Monte Carlo Tree Search & Stochastic Games

Outline

I. Monte Carlo tree search (MCTS)

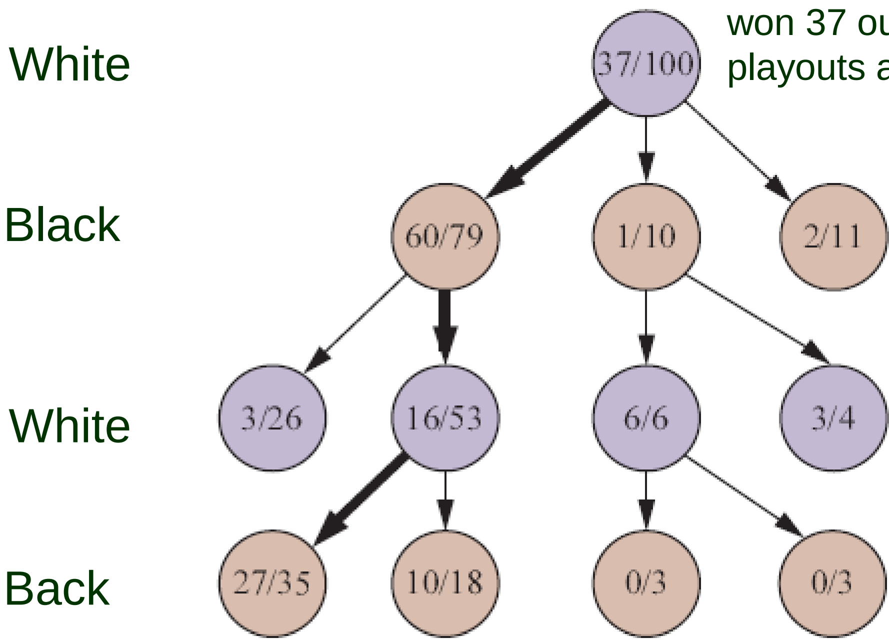
II. Stochastic games

* Figures/images are from the textbook site (or by the instructor).

I. An Iteration of MCTS – Step 1: Selection

Which move should Black make (at the root)?

Root: state just after the move by white, who has won 37 out of the 100 playouts at the node so far.



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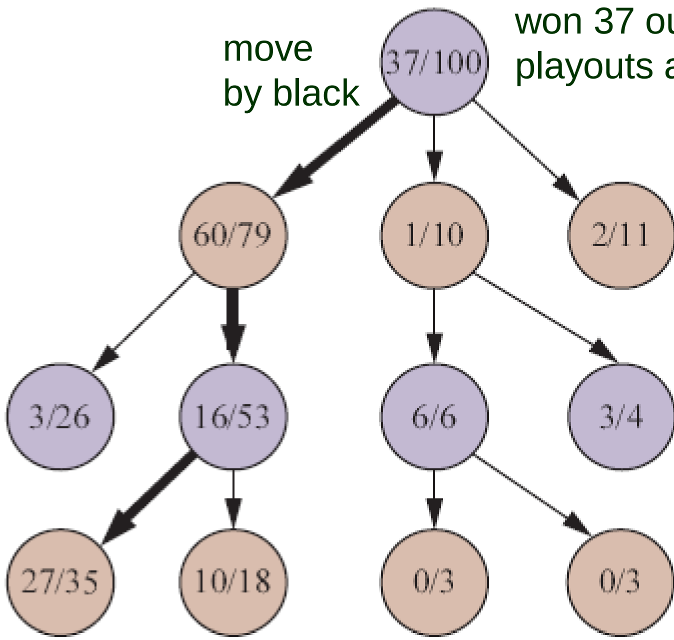
White

move
by black

Black

White

Back



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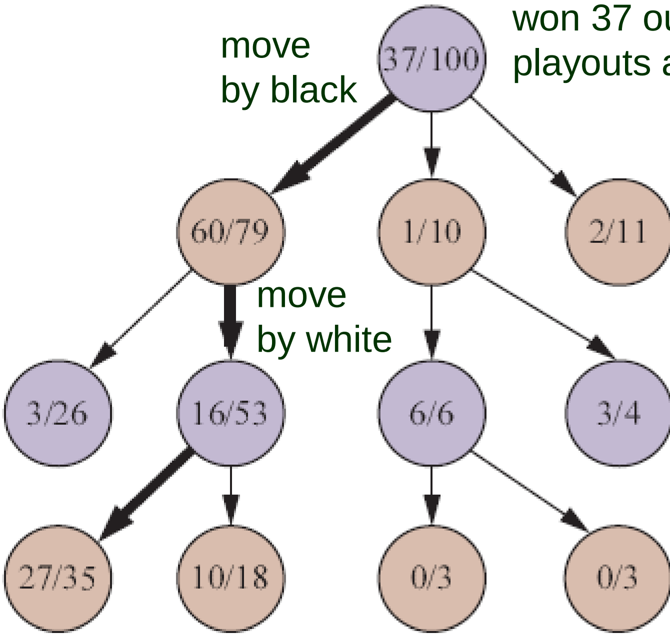

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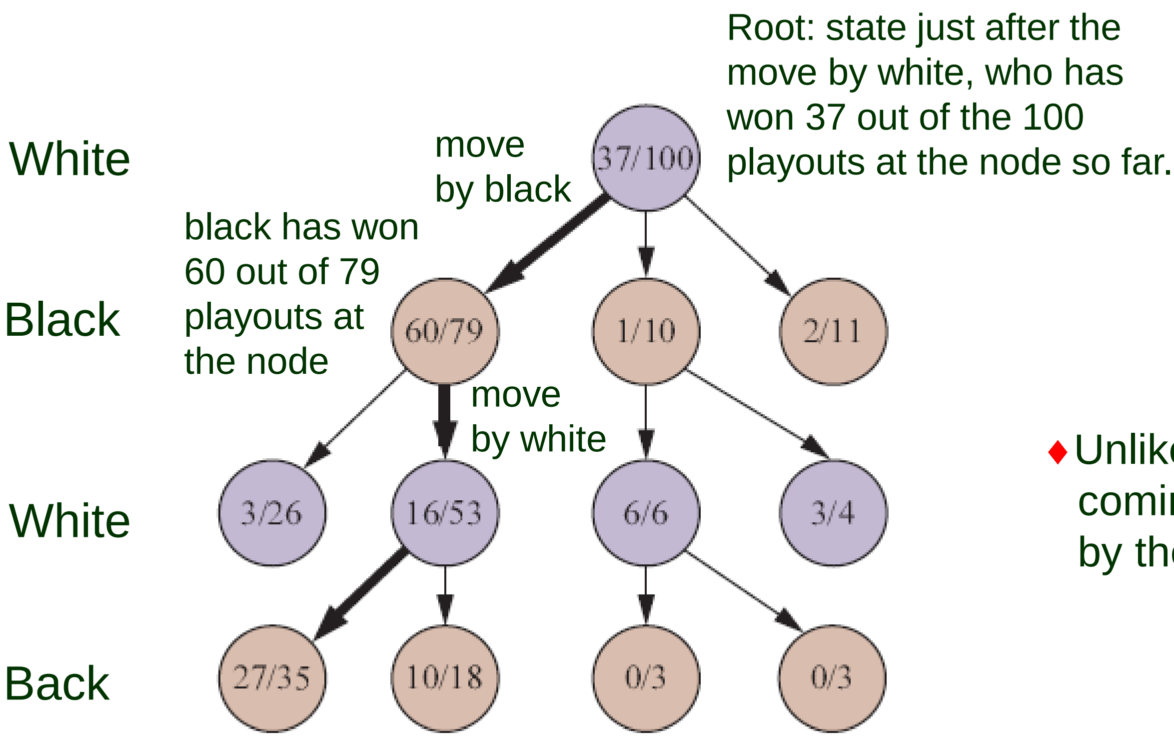

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move by black

black has won 60 out of 79 playouts at the node

Black

move by white

White

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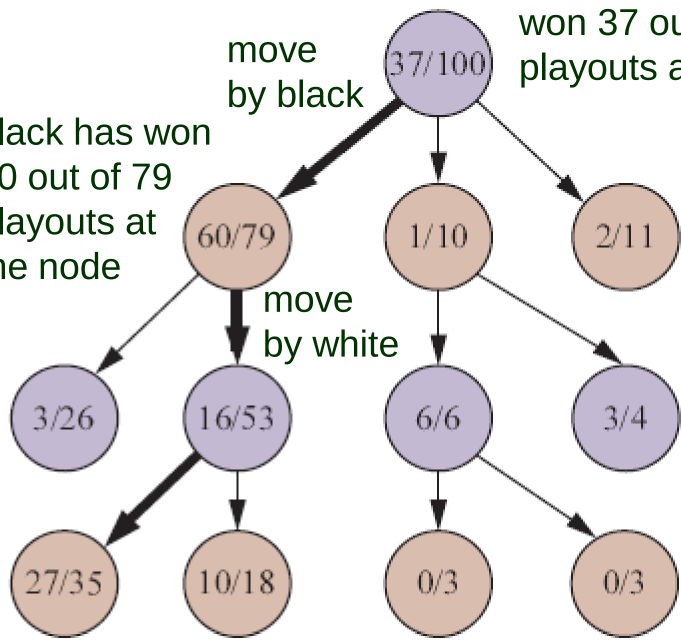

(a) Selection

Also reasonable to select for the purpose of exploration

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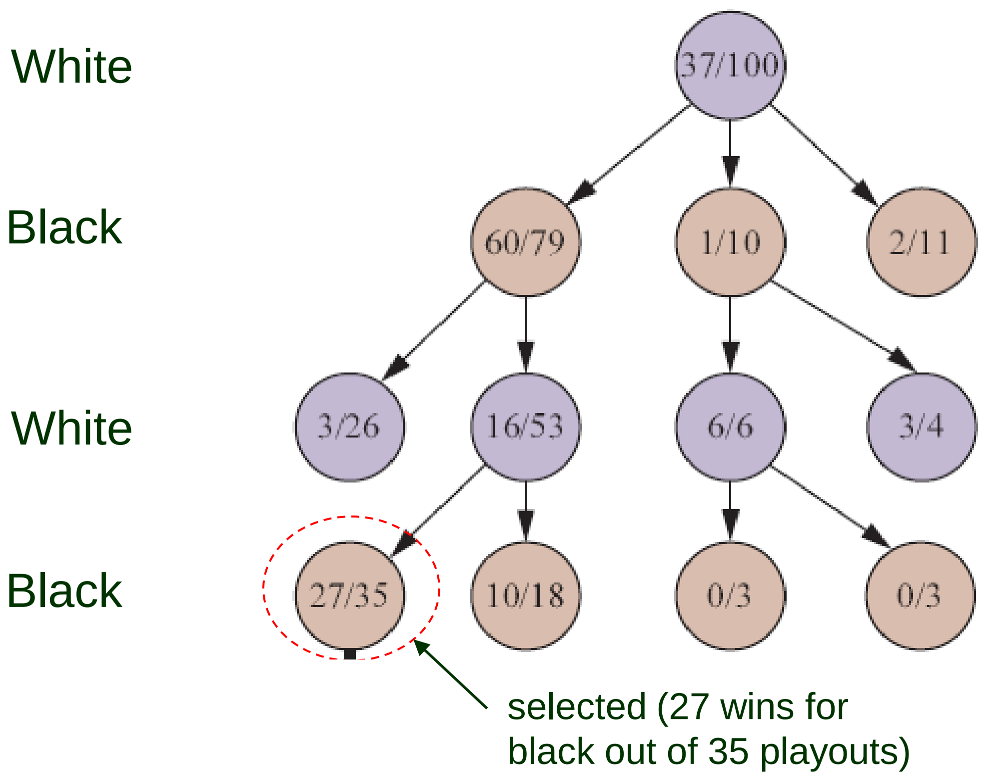
Steps 2 & 3: Expansion & Simulation



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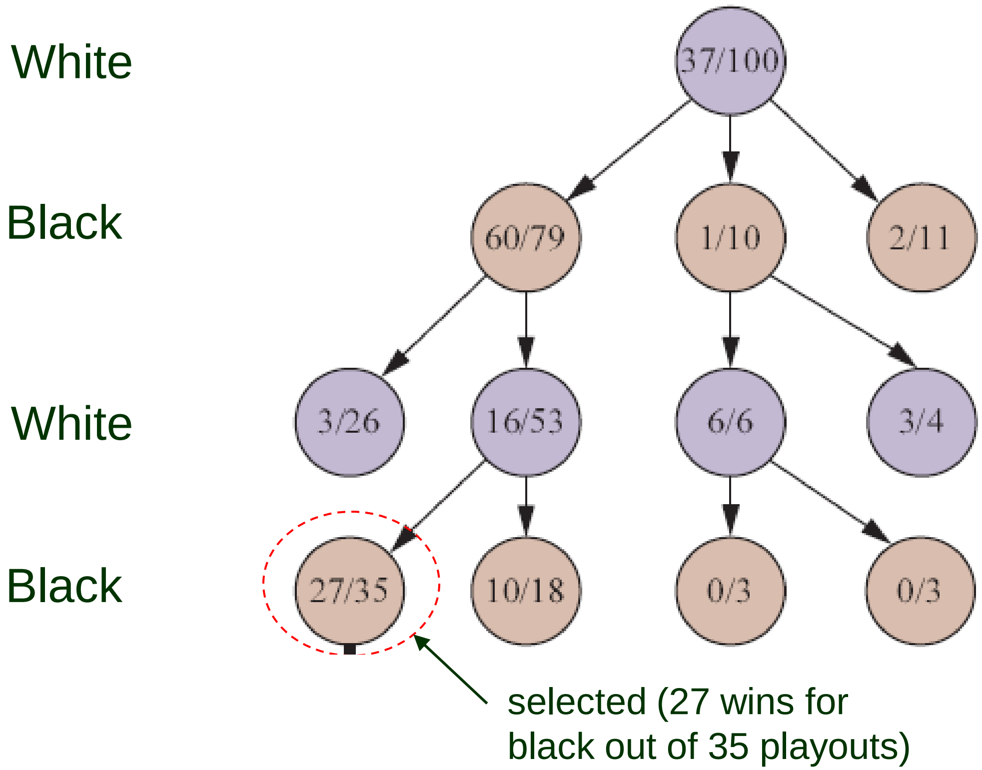

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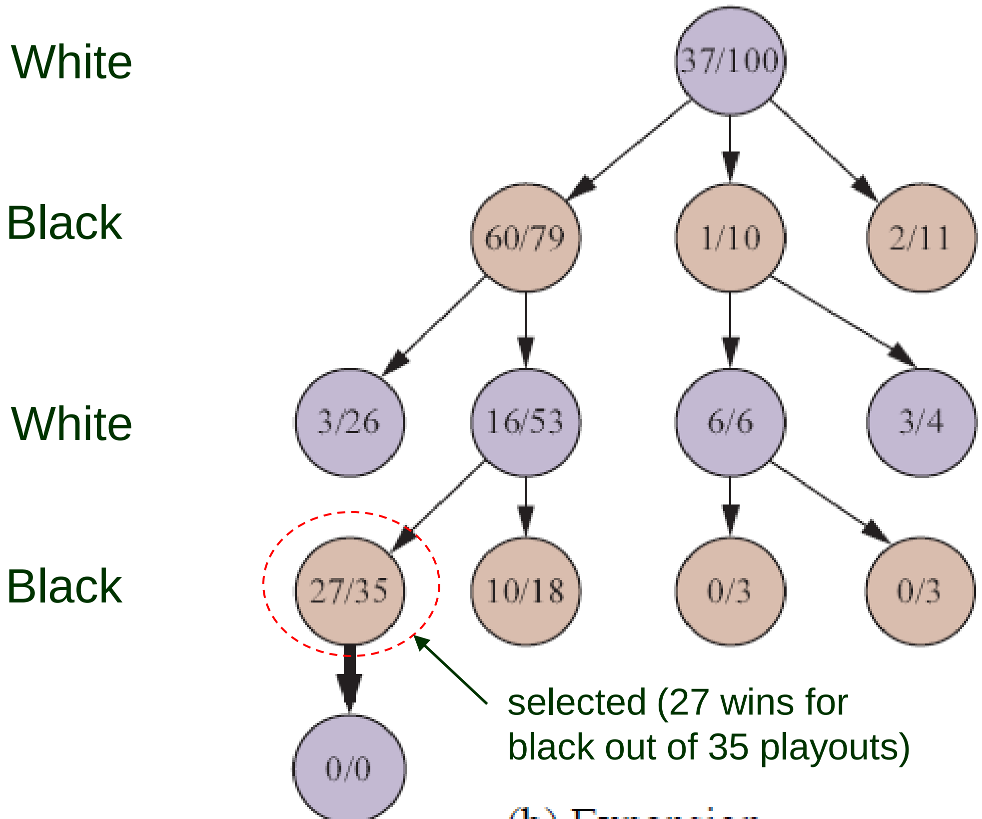

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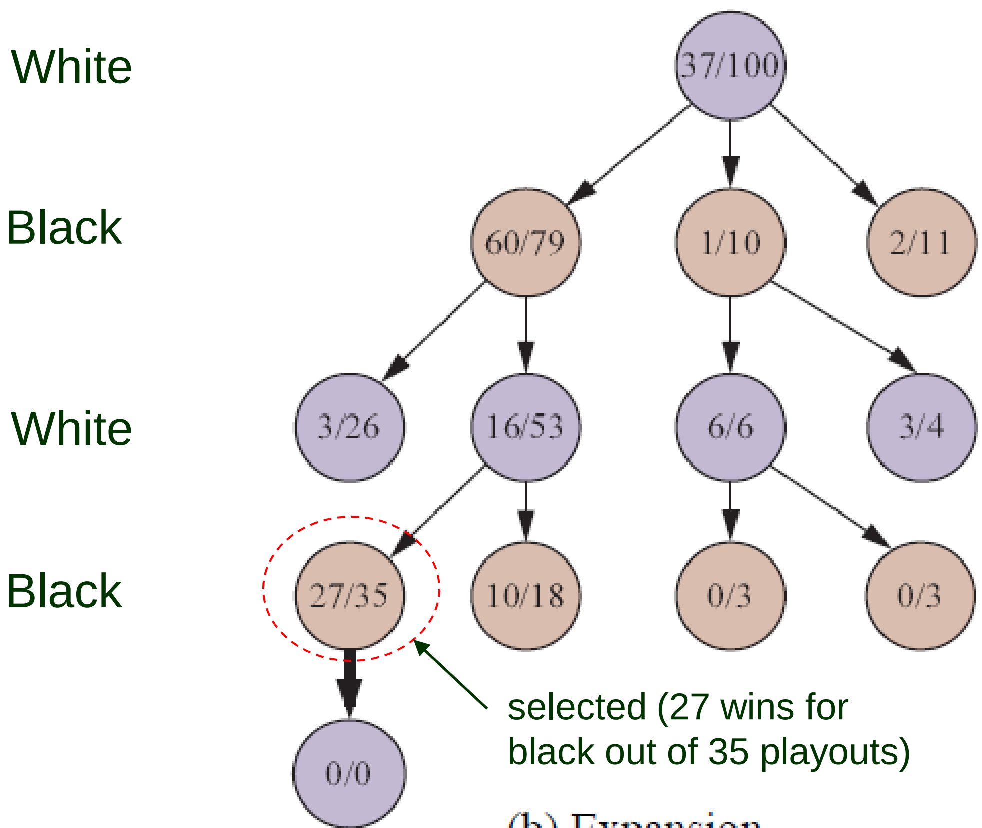

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- Perform a playout from the newly generated child node.

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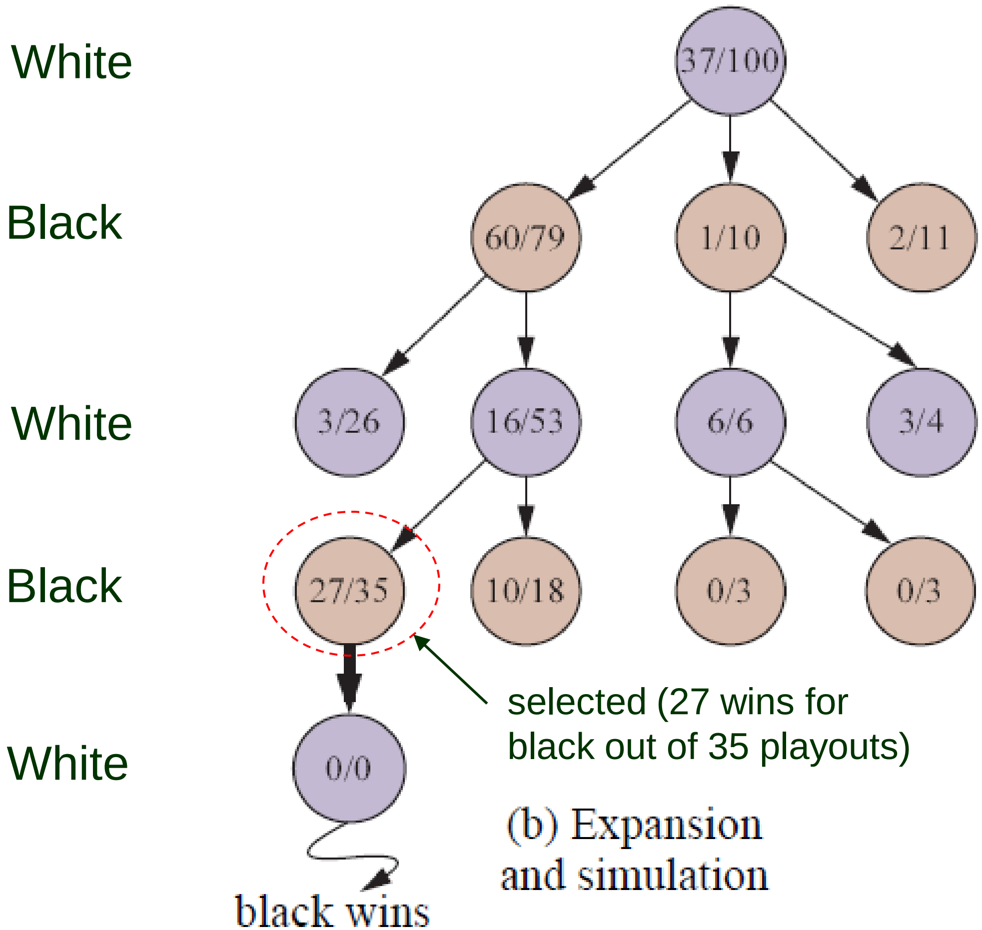

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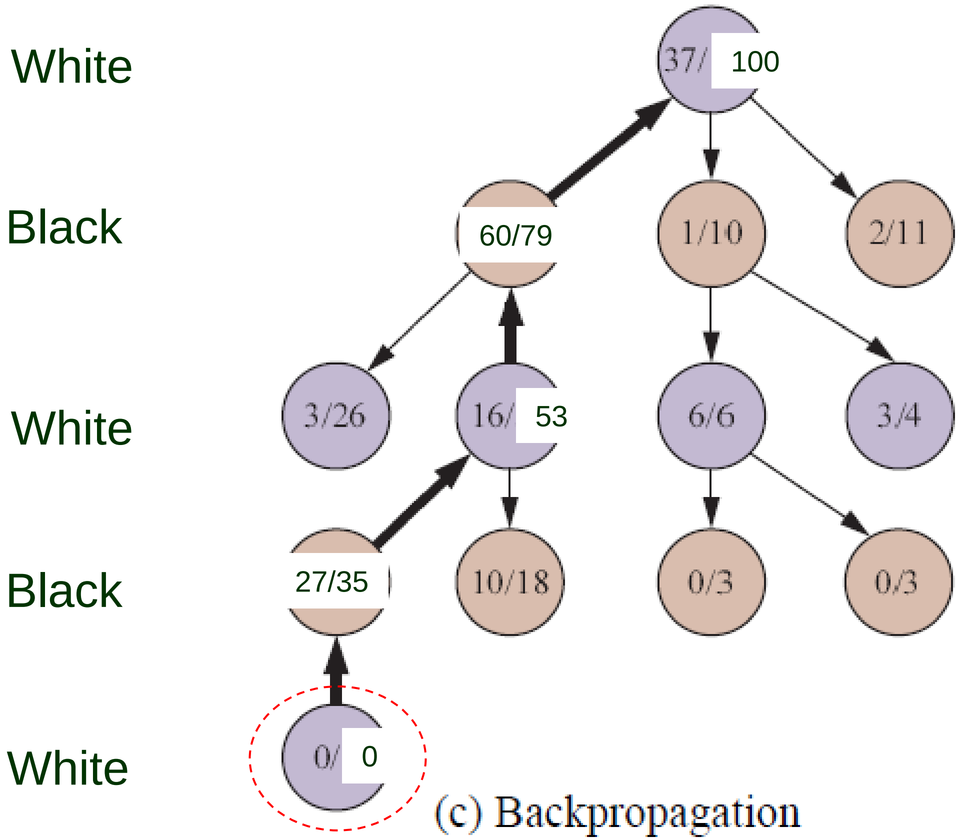

(c) Backpropagation

- Update all the nodes upward along the path until the root.

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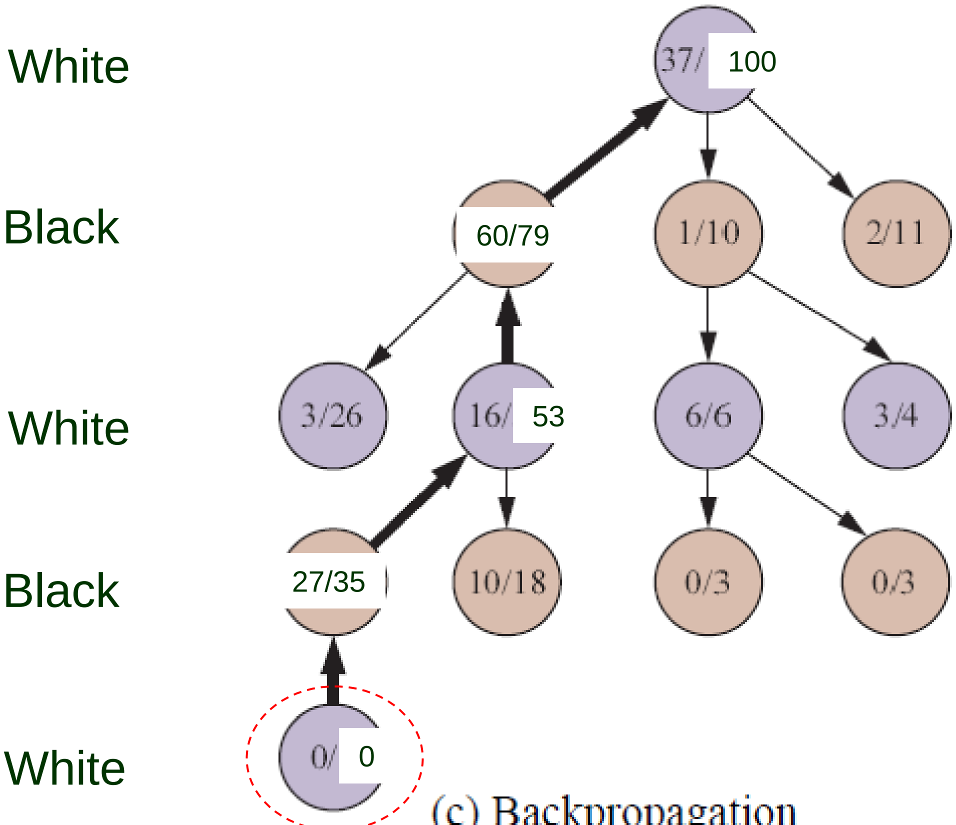

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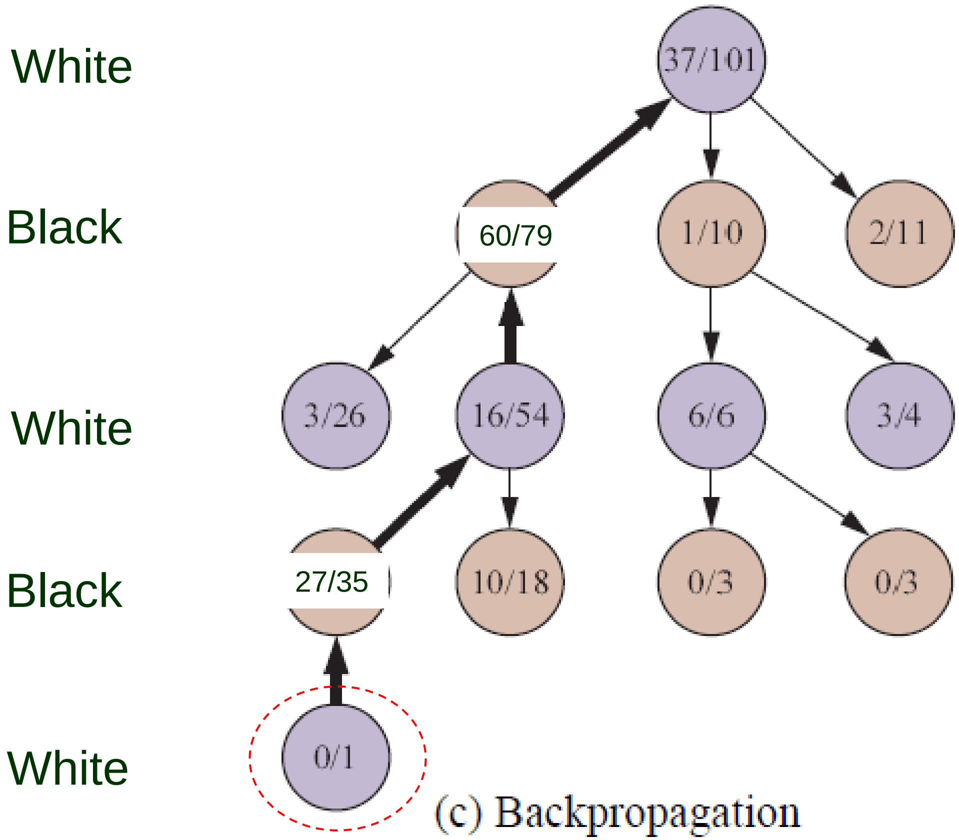

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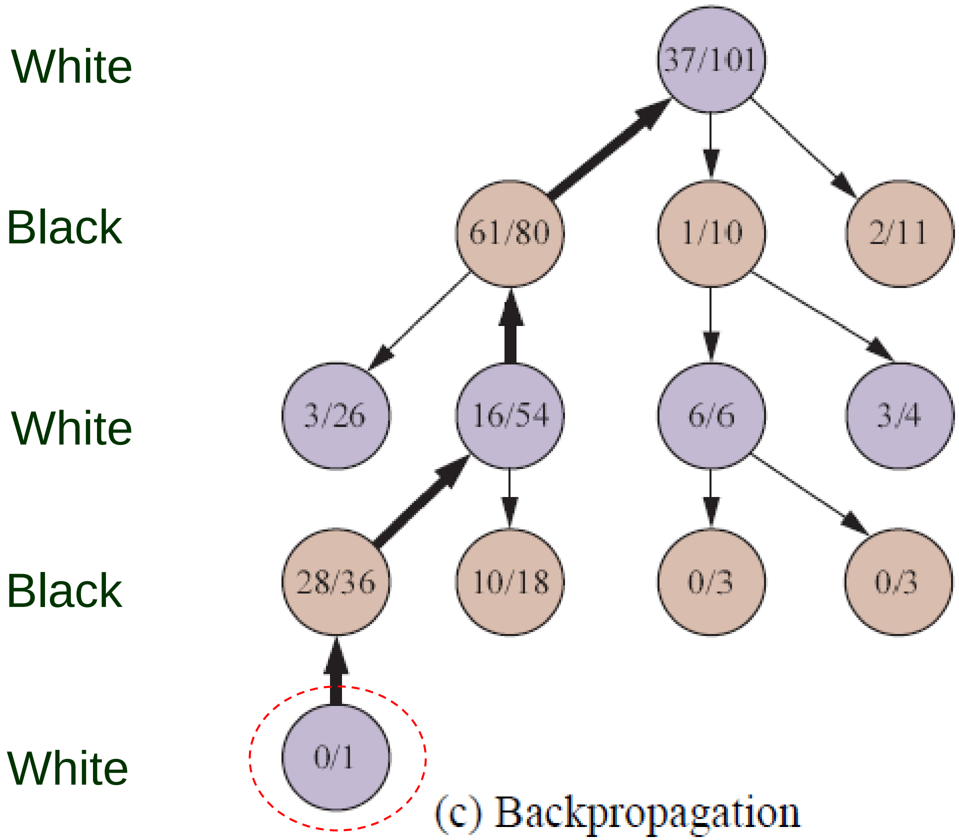

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- Update all the nodes upward along the path until the root.

Black wins this playout:

- ♦ At a white node, increment #playouts only.
- ♦ At a black node, increment #wins and #playouts.

Termination

- MCTS repeats the four steps (selection, expansion, simulation, back-propagation) in order until
  - a set number N of iterations have been performed, or
  - the allotted time has expired.
- It returns the move with the highest number of playouts.

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  - Since better moves are more likely to be chosen, the most promising move is expected to have the highest number of playouts.
  - A node with 65/100 wins is better than one with 2/3 wins (which has a lot of uncertainty).

Monte Carlo Tree Search Algorithm

```
function MONTE-CARLO-TREE-SEARCH(state) returns an action  // decide a move at state.
  tree ← NODE(state)    // initialize the tree with state at the root
  while IS-TIME-REMAINING() do  // each iteration expands the tree by one node.
    leaf ← SELECT(tree)               // the node to be expanded must be a leaf.
    child ← EXPAND(leaf)              // tree is expanded to the node child as a child of leaf.
    result ← SIMULATE(child)          // playout: moves are not recorded in the three.
    BACK-PROPAGATE(result, child)  // update nodes on the path upward to the root.
  return the move in ACTIONS(state) whose node has highest number of playouts
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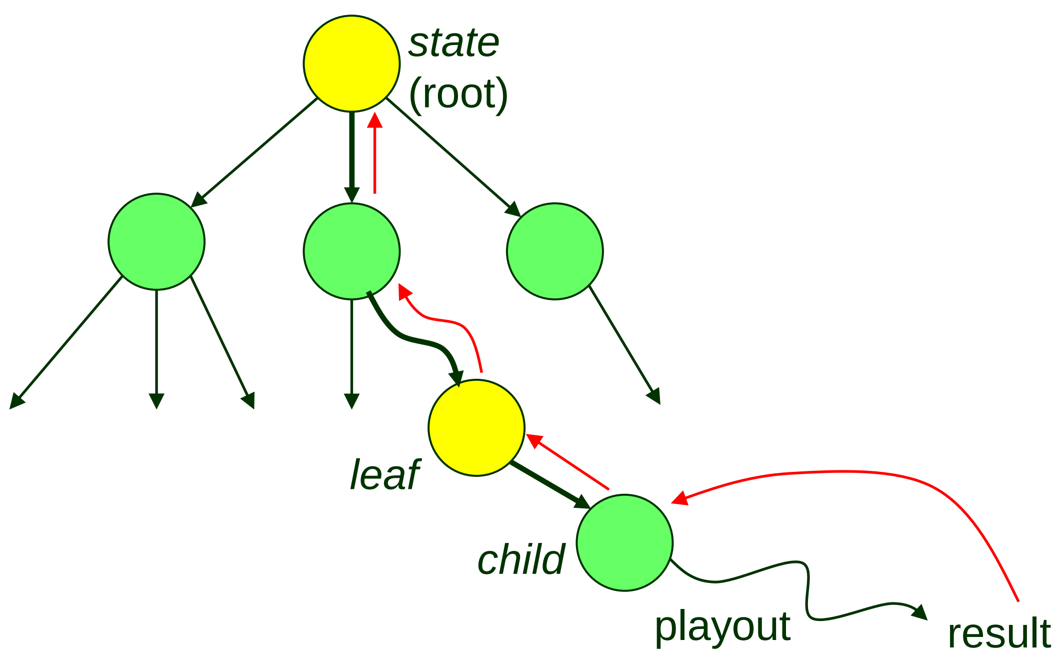

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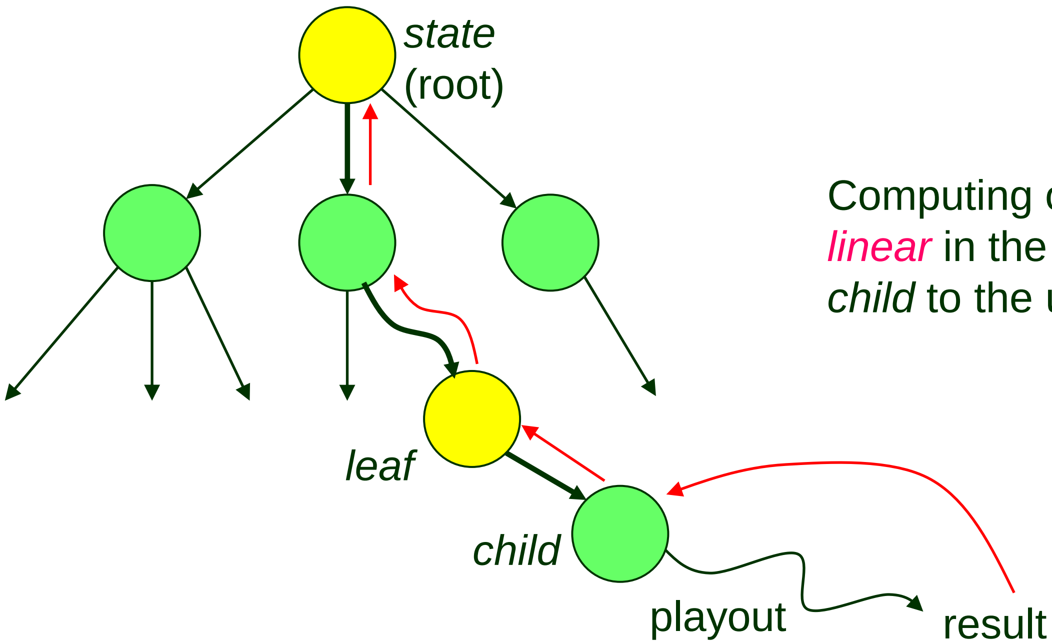
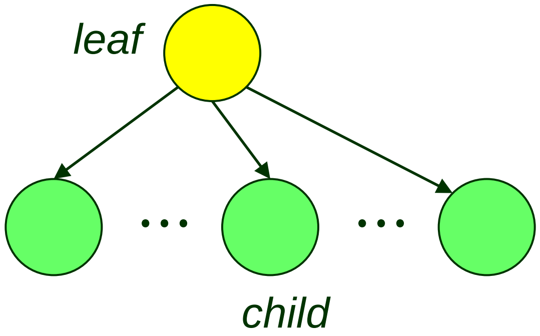


Computing one playout takes time *linear* in the length of the path from *child* to the utility node (result).

Three Issues

♦ *child* ← EXPAND(*leaf*)



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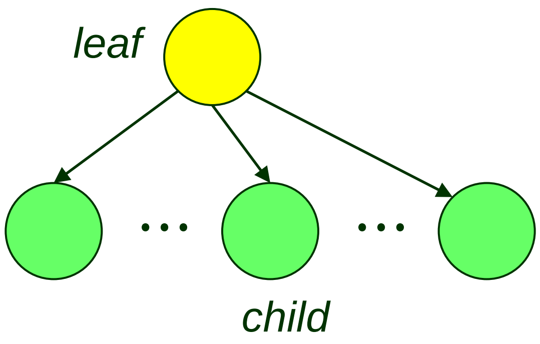

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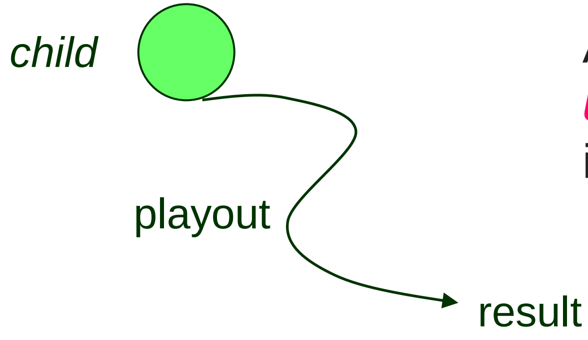

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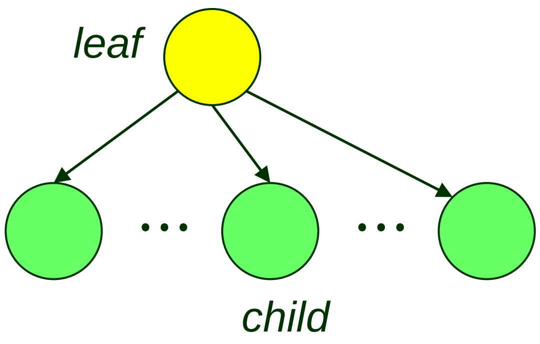

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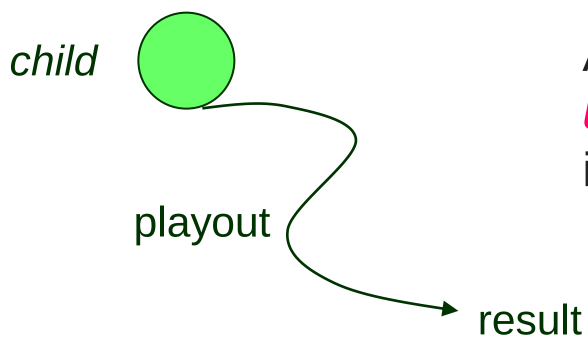

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- IS-TIME-REMAINING()

Pure Monte Carlo search does *N* simulations instead.

Ranking of Possible Moves

Upper confidence bound formula:

$$\mathrm{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}}$$

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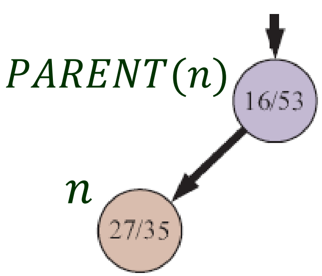

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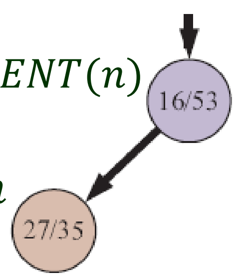
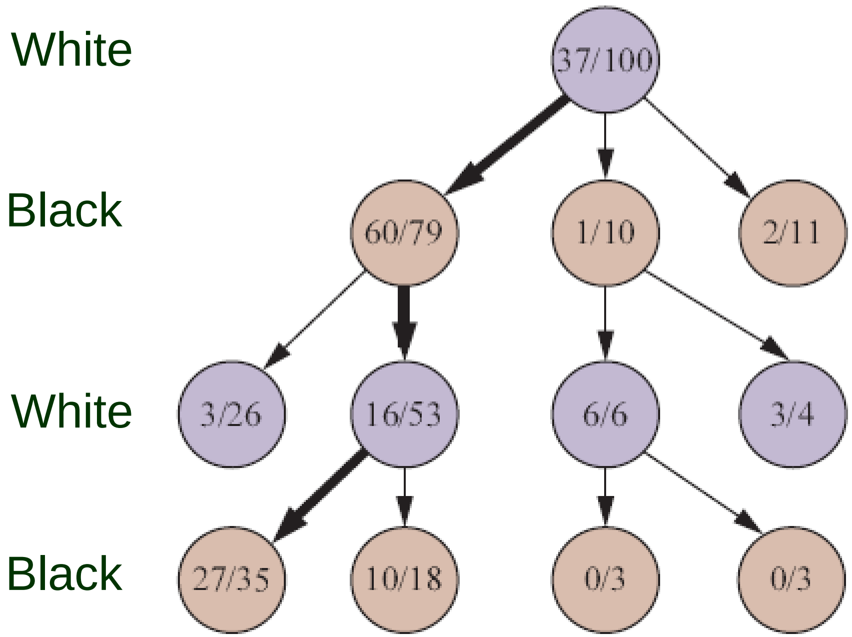
$PARENT(n)$



n

$$U(n) = 27$$
$$N(n) = 35$$
$$N\big(PARENT(n)\big) = 53$$
$$C = \sqrt{2}$$ (choice by a theoretical argument)

Constant C



(a) Selection

- Balances exploitation and exploration.
- Multiple values are tried and the one that performs the best is chosen.
  - $C = 1.4$

    The 60/79 node has the highest score.
  - $C = 1.5$

    The 2/11 node has the highest score.

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- MCTS is less desired than alpha-beta on a game like chess with low b and good evaluation function.

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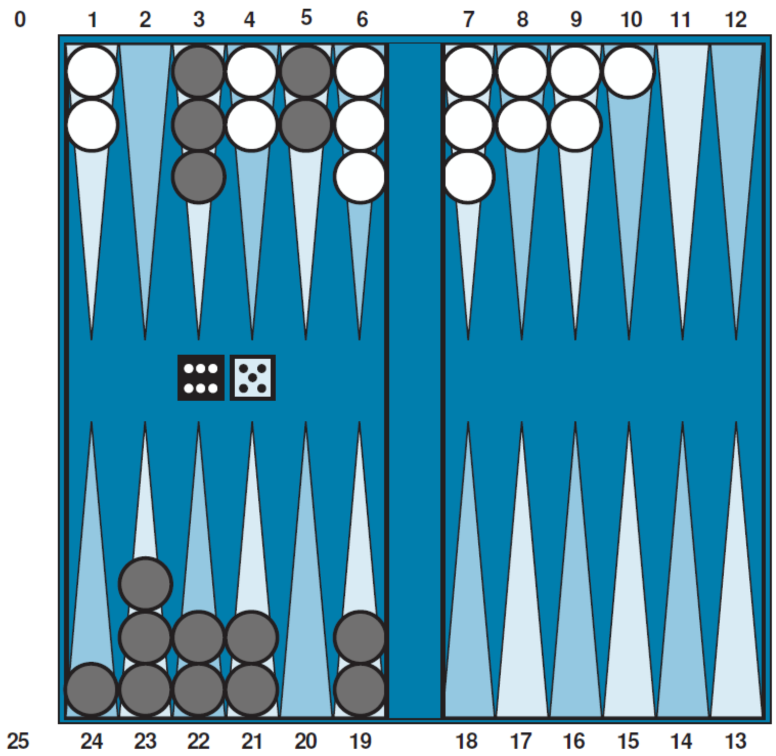

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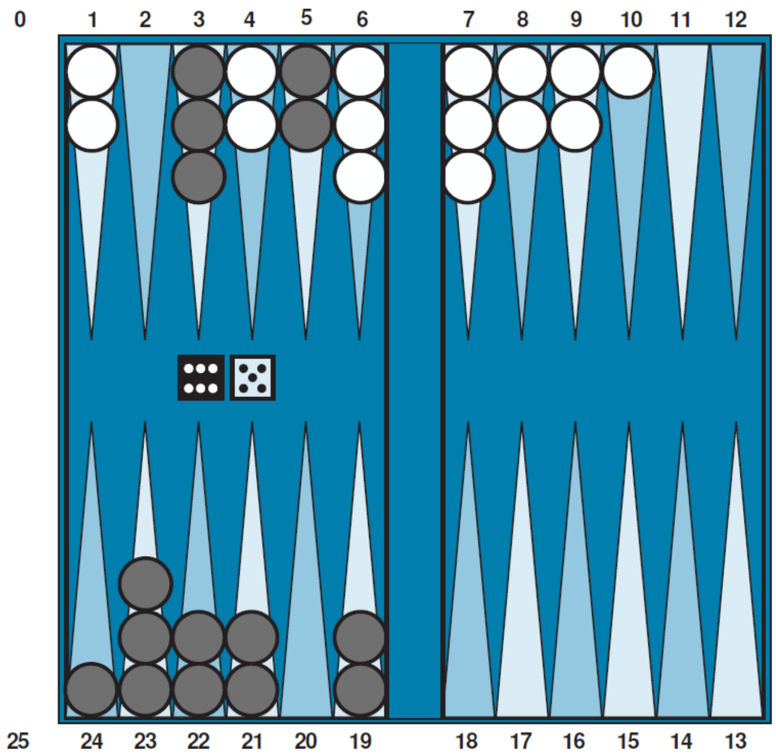

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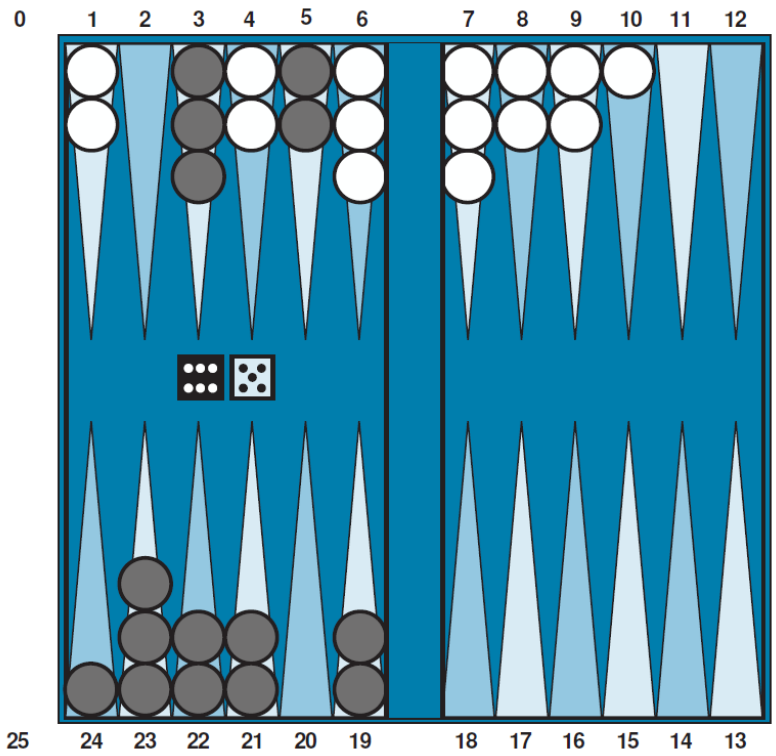

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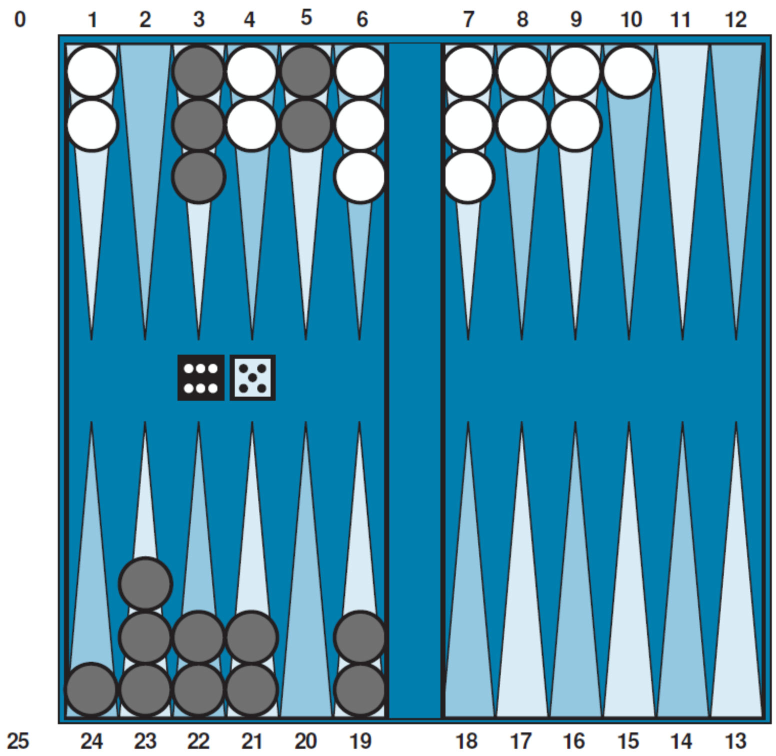

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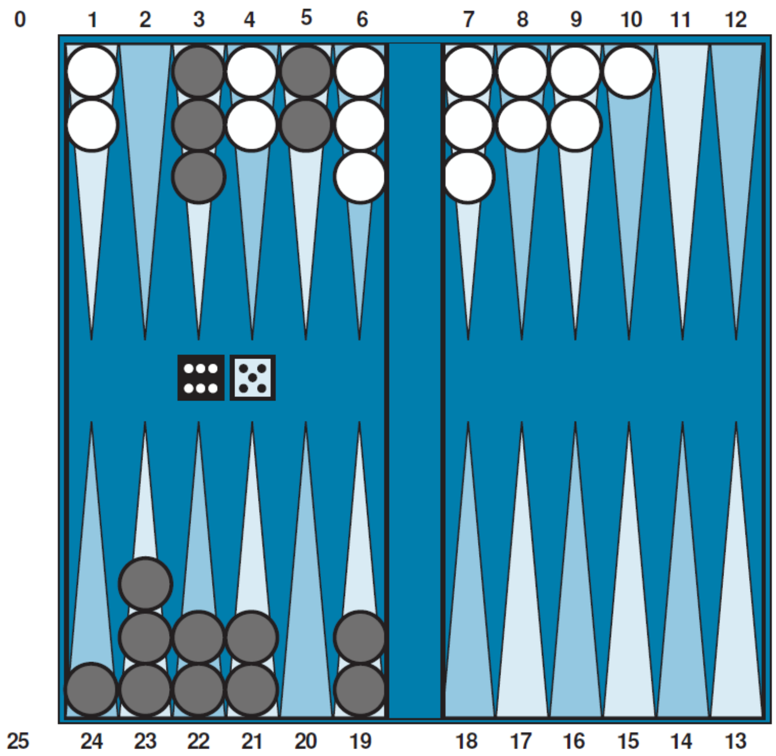

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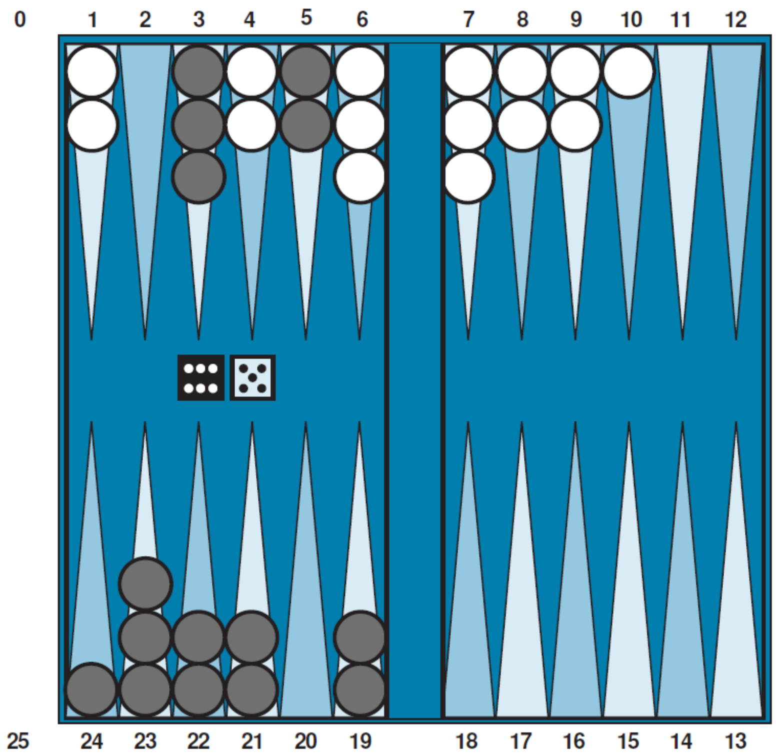

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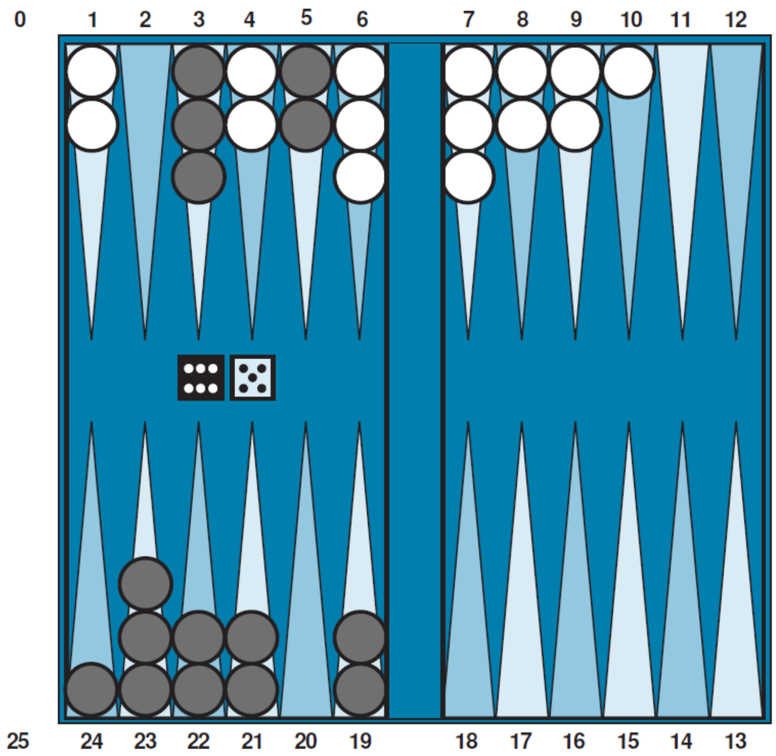

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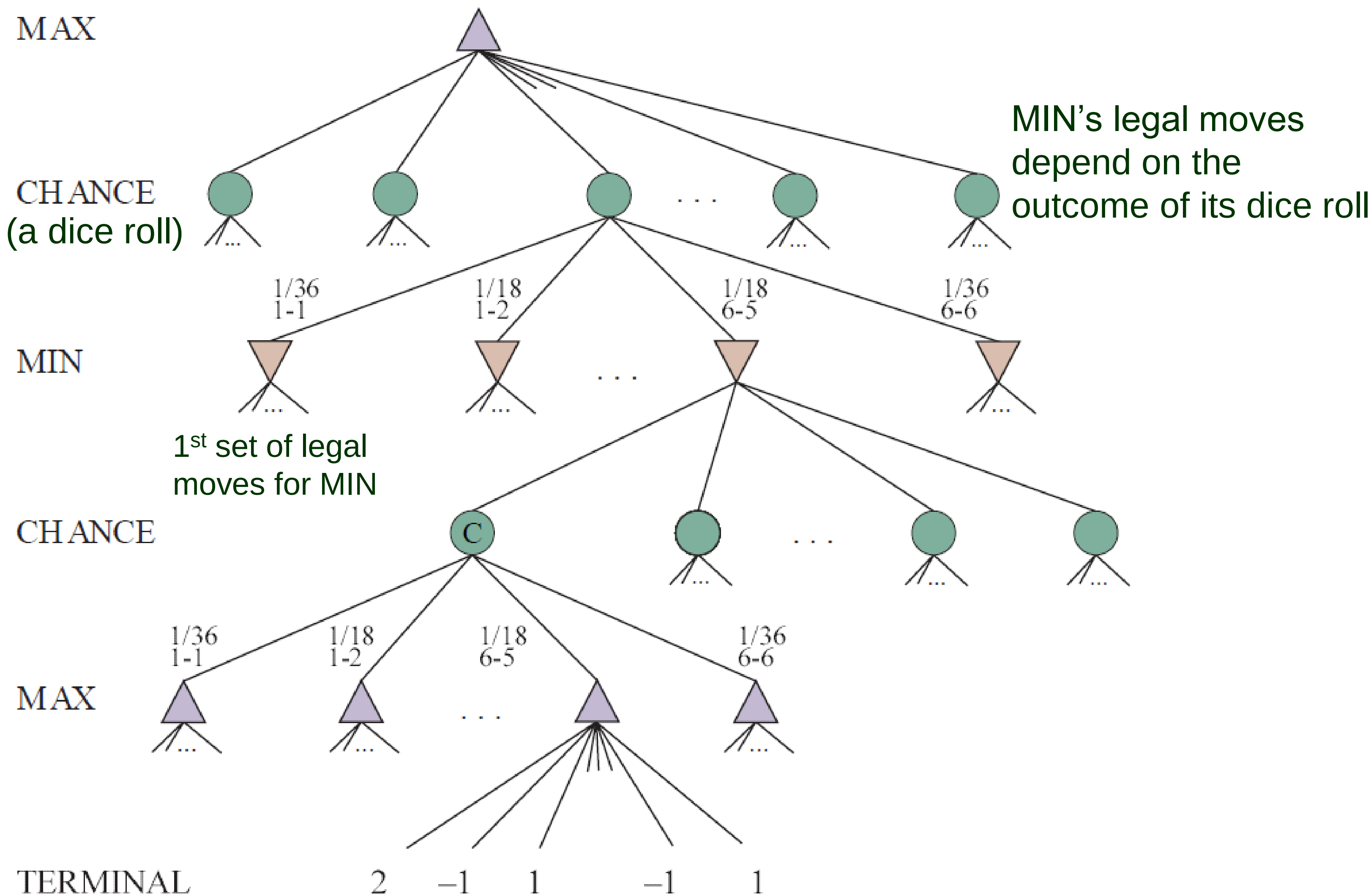
- Calculate expected value (called *expectiminimax* value) of a position.

Game Tree for a Backgammon Position



Expectiminimax Value

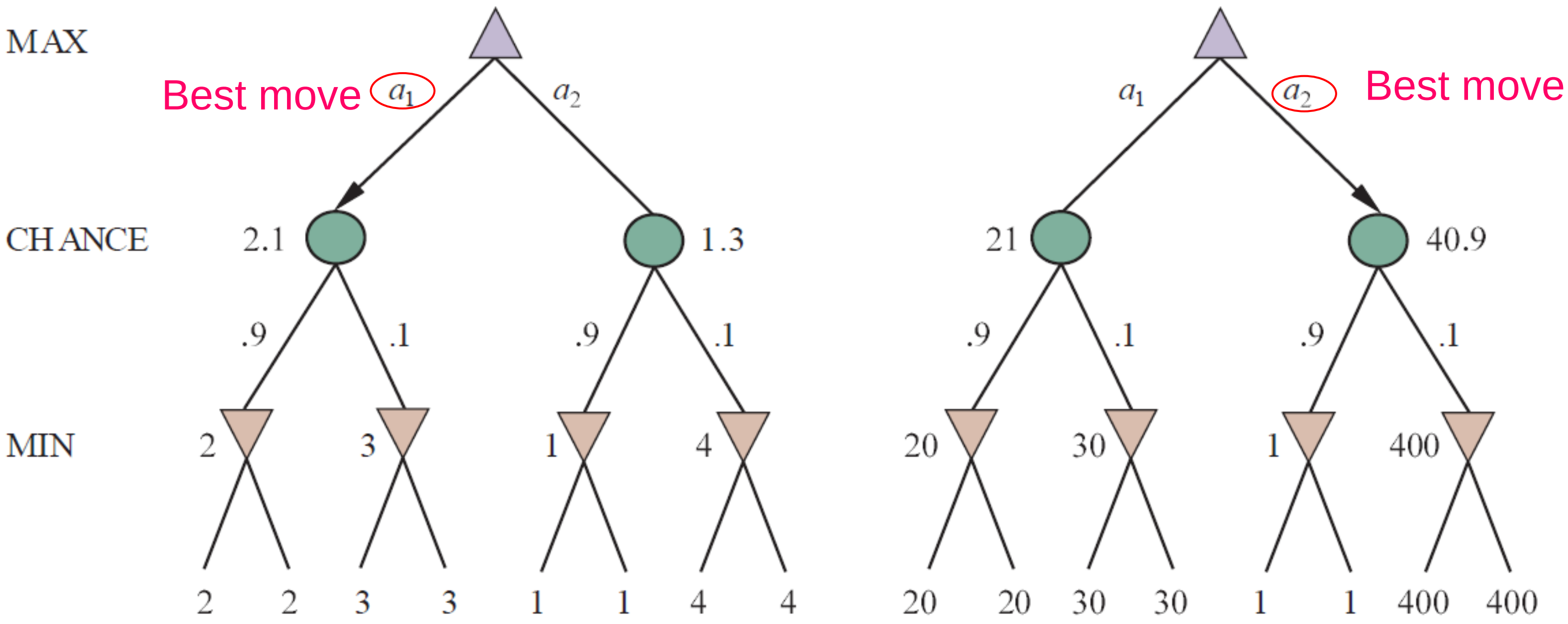
$$\text{EXPECTIMINIMAX}(s) = \begin{cases} \text{UTILITY}(s, \text{MAX}) & \text{if IS-TERMINAL}(s) \\ \max_a \text{EXPECTIMINIMAX}(\text{RESULT}(s,a)) & \text{if TO-MOVE}(s) = \text{MAX} \\ \min_a \text{EXPECTIMINIMAX}(\text{RESULT}(s,a)) & \text{if TO-MOVE}(s) = \text{MIN} \\ \boxed{\sum_r P(r)\ \text{EXPECTIMINIMAX}(\text{RESULT}(s,r))} & \text{if TO-MOVE}(s) = \text{CHANCE} \end{cases}$$

expected value

one possible dice roll

Evaluation Functions

Evaluation functions with the same order of leaf values can yield different move choices at a state.



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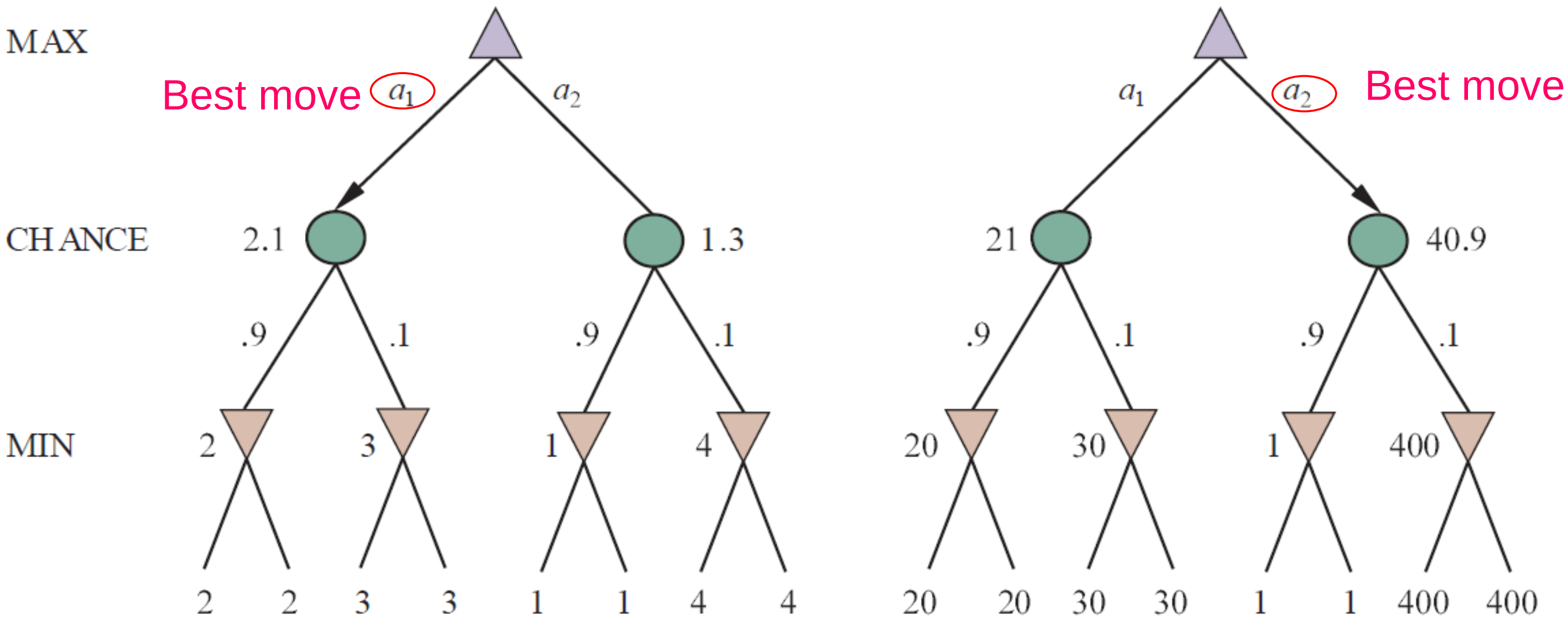

Alpha-beta pruning is still applicable if we can bound values on chance nodes.