(64) SUMMARY:

- Crystalline solids exhibit both rotational symmetry and translational periodicity, which is summarized by the space group of the crystal.
- Translational symmetry is described by a Bravais lattice, which repeats a fundamental pattern in the different independent directions of real space.
- Rotational symmetry must be compatible with Bravais lattices, which limits rotations to two-fold, three-fold, four-fold, and six-fold orders.
- Space groups can include screw rotations and glide reflections, which are rotations or reflections combined with translations that are not lattice translations.
- Space groups are symbolized using the international notation, which identifies the orientations of symmetry operations with respect to the unit cell axes of the crystal.
- Space groups are the product of two sets: the set of Bravais lattice translations and the set of essential symmetry operations. If the set of essential symmetry operations is a group, then the space group is symmorphic; if not, the space group in nonsymmorphic, which means that it contains screw rotations or glide reflections as members of the essential set.
- The wavefunctions for the electronic and vibrational states of crystals must conform to both the rotational and translational symmetry components.
- The group of Bravais lattice translations is abelian so all irreducible representations are one-dimensional, each labeled by a wavevector of reciprocal space.
- The Bravais lattice translations in real space define a corresponding lattice in reciprocal space. The region closest to one lattice point is called the first Brillouin zone.
- All necessary information concerning the symmetry characteristics of wavefunctions occurs for wavevectors in the first Brillouin zone.
- Rotations in real space effect rotational transformations of wavevectors. Identifying those rotations that take a wavevector to itself or an equivalent wavevector is essential to determine the irreducible representations of a space group.
- Bloch's theorem provides useful features for characterizing wavefunctions for crystals.

Group theory provides important understanding of the physical properties of crystalline solids as well as magnetic order, and such applications broaden the perspective on the types of groups that are needed.