Alpha-Beta Pruning & MCTS

Outline

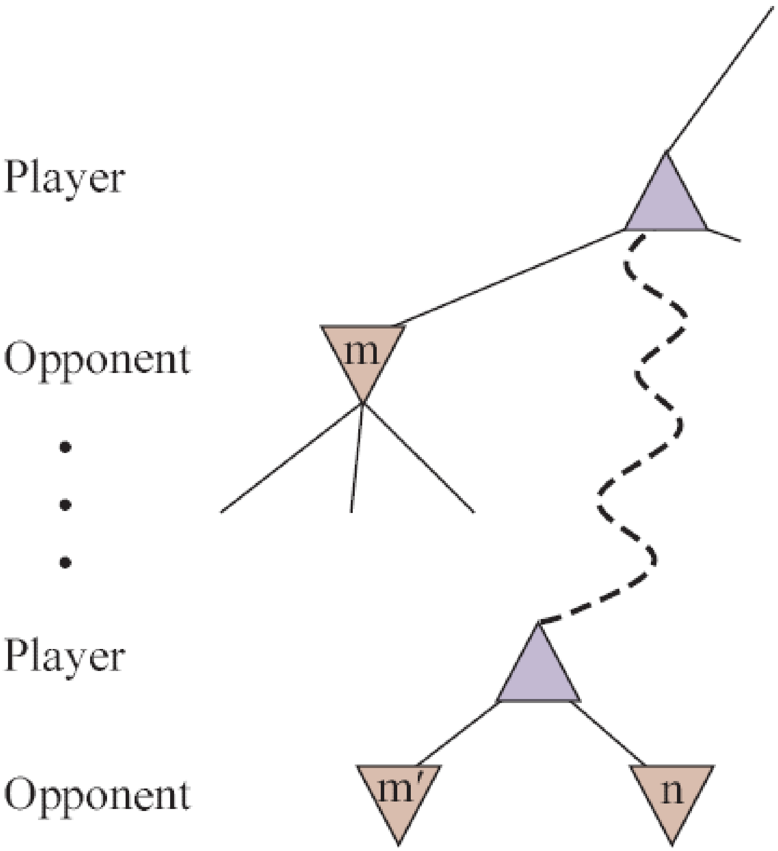
I. Alpha-beta pruning algorithm

II. Heuristic alpha-beta tree approach

III. Monte Carlo tree search (MCTS)

* Figures/images are from the textbook site (or by the instructor). Otherwise, the source is cited unless such citation would make little sense due to the triviality of generating the image.

I. Node Pruning



The player will not move to node n if it has a better choice

- either at the same level (e.g., m')
- or at any node (e.g., m) higher up in the tree.

Prune n once we have found enough about it to reach the above conclusion.

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- Update the values of α and β as the search goes along.
- Prune the remaining branches at a MIN node with current value $\leq \alpha$ or at a MAX node with current value $\geq \beta$.

Alpha-Beta Search Algorithm (3rd Edition of Textbook)

function ALPHA-BETA-SEARCH(*state*) **returns** an action
v ←MAX-VALUE(*state*, $-\infty, +\infty$)
return the action in ACTIONS(*state*) with value v

function MAX-VALUE (*state*,α, β) **returns** *a utility value*
if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
$v \leftarrow -\infty$
for each a in ACTIONS(*state*) **do**
v ←MAX(v, MIN-VALUE(RESULT(s, a),α, β))
if $v \geq \beta$ **then return** v
α ←MAX(α, v)
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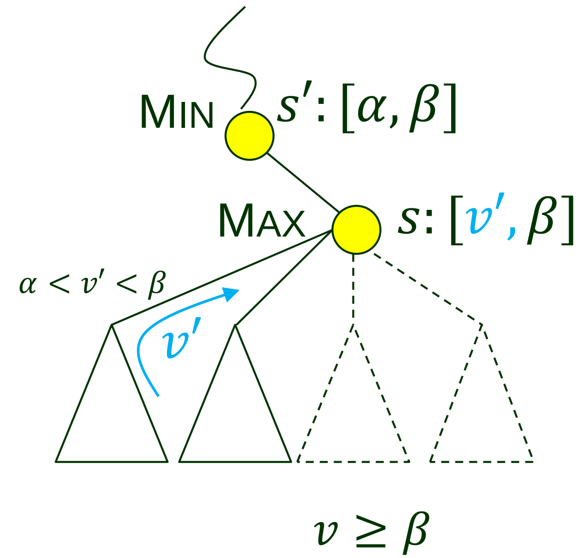

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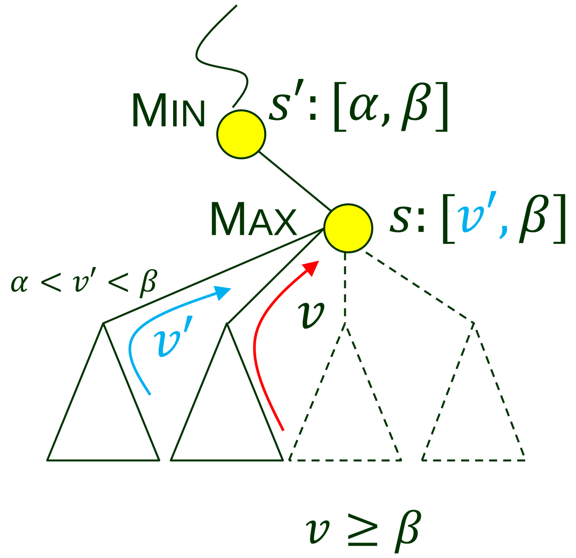

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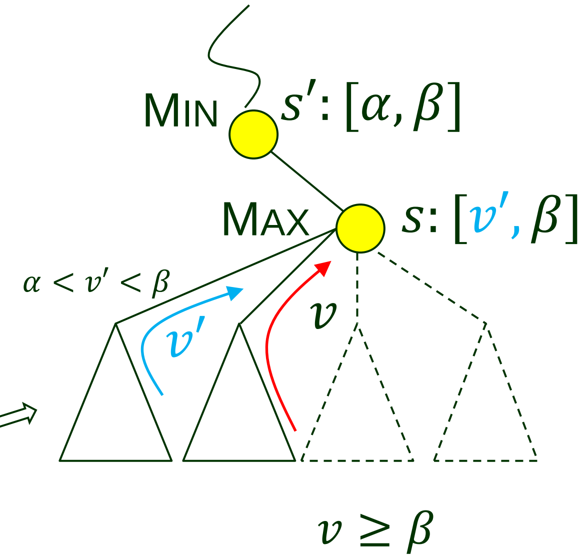

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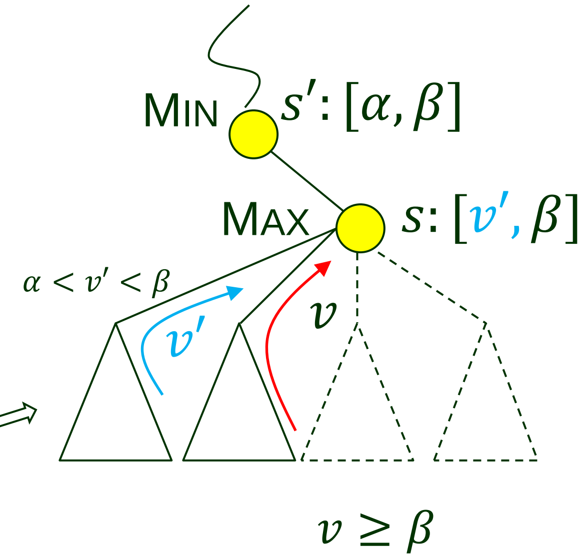

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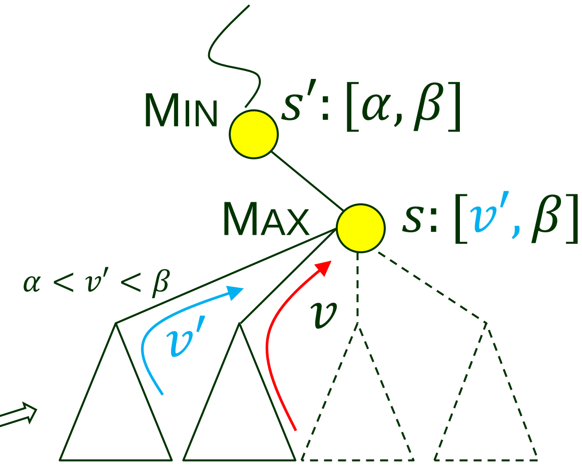

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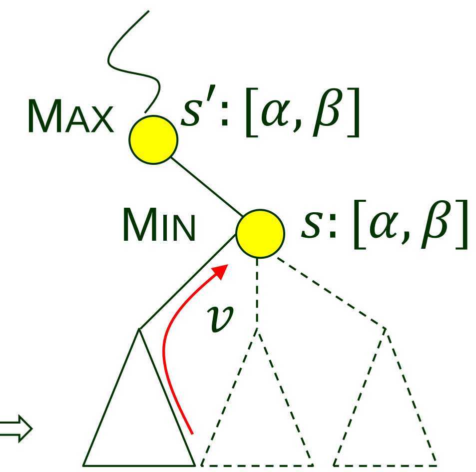

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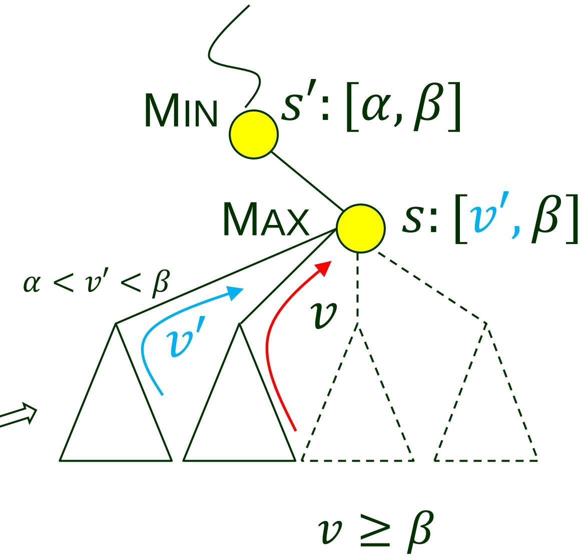

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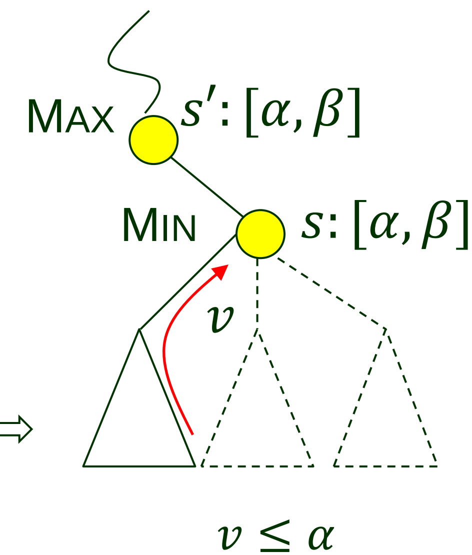

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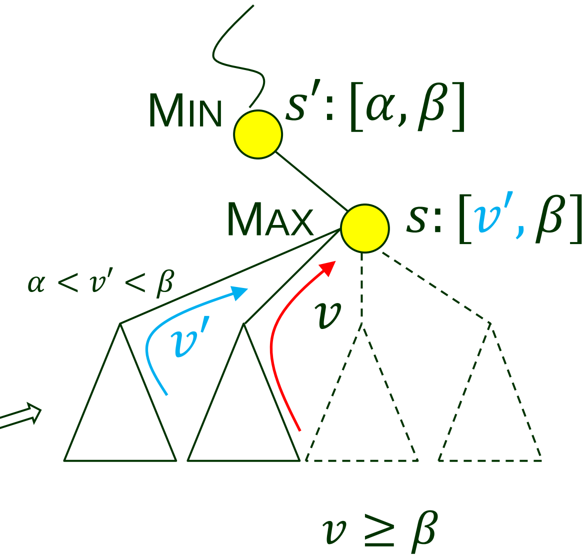

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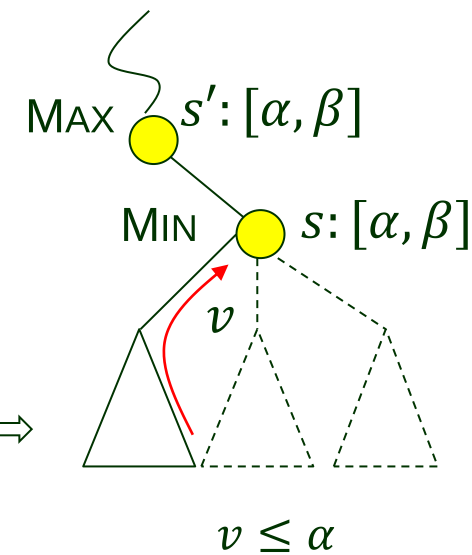
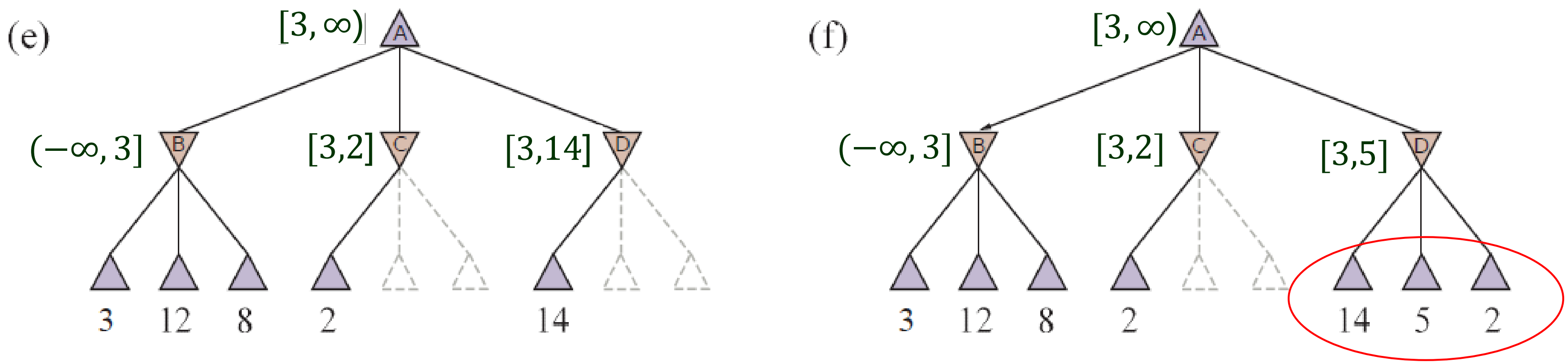


Move Ordering



Successors 14 and 5 would've been pruned had 2 been generated first.

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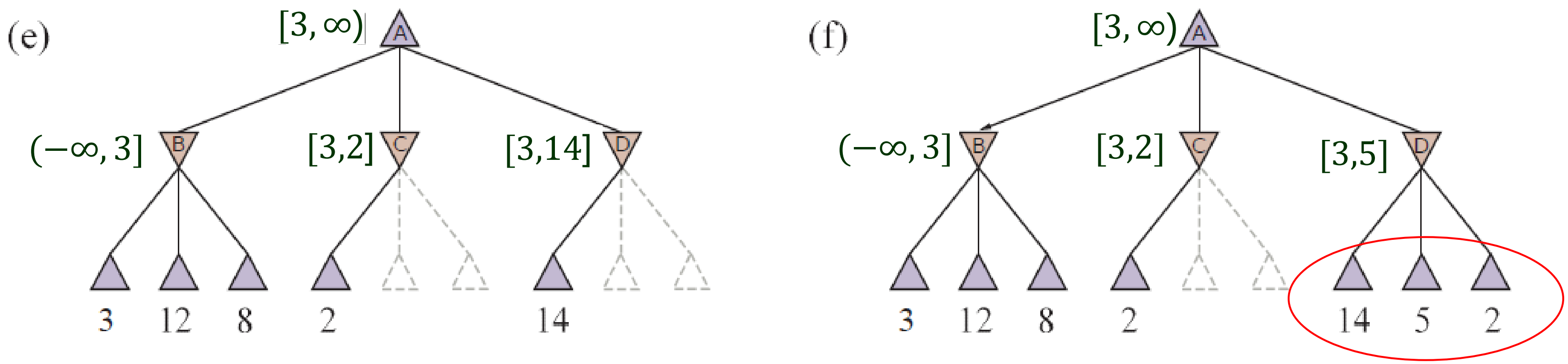

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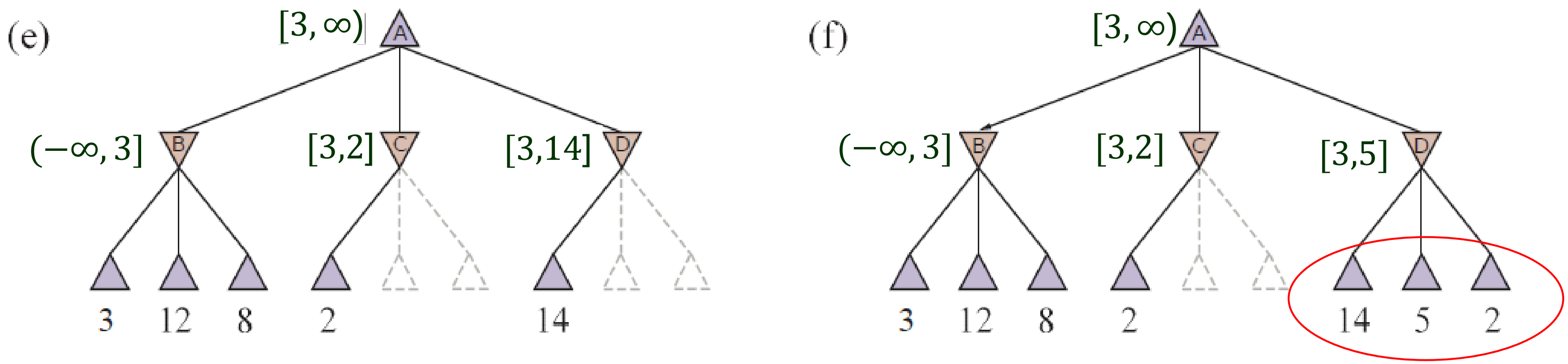

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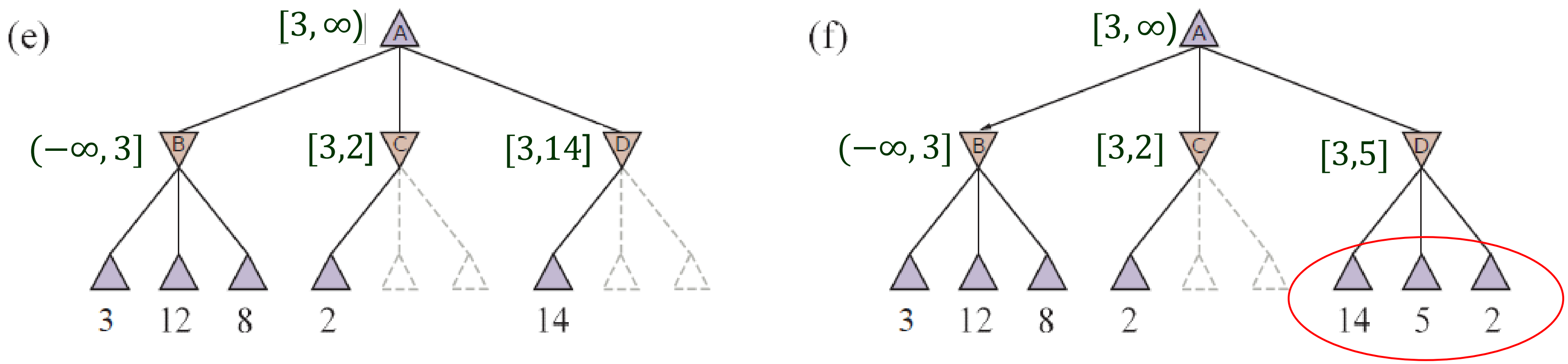

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- $O(b^{3m/4})$ nodes for random move ordering.

Two Strategies

- For chess, a simple ordering function (sequentially considering captures, threats, forward moves, backward moves) could get close to the best case.
- Even with alpha-beta pruning and clever move ordering, minimax won't work well enough for games like chess and Go due to their vast state spaces.

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- Type A (heuristic alpha-beta tree search) – chess
  - Considers all possible moves to a certain depth.
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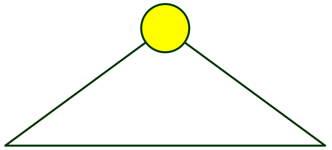

Explores wide & shallow portion of the search tree.

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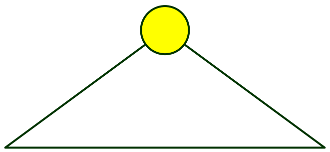
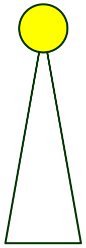


Explores wide & shallow portion of the search tree.

- Type B (Monte Carlo tree search) – Go
  - ♣ Ignore moves that look bad.
  - ♣ Follow promising lines "as far as possible".



Explores deep but narrow portion of the search tree.

II. Heuristic Alpha-Beta Tree Search

- Cut off the search early by applying a heuristic evaluation function.
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$$\text{H-MINIMAX}(s,d) =$$

$$\begin{cases} \text{EVAL}(s, \text{MAX}) & \text{if IS-CUTOFF}(s,d) \\ \max_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if TO-MOVE}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if TO-MOVE}(s) = \text{MIN} \end{cases}$$

Evaluation Functions

$\text{EVAL}(s, p)$ returns an estimate of the expected utility s to player p.

- $\text{EVAL}(s, p) = \text{UTILITY}(s, p)$ if s is terminal;
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Criteria:

- No excessive computation time.
- Strong correlation with actual chances of winning.

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- Too many categories and too much dependence on experience.

Eval Function: Feature Combination

Compute separate numerical contributions from each feature and combine them.

$$\boxed{\textsc{Eval}(s) = \underset{\substack{\uparrow \\ \text{weight}}}{w_1} \underset{\substack{\uparrow \\ \text{e.g., \#pawns in chess}}}{f_1}(s) + w_2 f_2(s) + \cdots + w_n f_n(s)}$$

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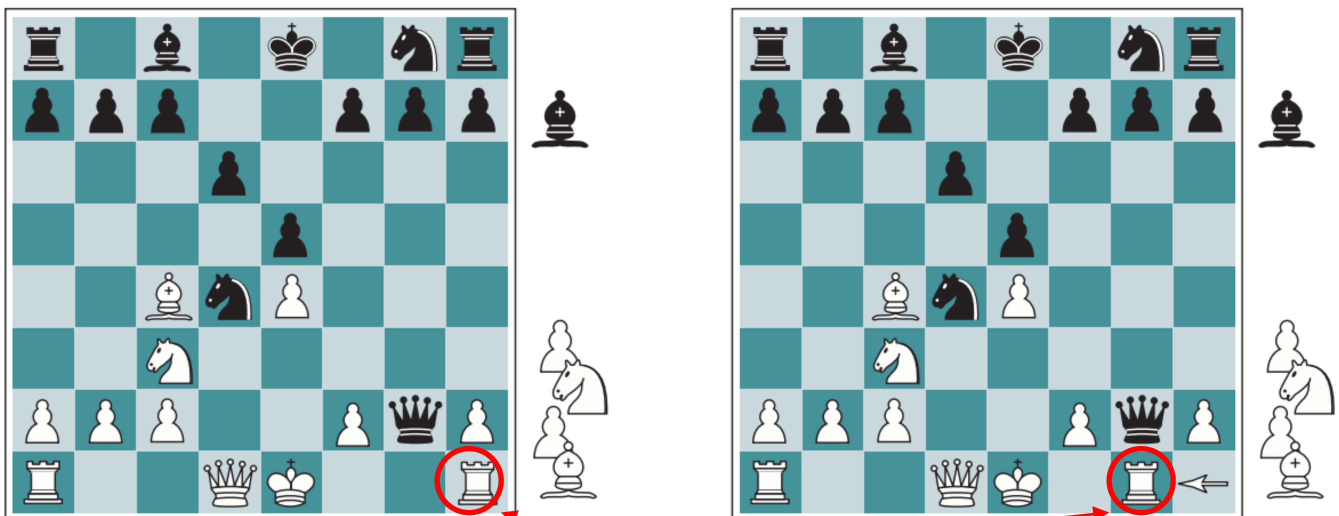

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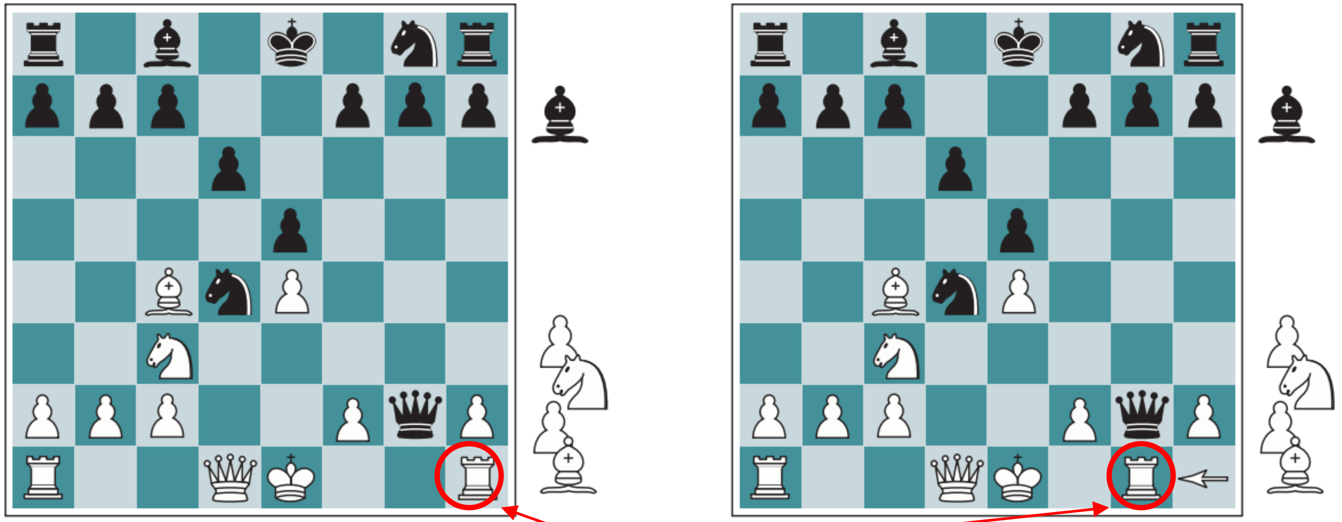

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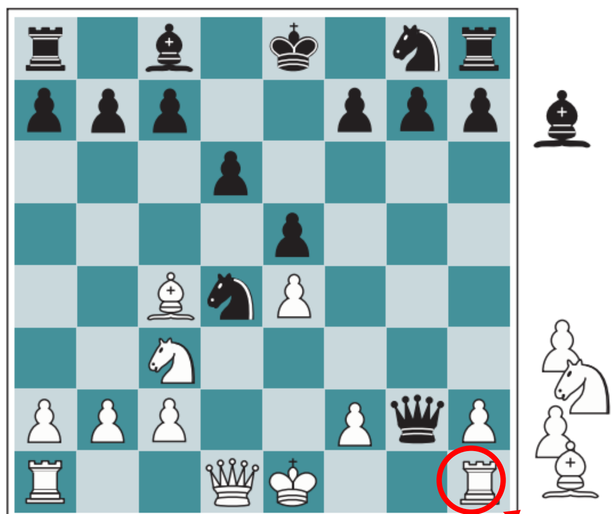

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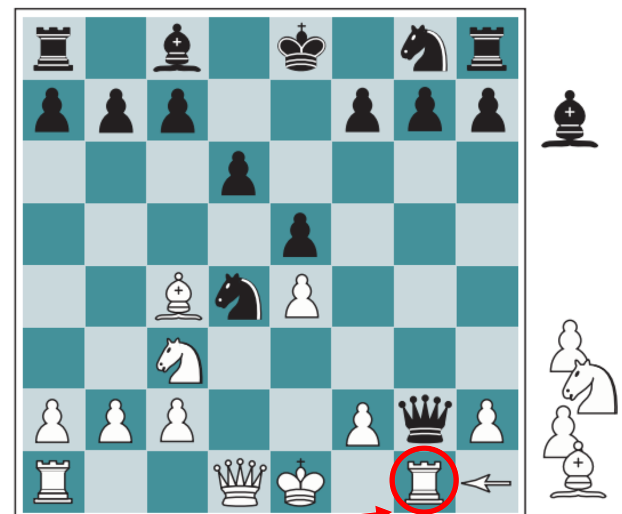

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White should win because its rook will capture the queen.

Eval Function (cont'd)

$$\boxed{\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s)}$$

♠ Assumes independent feature contributions.

♦ Use a nonlinear feature combination.

e.g., two bishops might be worth more than twice the value of a single bishop.

Cutting off Search

if $game.\cancel{\text{IS-TERMINAL}}(state)$ **then return** $game.\cancel{\text{UTILITY}}(state, player), null$

IS-CUTOFF EVAL

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Some strategies:

- Set a fixed depth limit d to control the amount of search.

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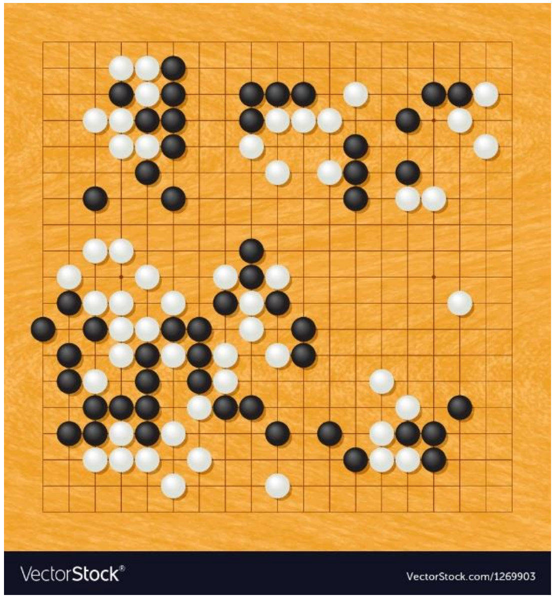
- Apply iterative deepening:

  When time runs out, returns the move selected by the deepest completed search.

Real-Time Decisions

- Minimax with alpha-beta pruning.
- Extensively tuned evaluation function.
- Pruning heuristics.
- A transposition table of repeated states and evaluations.
  - To avoid re-searching the game tree below that state.
- A large database of optimal opening and endgame moves.
  - Table lookup instead of search.
  - Chess endgames with up to 7 pieces solved.
- Minimax unsuccessful in Go.

III. Monte Carlo Tree Search



Two weaknesses of heuristic alpha-beta search on Go:

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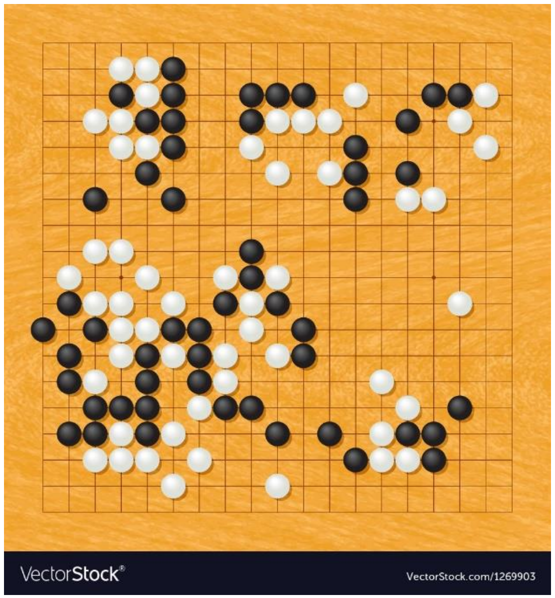

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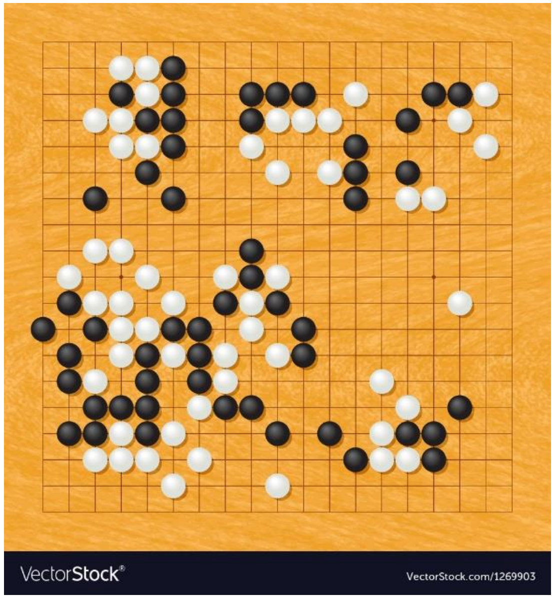

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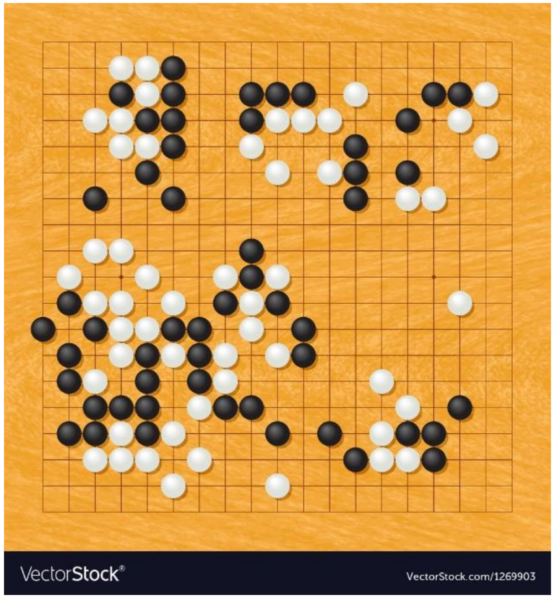

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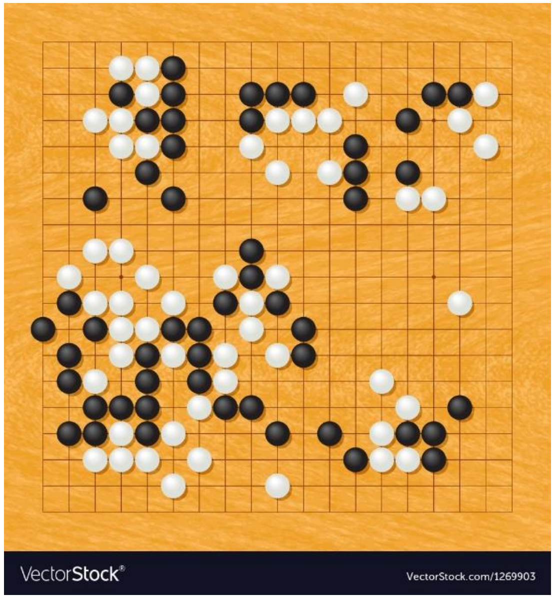

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Modern Go programs use Monte Carlo tree search (MCTS) instead of alpha-beta search.

Monte Carlo Method

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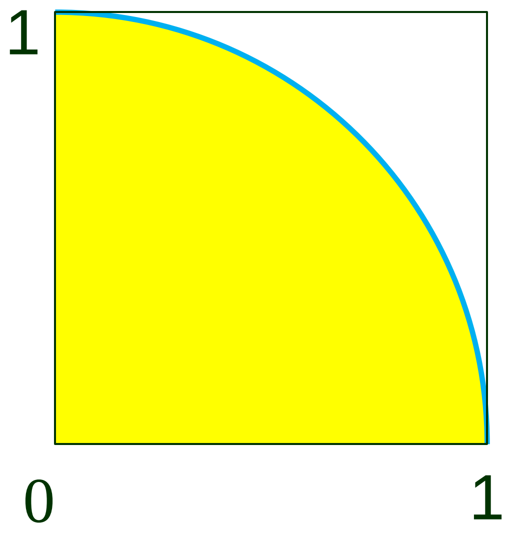

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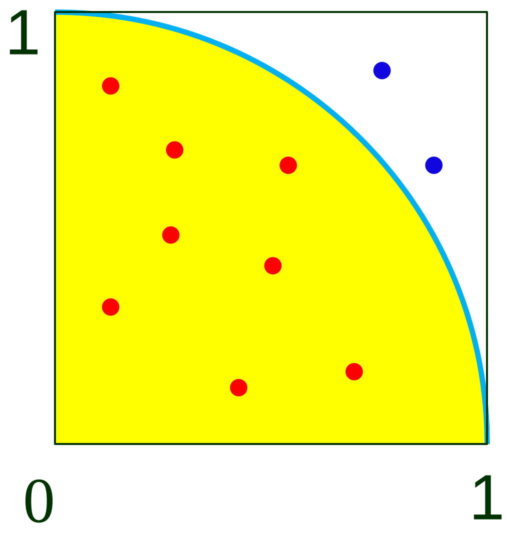

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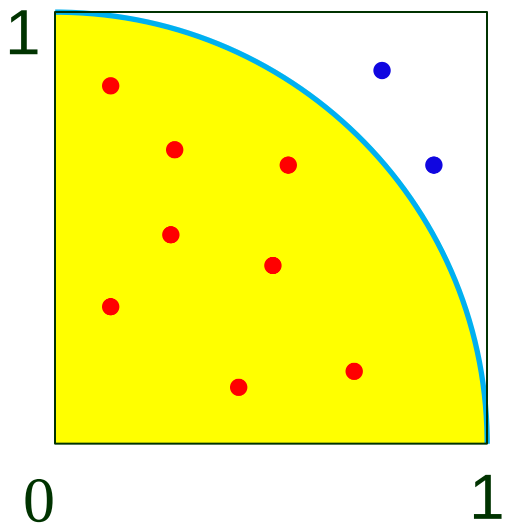

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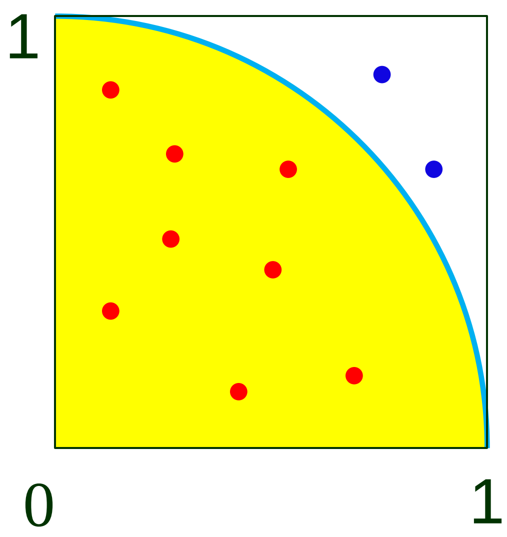

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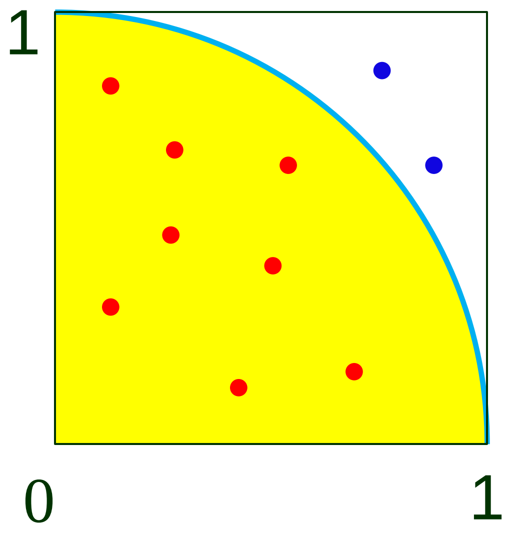

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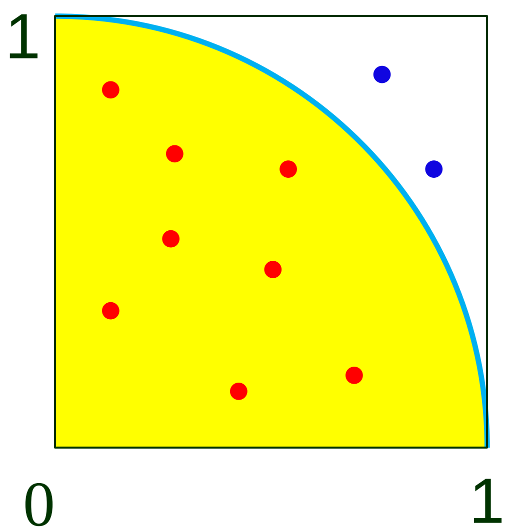

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For accuracy, many points should be generated.

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- No use of a heuristic evaluation function.
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A simulation (a playout or rollout) proceeds as below:

- Choose moves alternatively for the two players.
- Determine the outcome when a terminal position is reached.

Choice of a Move

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What is the best move if both players play well?

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- These policies are often *learned from self-play* using neural networks.

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How to improve? Need a selection policy that balances

- exploration of states that have few playouts, and
- exploitation of states that have done well in the past.