Monte Carlo Tree Search & Stochastic Games

Outline

I. Monte Carlo tree search (MCTS)

II. Stochastic games

* Figures/images are from the textbook site (or by the instructor).
I. An Iteration of MCTS – Step 1: Selection

Which move should Black make (at the root)?

Root: state just after the move by white, who has won 37 out of the 100 playouts at the node so far.

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Also reasonable to select for the purpose of exploration.

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Steps 2 & 3: Expansion & Simulation

(b) Expansion and simulation

(selected (27 wins for black out of 35 playouts)
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- Generate a new child of the selected node.

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- Perform a playout from the newly generated child node.

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Step 4: Back Propagation

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- At a black node, increment #wins and #playouts.
Termination

- MCTS repeats the four steps (selection, expansion, simulation, back-propagation) in order until
  - a set number $N$ of iterations have been performed, or
  - the allotted time has expired.

- It returns the move with the highest number of playouts.
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  Why not the highest ratio?
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  - A node with 65/100 wins is better than one with 2/3 wins (which has a lot of uncertainty).
Monte Carlo Tree Search Algorithm

```
function MONTE-CARLO-TREE-SEARCH(state) returns an action // decide a move at state.
    tree ← NODE(state)   // initialize the tree with state at the root
    while IS-TIME-REMAINING() do // each iteration expands the tree by one node.
        leaf ← SELECT(tree)   // the node to be expanded must be a leaf.
        child ← EXPAND(leaf)   // tree is expanded to the node child as a child of leaf.
        result ← SIMULATE(child)  // playout: moves are not recorded in the three.
        BACK-PROPAGATE(result, child) // update nodes on the path upward to the root.
    return the move in ACTIONS(state) whose node has highest number of playouts
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Computing one playout takes time linear in the length of the path from child to the utility node (result).
Three Issues

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- $\text{IS\text{-TIME\text{-REMAINING}}()}$

Pure Monte Carlo search does $N$ simulations instead.
Ranking of Possible Moves

Upper confidence bound formula:

$$\text{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\text{PARENT}(n))}{N(n)}}$$
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- **Exploration term:** \( U(n)/N(n) \) + \( C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}} \)

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- **Exploitation term:**
  - #playouts through the node

- **Exploration term:**
  - balance between exploitation and exploration

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Balance between exploitation and exploration
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\( U(n) = 27 \)
\( N(n) = 35 \)
\( N(PARENT(n)) = 53 \)

\( C = \sqrt{2} \) (choice by a theoretical argument)
Constant $C$

- Balances exploitation and exploration.
- Multiple values are tried and the one that performs the best is chosen.

- $C = 1.4$
  The 60/79 node has the highest score.

- $C = 1.5$
  The 2/11 node has the highest score.
More on MCTS

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- MCTS is less desired than alpha-beta on a game like chess with low $b$ and good evaluation function.
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- 1-1, ..., 6-6: probability $\frac{1}{36}$ each
- 1-2, ..., 1-6, 2-3, ..., 5-6: probability $\frac{1}{18}$ each
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15
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- Calculate expected value (called expectiminimax value) of a position.
Game Tree for a Backgammon Position

MIN’s legal moves depend on the outcome of its dice roll.

1\textsuperscript{st} set of legal moves for MIN

MAX

CHANCE
(a dice roll)
**Expectiminimax Value**

\[
\text{EXPECTIMINIMAX}(s) =
\begin{cases}
\text{UTILITY}(s, \text{MAX}) & \text{if Is-TERMINAL}(s) \\
\max_a \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) & \text{if } \text{TO-MOVE}(s) = \text{MAX} \\
\min_a \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) & \text{if } \text{TO-MOVE}(s) = \text{MIN} \\
\sum_r P(r) \text{EXPECTIMINIMAX}(\text{RESULT}(s, r)) & \text{if } \text{TO-MOVE}(s) = \text{CHANCE}
\end{cases}
\]

- one possible dice roll
- expected value
Evaluation Functions

Evaluation functions with the same order of leaf values can yield different move choices at a state.
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Alpha-beta pruning is still applicable if we can bound values on chance nodes.