Search for CSPs

Outline

I. Backtracking algorithm

II. Local search

III. CSP structure

* Figures/images are from the textbook site (or by the instructor).

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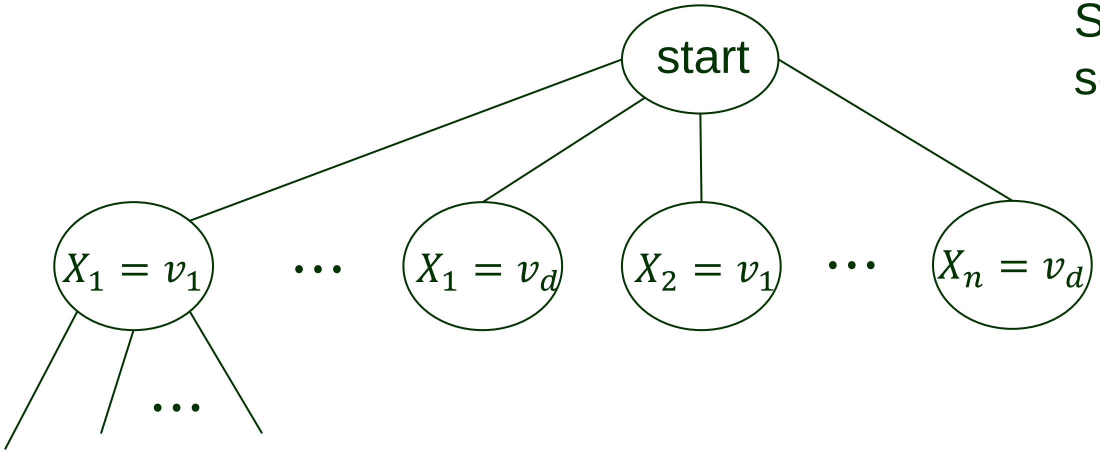

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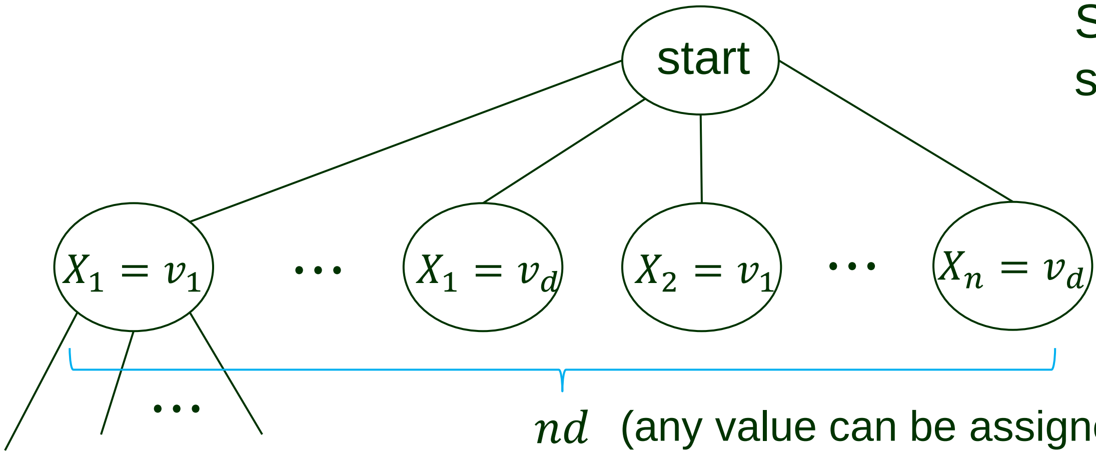

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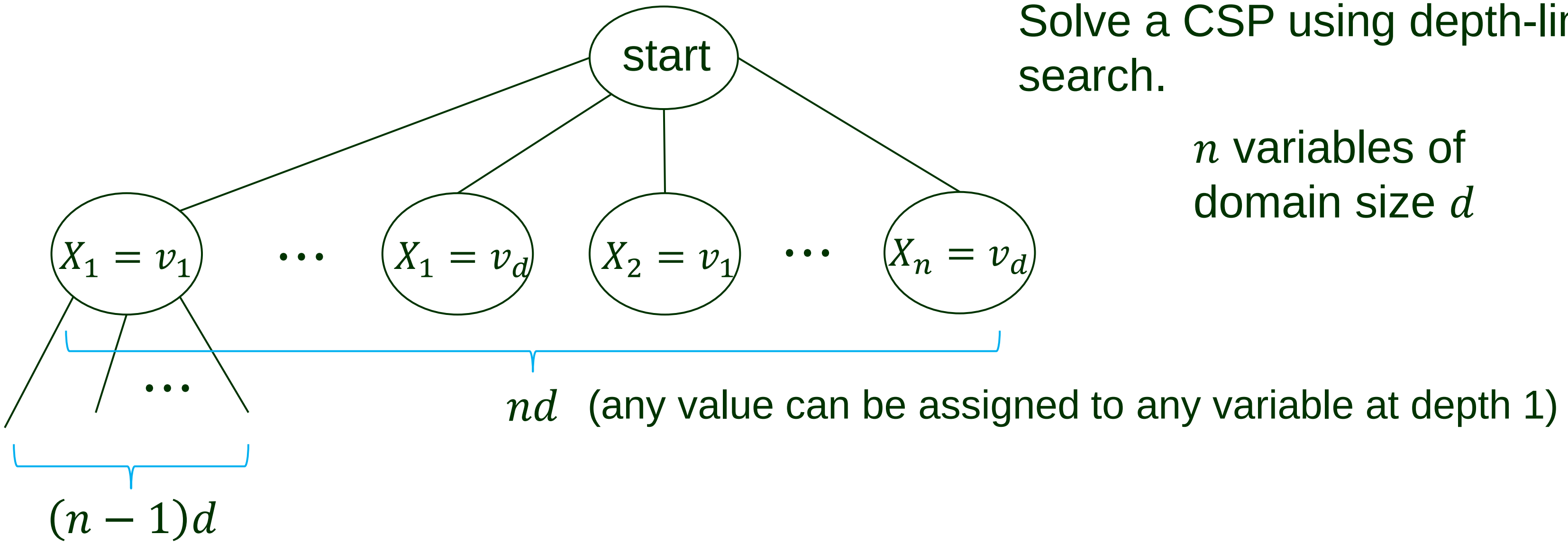

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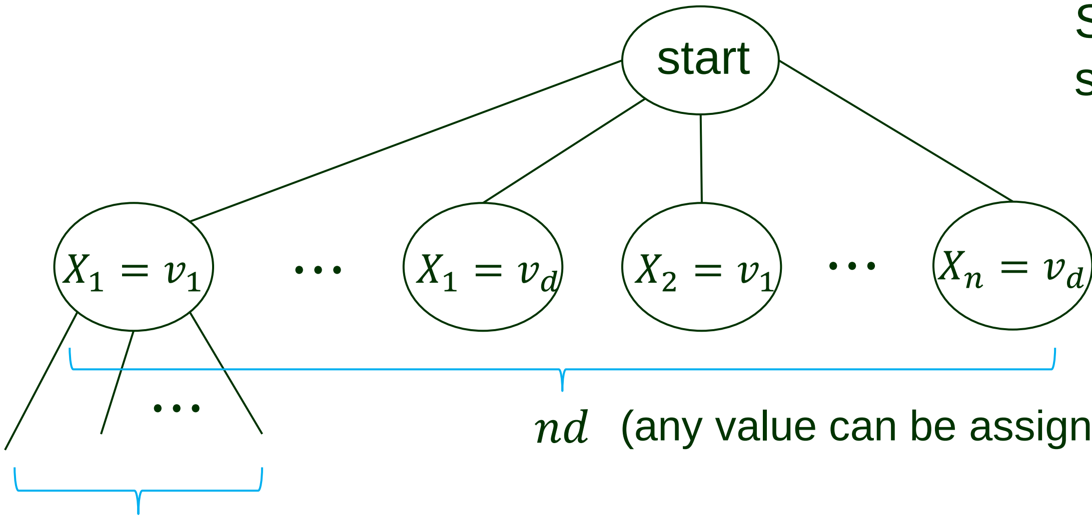

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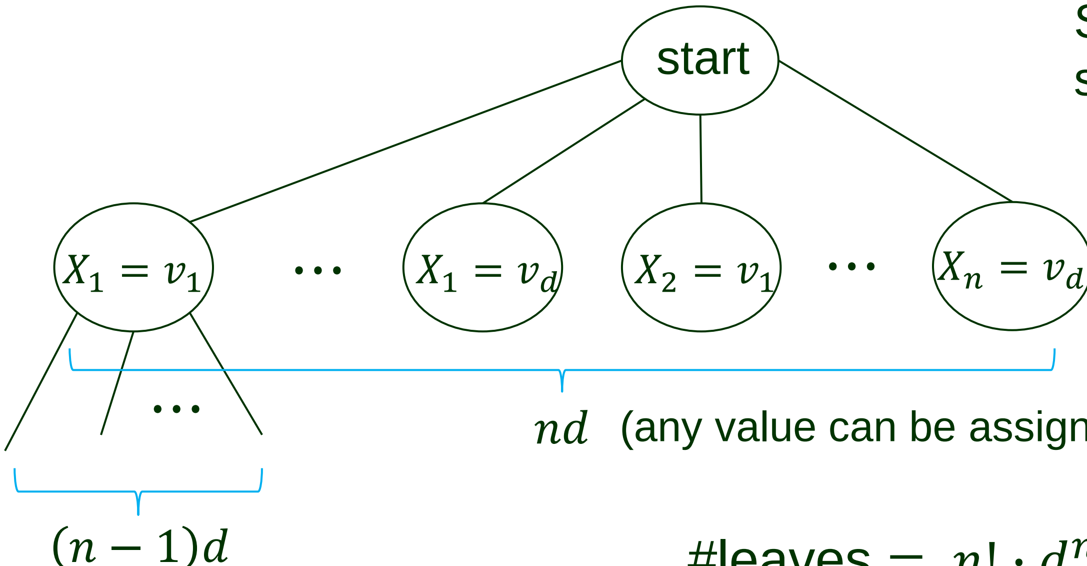

nd (any value can be assigned to any variable at depth 1)

Solve a CSP using depth-limited search.

n variables of domain size d

#leaves = $n! \cdot d^n$

But #assignments = d^n.

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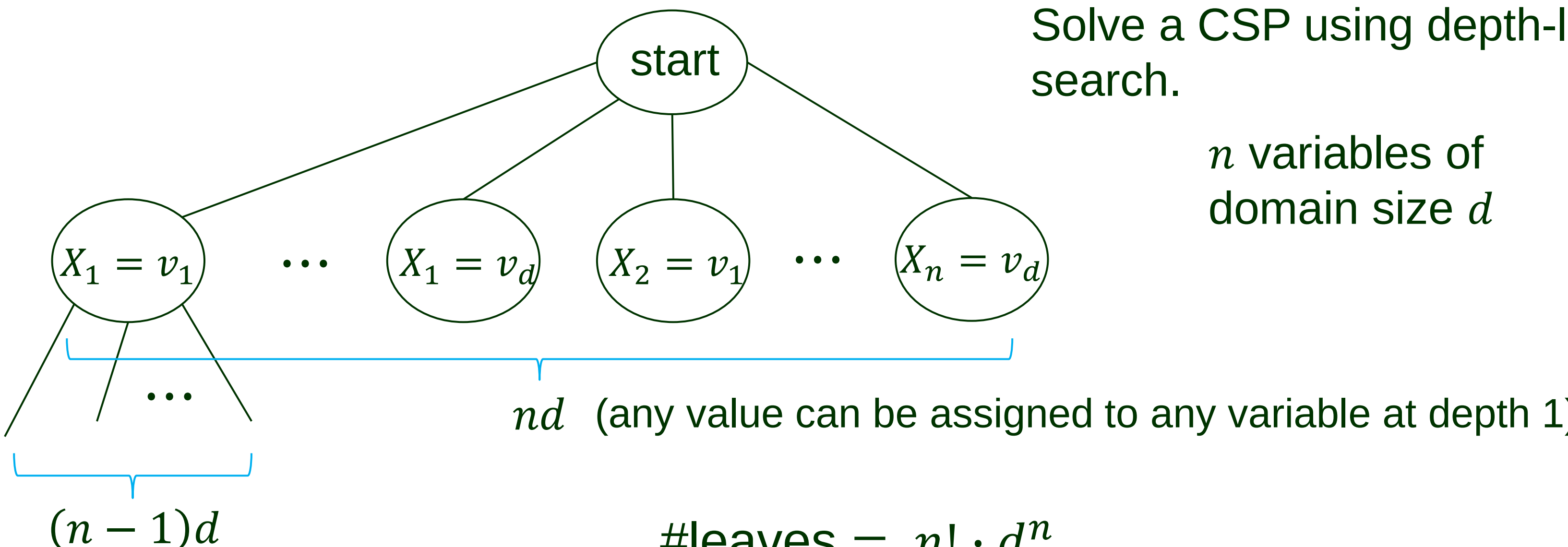

Solve a CSP using depth-limited search.

n variables of domain size d

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How to get back to d^n?

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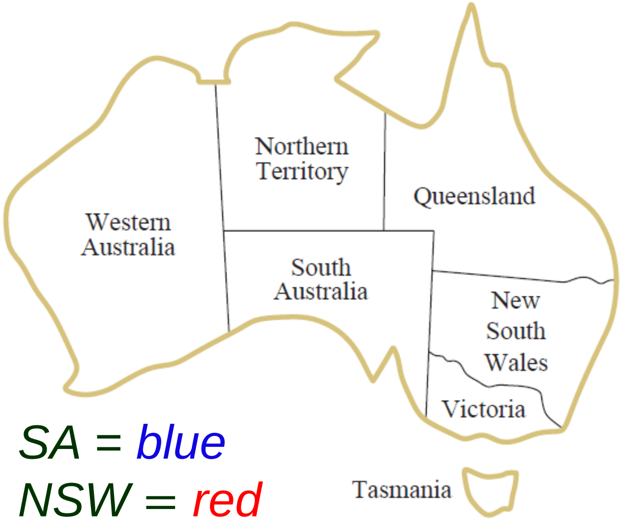

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Step 1: *NSW* = *red*
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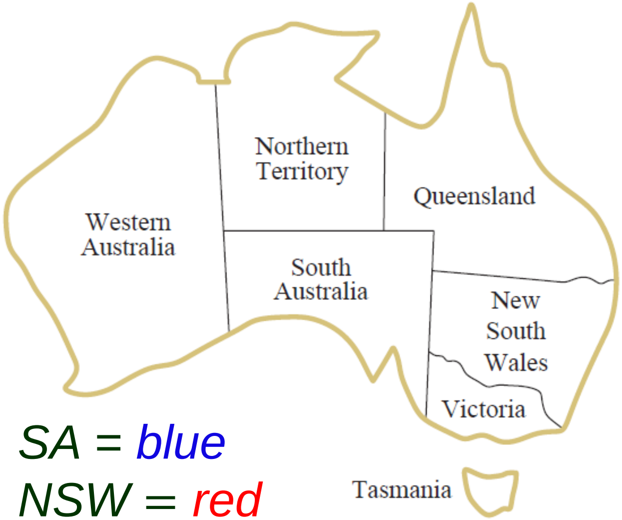

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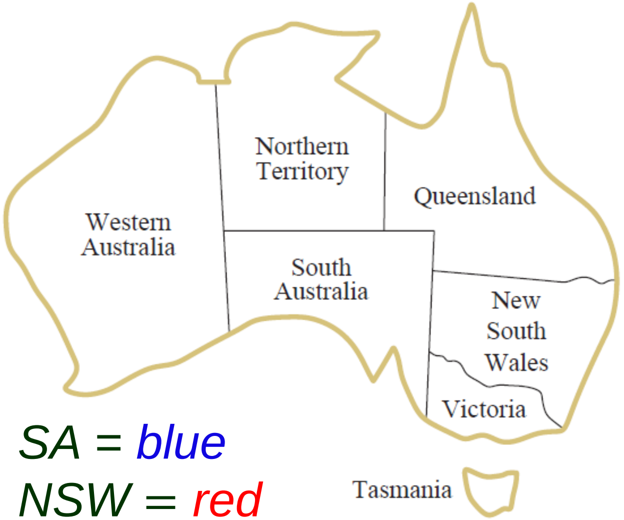

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SA = *blue*, *SA* = *red*, and *SA* = *green*

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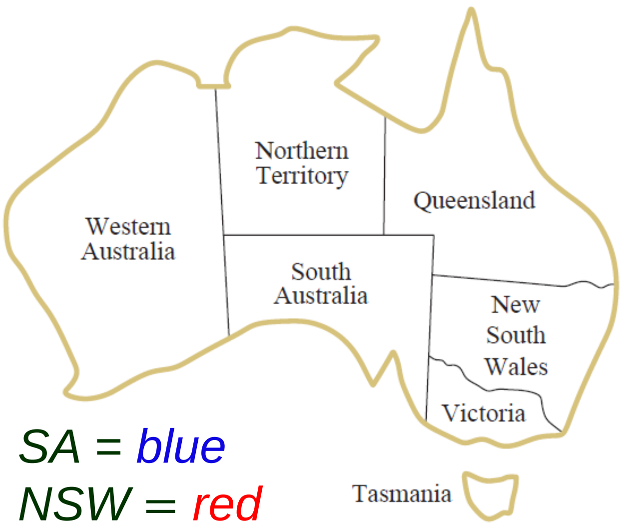

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SA = *blue* and *NSW* = *red*

Backtracking Algorithm

- Repeatedly chooses an unassigned variable X_i.
- Tries all values $v_j \in D_i$ (its domain).

  - Add $X_i = v_j$ to the partial solution (after consistency checking).

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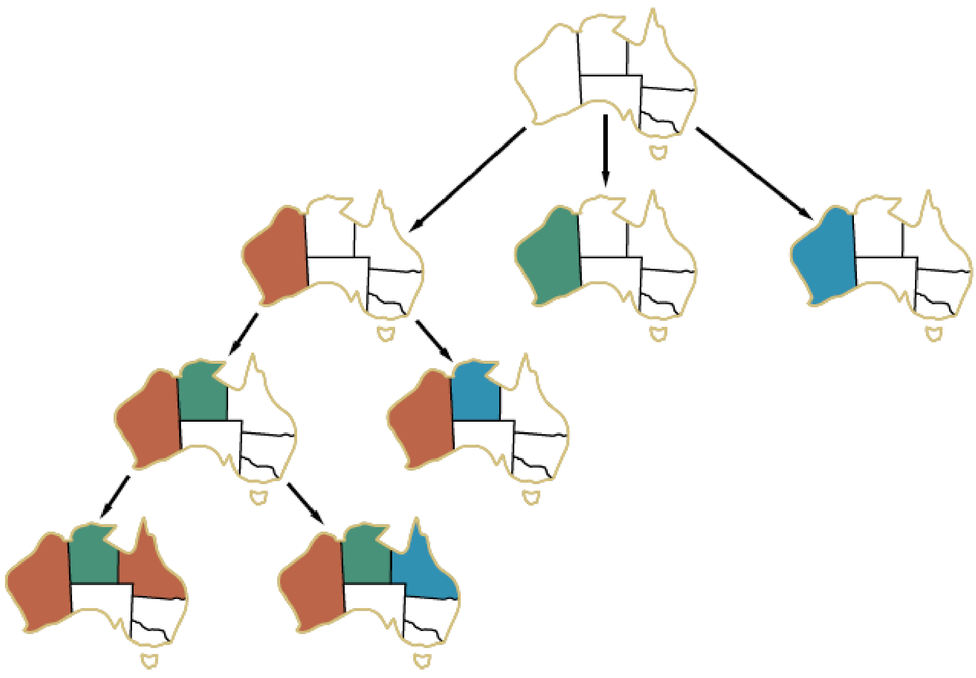

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Backtracking

```
function BACKTRACKING-SEARCH(csp) returns a solution or failure
  return BACKTRACK(csp, { })

function BACKTRACK(csp, assignment) returns a solution or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp, assignment)
  for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
     if value is consistent with assignment then
        add {var = value} to assignment
        inferences ← INFERENCE(csp, var, assignment)      // arc- or path-consistency, etc.
                                                          // forward checking, etc.
        if inferences ≠ failure then
           add inferences to csp
           result ← BACKTRACK(csp, assignment)
           if result ≠ failure then return result
           remove inferences from csp
        remove {var = value} from assignment
  return failure
```

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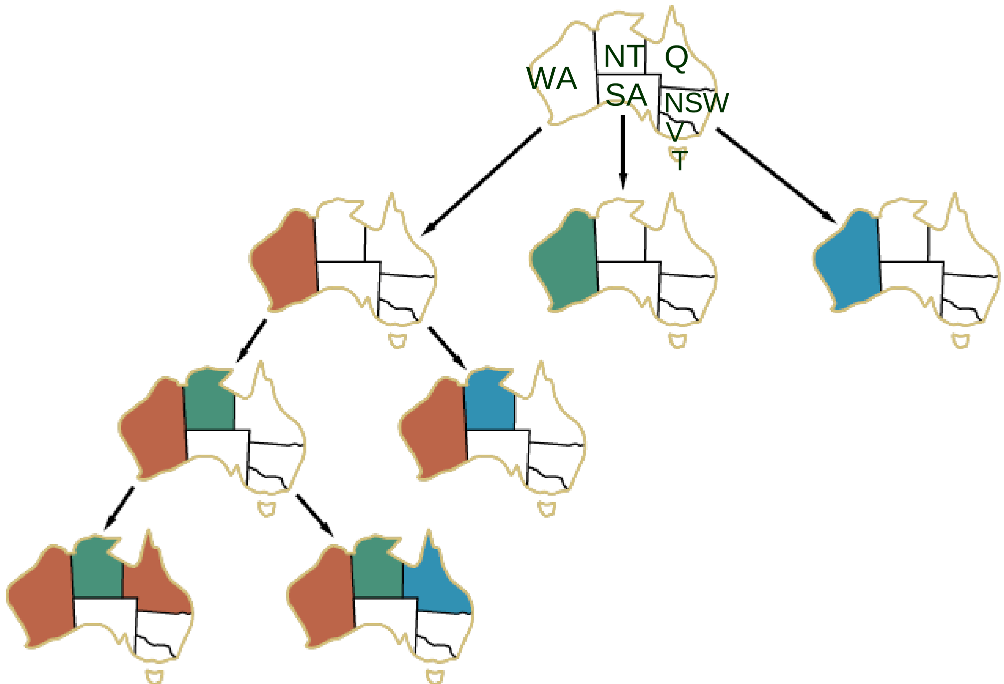

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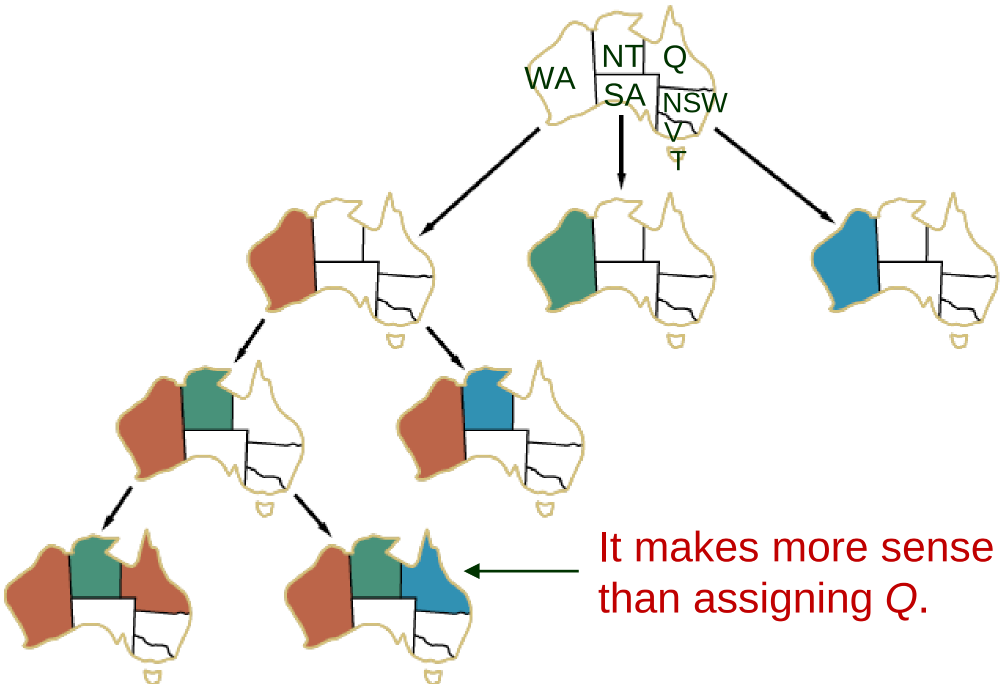

It makes more sense to assign $SA = blue$ than assigning *Q*.

MRV and Degree Heuristics

Minimum-remaining-values (MRV): Choose the variable with the *fewest "legal" values*.

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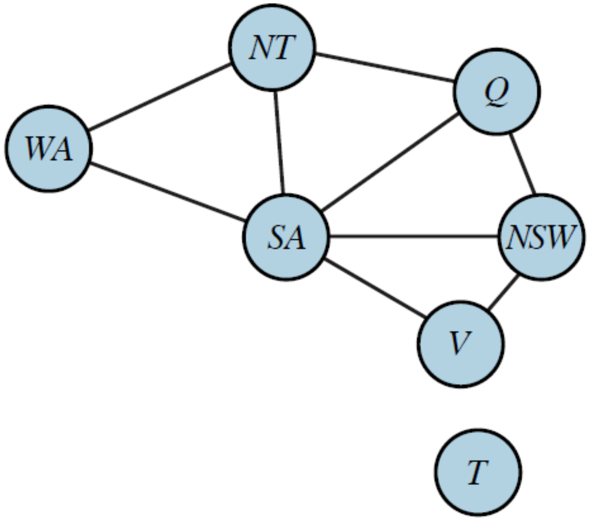

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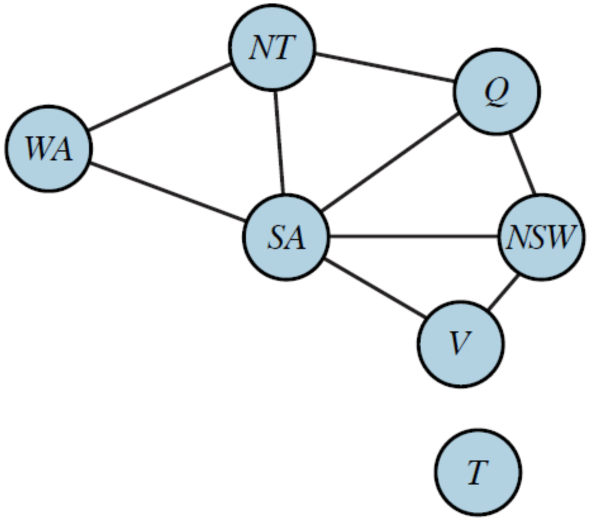

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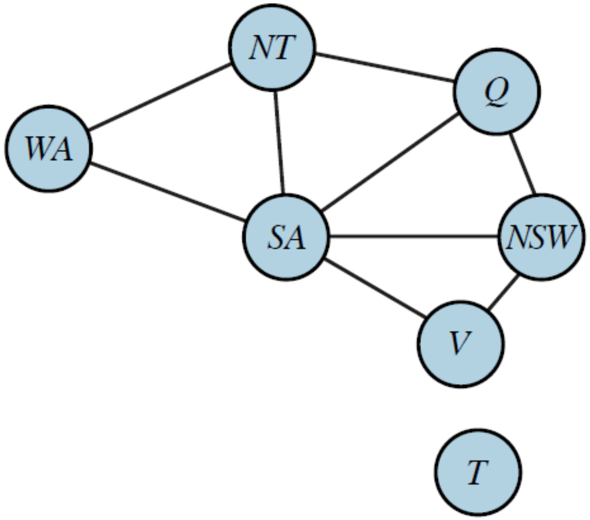

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  $\deg(SA) = 5$
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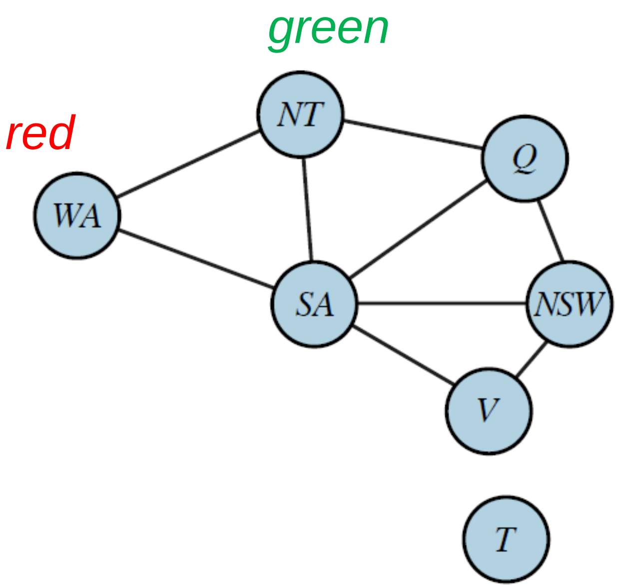
  $\Downarrow$

  Color *SA* first.

Least Constraining Value

For the selected variable, choose its value that *rules out the fewest choices* for the neighboring variables in the constraint graph.



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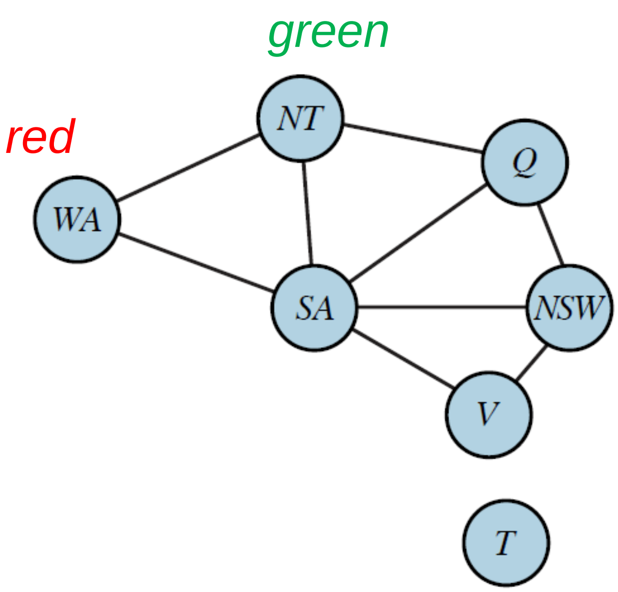

Which color to assign to *Q* next?

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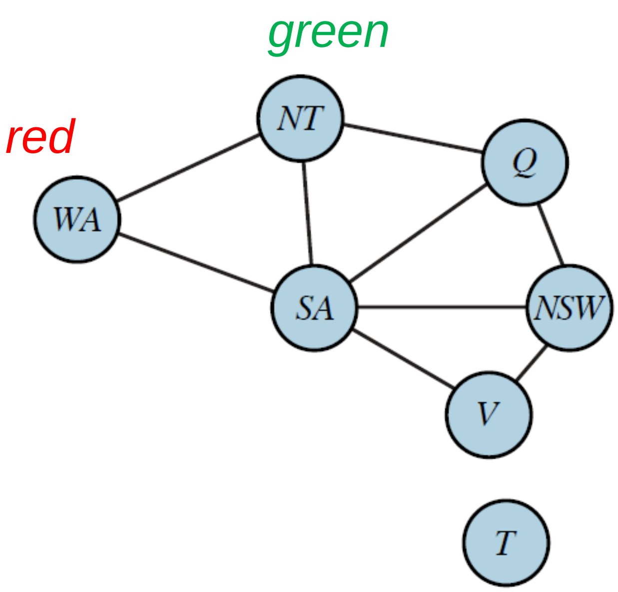

Which color to assign to *Q* next?

If *blue*, then SA would have no color left.

Choose *red*.

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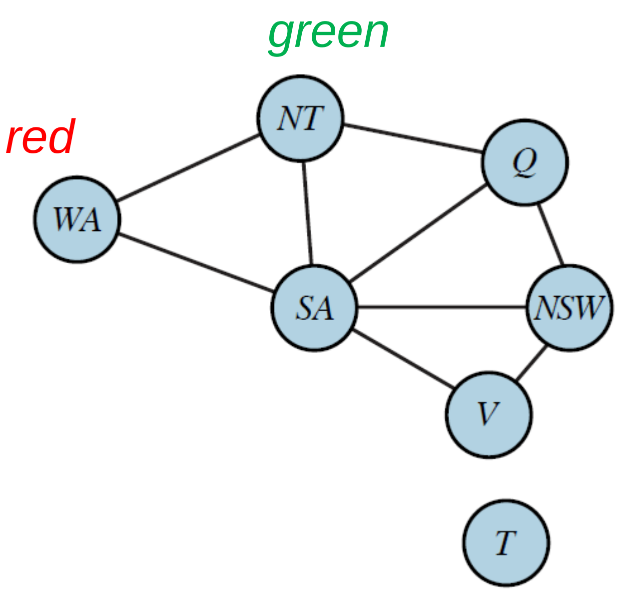

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The least-constraining-value heuristic tries to create the *maximum* room for subsequent variable assignments.

Variable vs. Value Selections

Variable order: *fail-first*.

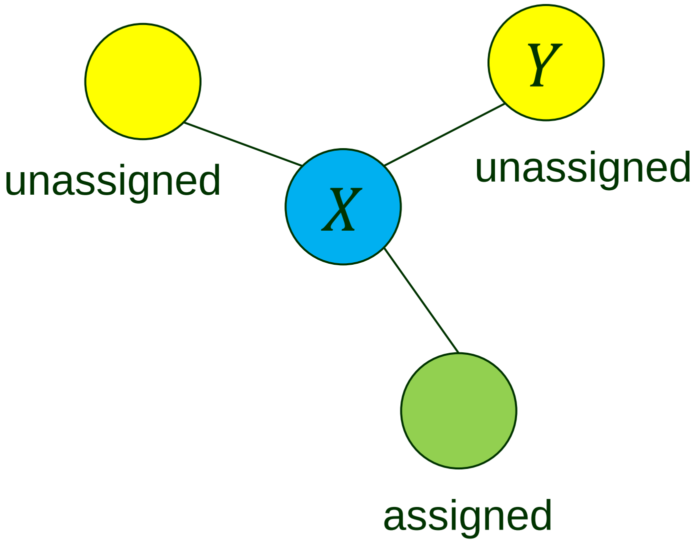
Fewer successful assignments to backtrack over.

Value order: *fail-last*.

- Only one solution needed.
- It makes sense to look for the most likely values first.

Forward Checking

Inference: Every new variable assignment opens the door for new domain reductions on neighboring variables.



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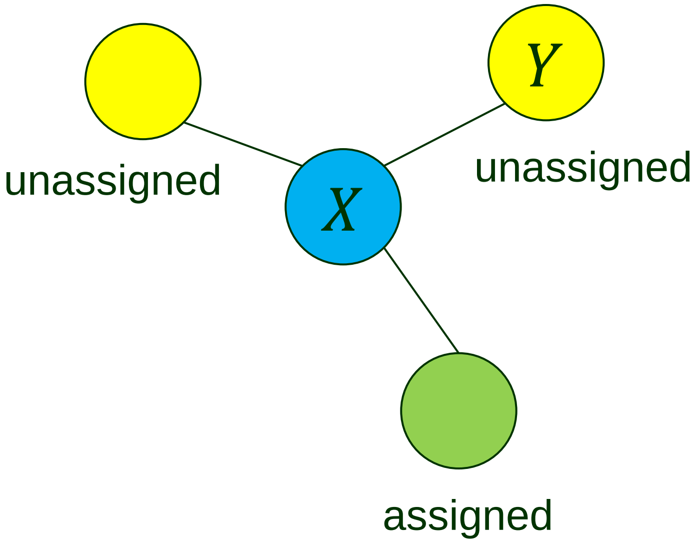

Assignment $X = v$

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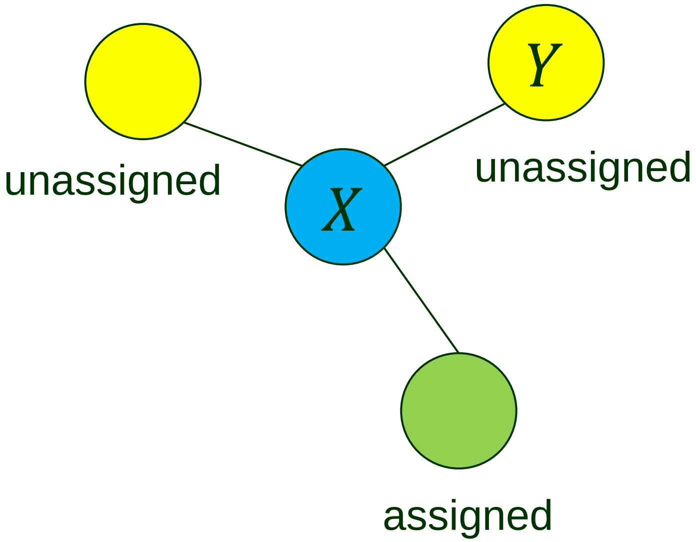
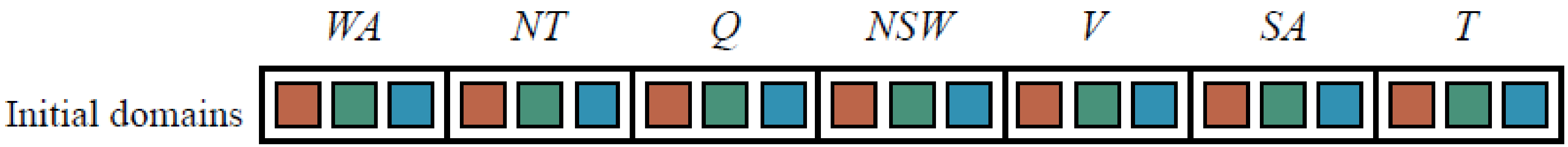


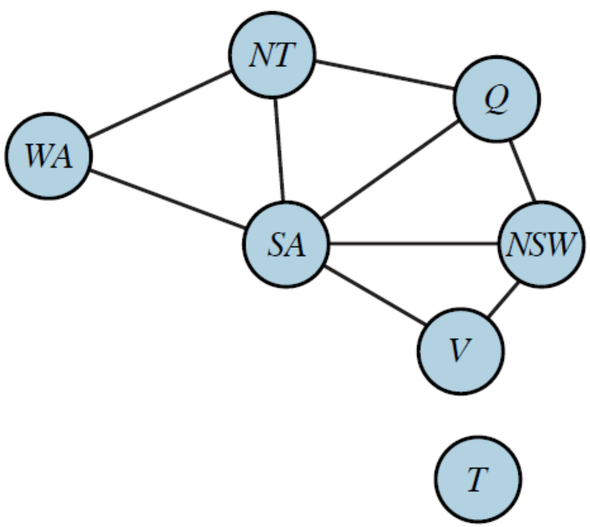
Assignment $X = v$

$\Downarrow$

For every unassigned Y connected to X , delete any value from Y's domain that is inconsistent with v.

Backtracking with Forward Checking





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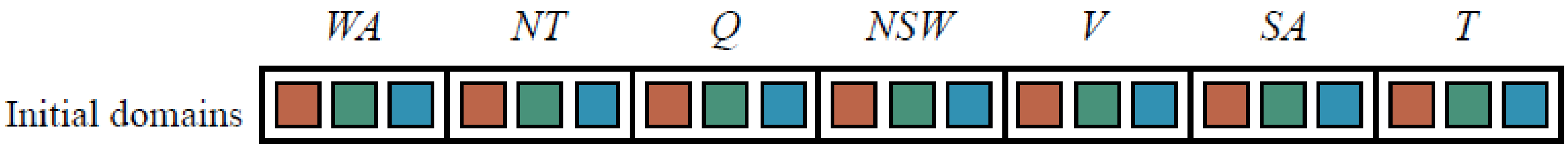
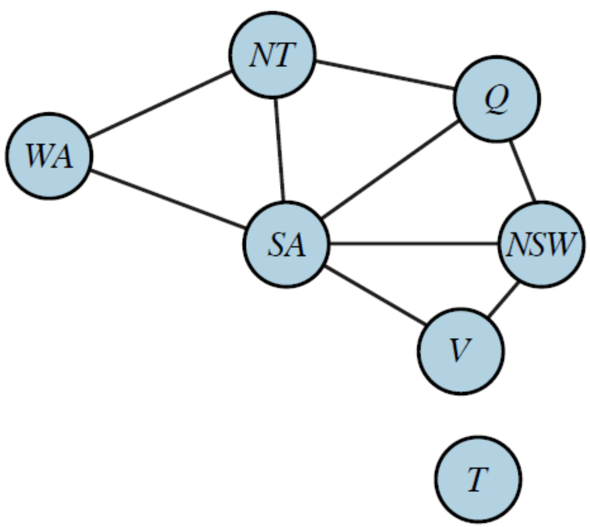
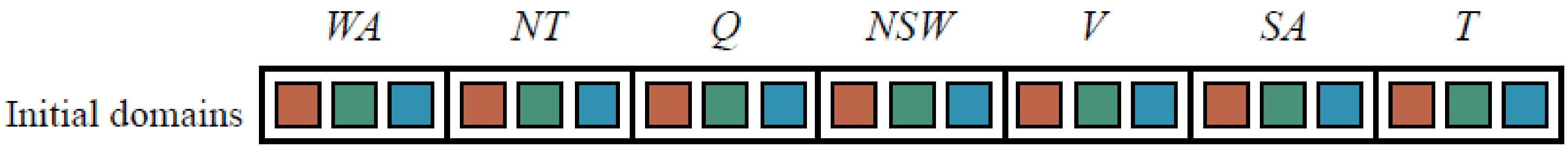
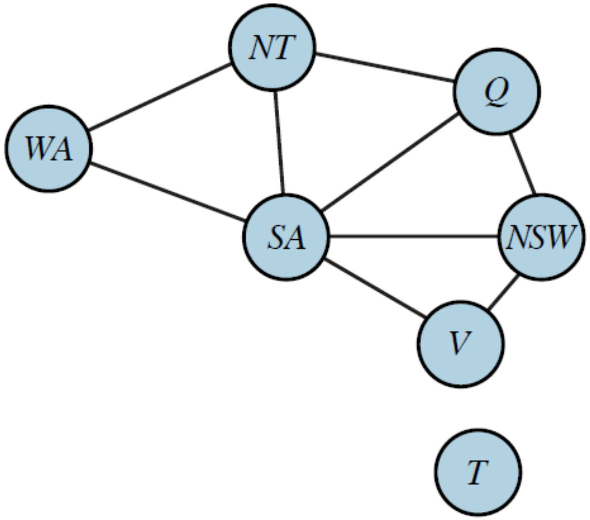




- *WA* = *red*

Backtracking with Forward Checking





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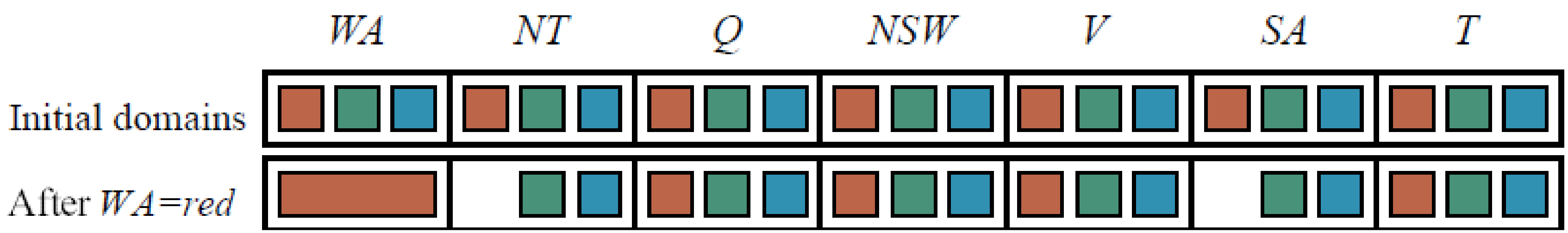

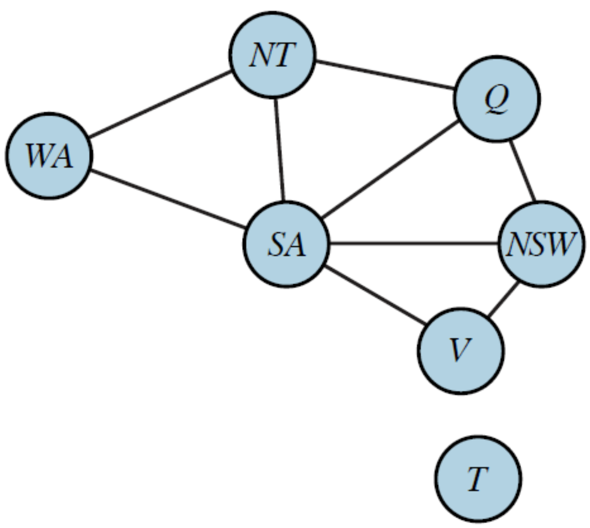

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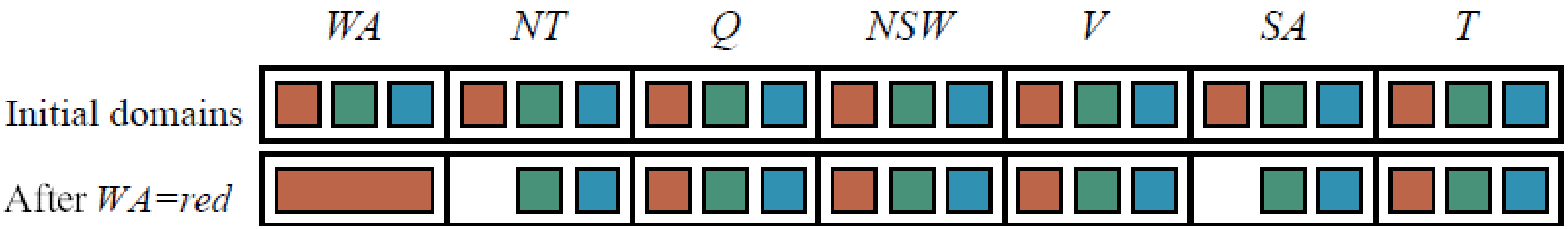

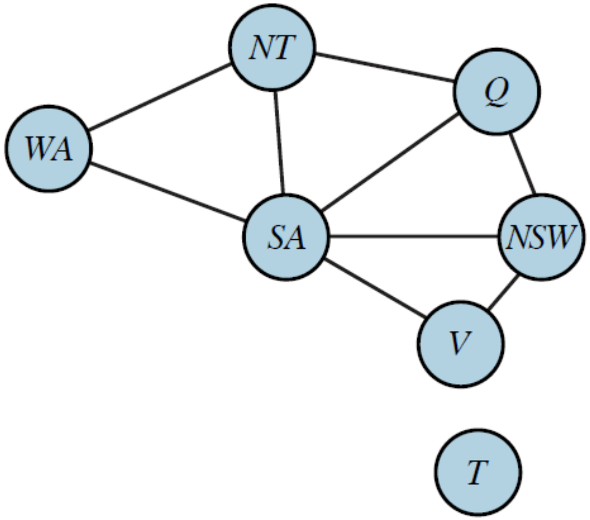

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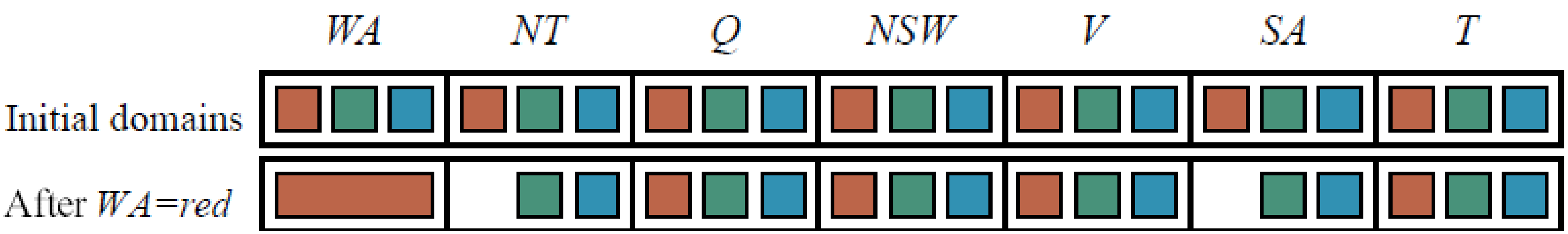

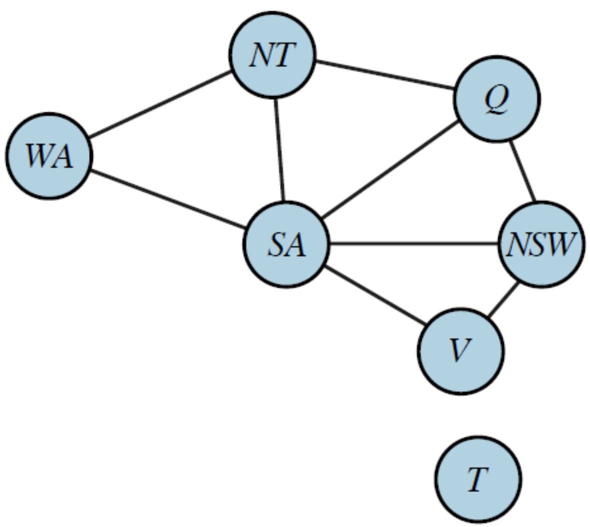

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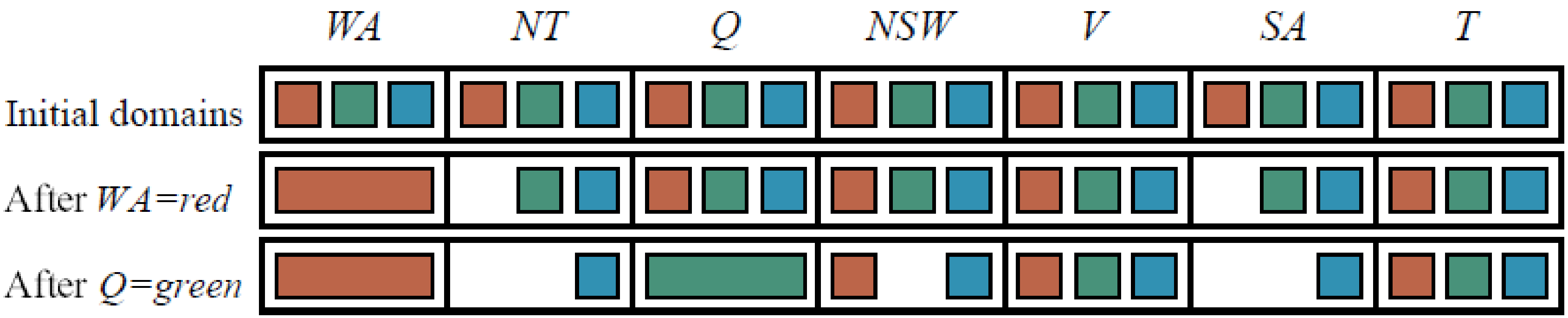

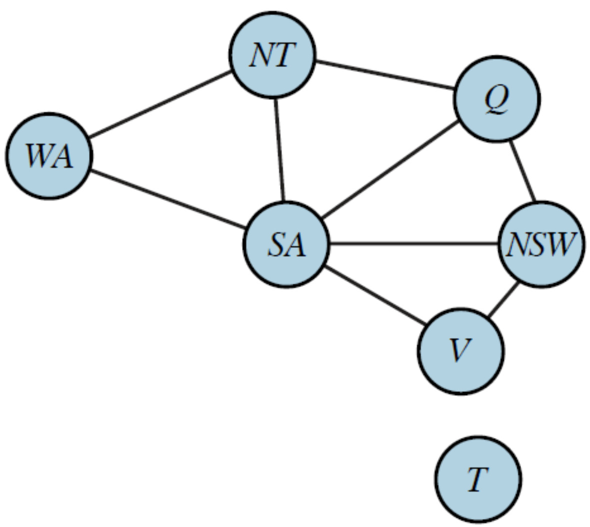

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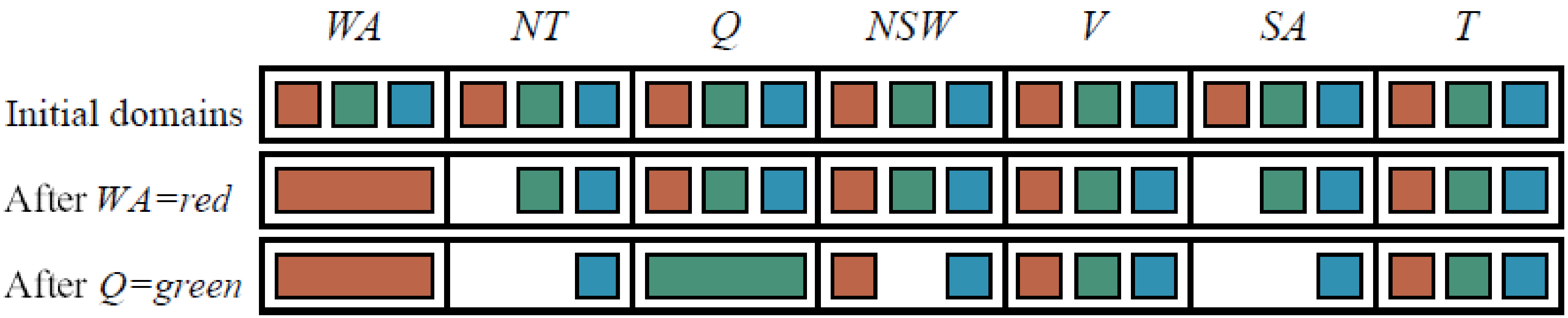

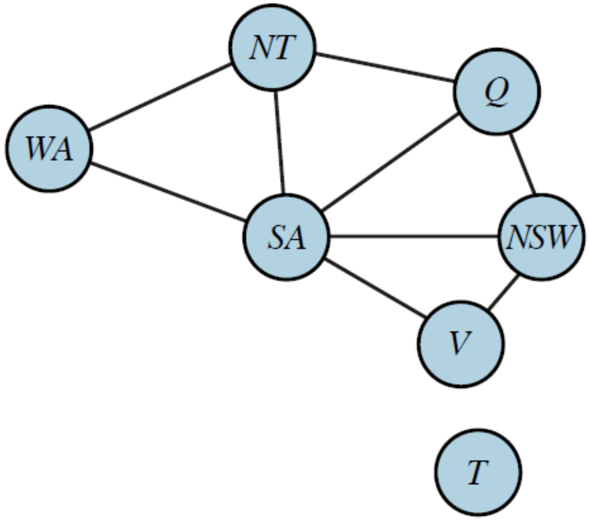

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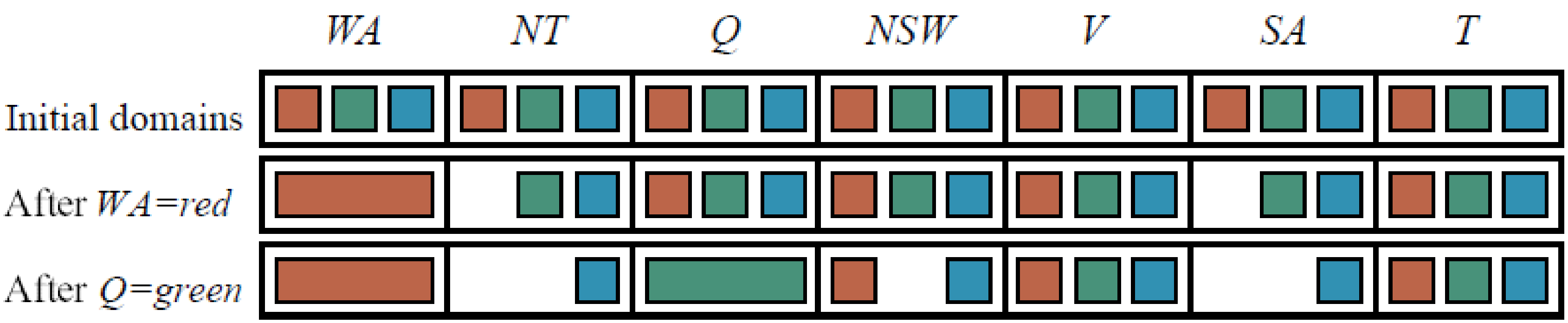

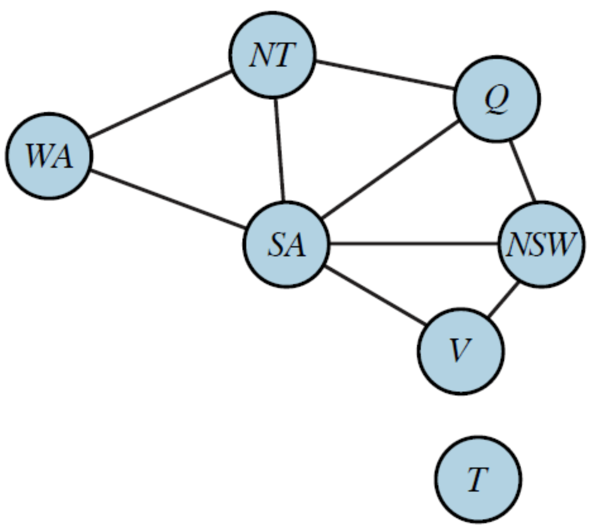

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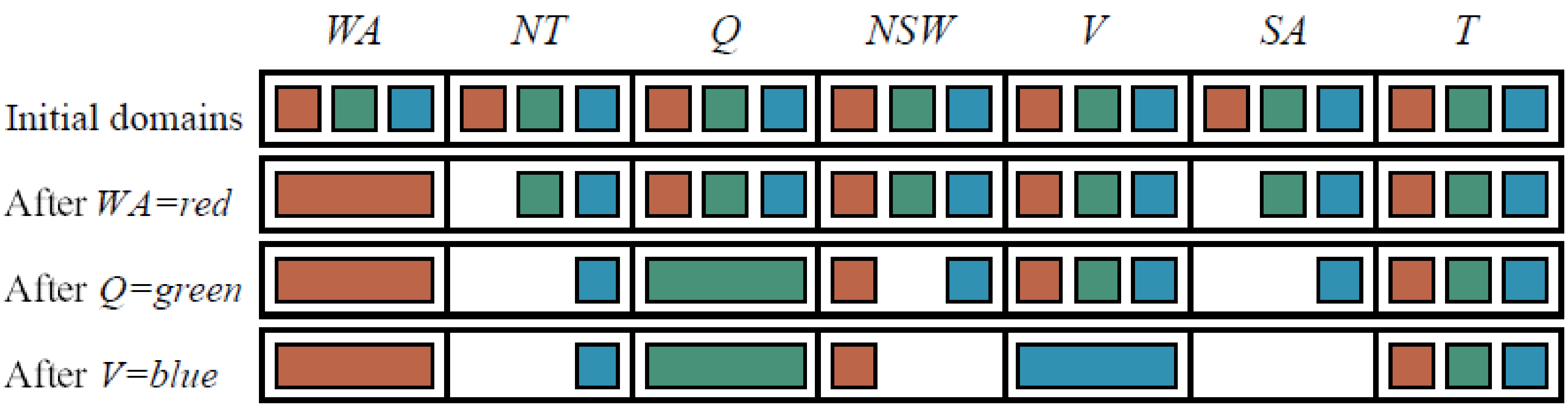

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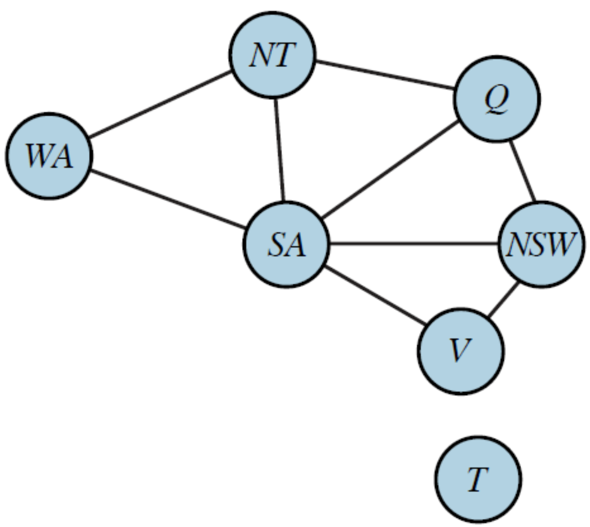

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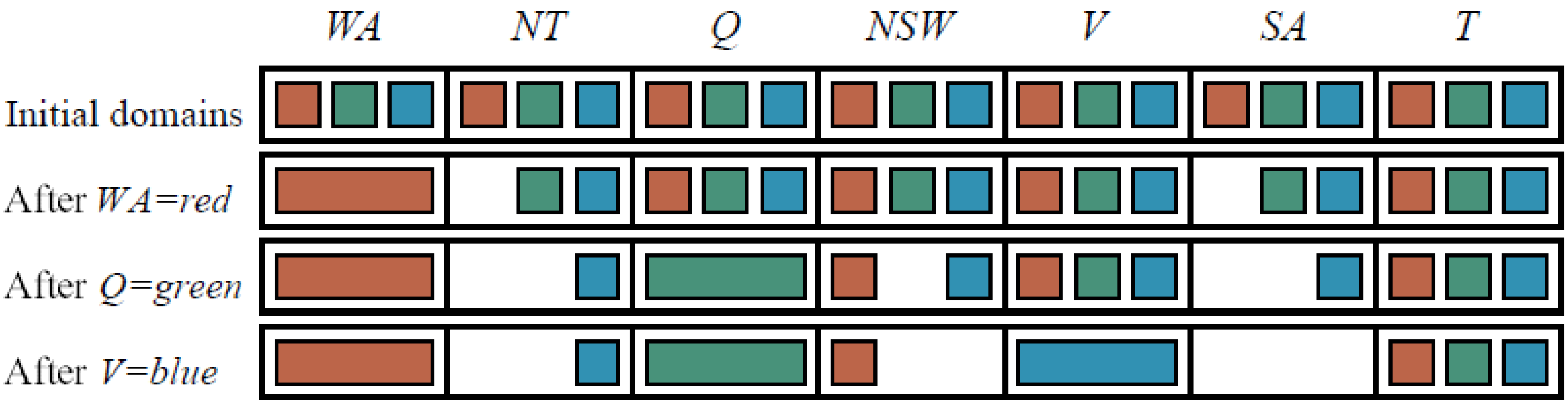
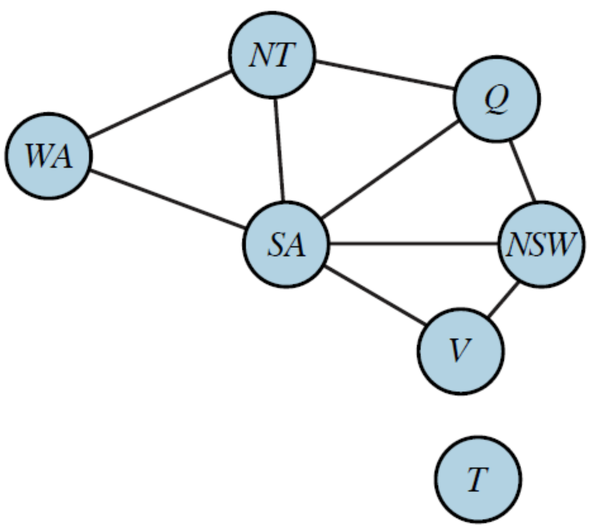




- *WA* = *red*

  Deletes *red* for *NT* and *SA*.

- *Q* = *green*

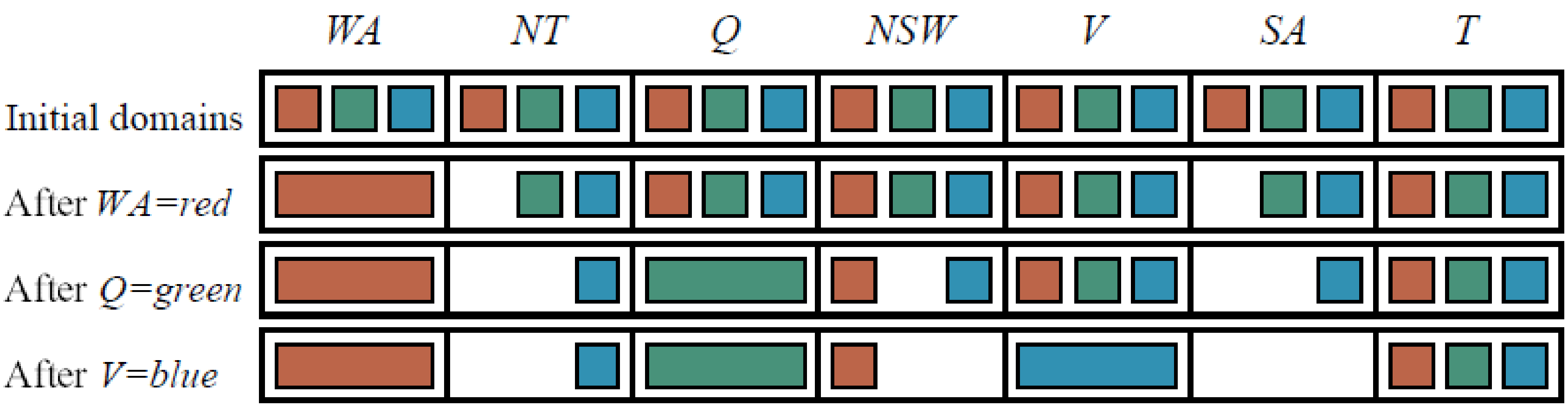
  Deletes *green* for *NT*, *SA* , *and NSW*.

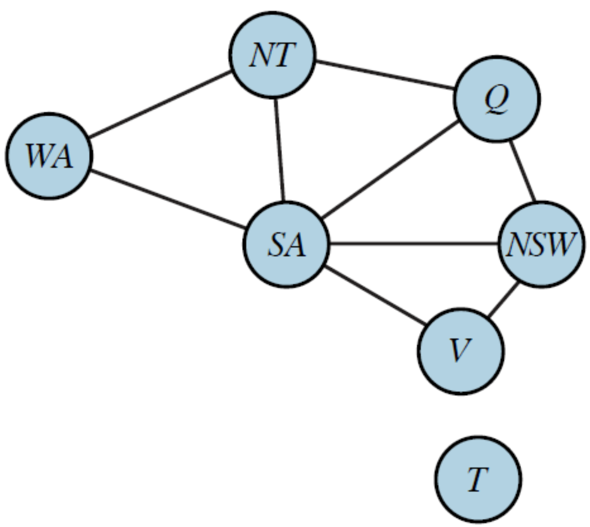
  NT & *SA* each has a single value.

- *V* = *blue*

  SA has no legal value.

Backtracking with Forward Checking





- *WA* = *red*

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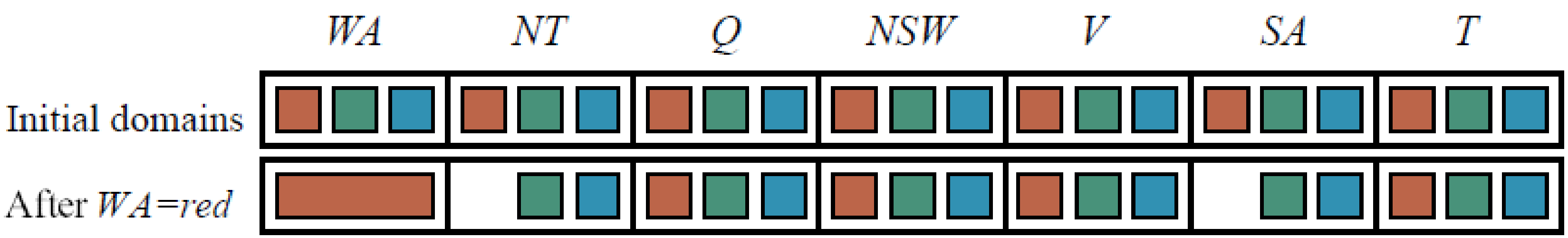
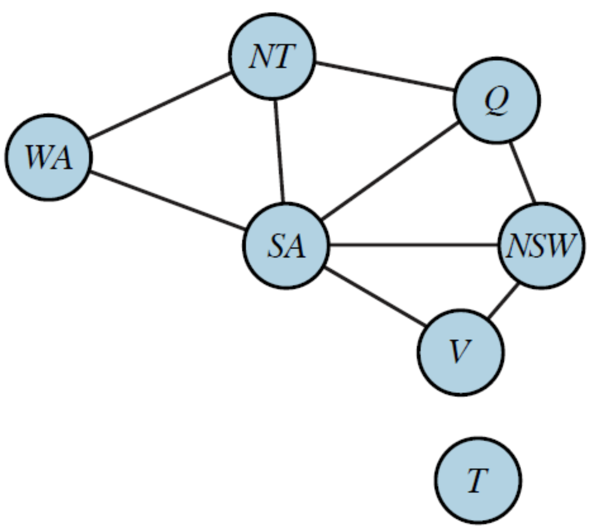
  SA has no legal value.

  Delete {*WA* = *red*, *Q* = *green*, *V* = *blue*}. Start backtracking.

Combining MRV and FC Heuristics

Search becomes more effective when they are combined.





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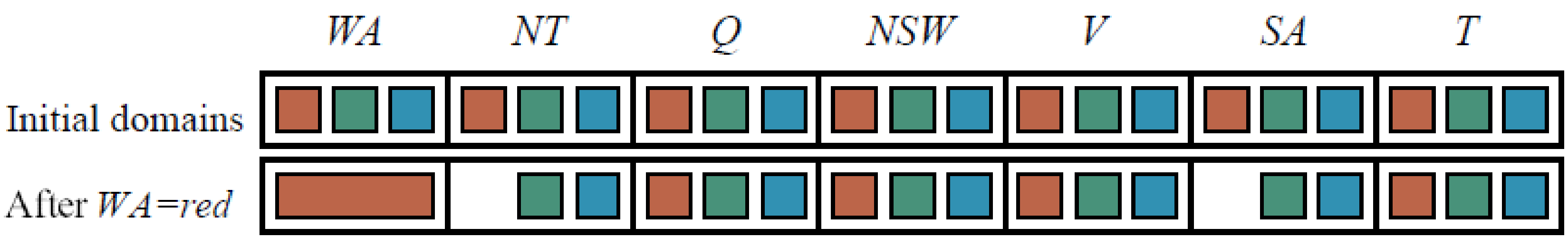

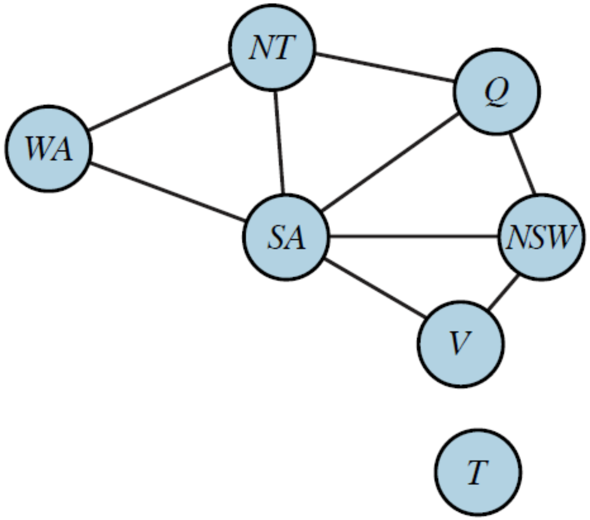

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Combination of MRV and FC can solve the 1000-queen problem.

II. Local Search

- Every state corresponds to a complete assignment.
- Search changes the value of one variable at a time.

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- Every state corresponds to a complete assignment.
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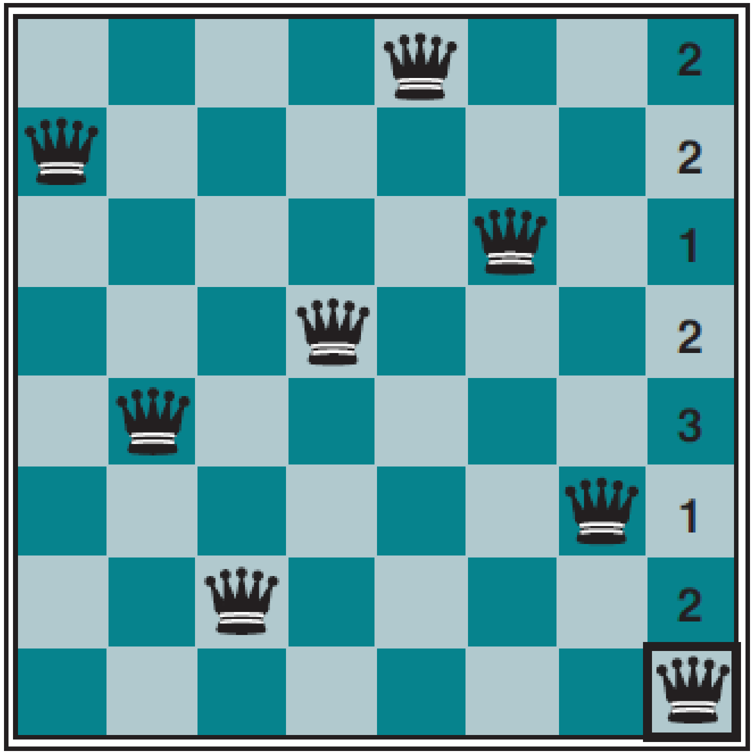
Min-conflicts heuristic:

- Start with a complete assignment.
- Randomly choose a conflicted variable.
- Select the value that results in the least conflicts with other variables.

Applying Min-conflicts to 8-Queen

Variable set: $\mathcal{X} = \{Q_1, Q_2, \ldots, Q_8\}$

Q_i: the row number of the queen placed in the ith column.



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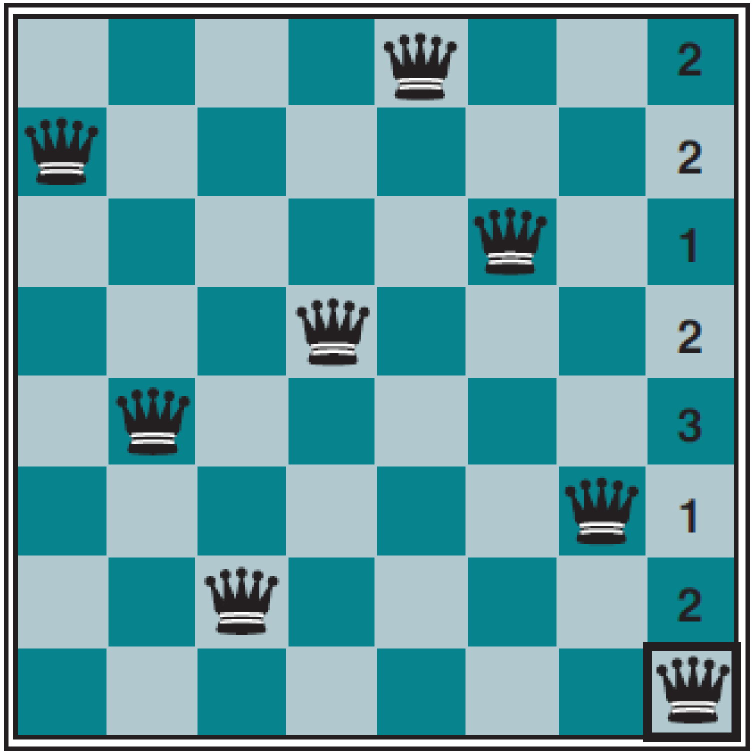

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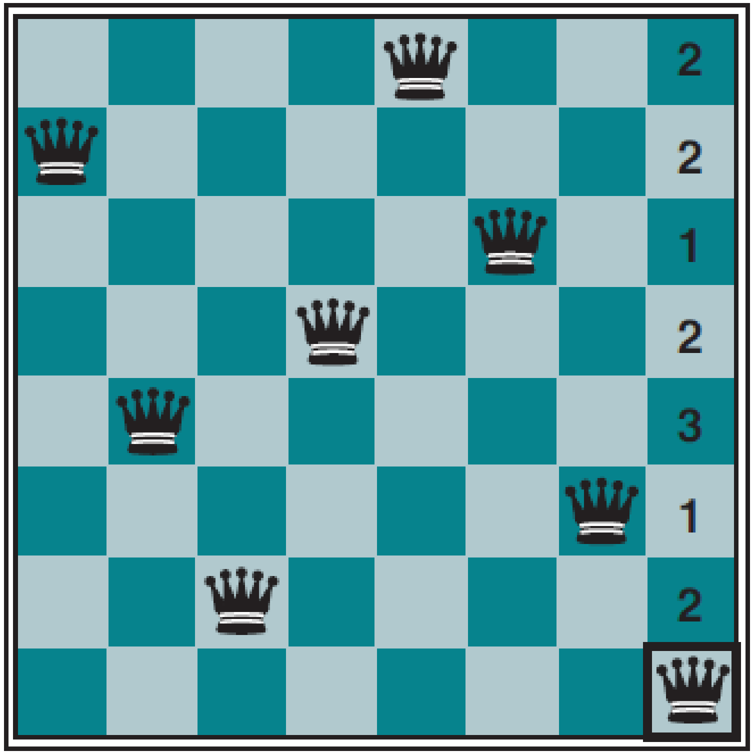

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- Q_8 out of the set $\{Q_4, Q_8\}$ of conflicted variables by a random choice.

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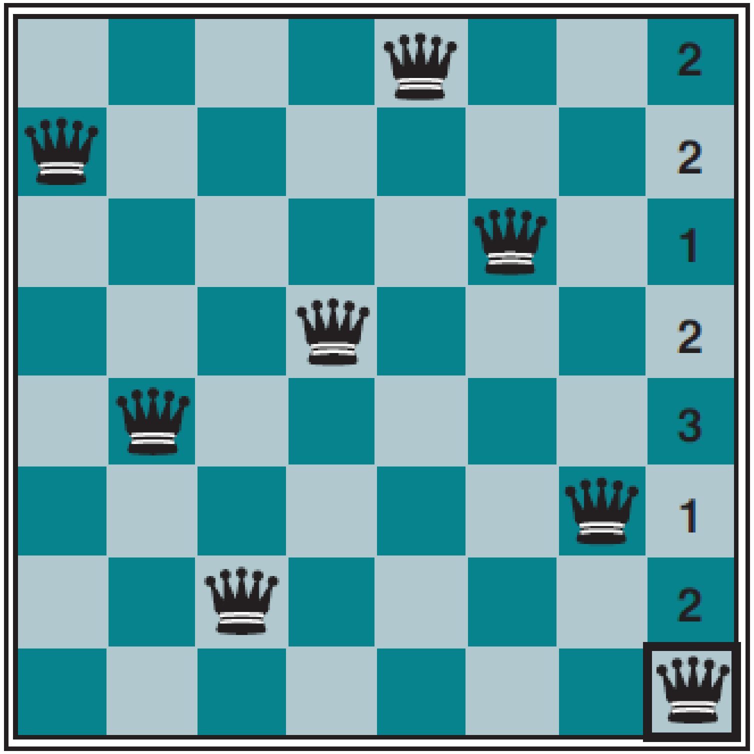

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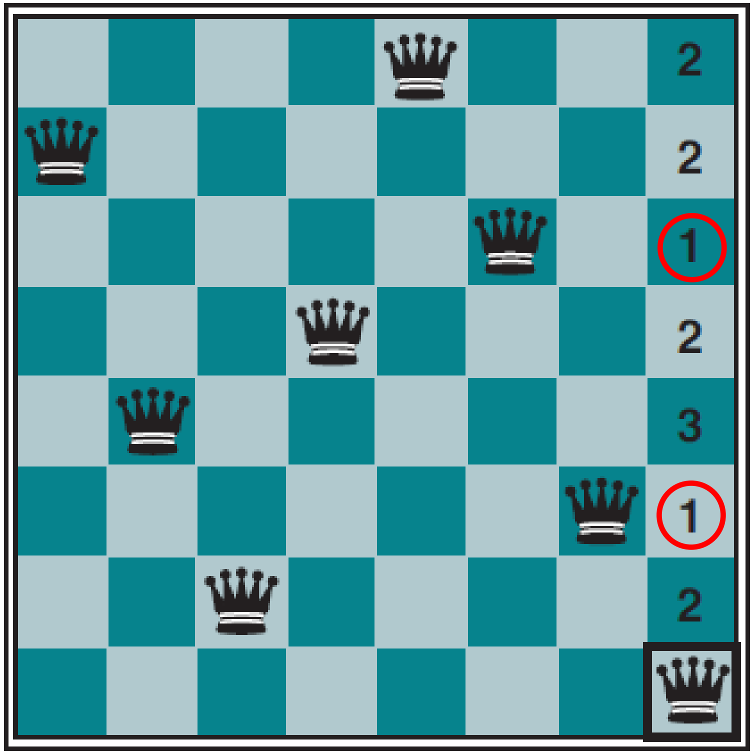

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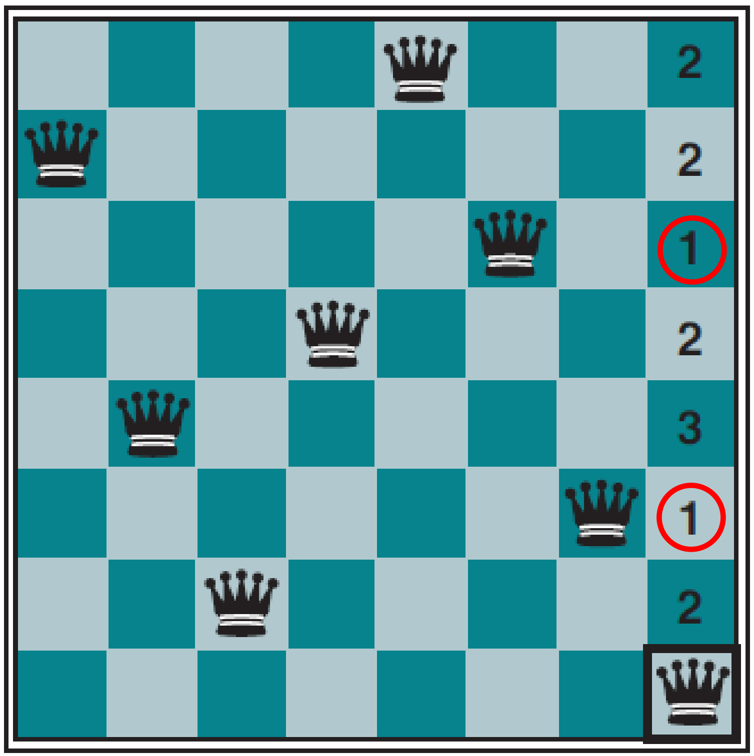

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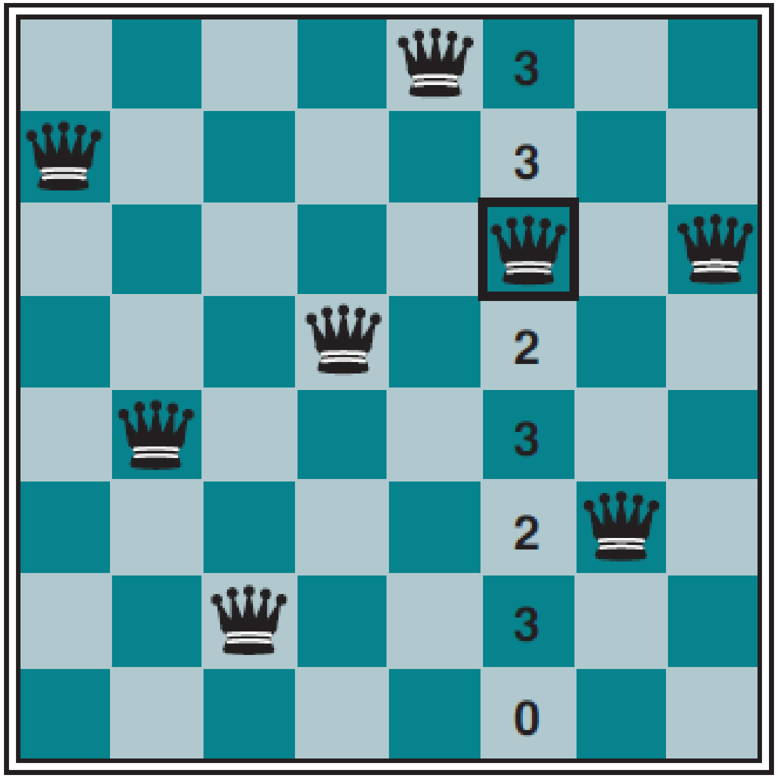
- Q_8 out of the set $\{Q_4, Q_8\}$ of conflicted variables by a random choice.
- The 8th queen in row 3 or 6 would violate only one constraint.
- Move the queen to, say, row 3.

8- and n-Queen problems



- Q_6 out of $\{Q_6, Q_8\}$.

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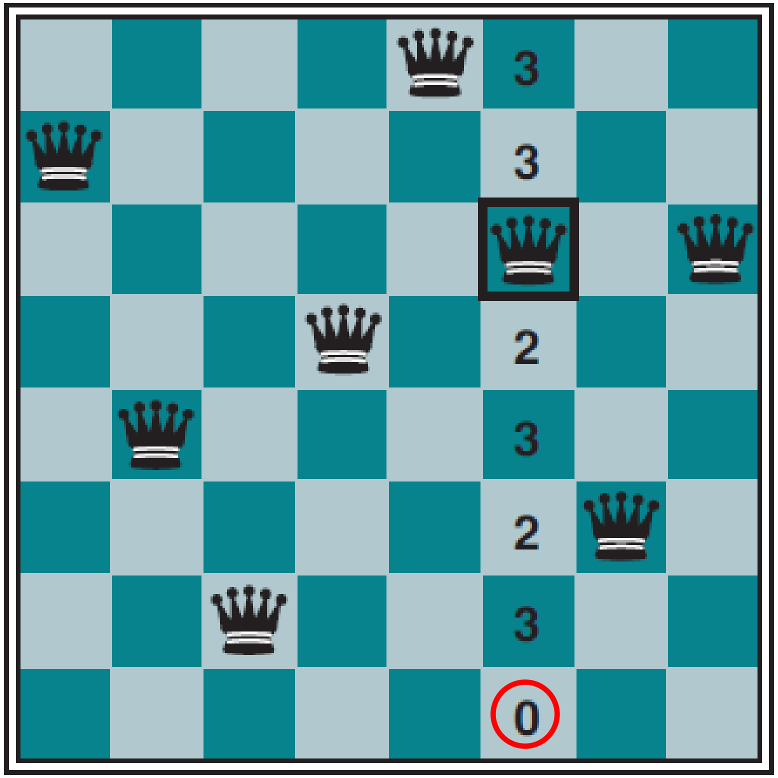

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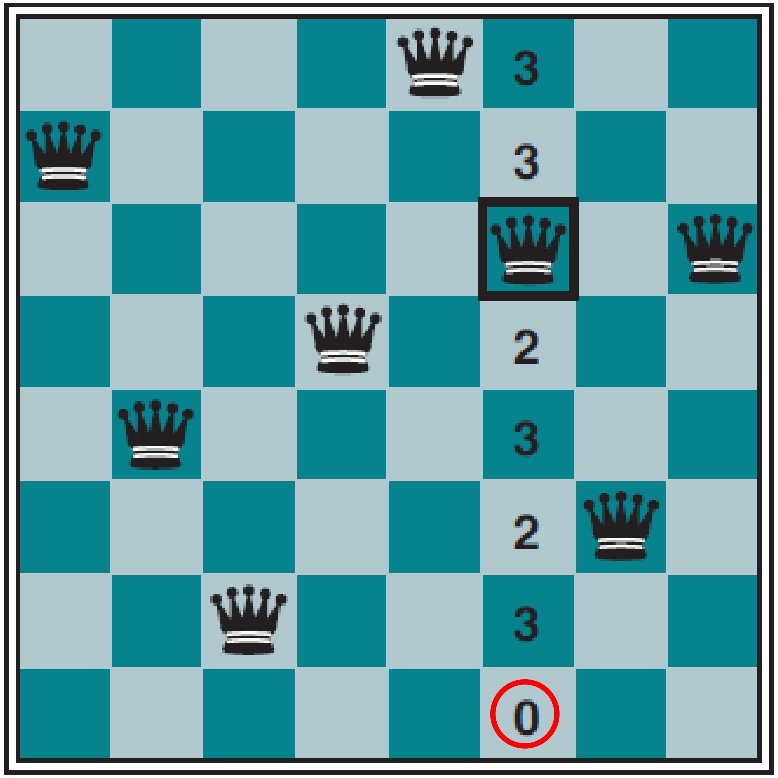

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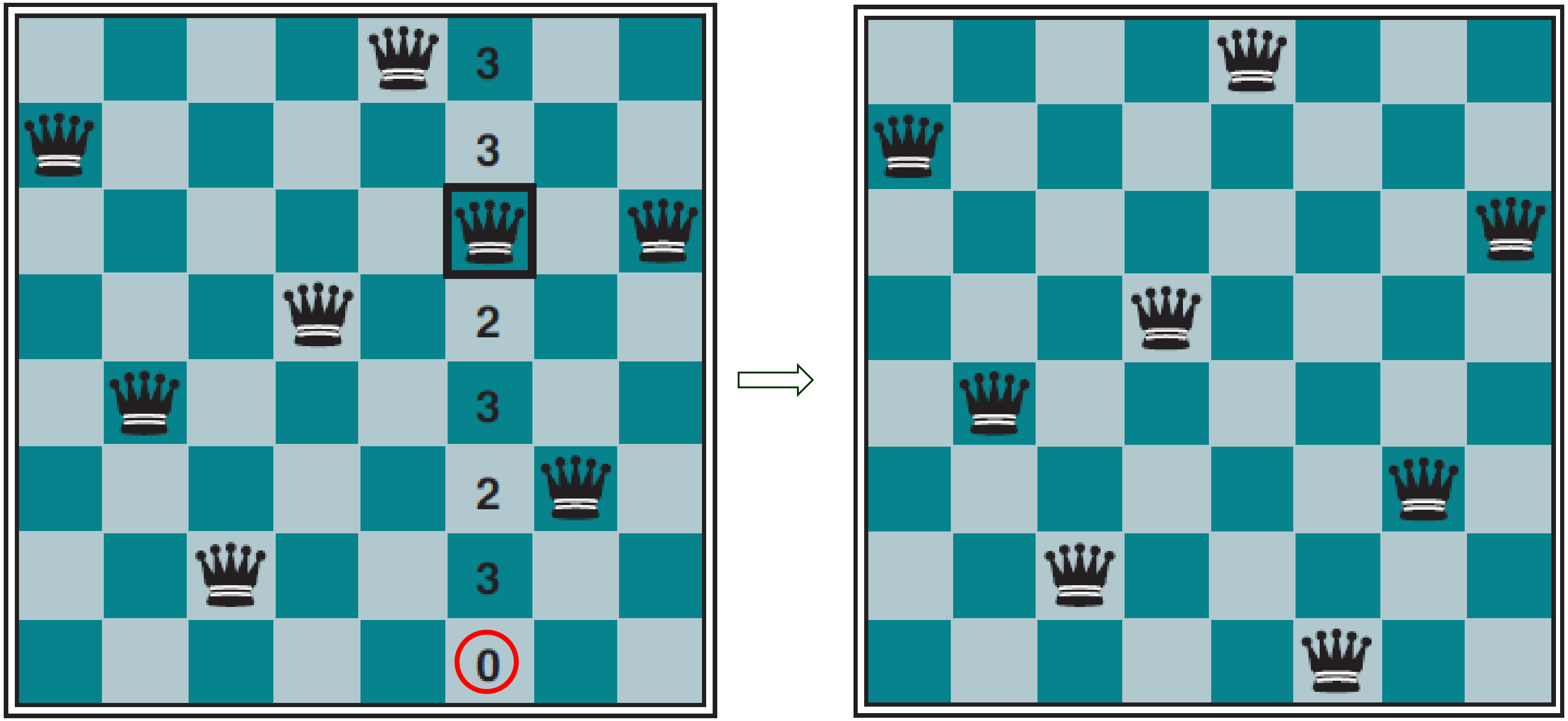

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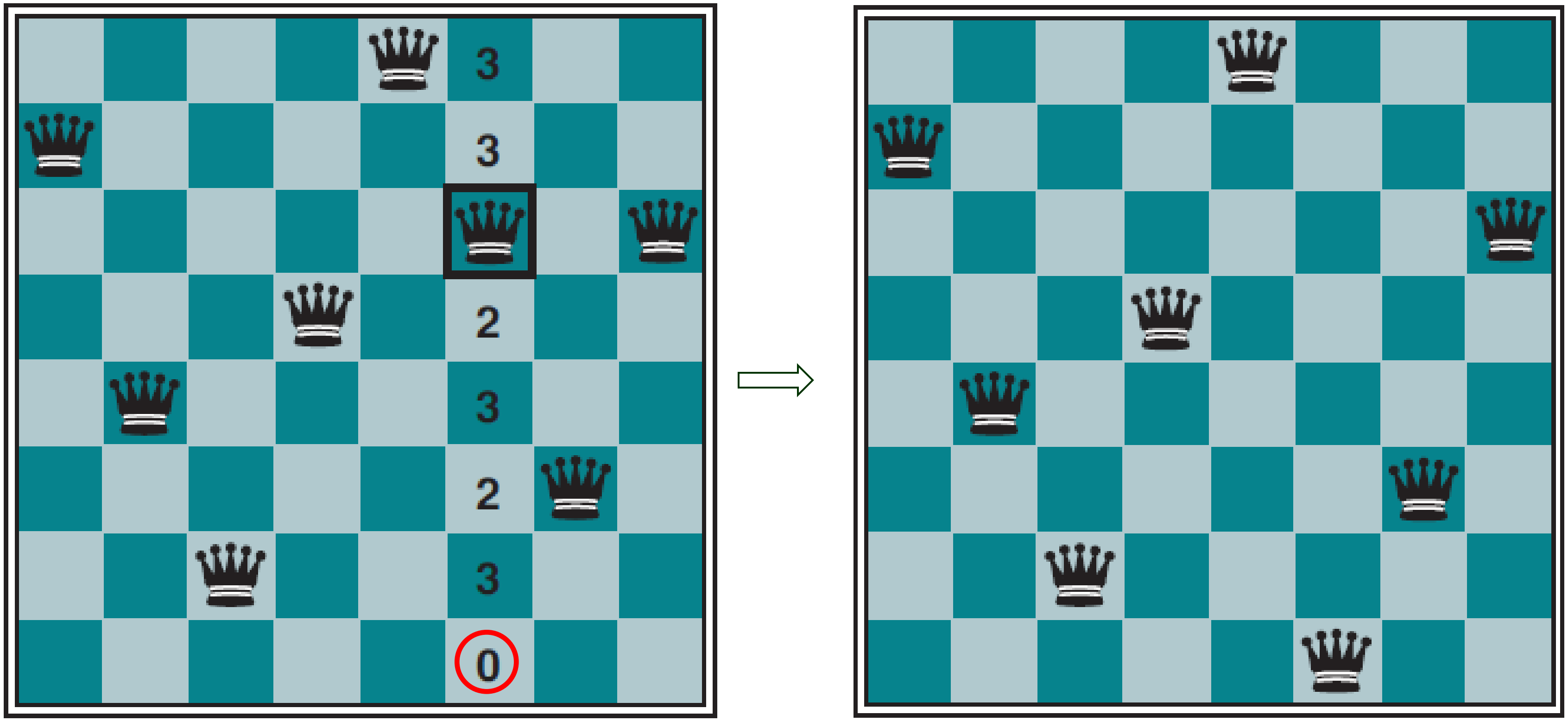

Solution

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Local Search: n-Queen and Beyond

- Run time of min-conflicts on n-queen is roughly independent of n.

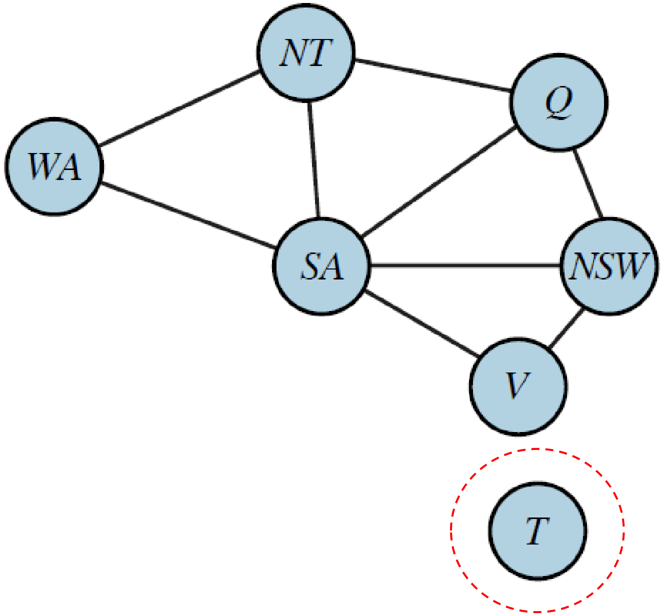
  10^6-queen problems are solved in an average of 50 steps (after the initial assignment).

- Ease of solving n-queen due to dense distribution of solutions throughout the state space.

- Min-conflicts also effective on hard problems such as observation scheduling for the Hubble Space Telescope.

- Local search is applicable in an online setting (e.g., repairing the scheduling of an airline's weekly activities – in the advent of bad weather).

III. The Structure of CSP Problems



Independent subproblems

- Connected components in the constraint graph.
- Each subproblem can be solved independently.

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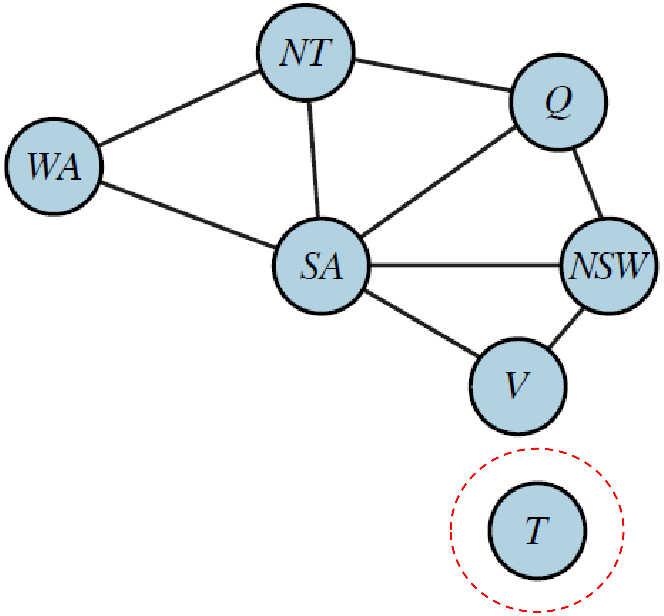

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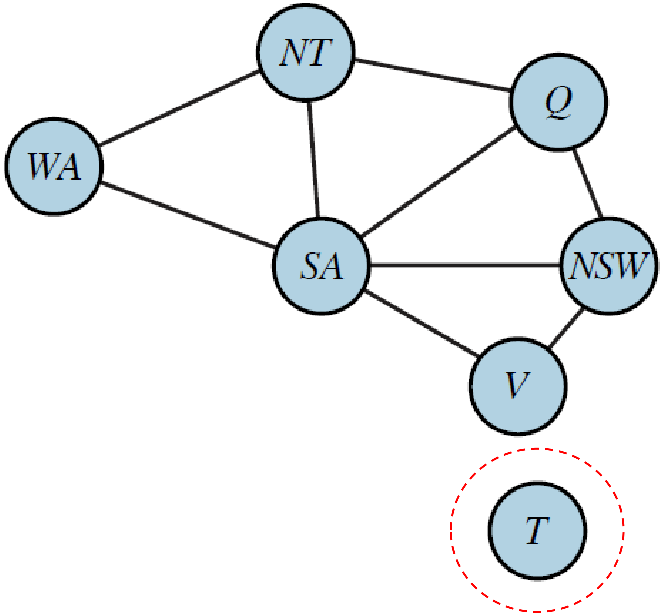

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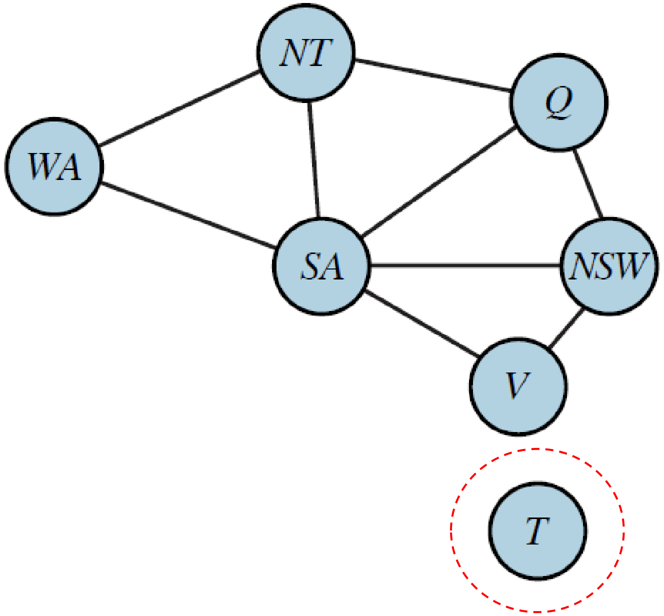

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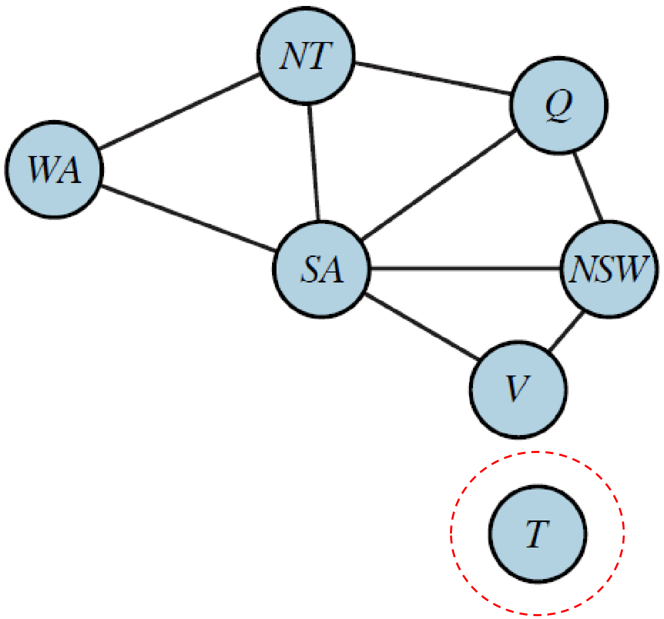

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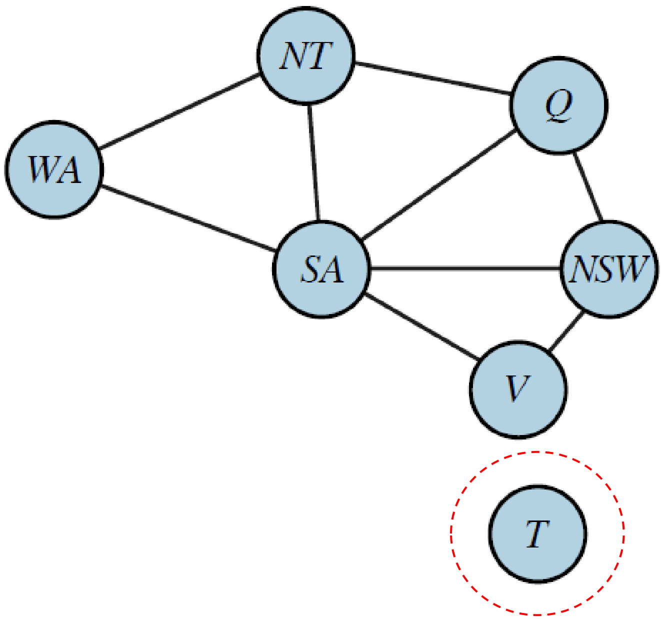

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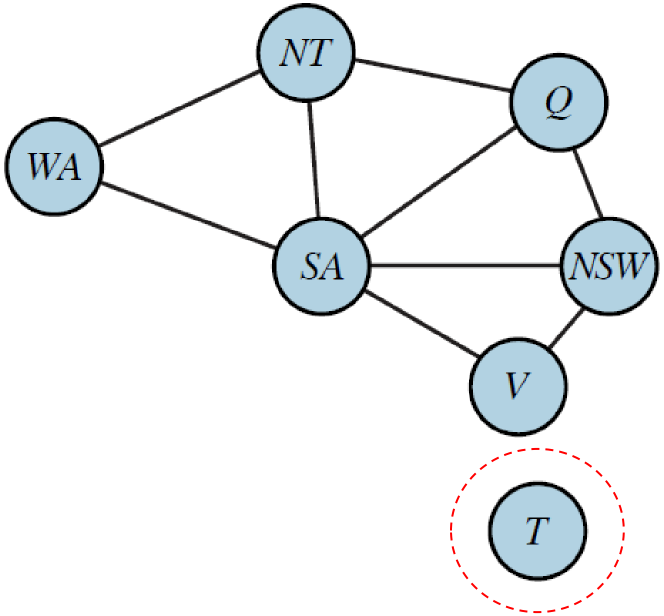

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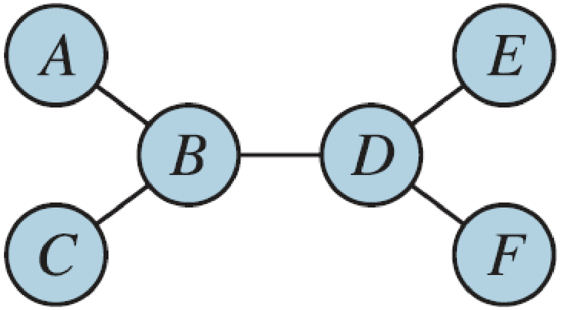
⟹ Total work $O(d^c n/c)$ Linear in n.

Tree-Structured CSPs

Constraint graph is a tree.

root



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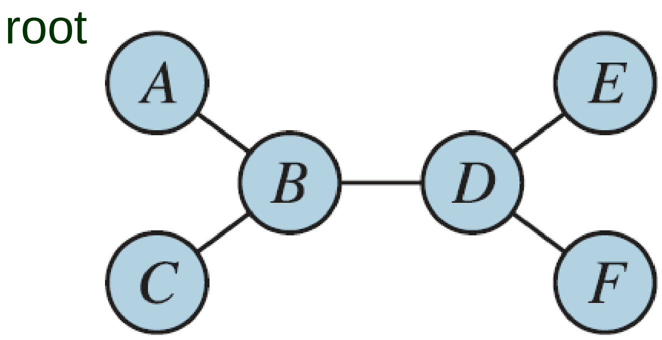

Solution:

- Generate a topological order of the variables.

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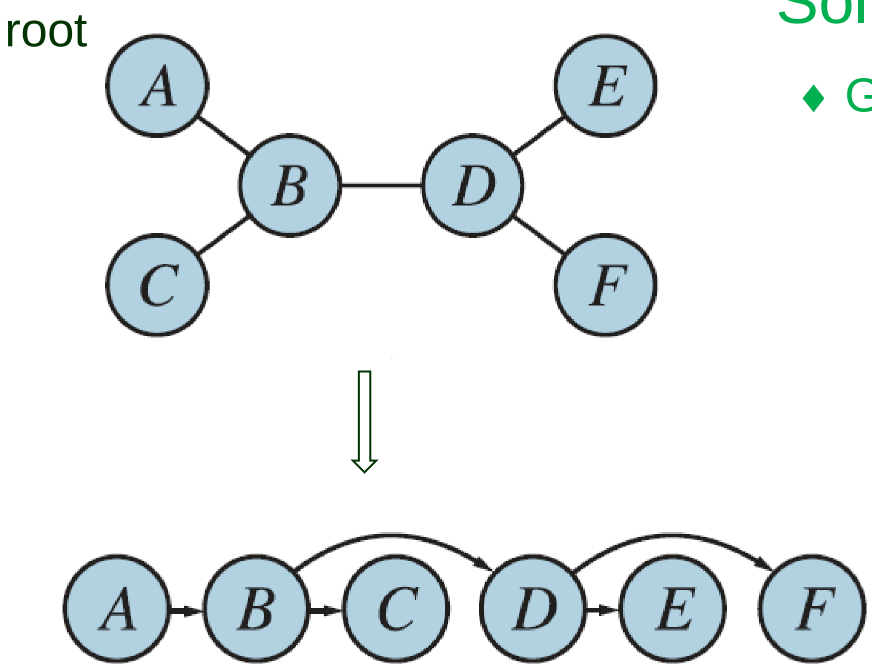

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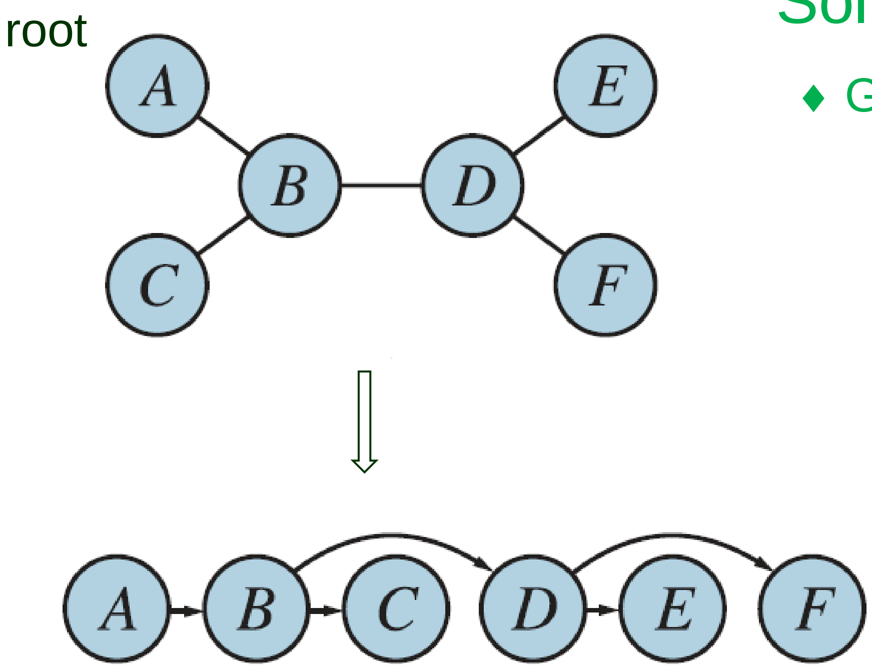

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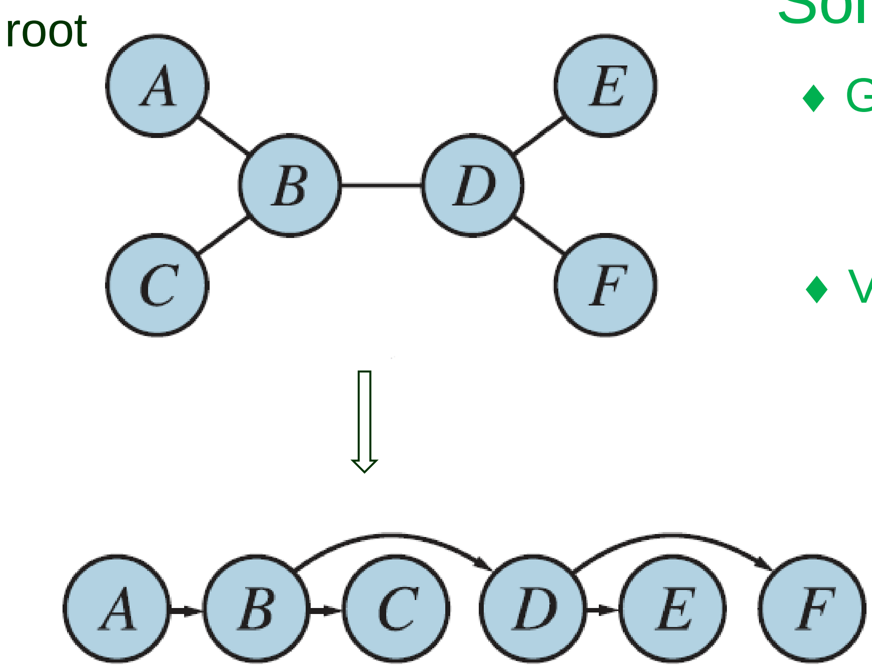

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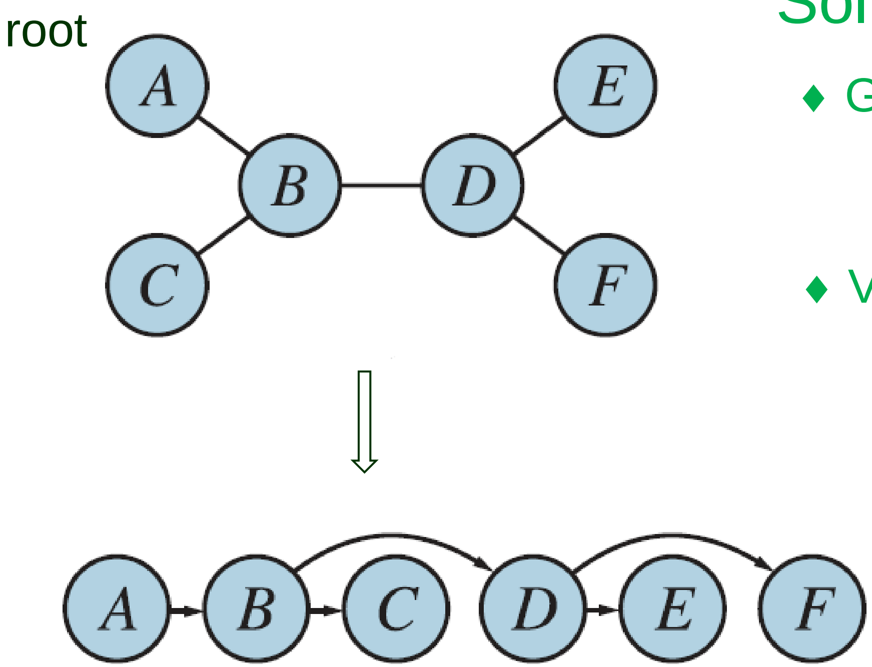

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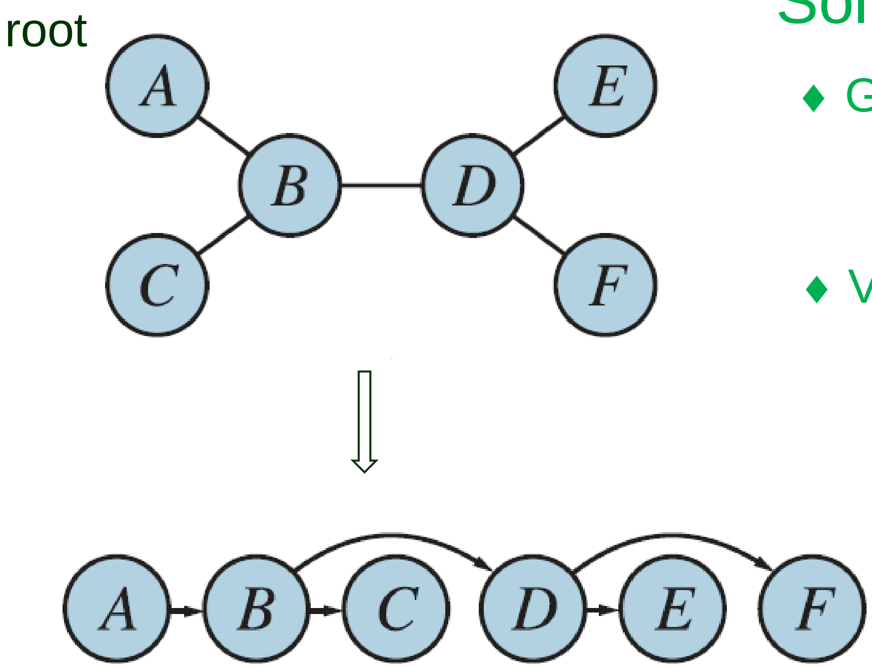

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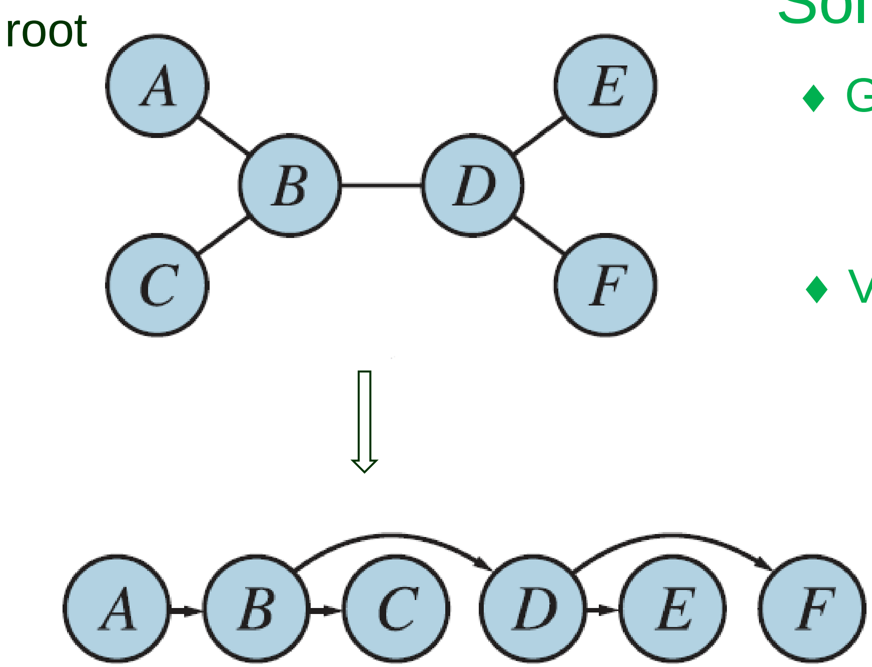

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- Finally, visit variables in the topological order again and choose any value from its reduced domain,

Tree-Structured CSPs

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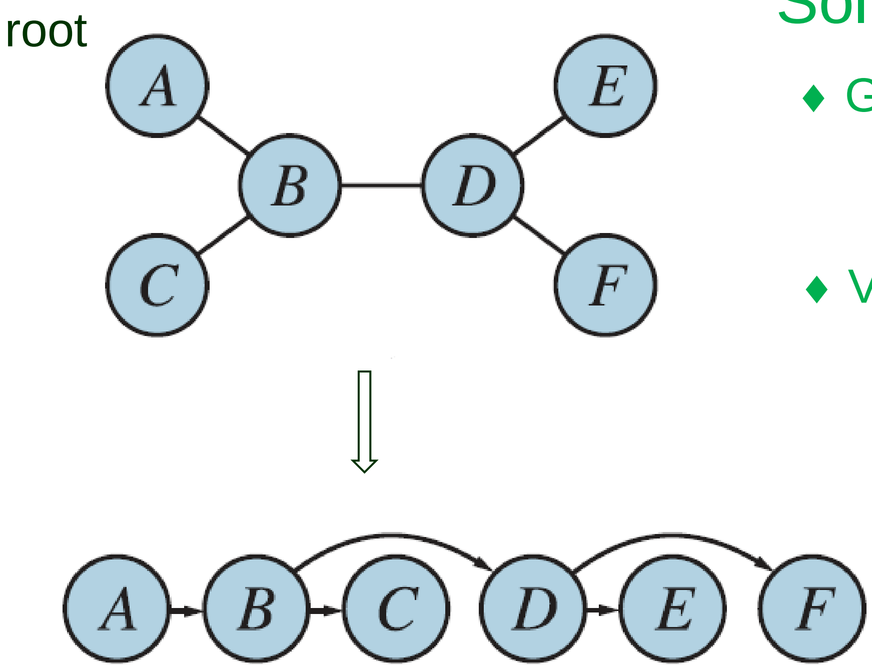

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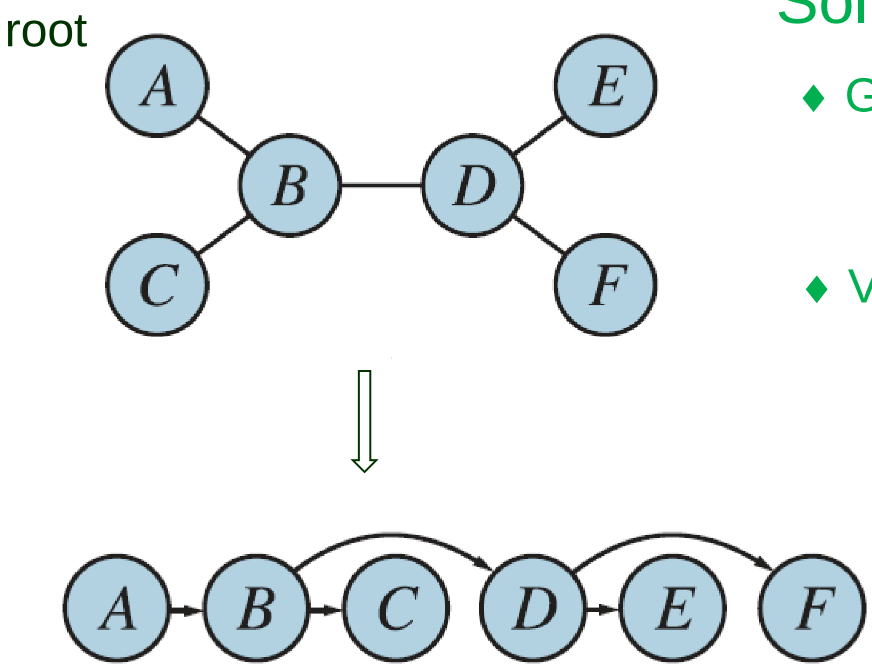

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- Finally, visit variables in the topological order again and choose any value from its reduced domain, $O(n)$

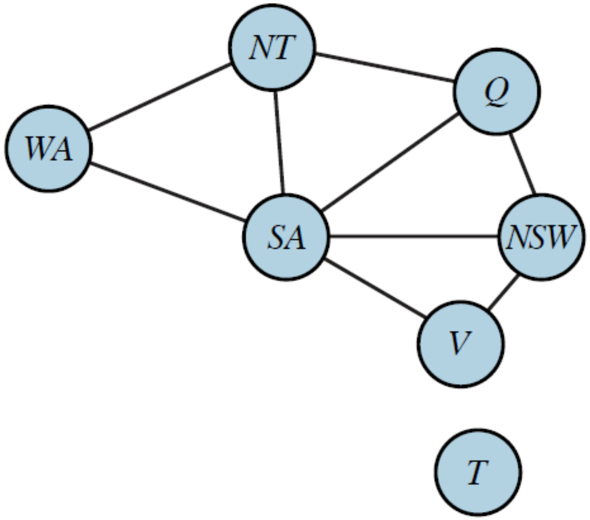
Tree CSP Solver

function TREE-CSP-SOLVER(*csp*) **returns** a solution, or *failure*
  inputs: *csp*, a CSP with components X, D, C

  $n \leftarrow$ number of variables in X
  assignment $\leftarrow$ an empty assignment
  root $\leftarrow$ any variable in X
  $X \leftarrow$ TOPOLOGICALSORT(X, *root*)
  for $j = n$ **down to** 2 **do**
    MAKE-ARC-CONSISTENT(PARENT(X_j), X_j)
    if it cannot be made consistent **then return** *failure*
  for $i = 1$ **to** n **do**
    assignment$[X_i] \leftarrow$ any consistent value from D_i
    if there is no consistent value **then return** *failure*
  return *assignment*

Cutset Conditioning

Reduce a constraint graph to a tree (or a forest) by assigning values to some variables.



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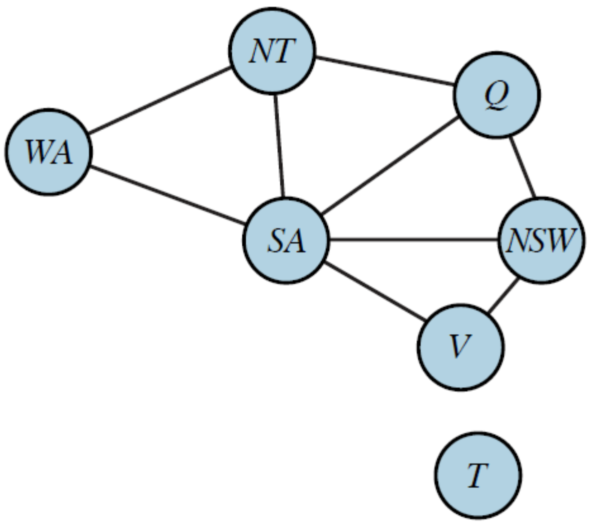
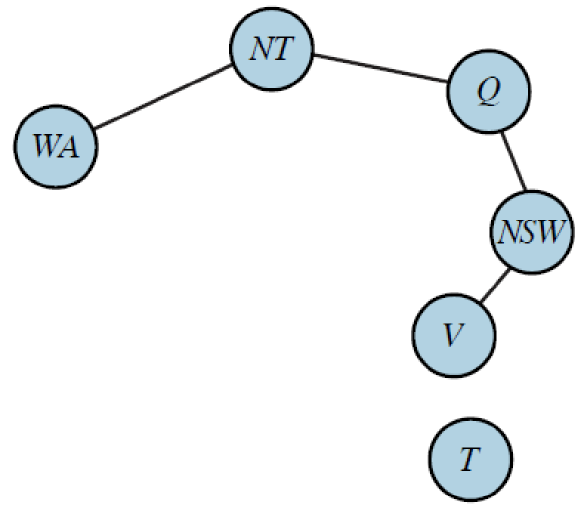


Assign a value to *SA* and remove the node.



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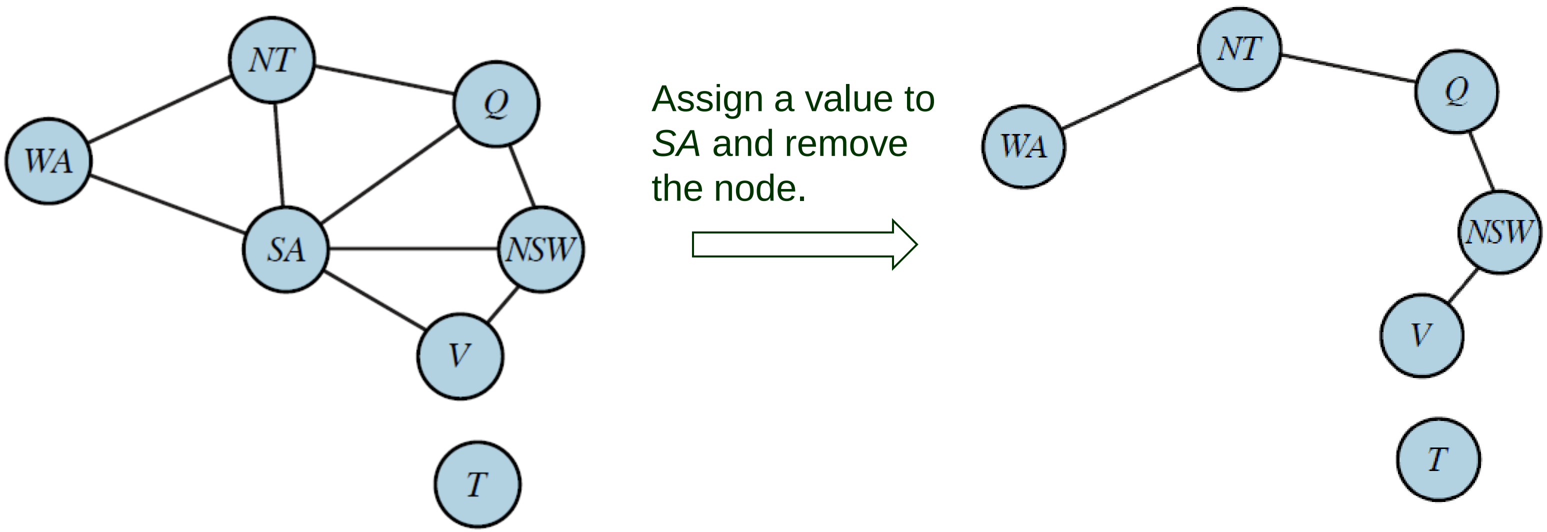

1. Choose a subset $S \subset \mathcal{X}$ of variables whose removals reduce the constraint graph to a tree (or a forest).

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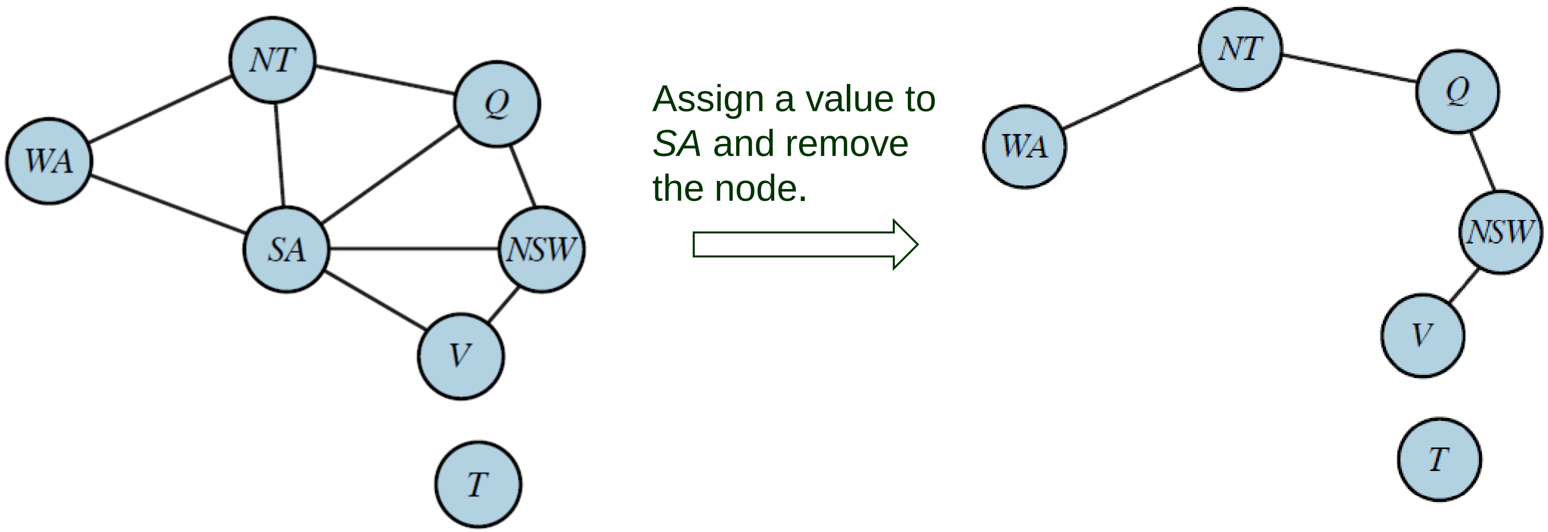

1. Choose a subset $S \subset \mathcal{X}$ of variables whose removals reduce the constraint graph to a tree (or a forest).

2. For every consistent assignment A to variables in S:
   - remove from the domain of every $X \in \mathcal{X} \setminus S$ all values that are inconsistent with A.
   - return the solution to the reduced CSP (if exists) along with A.

Tree Decomposition

Transform the constraint graph into a tree where each node consists of a set of variables such that

- Every variable X must appear in at least one tree node n.
- Two variables X, Y sharing a constraint must appear together in at least one node n.
- If X appears in two nodes n_1 and n_2, it must appear in every node on the path connecting n_1 and n_2.

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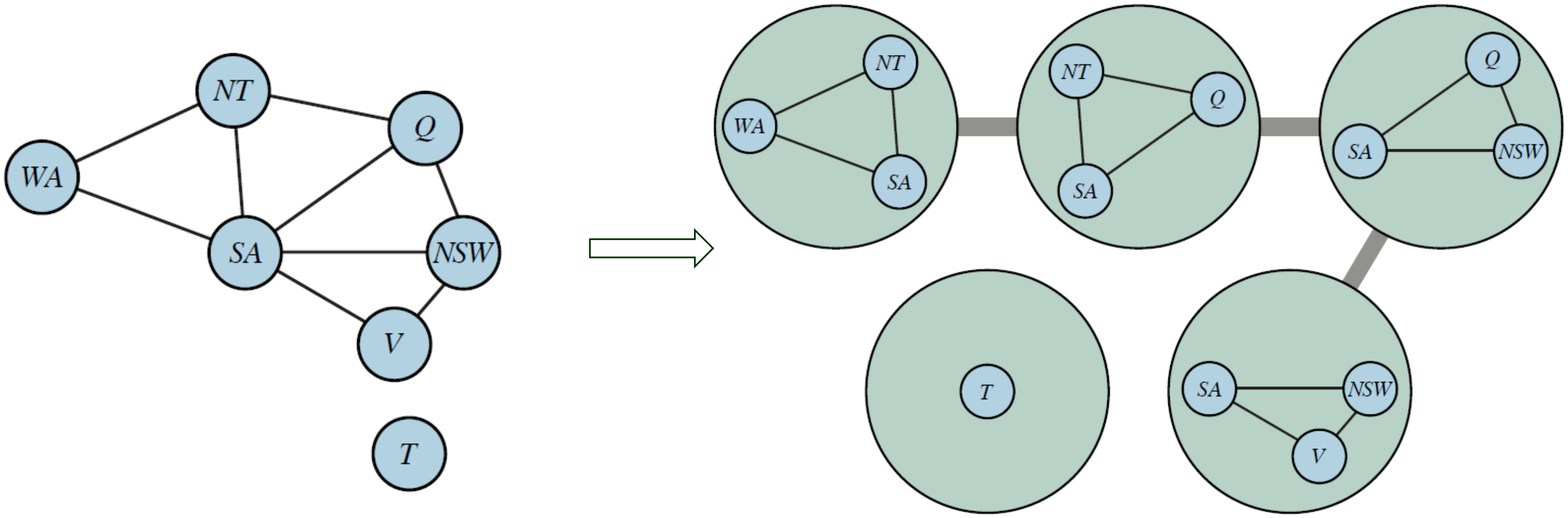

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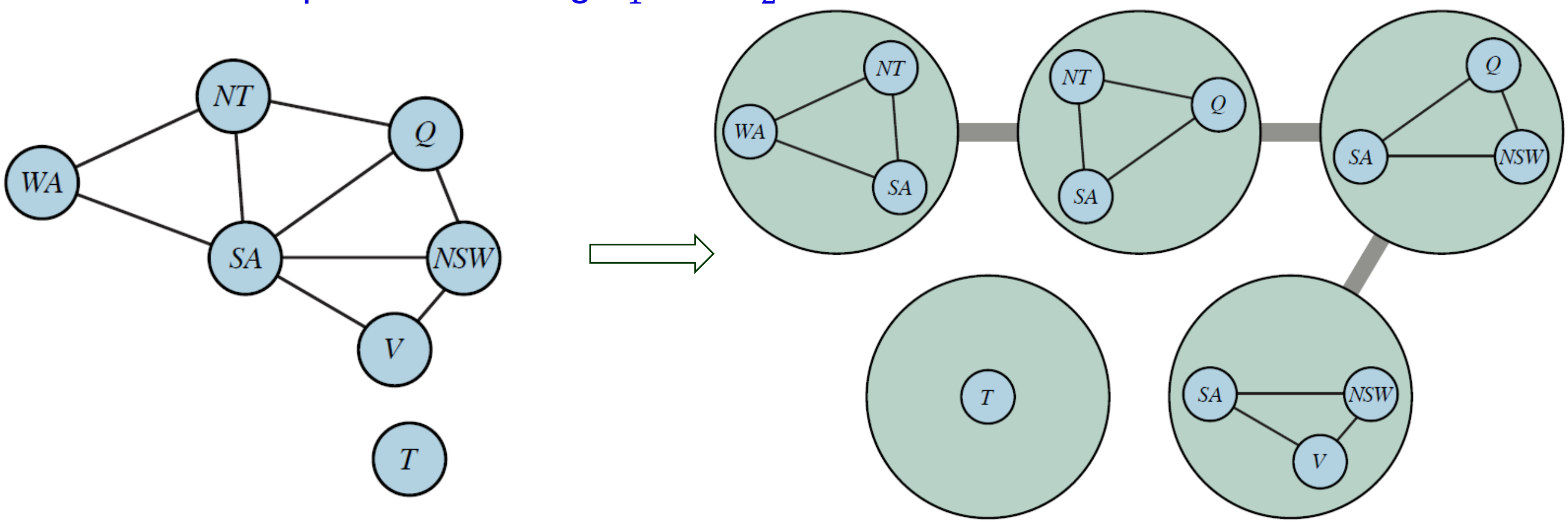

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A variable must have the same value everywhere it appears.

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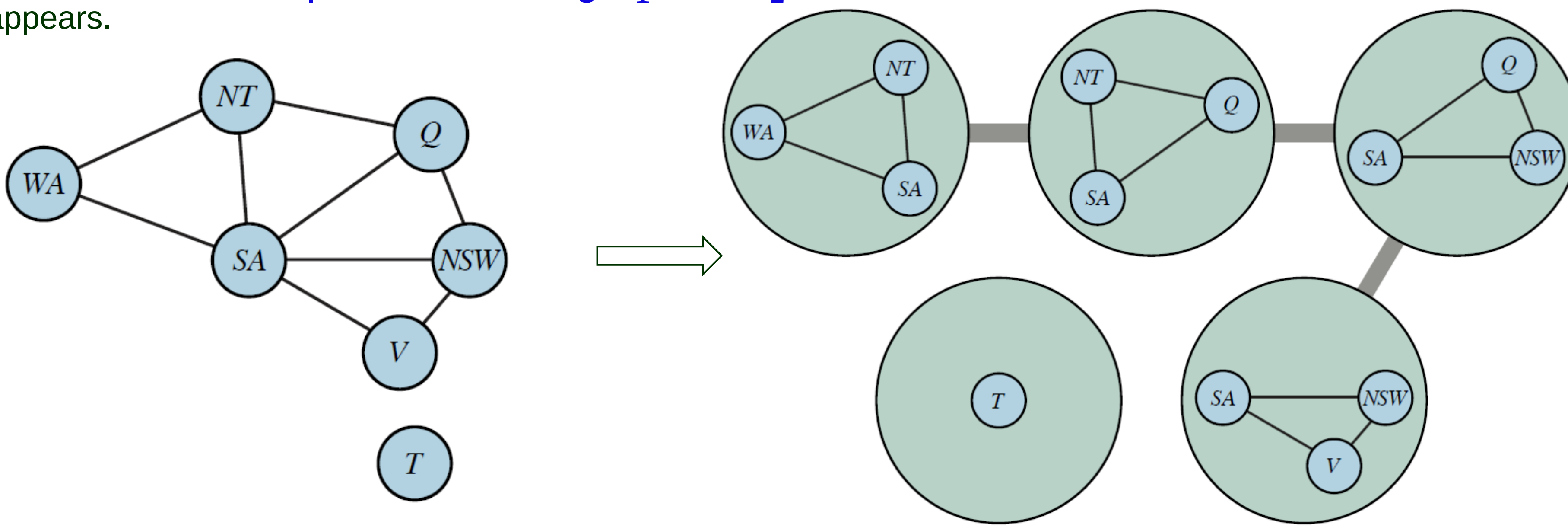
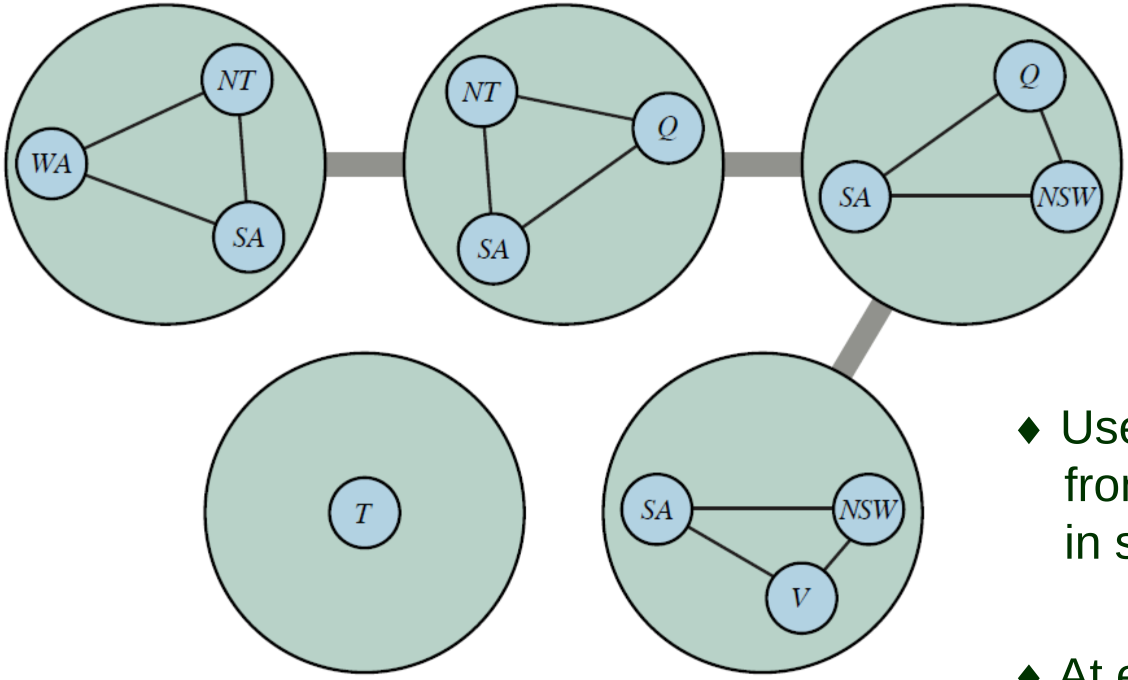


Solution After Tree Decomposition



- Use the CSP tree solver to move from one tree node to the next in some topological order.
- At each tree node, solve the CSP subproblem represented at that node.

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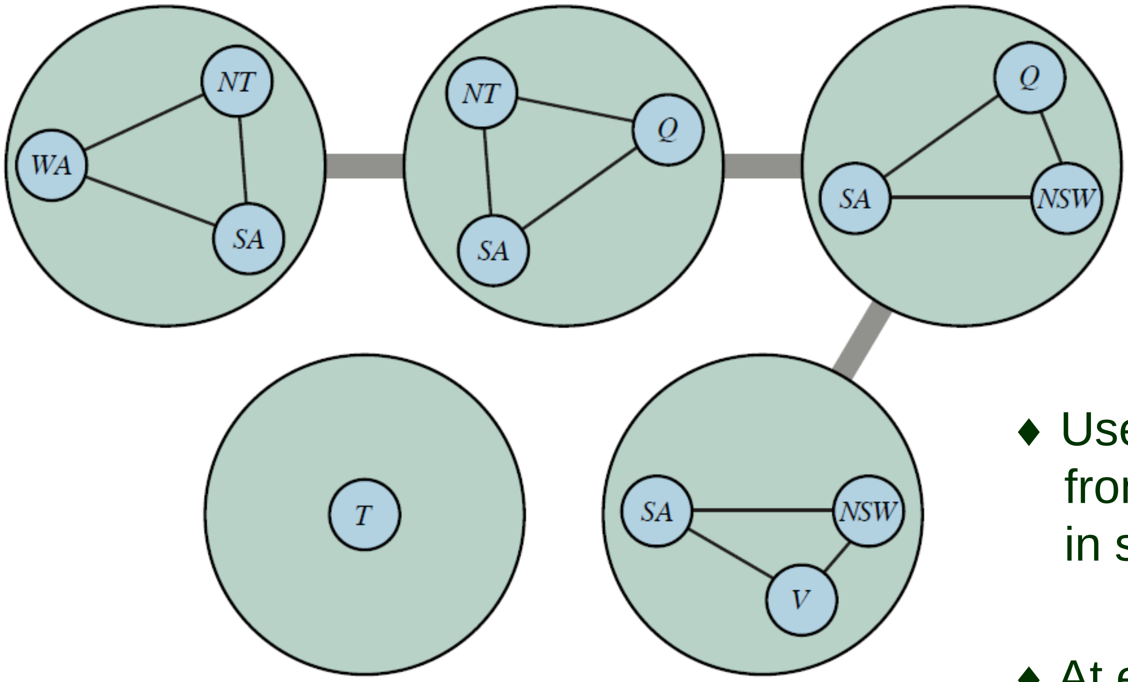

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