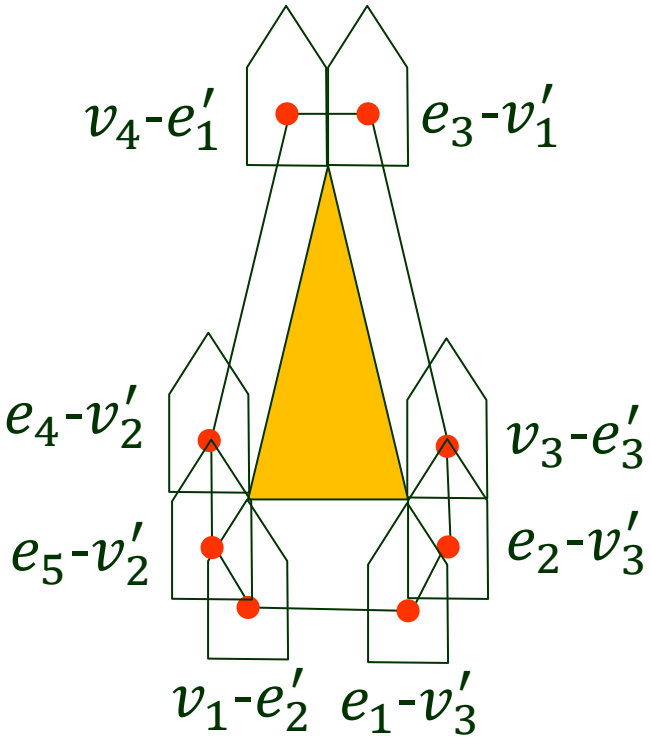

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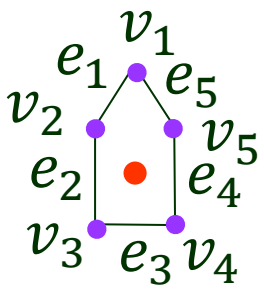
C-obstacle for a Translating Robot

Robot



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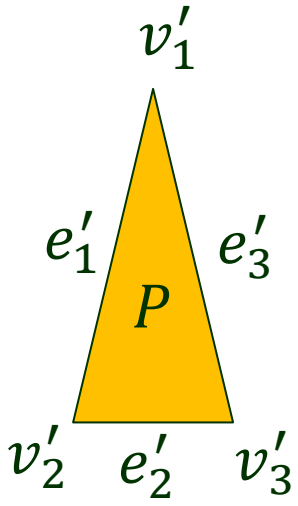

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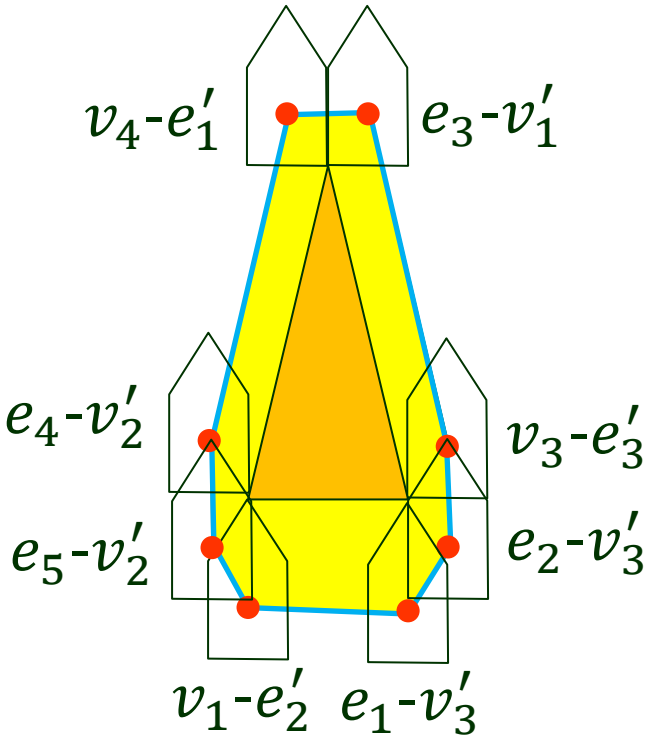

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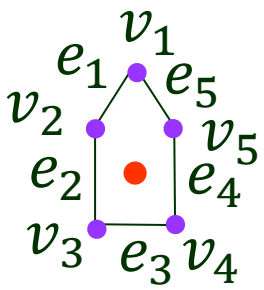

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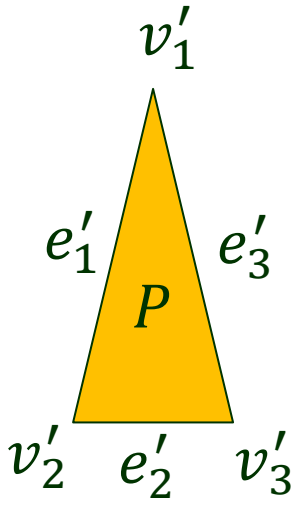

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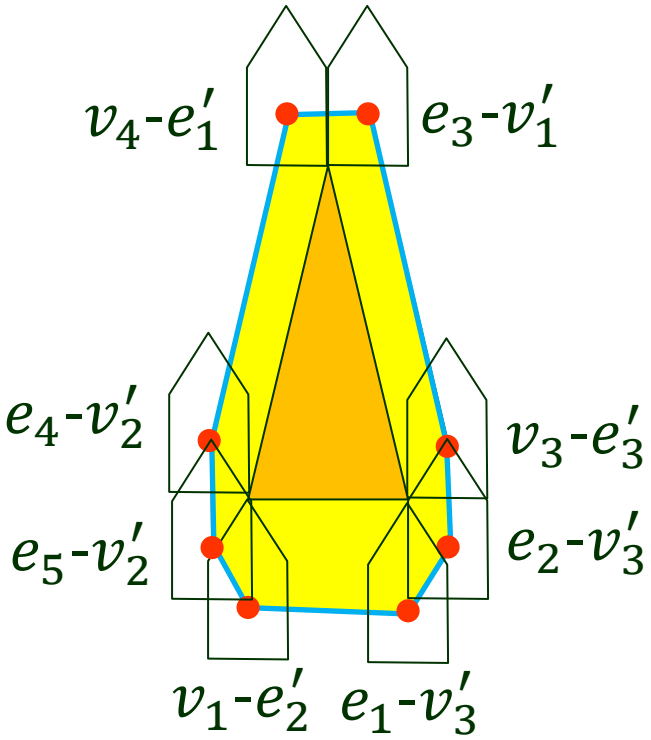

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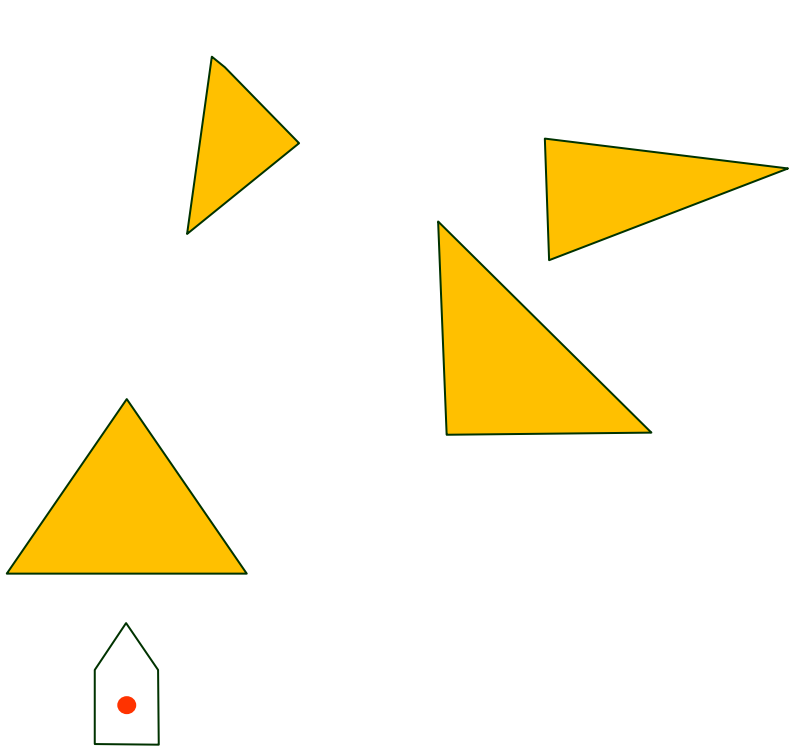
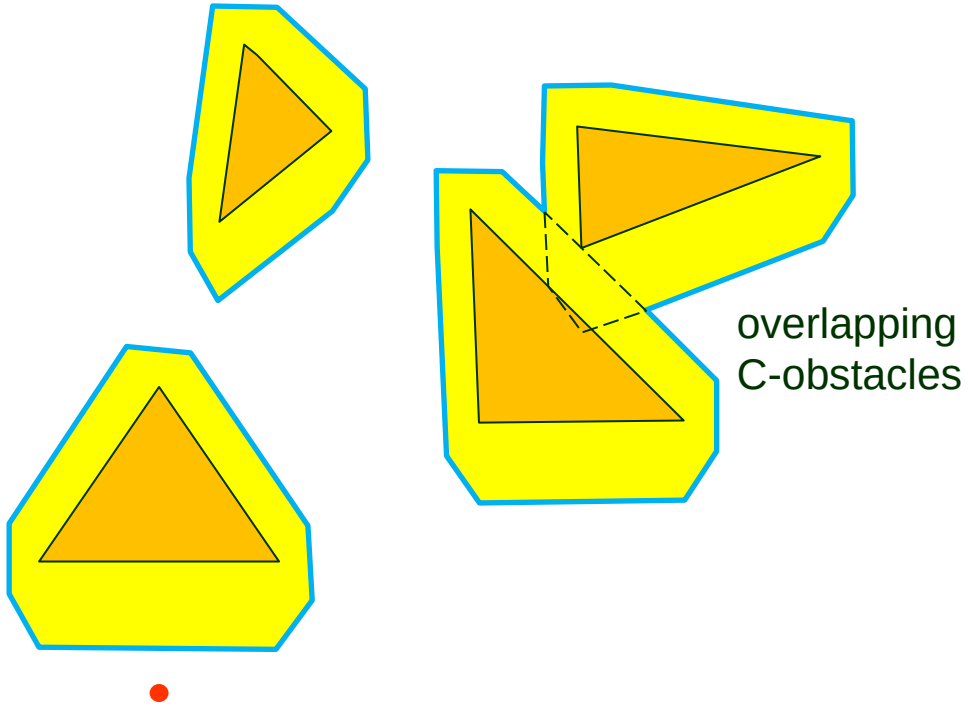
$$C_{forb}(R,S) = \bigcup_{P \in S} O(P)$$

Path

Work space:		C-space:
robot	↦	point
obstacle	↦	C-obstacles (possibly joined)
path	↦	curve





Path

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robot ↦ point

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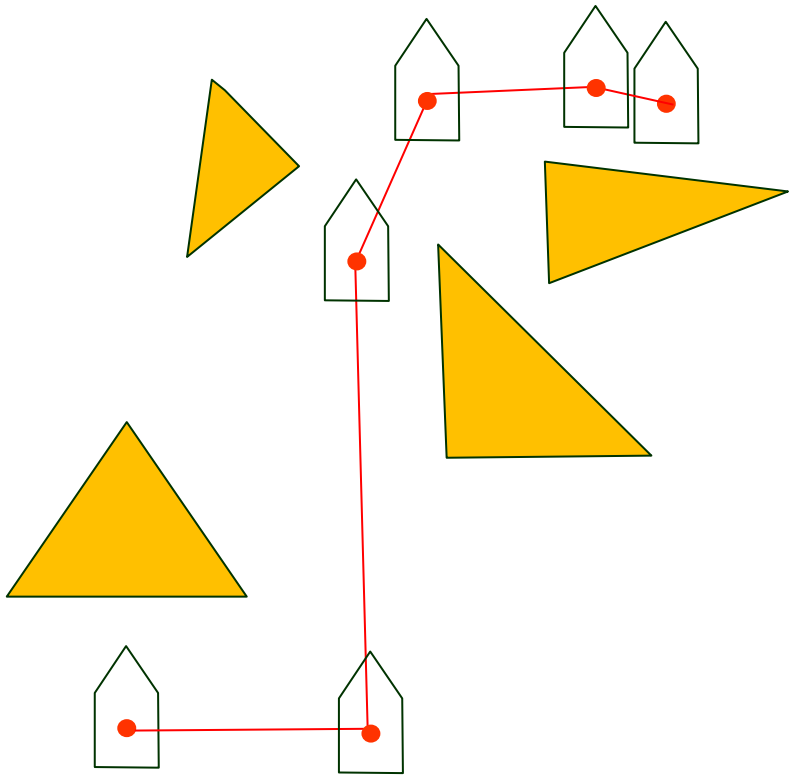

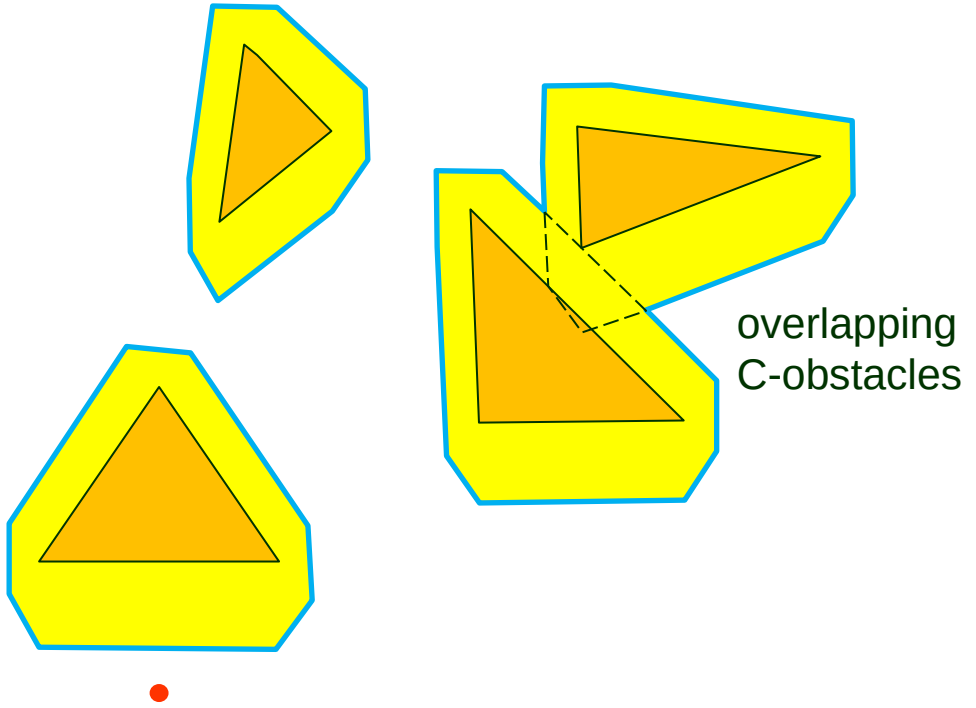

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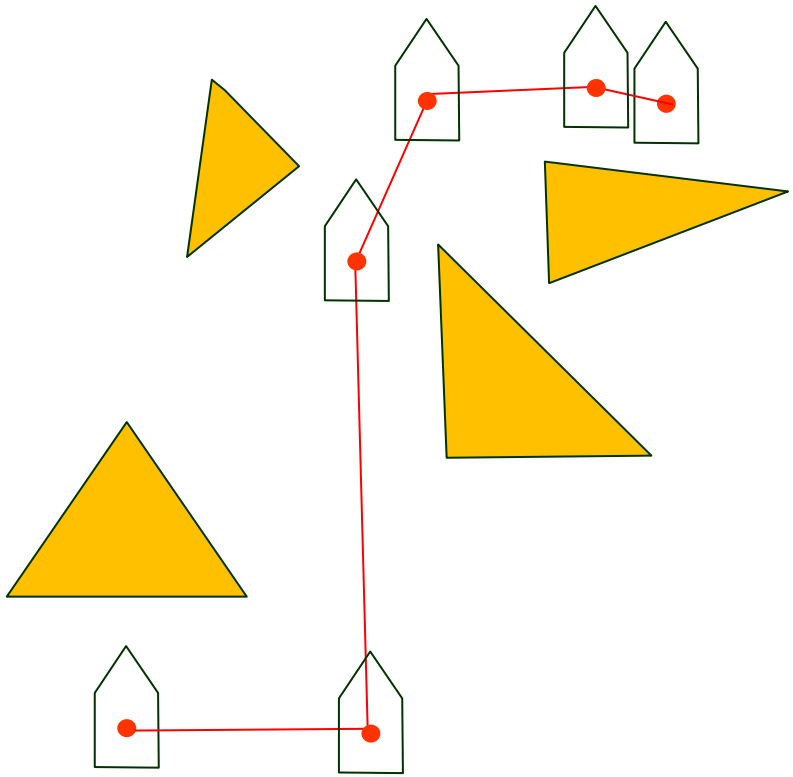

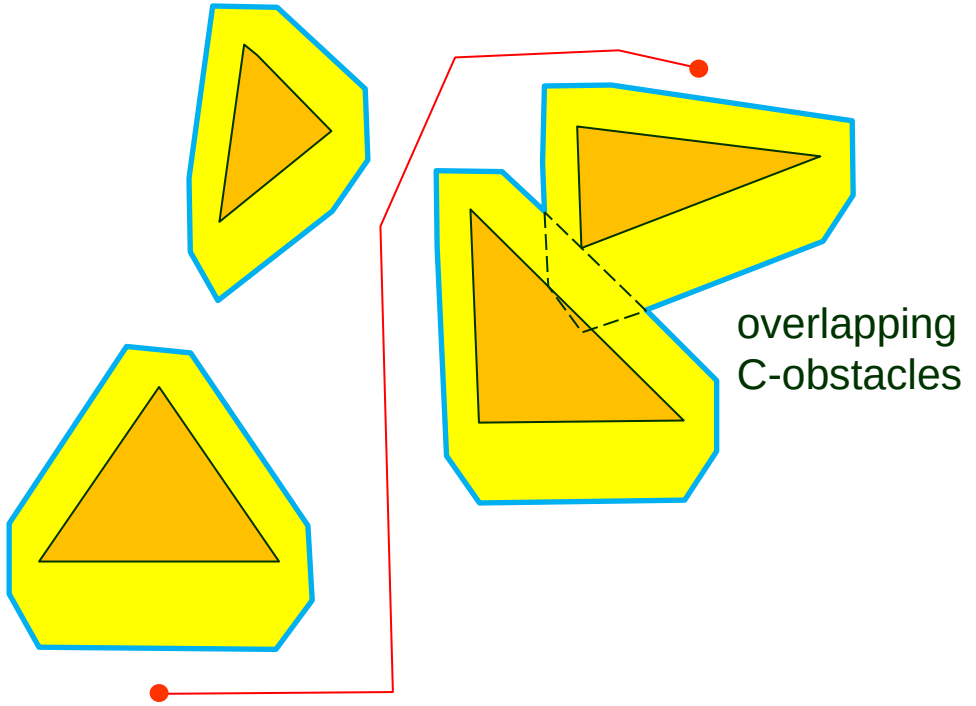

Point Robot

Work space = C-space & every obstacle = its C-obstacle

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- Obstacles $P_1, P_2, \ldots, P_t$ have disjoint interiors.

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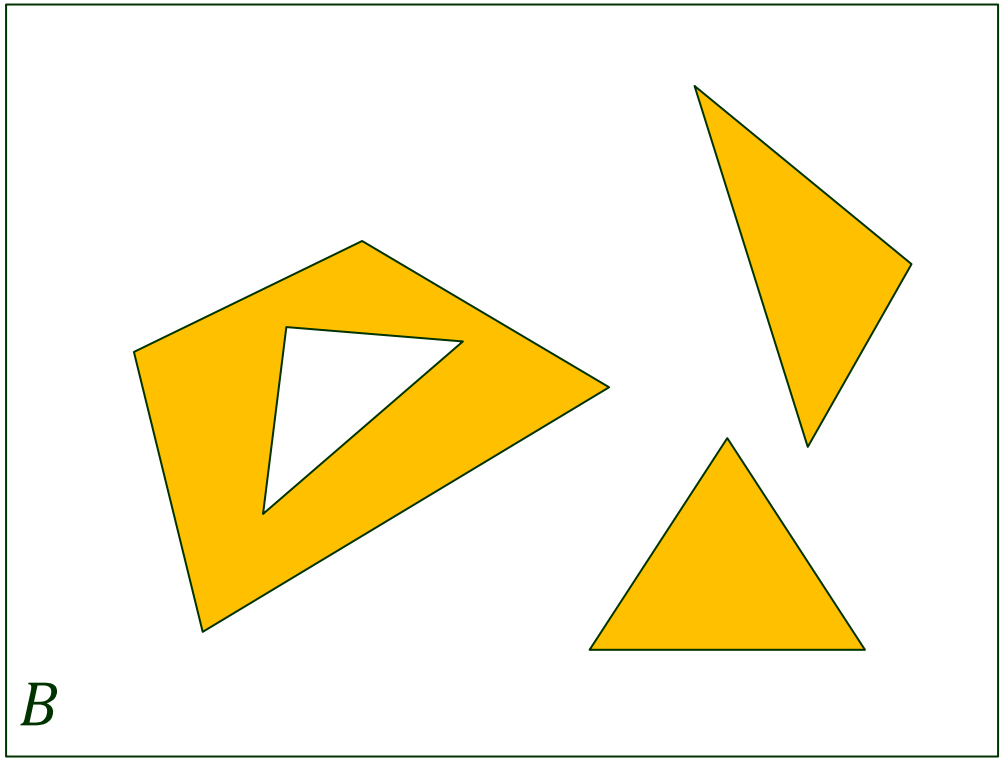
- Bounding box B contains all obstacles.



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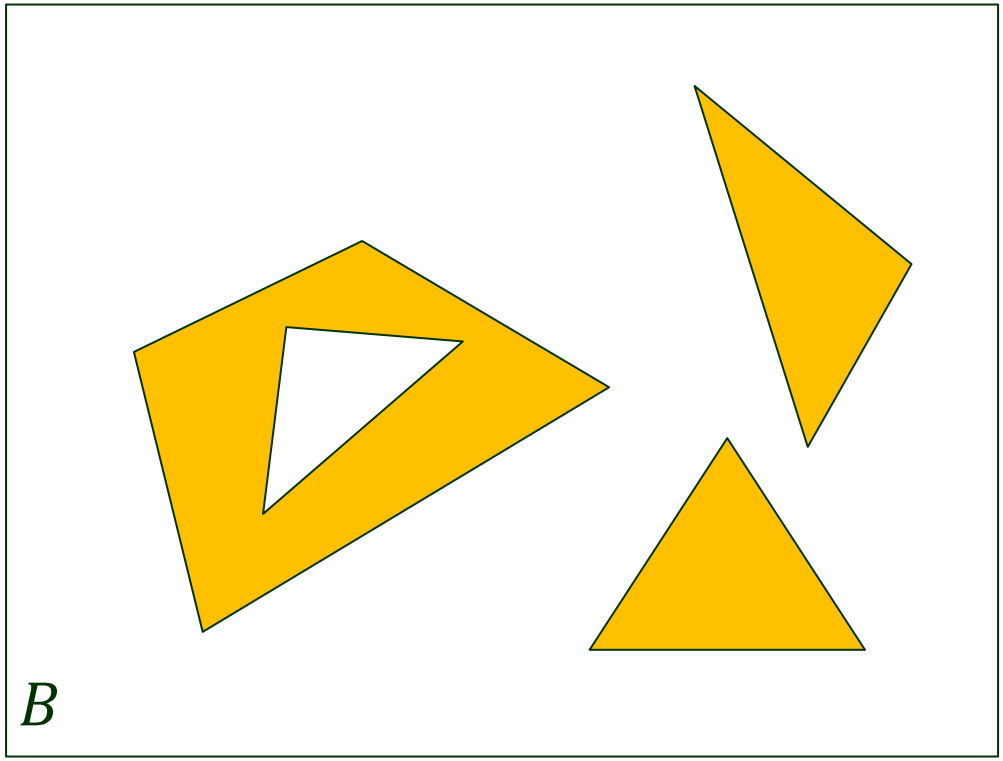
$$C_{free} = B \setminus \bigcup_{i=1}^{t} P_i$$



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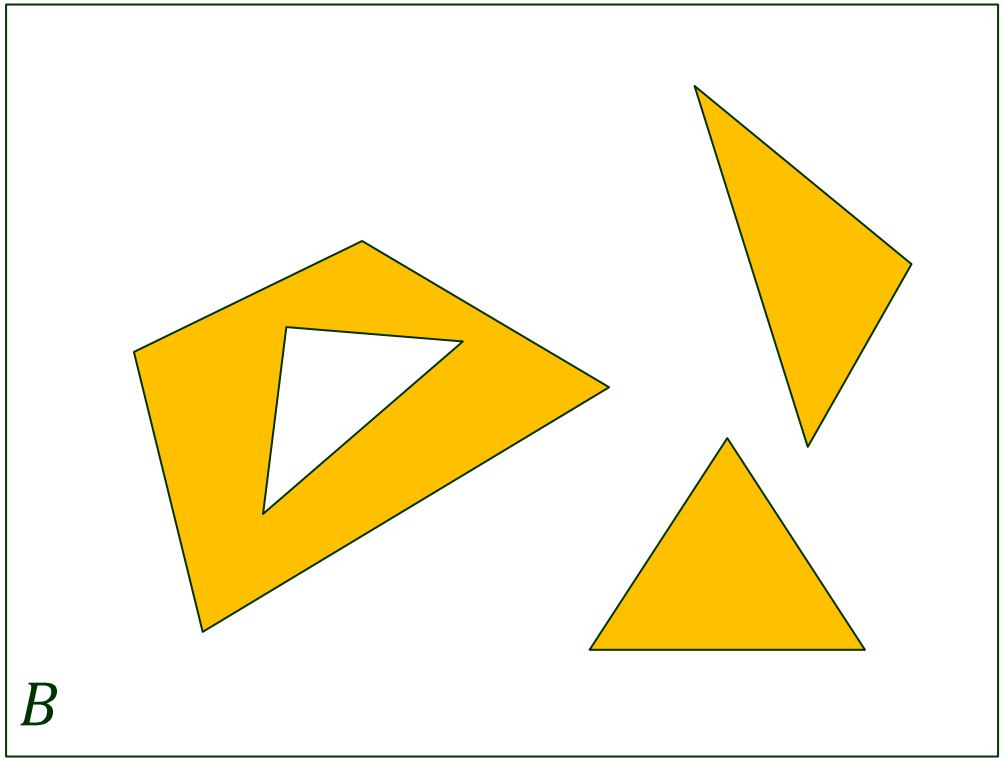
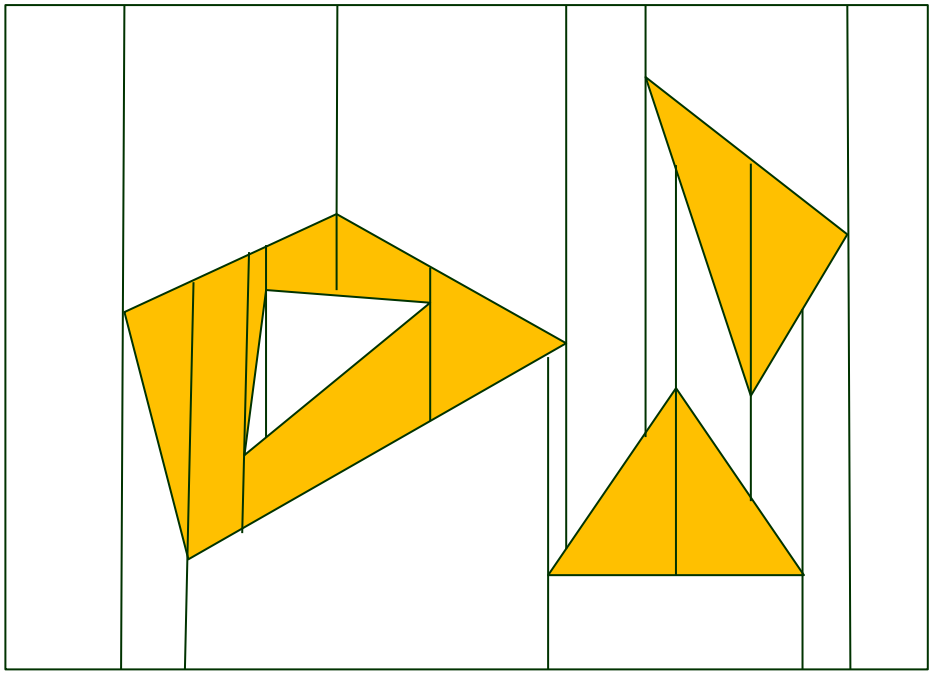
$$C_{free} = B \setminus \bigcup_{i=1}^{t} P_i$$

  - Also the free work space.
  - Possibly disconnected.



C_{free} as a Trapezoidal Map



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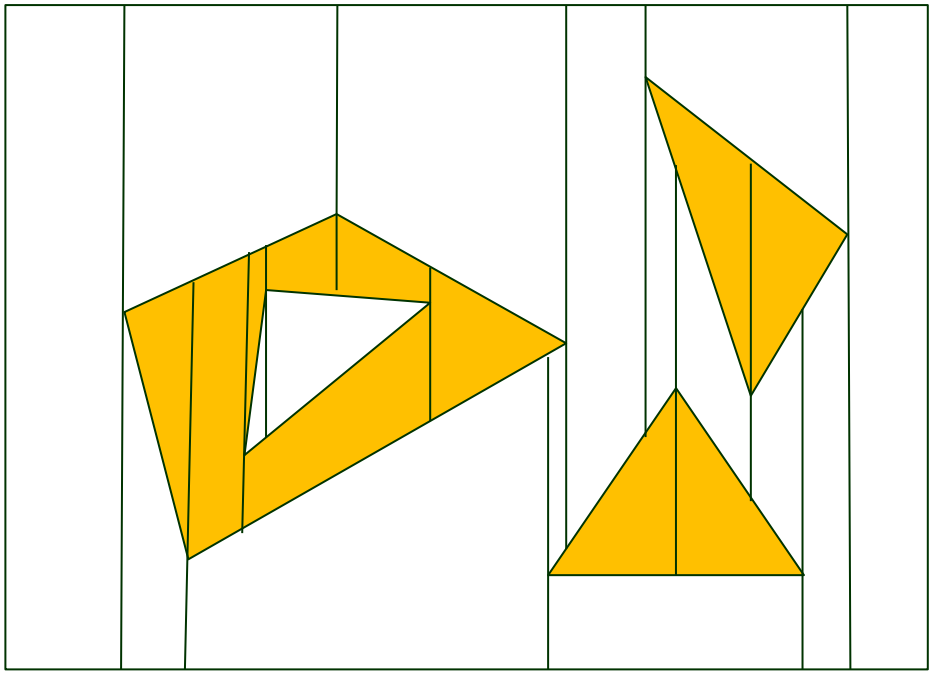
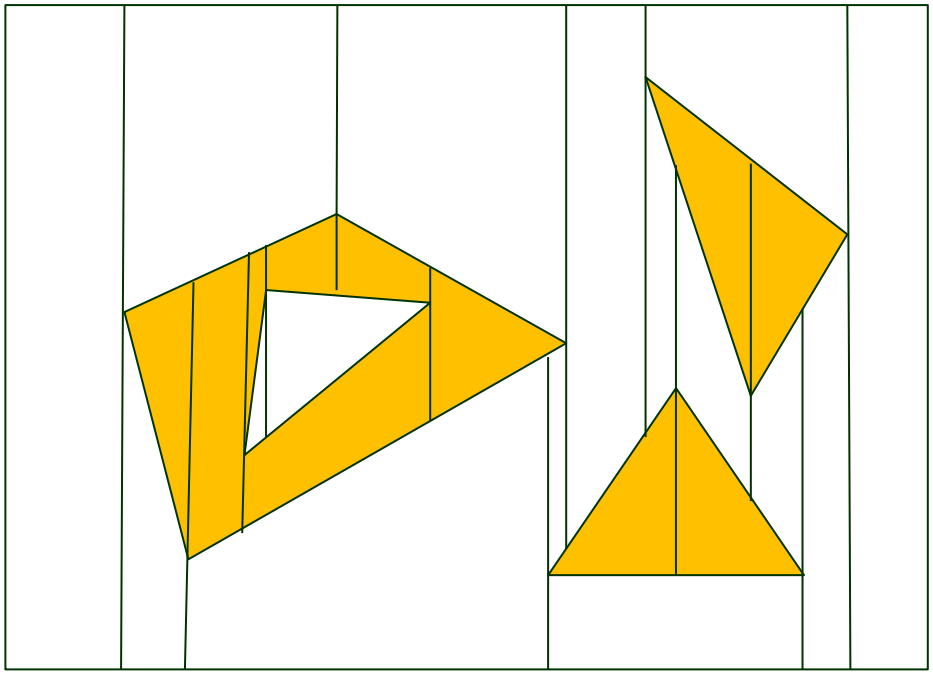


Remove trapezoids lying inside the obstacles

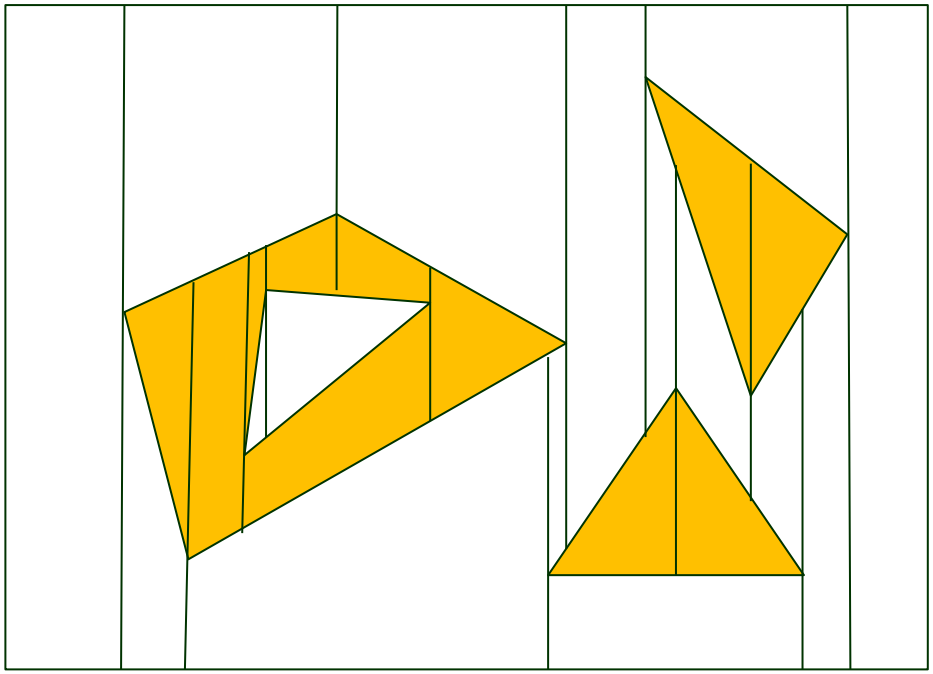
C_{free} as a Trapezoidal Map



Remove trapezoids lying inside the obstacles

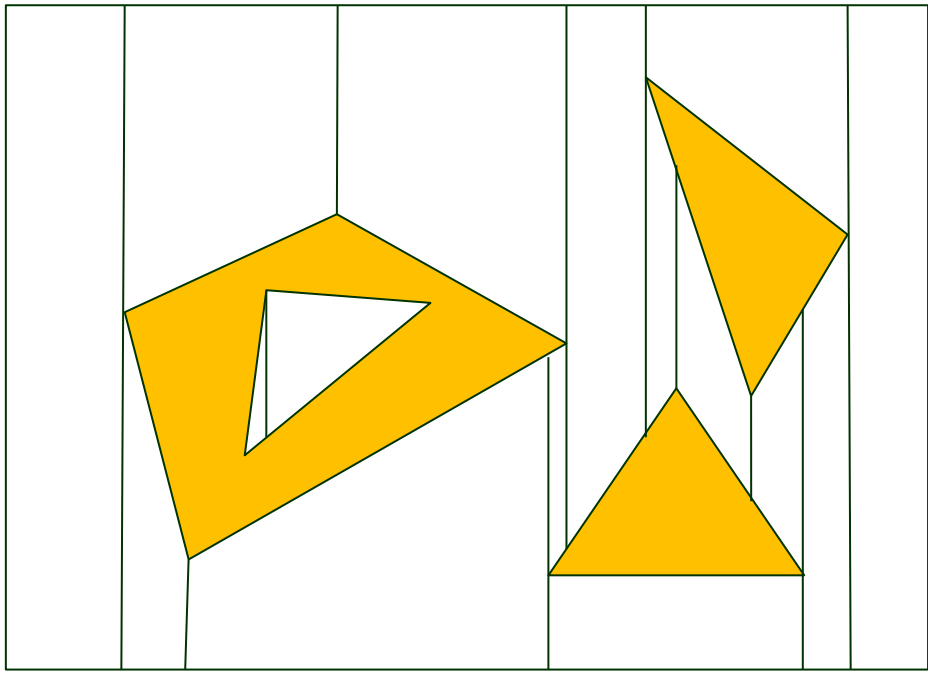
A trapezoid is removed if its top edge bounds an obstacle from above.

C_{free} as a Trapezoidal Map



Remove trapezoids lying inside the obstacles

$T(C_{free})$



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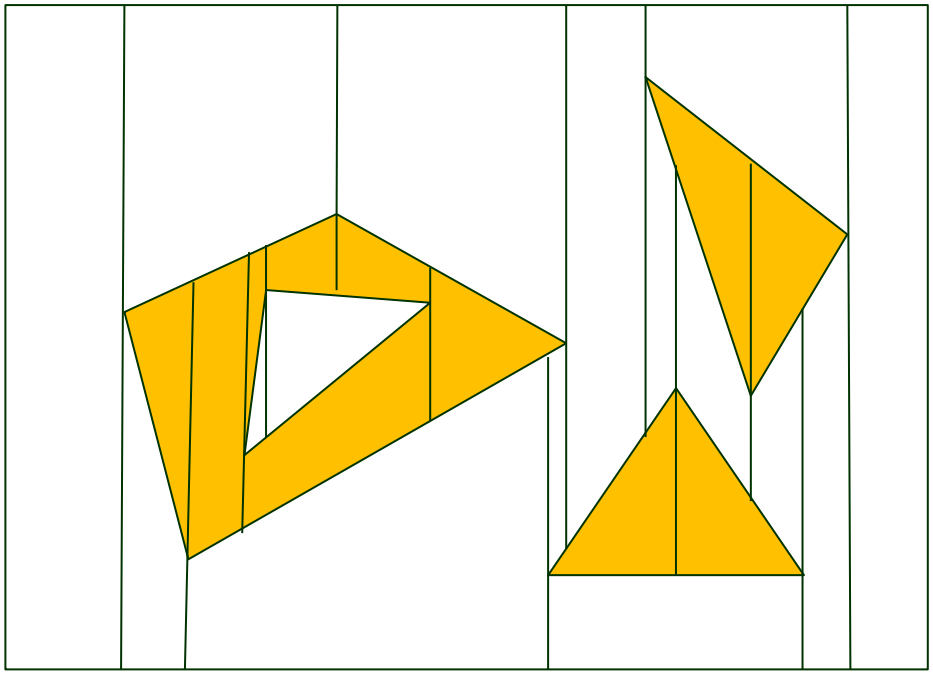

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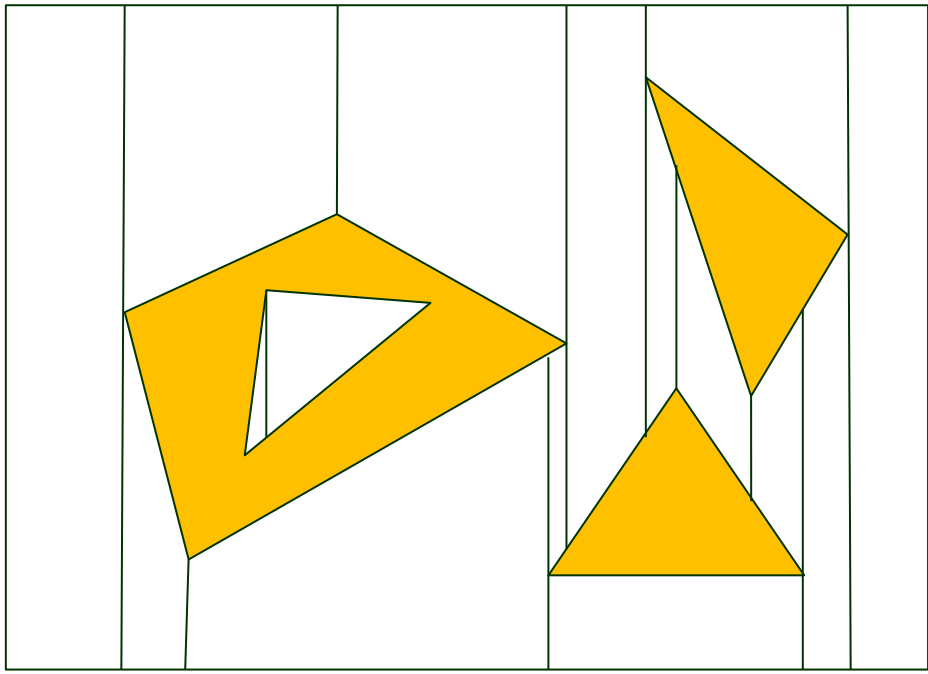
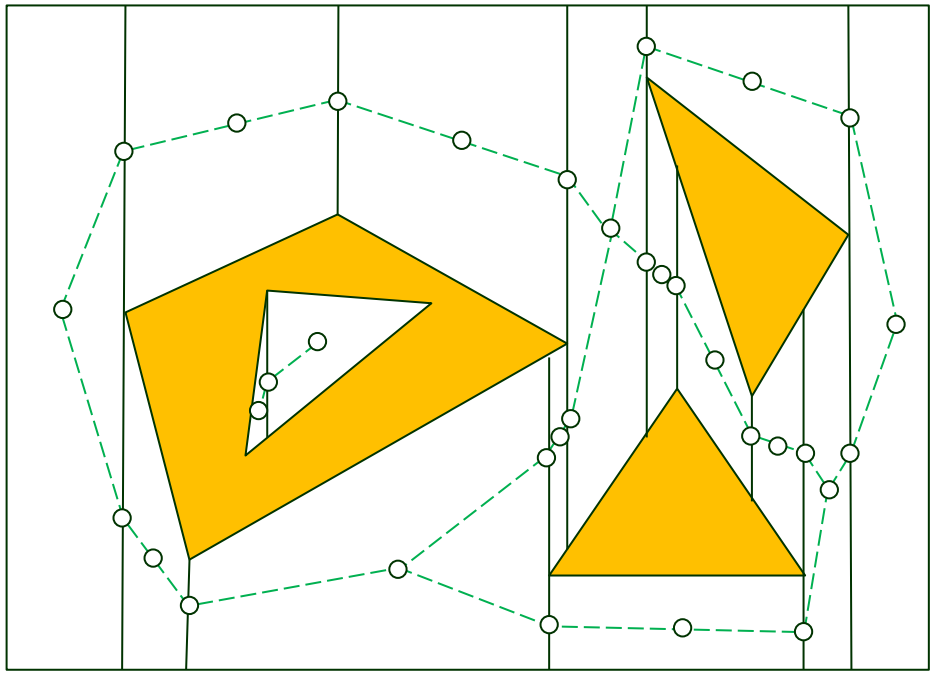


A trapezoid is removed if its top edge bounds an obstacle from above.

$O(n \log n)$ expected time.

III. Roadmap



A *planar graph* embedded in C_{free} and formed by adding

- one node in the center of each trapezoid;
- one node in the middle of each extension;
- an arc between two nodes if one is in the center of a trapezoid while the other is on the boundary.

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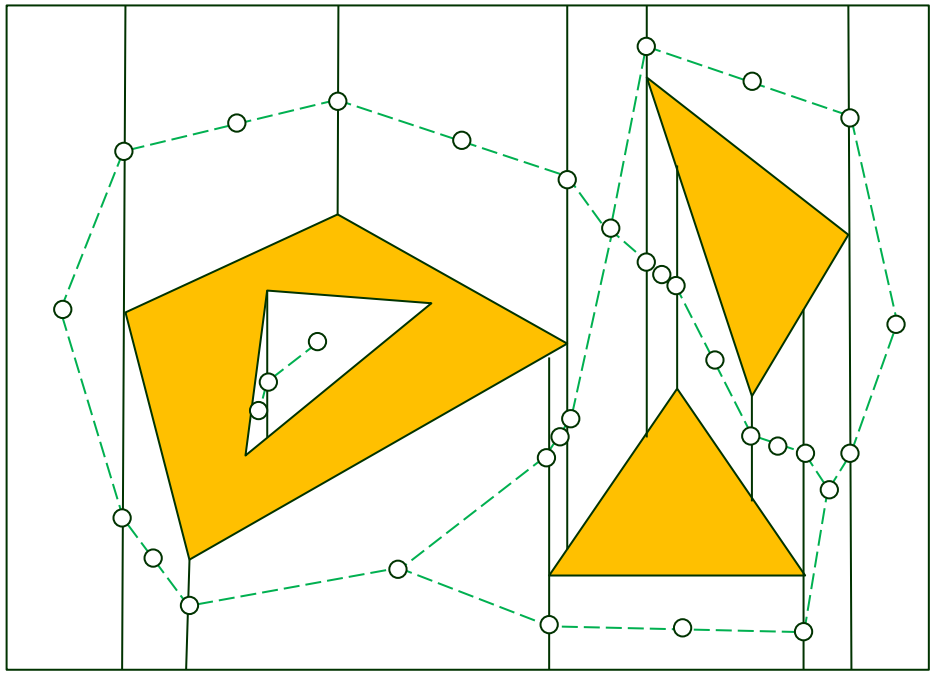

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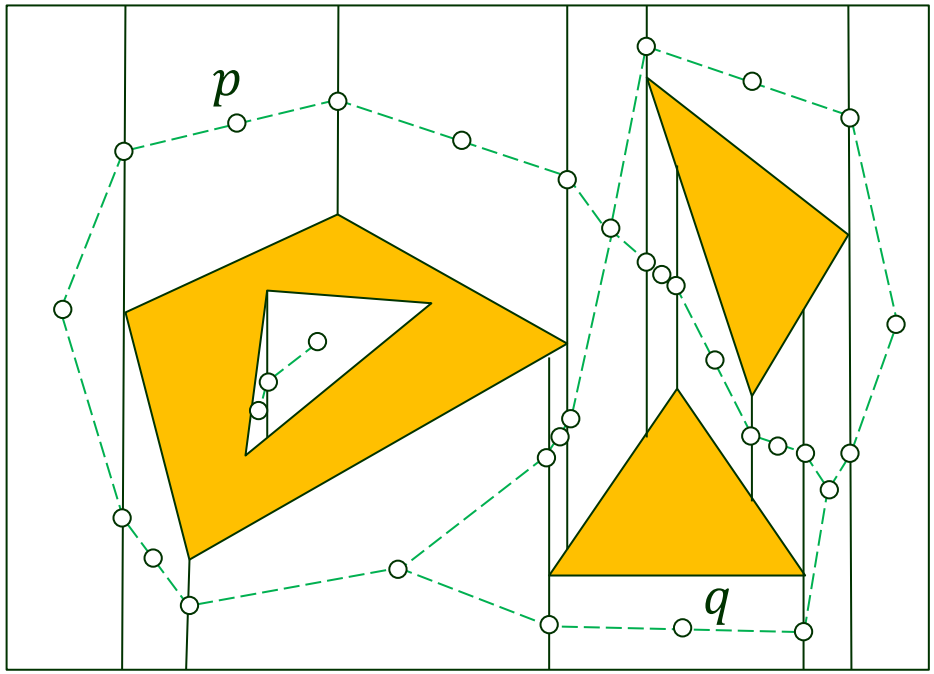
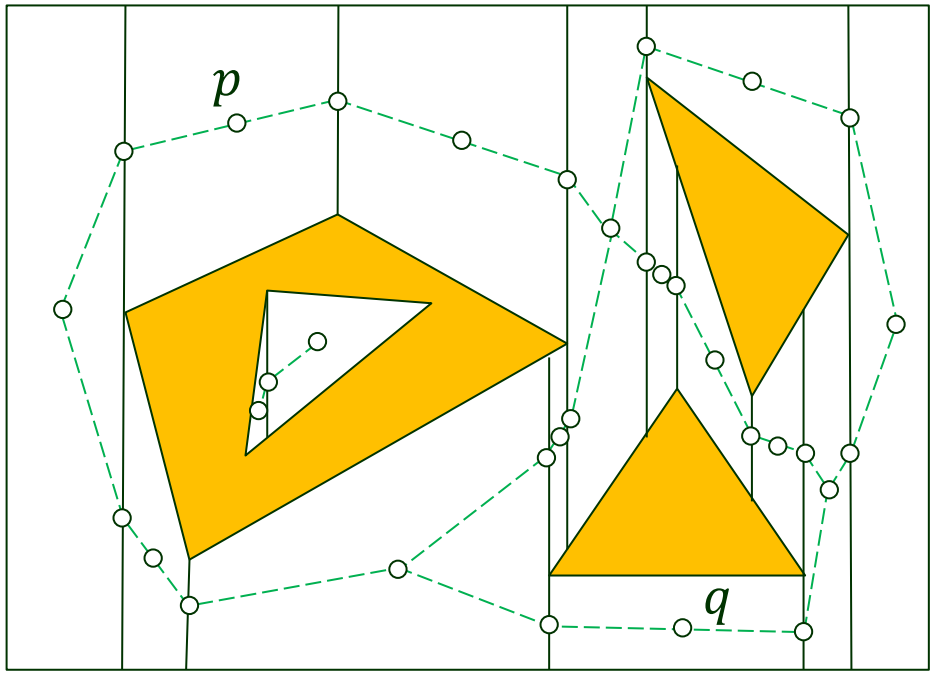
The roadmap is constructed by traversing the DCEL of $T(C_{free})$.

Roadmap Construction



Traverse the DCEL of the trapezoidal map $T(C_{free})$.

Roadmap Construction



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$O(n)$

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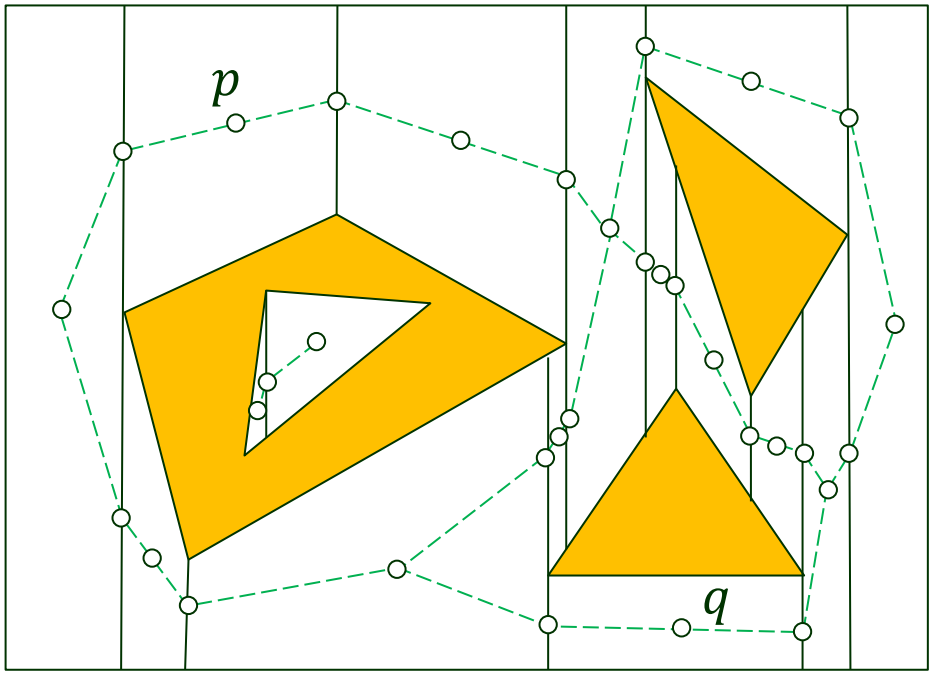
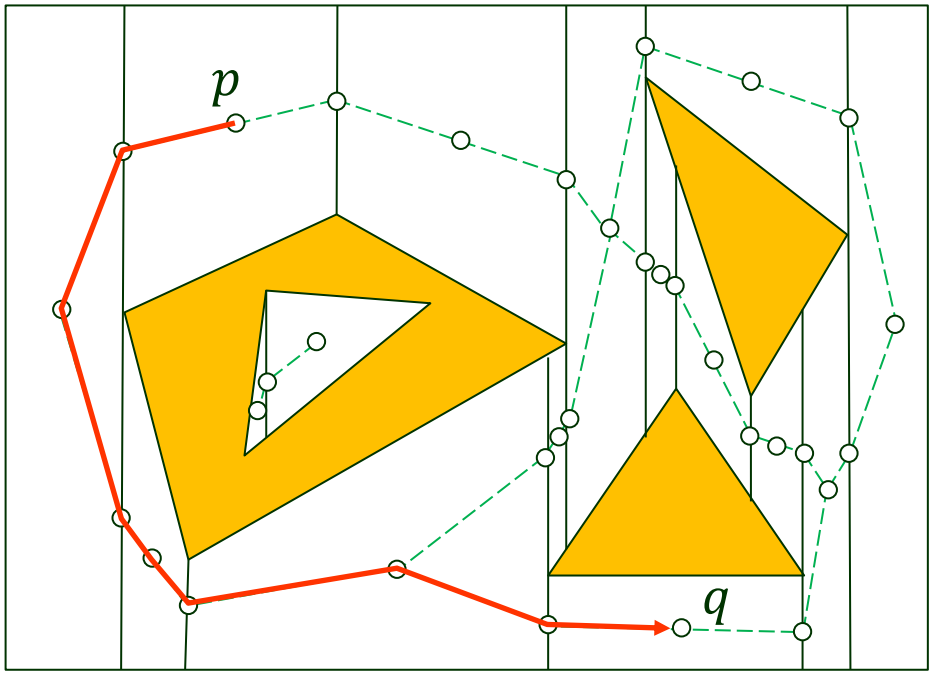


Traverse the DCEL of the trapezoidal map $T(C_{free})$.

$O(n)$

We can navigate from the node representing the center of one trapezoid to that representing the center of any other reachable trapezoid.

Roadmap Construction



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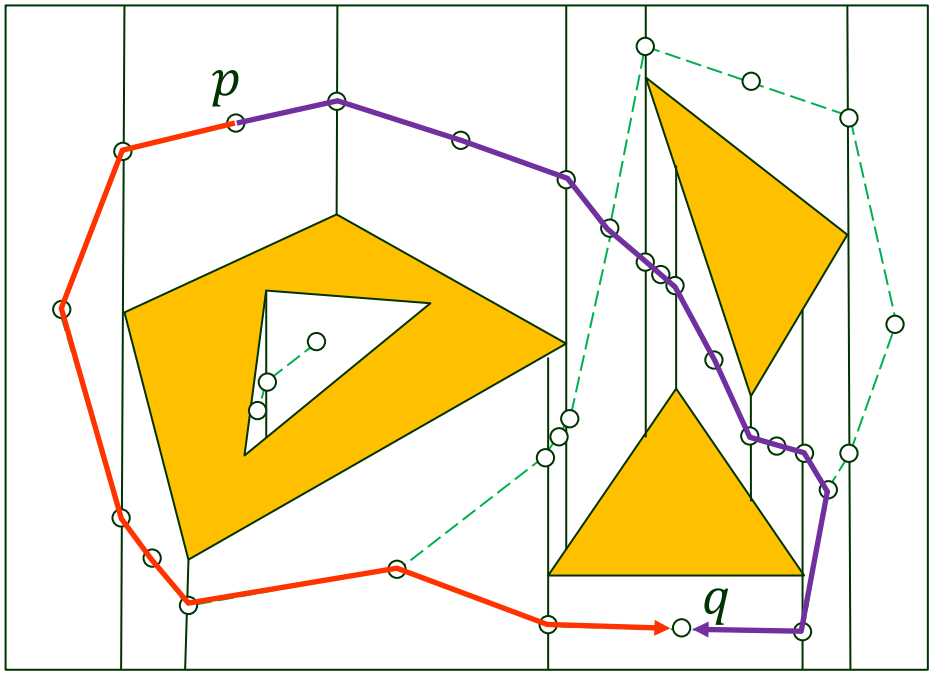

Traverse the DCEL of the trapezoidal map $T(C_{free})$.

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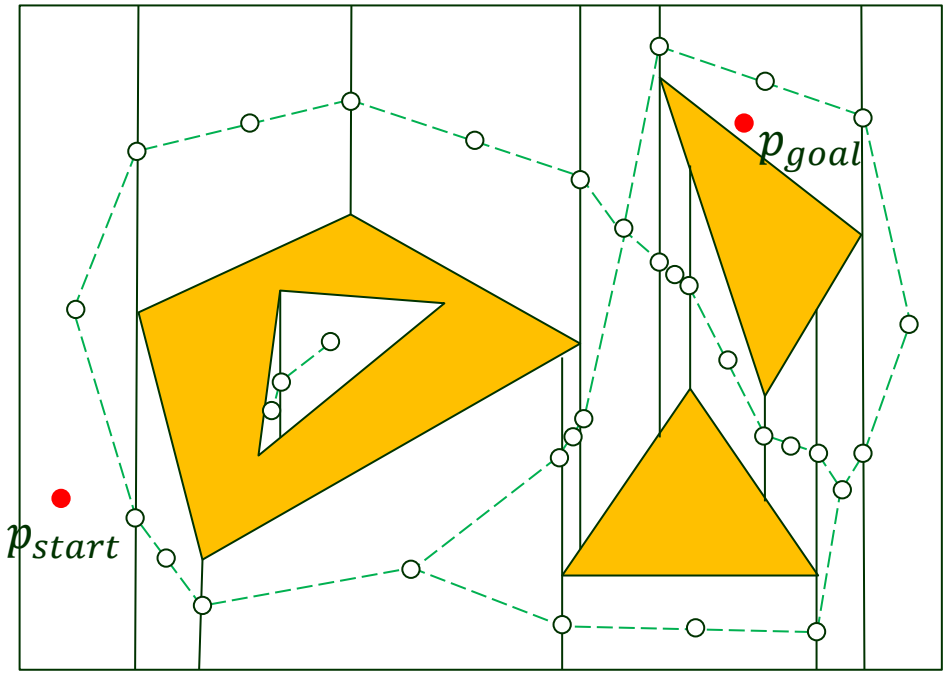
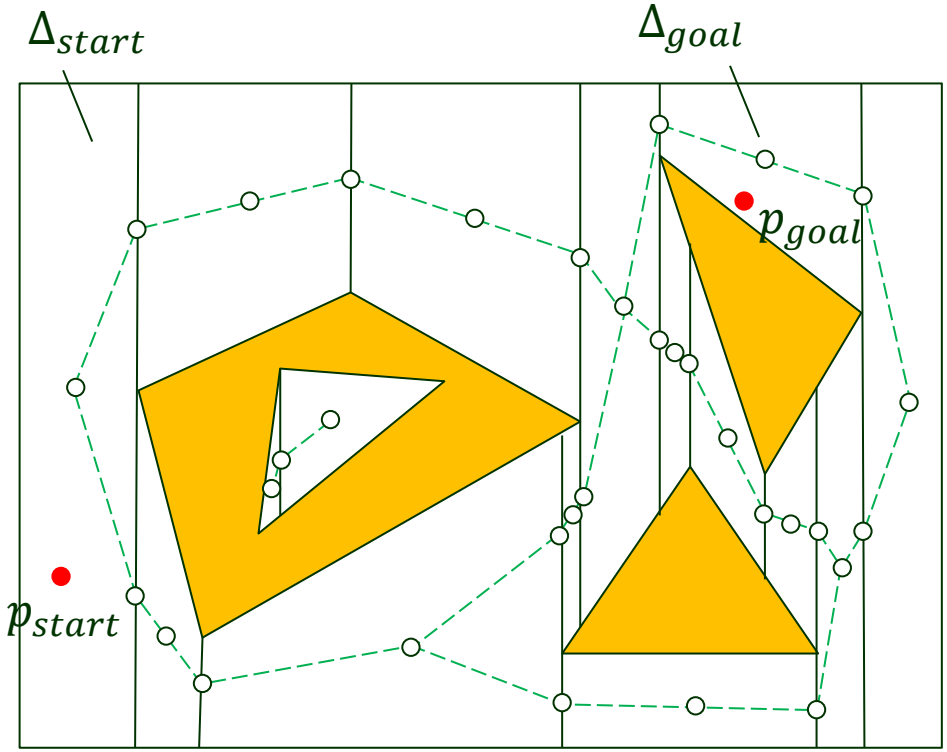
Motion Planning for a Point Robot



Input: starting position p_{start} and goal position p_{goal}.

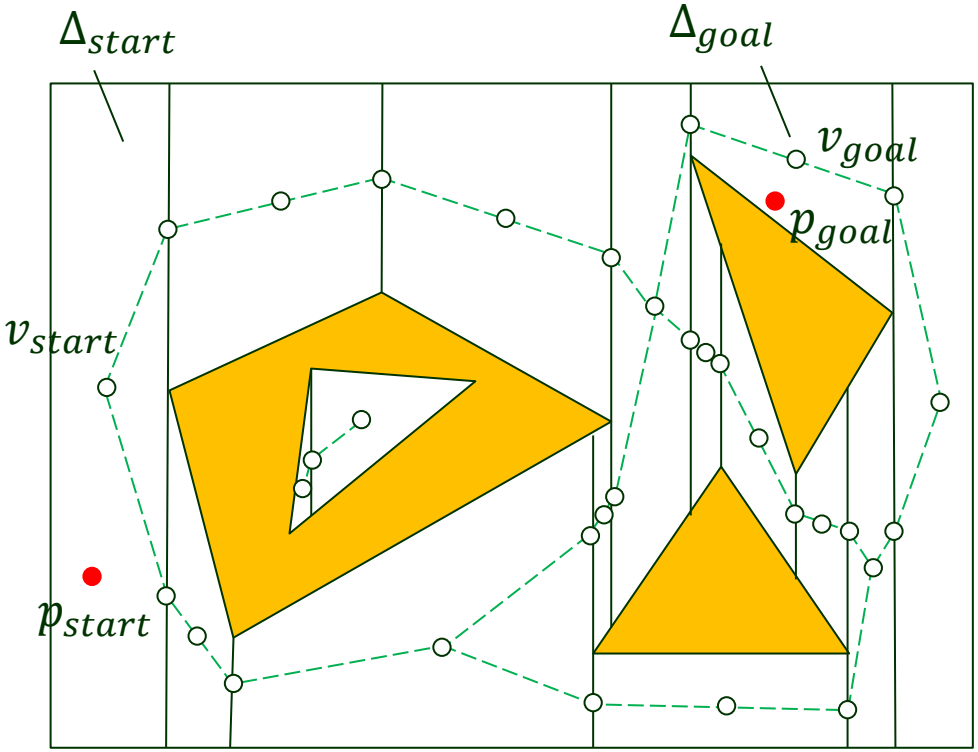
Motion Planning for a Point Robot



Input: starting position p_{start} and goal position p_{goal}.

- Determine the trapezoids Δ_{start} and Δ_{goal} containing p_{start} and p_{goal}, respectively.

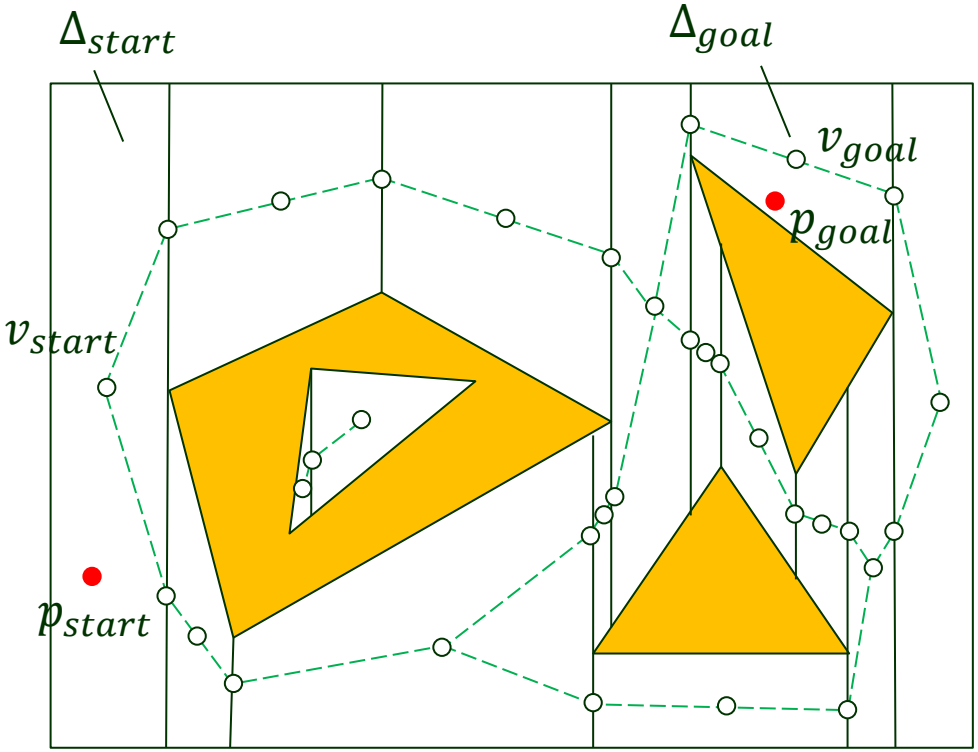
Motion Planning for a Point Robot



Input: starting position p_{start} and goal position p_{goal}.

- Determine the trapezoids Δ_{start} and Δ_{goal} containing p_{start} and p_{goal}, respectively.
- Let v_{start} and v_{goal} be the nodes at the centers of Δ_{start} and Δ_{goal}.

Motion Planning for a Point Robot



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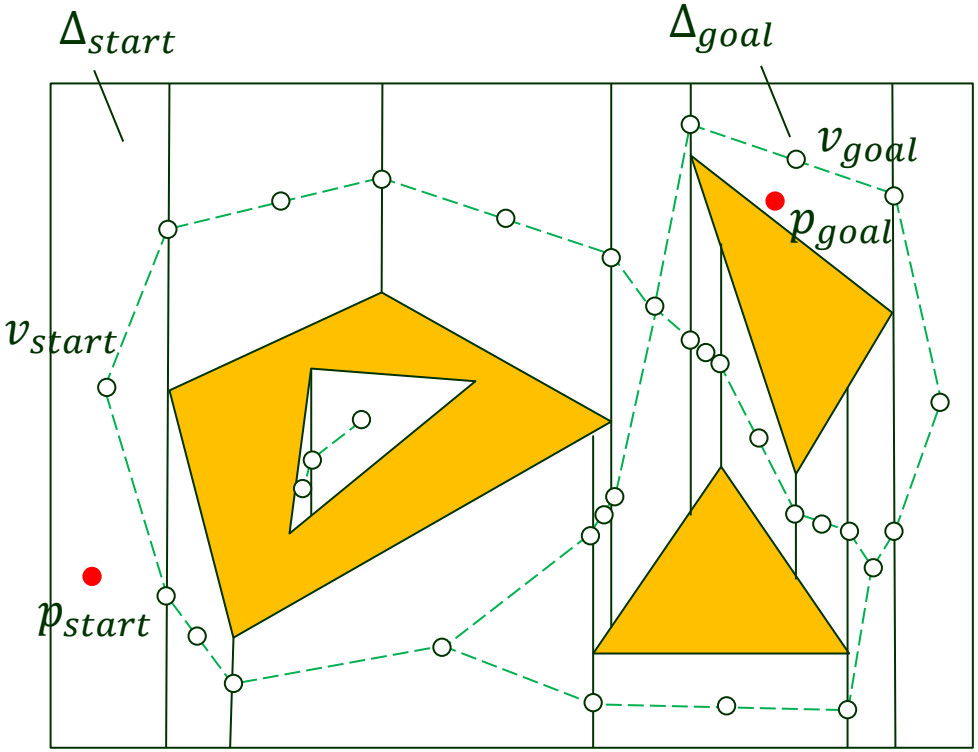
- Three parts of the path $p_{start} \rightsquigarrow p_{goal}$:

Motion Planning for a Point Robot



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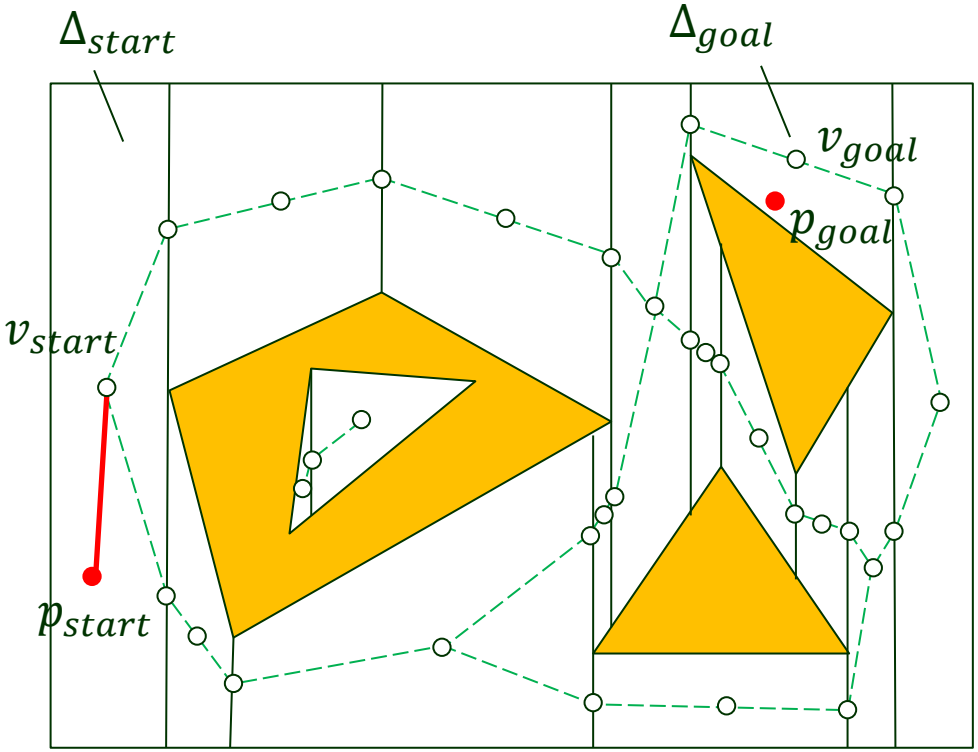
- Let v_{start} and v_{goal} be the nodes at the centers of Δ_{start} and Δ_{goal}.
- Three parts of the path $p_{start} \rightsquigarrow p_{goal}$:
  - straight line segment $\overline{p_{start}v_{start}}$;

Motion Planning for a Point Robot



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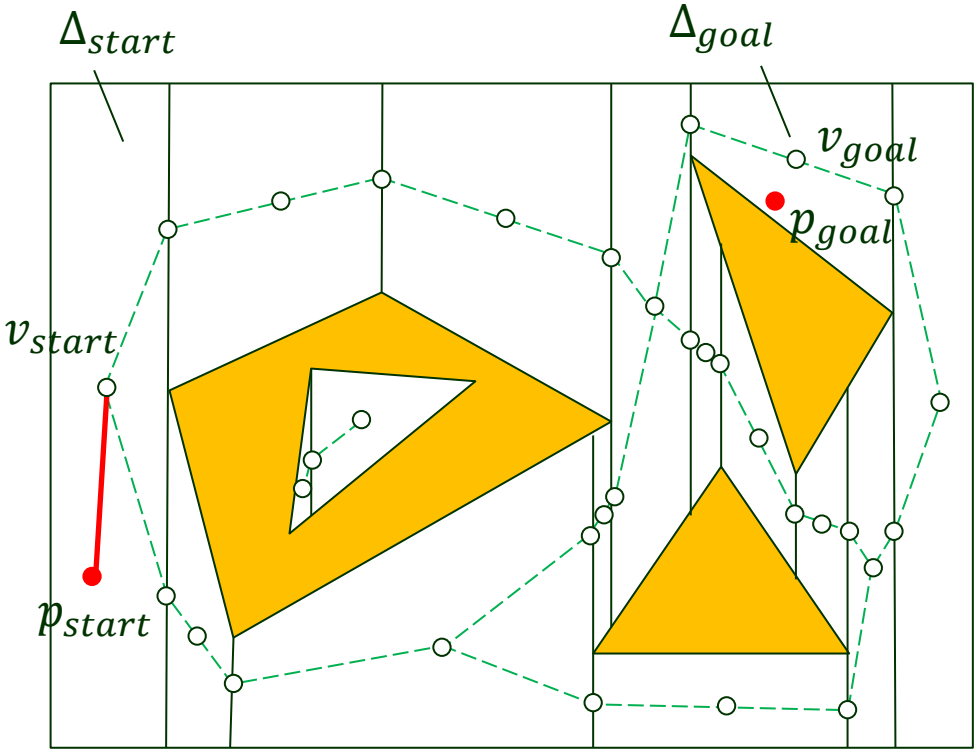
Motion Planning for a Point Robot



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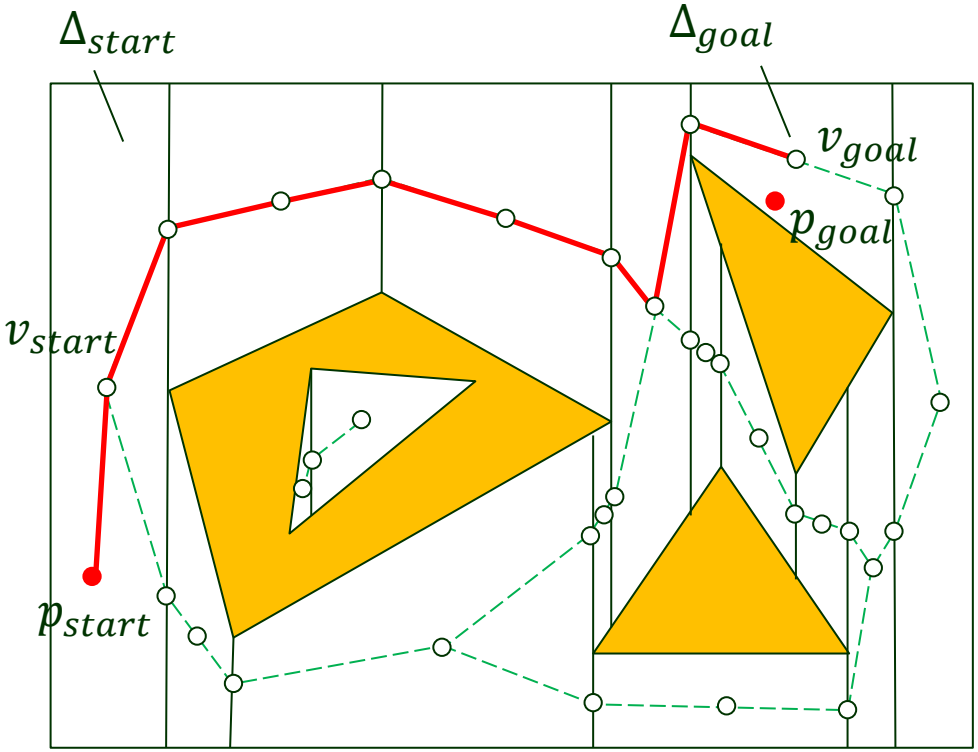

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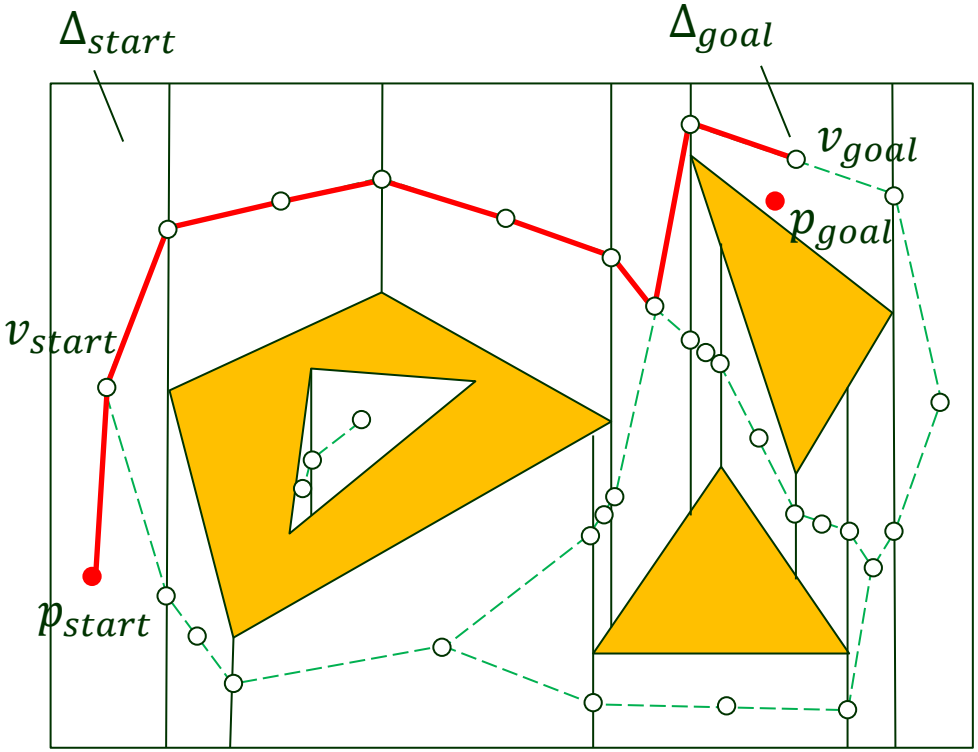

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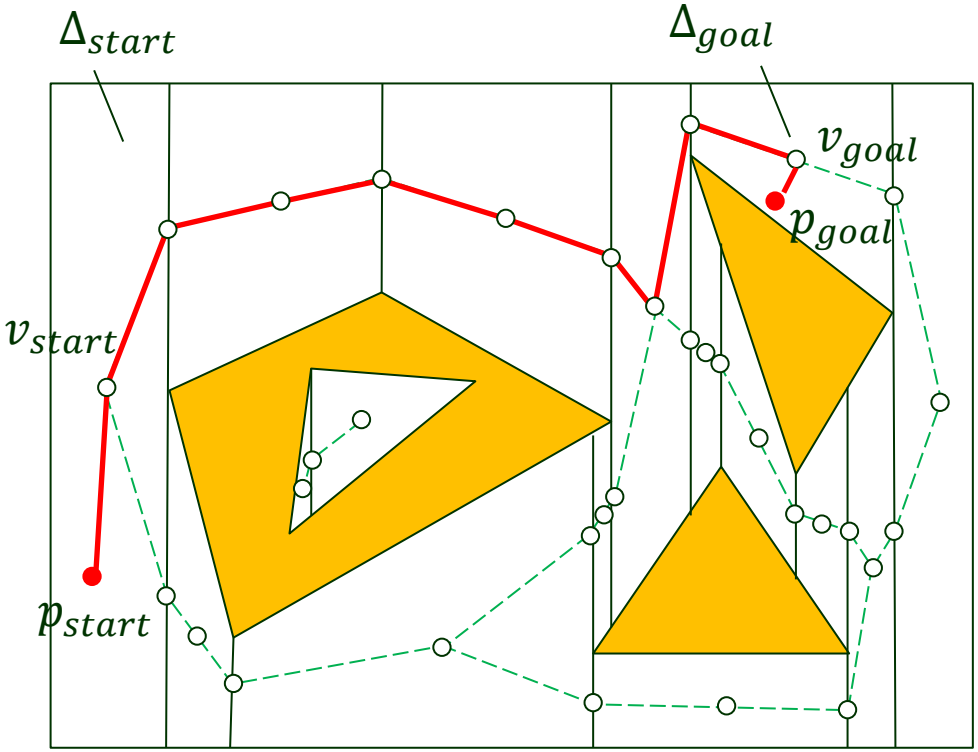

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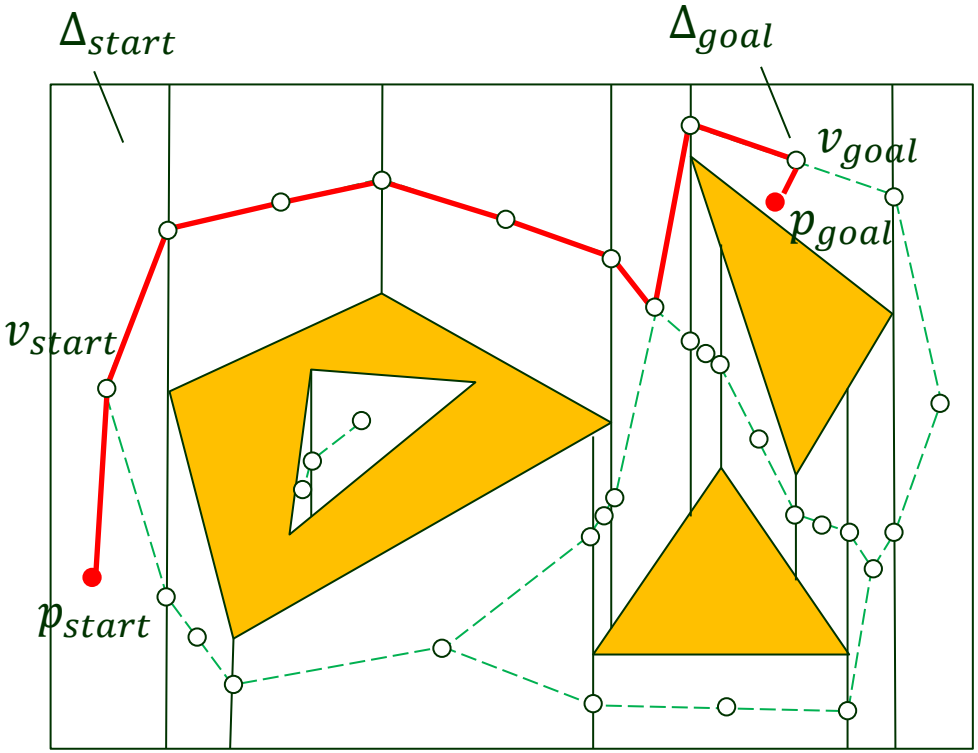
Collison-Free Path



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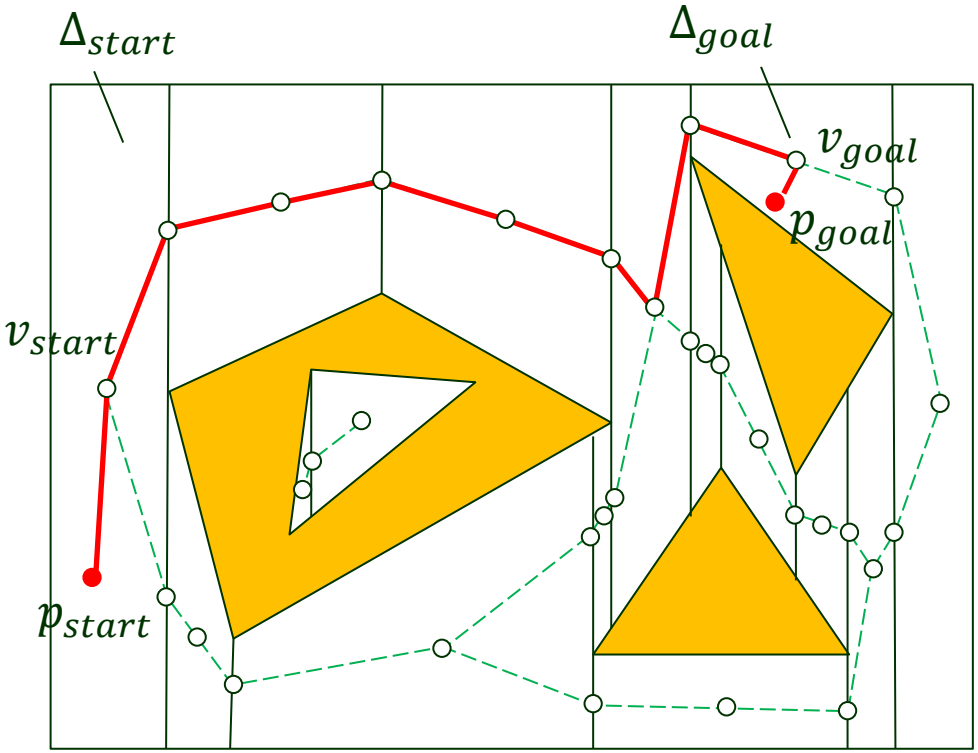

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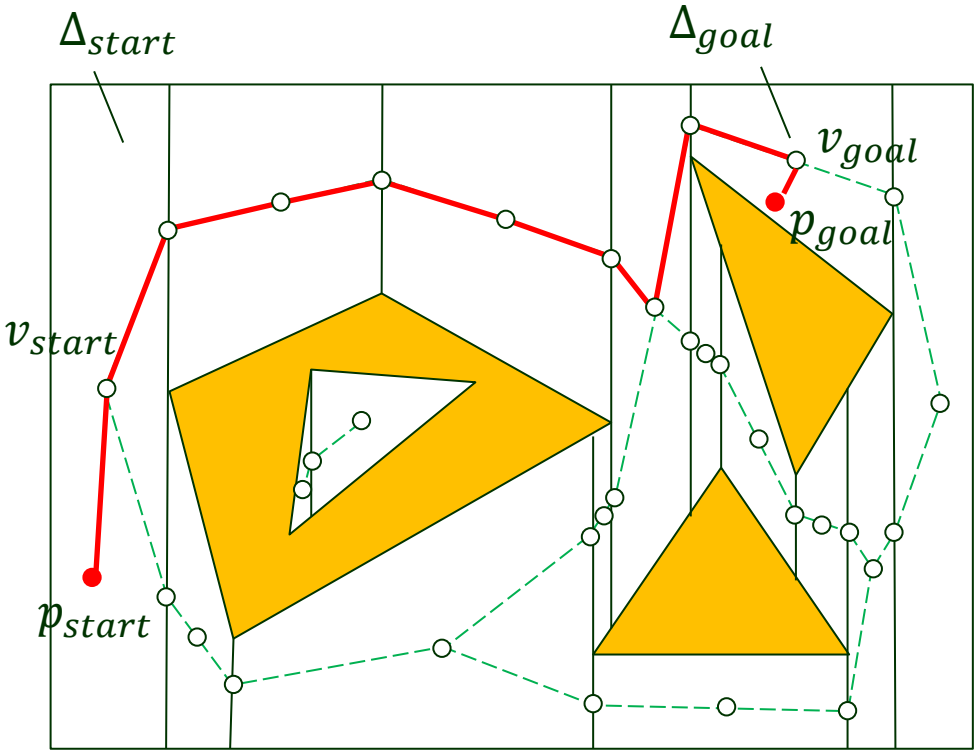

Correctness hinges on two questions:

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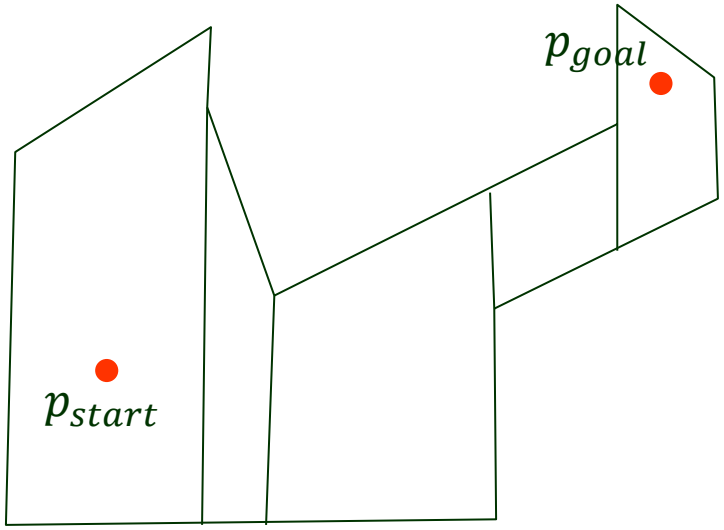
Yes, because it consists of segments inside trapezoids (which are in C_{free}).

Completeness

- Does the algorithm always find a collision-free path whenever one exists?



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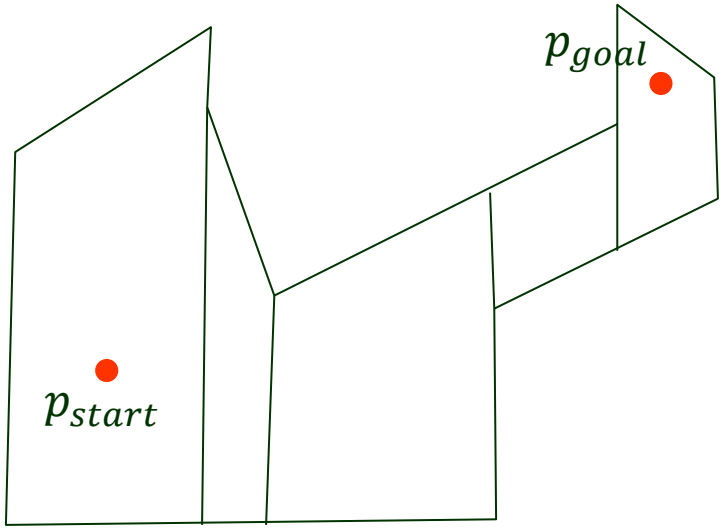

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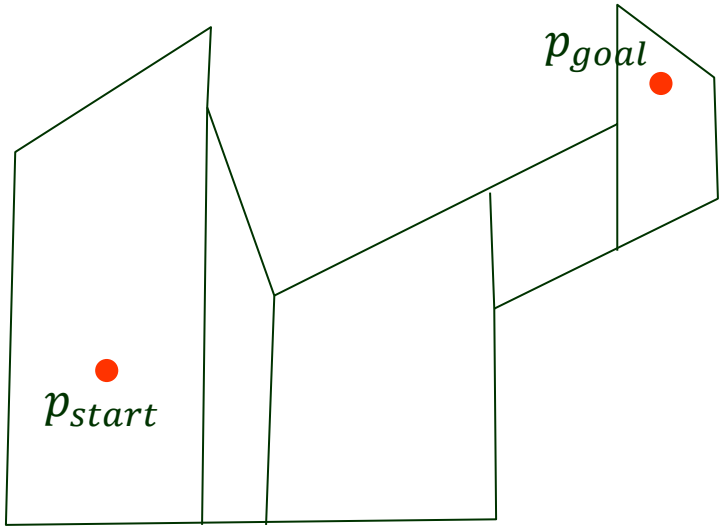

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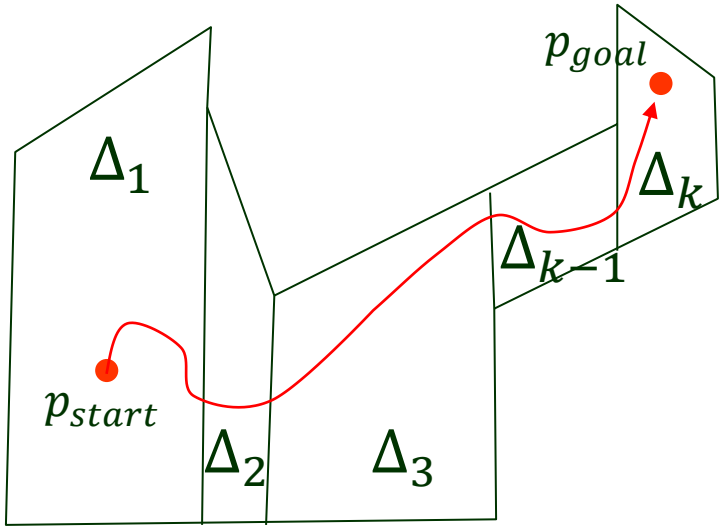

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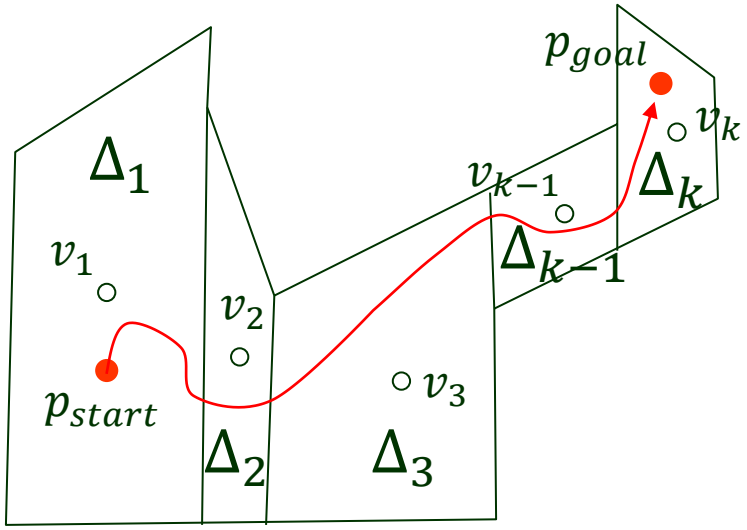

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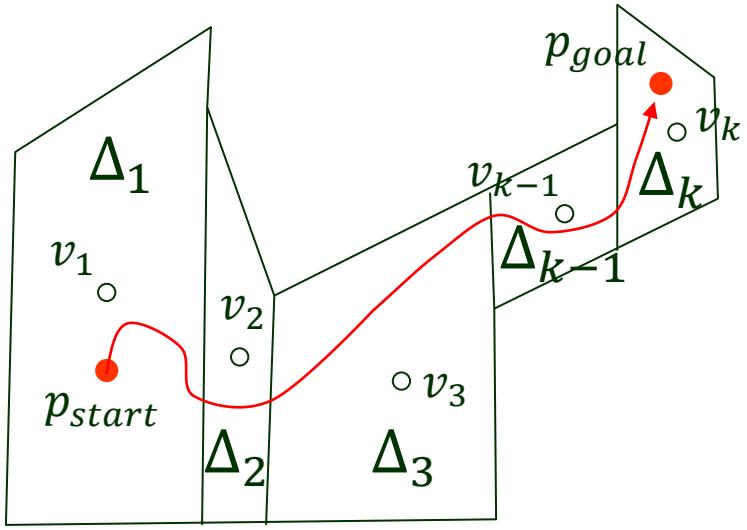

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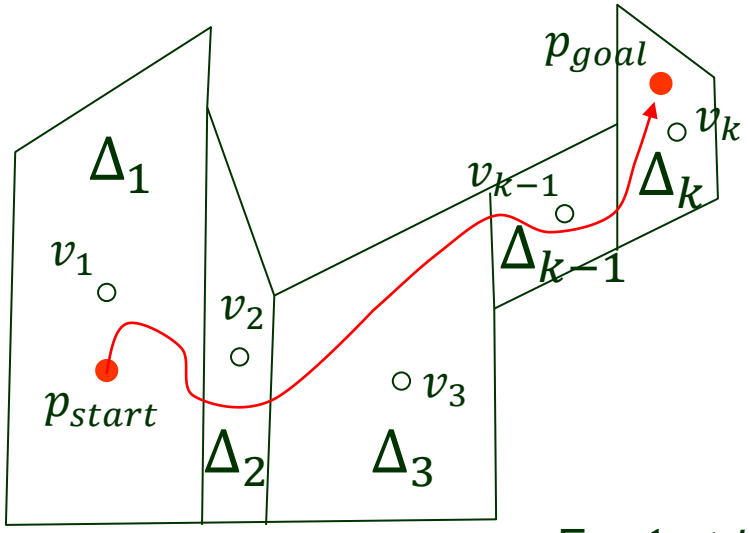

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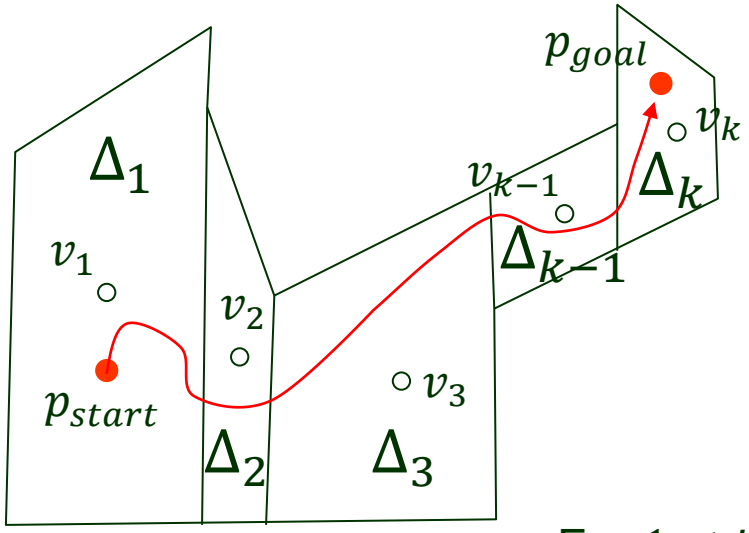

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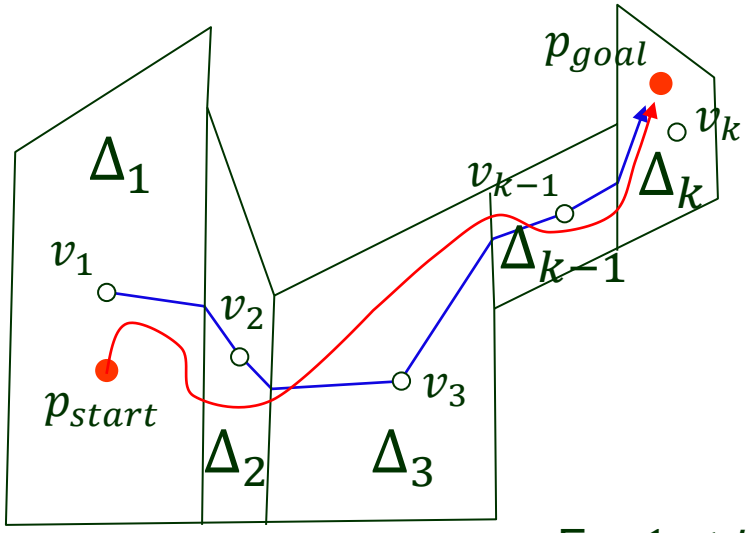

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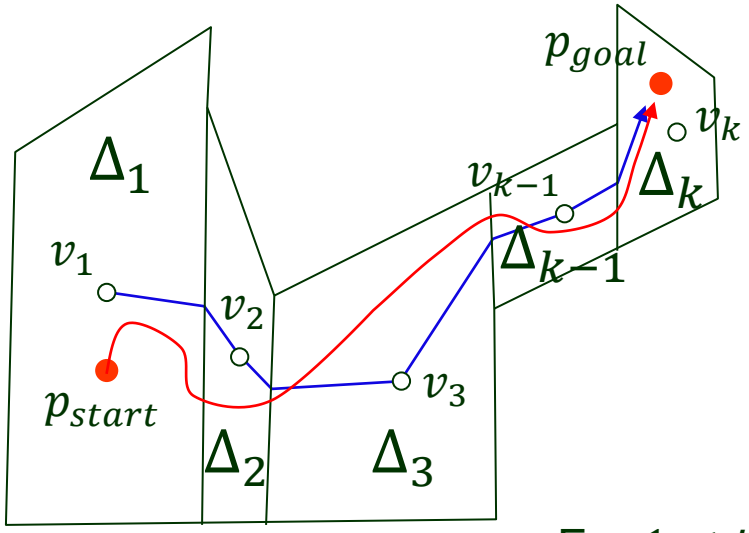

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Theorem A set S of polygonal obstacles with n edges can be preprocessed in $O(n \log n)$ expected time such that a collision-free path for a point robot between any start and goal positions can be computed in $O(n)$ time if it exists.