Temperature

Ruslan Prozorov

31 August 2009

Physics 590B
what is temperature?

• **bodily sensation (hot vs cold)**
  - Why **cats** feel so warm? A cat's normal temperature ranges from 100.5 to 102.5 degrees Fahrenheit.
  - In comparison, normal **human body** temperature ranges from 97.8 to 99.1 degrees Fahrenheit. Anything above 100 F is considered a fever.
  - **Dogs** generally have a temperature similar to that of cats at around 102 degrees Fahrenheit.
  - **Rabbits** have a higher temperature of 103.1 degrees Fahrenheit.
  - Male horses (**stallions**) have a slightly lower temperature, at 99.7 degrees Fahrenheit, than female horses (**mares**), at 100 degrees Fahrenheit.
  - **Chicken** – 107 F
what is temperature?

• On the **microscopic scale**, temperature is defined as the average kinetic energy of a particle in the system per degree of freedom. On the **macroscopic scale**, temperature is the unique physical property that determines the direction of heat flow between two objects placed in thermal contact.

• Internal energy, $U$, and entropy, $\sigma$, are the fundamental parameters describing any system. The temperature is DEFINED as

\[
\frac{1}{\tau} = \left( \frac{\partial \sigma}{\partial U} \right)_{N,V}
\]

the conventional temperature $\tau = k_B T$

and conventional entropy $S = k_B \sigma$
we’ve heard of...

- Nernst law?
- Arrhenius thermal activation?
- Boltzmann constant?
- Kelvin(s)?
- Celcius?
- Rankine scale?
Walther Nernst  Ludwig Boltzmann  Svante Arrhenius
The entropy

$$\sigma = \ln W$$

$W$ is the *Wahrscheinlichkeit*, or number of possible microscopic quantum states corresponding to the macroscopic state of a system — number of (unobservable) "ways" the (observable) thermodynamic state of a system can be realized by assigning different positions and momenta to the various molecules.
Ludwig Eduard Boltzmann (February 20, 1844 – September 5, 1906) was an Austrian physicist
S = k \log W

LUDWIG BOLTZMANN
1844–1906

DR. PHILPAULA BOLTZMANN
geb. CHIARI
1871–1977

ARTHUR BOLTZMANN
DIPLINO. DR. PHIL. HOFRAT
1881–1952

LUDWIG BOLTZMANN
1923–1945

HINRIETTE BOLTZMANN
EHE. EDLE von AUGENTLER
1854–1938

FELDBRINNER NACHTRÄG.
GEFALLEN BEI SMOLNISKI
laws of thermodynamics

• 0. if two systems are in equilibrium with a third system, they must be in equilibrium
• 1. heat is form of energy. Energy is conserved.
• 2. entropy of a closed system increases until it reaches maximum (at equilibrium)
• 3. entropy approaches constant value when temperature goes to zero
thermal fluctuations: the “evil” force

• Boltzmann constant

\[ k_B = \frac{R}{N_A} \]

• a way to reconcile the mathematical stat mech with observable temperature and the universal gas law

\[ pV = nRT \]

• thermal fluctuations – randomizing force

\[ k_B T \]
The Absolute Zero

- At absolute zero the entropy is minimal and the system is in its ground state. This includes quantum mechanical zero-point energy.
- At $T=0$, the molecular motion does not cease but does not have enough energy to be transferred to other systems. At $T=0$ molecular energy is minimal.
developers of the temperature scales

Daniel Gabriel Fahrenheit (1686-1736)
Anders Celsius (1701 – 1744)
Kelvin, Lord William Thomson (1824-1907)
William John MacQuorn Rankine (1826 – 1872)
early temperature scales

• In 1724 Daniel Gabriel Fahrenheit produced a temperature scale which now (slightly adjusted) bears his name. He could do this because he manufactured thermometers, using mercury (which has a high coefficient of expansion) for the first time and the quality of his production could provide a finer scale and greater reproducibility, leading to its general adoption.

• In 1742 Anders Celsius proposed a scale with zero at the boiling point and 100 degrees at the melting point of water, though the scale which now bears his name has them the other way around
The **kelvin** (symbol: K) is a unit increment of temperature and is one of the seven SI base units. The **Kelvin scale** is a thermodynamic (absolute) temperature scale where absolute zero, the theoretical absence of all thermal energy, is zero (0 K). The Kelvin scale and the kelvin are named after the British physicist and engineer William Thomson, 1st Baron Kelvin (1824–1907), who wrote of the need for an “absolute thermometric scale”.

The kelvin unit and its scale, by international agreement, are defined by two points: absolute zero, and the triple point of Vienna Standard Mean Ocean Water.

The single combination of pressure and temperature at which water, ice, and water vapor can coexist in a stable equilibrium occurs at exactly 273.16 K (0.01 °C) and a partial vapor pressure of 611.73 pascals (6.1173 millibars, 0.0060373057 atm).
## Kelvin

<table>
<thead>
<tr>
<th></th>
<th><strong>from Kelvin</strong></th>
<th><strong>to Kelvin</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Celsius</strong></td>
<td>$[^\circ\text{C}] = [K] - 273.15$</td>
<td>$[K] =[^\circ\text{C}] + 273.15$</td>
</tr>
<tr>
<td><strong>Fahrenheit</strong></td>
<td>$[^\circ\text{F}] = [K] \times \frac{9}{5} - 459.67$</td>
<td>$[K] = ([^\circ\text{F}] + 459.67) \times \frac{5}{9}$</td>
</tr>
<tr>
<td><strong>Rankine</strong></td>
<td>$[\text{R}] = [K] \times \frac{9}{5}$</td>
<td>$[K] = [\text{R}] \times \frac{5}{9}$</td>
</tr>
<tr>
<td><strong>Delisle</strong></td>
<td>$[^\circ\text{De}] = (373.15 - [K]) \times \frac{3}{2}$</td>
<td>$[K] = 373.15 -[^\circ\text{De}] \times \frac{2}{3}$</td>
</tr>
<tr>
<td><strong>Newton</strong></td>
<td>$[^\circ\text{N}] = ([K] - 273.15) \times \frac{33}{100}$</td>
<td>$[K] =[^\circ\text{N}] \times \frac{100}{33} + 273.15$</td>
</tr>
<tr>
<td><strong>Réaumur</strong></td>
<td>$[^\circ\text{Ré}] = ([K] - 273.15) \times \frac{4}{5}$</td>
<td>$[K] =[^\circ\text{Ré}] \times \frac{5}{4} + 273.15$</td>
</tr>
<tr>
<td><strong>Romer</strong></td>
<td>$[^\circ\text{Rō}] = ([K] - 273.15) \times \frac{21}{40} + 7.5$</td>
<td>$[K] = ([^\circ\text{Rō}] - 7.5) \times \frac{40}{21} + 273.15$</td>
</tr>
</tbody>
</table>
## International Temperature Scale of 1990 (ITS-90)

<table>
<thead>
<tr>
<th>Substance and its state</th>
<th>Defining point in kelvins (range)</th>
<th>Defining point in degrees Celsius (range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor-pressure / temperature relation of helium-3 (by equation)</td>
<td>(0.65 to 3.2)</td>
<td>(~272.50 to ~269.95)</td>
</tr>
<tr>
<td>Vapor-pressure / temperature relation of helium-4 below its lambda point (by equation)</td>
<td>(1.25 to 2.1768)</td>
<td>(~271.90 to ~270.9732)</td>
</tr>
<tr>
<td>Vapor-pressure / temperature relation of helium-4 above its lambda point (by equation)</td>
<td>(2.1768 to 5.0)</td>
<td>(~270.9732 to ~268.15)</td>
</tr>
<tr>
<td>Vapor-pressure / temperature relation of helium (by equation)</td>
<td>(3 to 5)</td>
<td>(~270.15 to ~268.15)</td>
</tr>
<tr>
<td>Triple point of hydrogen</td>
<td>13.8033</td>
<td>~259.3467</td>
</tr>
<tr>
<td>Triple point of neon</td>
<td>24.5561</td>
<td>~248.5939</td>
</tr>
<tr>
<td>Triple point of oxygen</td>
<td>54.3584</td>
<td>~218.7916</td>
</tr>
<tr>
<td>Triple point of argon</td>
<td>83.8058</td>
<td>~189.3442</td>
</tr>
<tr>
<td>Triple point of mercury</td>
<td>234.3156</td>
<td>~38.8344</td>
</tr>
<tr>
<td>Triple point of water(^1)</td>
<td>273.16</td>
<td>0.01</td>
</tr>
<tr>
<td>Melting point(^2) of gallium</td>
<td>302.9146</td>
<td>29.7646</td>
</tr>
<tr>
<td>Freezing point(^2) of indium</td>
<td>429.7485</td>
<td>156.5985</td>
</tr>
<tr>
<td>Freezing point(^2) of tin</td>
<td>505.078</td>
<td>231.928</td>
</tr>
<tr>
<td>Freezing point(^2) of zinc</td>
<td>692.677</td>
<td>419.527</td>
</tr>
<tr>
<td>Freezing point(^2) of aluminum</td>
<td>933.473</td>
<td>660.333</td>
</tr>
<tr>
<td>Freezing point(^2) of silver</td>
<td>1234.93</td>
<td>961.78</td>
</tr>
<tr>
<td>Freezing point(^2) of gold</td>
<td>1337.33</td>
<td>1064.18</td>
</tr>
<tr>
<td>Freezing point(^1) of copper</td>
<td>1357.77</td>
<td>1084.62</td>
</tr>
<tr>
<td>Date</td>
<td>Investigator</td>
<td>Country</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>1860</td>
<td>Kirk</td>
<td>Scotland</td>
</tr>
<tr>
<td>1877</td>
<td>Cailletet</td>
<td>France</td>
</tr>
<tr>
<td>1884</td>
<td>Wroblewski &amp; Olzewski</td>
<td>Poland</td>
</tr>
<tr>
<td>1898</td>
<td>Dewar</td>
<td>England</td>
</tr>
<tr>
<td>1908</td>
<td>Kamerlingh-Onnes</td>
<td>Netherlands</td>
</tr>
<tr>
<td>1927</td>
<td>Simon</td>
<td>Germany &amp; England</td>
</tr>
<tr>
<td>1933</td>
<td>Giauque &amp; MacDougall</td>
<td>U.S.</td>
</tr>
<tr>
<td>1934</td>
<td>Kapitza</td>
<td>England &amp; U.S.S.R.</td>
</tr>
<tr>
<td>1946</td>
<td>Collins</td>
<td>U.S.</td>
</tr>
<tr>
<td>1956</td>
<td>Simon &amp; Kurti</td>
<td>England</td>
</tr>
<tr>
<td>1960</td>
<td>Kurti</td>
<td>England</td>
</tr>
</tbody>
</table>
### Comparison of temperature scales

<table>
<thead>
<tr>
<th>Comment</th>
<th>Kelvin K</th>
<th>Celsius °C</th>
<th>Fahrenheit °F</th>
<th>Rankine °R</th>
<th>Delisle °D</th>
<th>Newton °R, (°Ré, °Re)</th>
<th>Réaumer °Rø (°Rø)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute zero</td>
<td>0</td>
<td>-273.15</td>
<td>-459.67</td>
<td>0</td>
<td>559.725</td>
<td>-90.14</td>
<td>-218.52</td>
</tr>
<tr>
<td>Lowest recorded natural temperature on Earth (Vostok, Antarctica - July 21, 1983)</td>
<td>184</td>
<td>-89</td>
<td>-128.2</td>
<td>331.47</td>
<td>283.5</td>
<td>-29.37</td>
<td>-71.2</td>
</tr>
<tr>
<td>Celsius / Fahrenheit's &quot;cross-over&quot; temperature</td>
<td>233.15</td>
<td>-40</td>
<td>-40</td>
<td>419.67</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Fahrenheit's ice/salt mixture</td>
<td>255.37</td>
<td>-17.78</td>
<td>0</td>
<td>459.67</td>
<td>176.67</td>
<td>-5.87</td>
<td>-14.22</td>
</tr>
<tr>
<td><strong>Water freezes</strong> (at standard pressure)</td>
<td>273.15</td>
<td>0</td>
<td>32</td>
<td>491.67</td>
<td>150</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Average surface temperature on Earth</td>
<td>288</td>
<td>15</td>
<td>59</td>
<td>518.67</td>
<td>127.5</td>
<td>4.95</td>
<td>12</td>
</tr>
<tr>
<td>Average human body temperature</td>
<td>309.95</td>
<td>36.8</td>
<td>98.24</td>
<td>557.91</td>
<td>94.8</td>
<td>12.144</td>
<td>29.44</td>
</tr>
<tr>
<td>Highest recorded surface temperature on Earth (Al Azizyah, Libya - September 13, 1922)</td>
<td>331</td>
<td>58</td>
<td>136.4</td>
<td>596.07</td>
<td>63</td>
<td>19.14</td>
<td>46.4</td>
</tr>
<tr>
<td><strong>Water boils</strong> (at standard pressure)</td>
<td>373.1339</td>
<td>99.9839</td>
<td>211.97102</td>
<td>671.64102</td>
<td>0</td>
<td>33</td>
<td>80</td>
</tr>
<tr>
<td>Titanium melts</td>
<td>1941</td>
<td>1668</td>
<td>3034</td>
<td>3494</td>
<td>-2352</td>
<td>550</td>
<td>1334</td>
</tr>
<tr>
<td>The surface of the Sun</td>
<td>5800</td>
<td>5526</td>
<td>9980</td>
<td>10440</td>
<td>8140</td>
<td>1823</td>
<td>4421</td>
</tr>
<tr>
<td><strong>Note:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
temperature ranges

Figure 7.11 Types of thermometer suitable for use below 1 K, indicating the approximate temperature range over which they are used.
high temperatures - briefly

Thermocouples up to \( \sim 1500 \) K
Thermometers: primary and secondary

The word thermometer (in its French form) first appeared in 1624 in *La Récration Mathématique* by J. Leurechon, who describes one with a scale of 8 degrees.
Primary Thermometers

A reasonable question at this point is: “What is a primary thermometer?” The answer: it is a thermometer that is based on a well understood physical principle and is reliable. In other words, it gives us the absolute temperature and gives repeatable results.

Several temperature scales for the superfluid $^3$He regime have been proposed; four of major interest are:

- The Helsinki scale: based on nuclear orientation thermometry.
- The La Jolla scale: based on noise thermometry.
- The Cornell scale: based on $^3$He melting curve thermometry.
- The Greywall scale: based on the properties of liquid $^3$He.

The four principal methods of primary thermometry are:

1. Gas thermometry in which the pressure is measured, based on the equation of state for an ideal gas, $PV = nRT$ (where $n$ is the number of moles of gas) with small corrections for non-ideality in a real gas such as helium.
2. Acoustic gas thermometry dependent on measuring the sound velocity in a gas, e.g. helium, and subject to the same virial corrections as gas thermometry (dielectric constant gas thermometry is similar).
3. Electrical noise in a resistor of $R$ ohms using the mean square voltage $\overline{V^2} = 4kT R \Delta f$ where $\Delta f$ is the frequency bandwidth.
4. Total black body radiation measurement.
Helium Vapor Pressure Thermometry

### Table 3.1
Defining fixed points of ITS-90 with estimates of their uncertainty $\Delta T$ (Quinn 1990; Preston-Thomas 1990)

<table>
<thead>
<tr>
<th>Fixed points</th>
<th>$T_{90}$ (K)</th>
<th>$\Delta T$ (mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He $^3$He vap. press.</td>
<td>3–5</td>
<td>0.5</td>
</tr>
<tr>
<td>c-$^3$He t.p.</td>
<td>15.8033</td>
<td>0.5</td>
</tr>
<tr>
<td>Ne t.p.</td>
<td>24.5863</td>
<td>0.5</td>
</tr>
<tr>
<td>O$_2$ t.p.</td>
<td>34.3384</td>
<td>1</td>
</tr>
<tr>
<td>Ar t.p.</td>
<td>83.0058</td>
<td>1.5</td>
</tr>
<tr>
<td>Hg t.p.</td>
<td>234.3156</td>
<td>1.5</td>
</tr>
<tr>
<td>Water t.p.</td>
<td>273.16</td>
<td>0</td>
</tr>
<tr>
<td>Ge t.p.</td>
<td>302.9146</td>
<td>1</td>
</tr>
<tr>
<td>Pt t.p.</td>
<td>429.7485</td>
<td>3</td>
</tr>
<tr>
<td>At higher temps Sn, Zn, Al, etc. (m.p. and t.p. at pressure of 101325 Pa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superconductor</td>
<td>$T_c$ (K)</td>
<td>Width (mK)</td>
</tr>
<tr>
<td>W</td>
<td>0.016</td>
<td>0.7</td>
</tr>
<tr>
<td>Be</td>
<td>0.023</td>
<td>0.2</td>
</tr>
<tr>
<td>Ir</td>
<td>0.049</td>
<td>0.8</td>
</tr>
<tr>
<td>Au</td>
<td>0.1665</td>
<td>0.3</td>
</tr>
<tr>
<td>Au in Ag</td>
<td>0.2165</td>
<td>0.4</td>
</tr>
<tr>
<td>Cd</td>
<td>0.3499</td>
<td>0.2–0.8</td>
</tr>
<tr>
<td>Zn</td>
<td>0.8310</td>
<td>2.5–10</td>
</tr>
<tr>
<td>Al</td>
<td>1.1796</td>
<td>1.5–4</td>
</tr>
<tr>
<td>Ir</td>
<td>3.4145</td>
<td>0.5–2.5</td>
</tr>
<tr>
<td>Pb</td>
<td>7.1996</td>
<td>0.6–2</td>
</tr>
</tbody>
</table>

Note: The lower section shows some superconducting transition temperatures, $T_c$ (with width of transition), for metals encapsulated in SRM 767 and SRM 768 (Quinn 1990, p. 183).

Some of the defining fixed points and the uncertainty $\Delta T$ of their thermodynamic temperatures are listed in Table 3.1, together with some secondary points. The latter are superconducting transitions of Standard Reference Materials (SRMs) produced by the National Bureau of Standards (now NIST) which are of cryogenic interest. See also Storm et al. (2000) for details of a new reference device.

The unit of Celsius temperature is the degree Celsius, symbol °C, which is by definition equal in magnitude to the kelvin. The relationship between $T_{90}$ and $T_{90}$ is the same as that between $T$ and $t$, i.e., $T_{90} = T_{90} - 273.15$.

The ITS-90 equations for the vapour pressures of $^3$He and $^4$He, and the temperature ranges within which they are valid are given by (Quinn 1990; Preston-Thomas 1990)

$$T_{90} = A_0 + \sum_{i=1}^{9} A_i \left( \frac{\ln p}{C} \right)^i,$$

where $p$ is in Pa and the values of constants $A_0$, $A_i$, $B$, $C$ are given in Table 3.2 for the specified ranges (from Preston-Thomas 1990, p. 14).

### Table 3.2
Values of constants for the ITS-90 helium vapour pressure equations

<table>
<thead>
<tr>
<th>Constant</th>
<th>$^3$He</th>
<th>$^4$He</th>
<th>$^4$He</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.65–3.2 K</td>
<td>1.25–2.1768 K</td>
<td>2.1768–5.0 K</td>
</tr>
<tr>
<td>$A_0$</td>
<td>1.053447</td>
<td>1.392408</td>
<td>3.146631</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.980106</td>
<td>0.527153</td>
<td>1.357655</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.676380</td>
<td>0.166756</td>
<td>0.413923</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.372692</td>
<td>0.050988</td>
<td>0.091159</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.151656</td>
<td>0.026514</td>
<td>0.016349</td>
</tr>
<tr>
<td>$A_5$</td>
<td>-0.002263</td>
<td>-0.001975</td>
<td>0.001826</td>
</tr>
<tr>
<td>$A_6$</td>
<td>-0.006596</td>
<td>-0.017976</td>
<td>-0.004325</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.088966</td>
<td>0.005409</td>
<td>-0.004973</td>
</tr>
<tr>
<td>$A_8$</td>
<td>-0.004770</td>
<td>0.013259</td>
<td>0</td>
</tr>
<tr>
<td>$A_9$</td>
<td>-0.054943</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B$</td>
<td>7.3</td>
<td>5.6</td>
<td>10.3</td>
</tr>
<tr>
<td>$C$</td>
<td>4.3</td>
<td>2.9</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Helium melting curve thermometer

Figure 5.7 (top) The $^3$He melting curve and Figure 5.8 (bottom) Entropy of $^3$He in the liquid and solid phases (Beets, 1989)
Nuclear spin $I$ splits in a magnetic field into $2I+1$ hyperfine levels. $^{60}\text{Co}$ is substituted into $^{59}\text{Co}$ ($B_s=10$ Telsa), which gives

$$\Delta E / k_B = 3.1 \text{ mK}$$

The relative occupation of each level is given by the Boltzmann factor. Each level has its own anisotropic $\gamma$-decay properties. $W(\theta, T)$ can be computed.
Coulomb blockade thermometer

- Coulomb blockade, named after Charles-Augustin de Coulomb, is the increased resistance at small bias voltages of an electronic device comprising at least one low-capacitance tunnel junction.
- A tunnel junction is, in its simplest form, a thin insulating barrier between two conducting electrodes.
- According to the laws of classical electrodynamics, no current can flow through an insulating barrier. According to the laws of quantum mechanics, however, there is a nonvanishing (larger than zero) probability for an electron on one side of the barrier to reach the other side (see quantum tunnelling). When a bias voltage is applied, this means that there will be a current flow. In first-order approximation, that is, neglecting additional effects, the tunnelling current will be proportional to the bias voltage. In electrical terms, the tunnel junction behaves as a resistor with a constant resistance, also known as an ohmic resistor. The resistance depends exponentially on the barrier thickness. Typical barrier thicknesses are on the order of one to several nanometers.
- An arrangement of two conductors with an insulating layer in between not only has a resistance, but also a finite capacitance. The insulator is also called dielectric in this context, the tunnel junction behaves as a capacitor.
- Due to the discreteness of electrical charge, current flow through a tunnel junction is a series of events in which exactly one electron passes (tunnels) through the tunnel barrier (we neglect events in which two electrons tunnel simultaneously). The tunnel junction capacitor is charged with one elementary charge by the tunnelling electron, causing a voltage buildup \( U = e/C \), where \( e \) is the elementary charge of 1.6×10⁻¹⁹ coulomb and \( C \) the capacitance of the junction. If the capacitance is very small, the voltage buildup can be large enough to prevent another electron from tunnelling. The electrical current is then suppressed at low bias voltages and the resistance of the device is no longer constant. The increase of the differential resistance around zero bias is called the Coulomb blockade.
- In order for the Coulomb blockade to be observable, the temperature has to be low enough so that the characteristic charging energy (the energy that is required to charge the junction with one elementary charge) is larger than the thermal energy of the charge carriers. For capacitances above 1 femtofarad (10⁻¹⁵ farad), this implies that the temperature has to be below about 1 kelvin. This temperature range is routinely reached for example by dilution refrigerators.
- To make a tunnel junction in plate condenser geometry with a capacitance of 1 femtofarad, using an oxide layer of electric permeability 10 and thickness one nanometer, one has to create electrodes with dimensions of approximately 100 by 100 nanometers. This range of dimensions is routinely reached for example by electron beam lithography and appropriate pattern transfer technologies, like the Niemeyer-Dolan technique, also known as shadow evaporation technique.
- Another problem for the observation of the Coulomb blockade is the relatively large capacitance of the leads that connect the tunnel junction to the measurement electronics.
CBT is based on electric conductance characteristics of tunnel junction arrays. The physical basis of CBT is thoroughly explained elsewhere and only a short overview is given here.

The conductance of a tunnel junction array is determined by three energy contributions: the thermal energy $k_B T$ at temperature $T$, the electric potential energy $eV$ at bias voltage $V$, and the charging energy $e_C = e^2/2C_{\text{eff}}$, where $C_{\text{eff}}$ is the effective capacitance of the array. In the high temperature limit where $k_B T \gg e_C$ the dynamic conductance of a junction array can be expressed as

$$G/G_T = 1 - (e_C/k_B T)g(eV/Nk_B T),$$

where $G_T$ is the asymptotic conductance at high bias voltage and $N$ is the number of junctions in series. The function $g$, which is nearly Gaussian shaped, is defined by

$$g(x) = \frac{[x \sinh(x) - 4 \sinh^2(x/2)]}{[8 \sinh^4(x/2)].}$$

The parameter $V_{1/2} = 5.439/Nk_B T/e$, the full width at half minimum of the conductance dip described by Eq. (1), provides the primary thermometric quantity. There is a direct proportionality to $T$ in $V_{1/2}$ via the constants of nature, $e$ and $k_B$, $N$, and the numerical factor 5.439... originating from the shape of the function of Eq. (2). Thus, one has to know just the number of junctions in the array to extract the absolute temperature from the measured $V_{1/2}$. Equation (1) is strictly valid only when $e_C/k_B T \rightarrow 0$. A small linear correction must be applied to the measured $V_{1/2}$ due to the nonzero depth of the dip. This correction has been calculated analytically.

FIG. 2. Optical microscope images of CBT sensors. (a) Two of the five parallel arrays of 20 junctions of a CBT-MT for temperatures 1 K < $T$ < 30 K. The length of the arrays is 13 μm (b) Two parallel arrays of a sensor of LT-type for temperatures 20 mK < $T$ < 1.5 K with 20 junctions in series. The length of the arrays is 160 μm.

FIG. 7. Primary thermometer signal, the differential resistance $dV/dI$, measured at 12 different magnetic fields oriented parallel to the aluminum films in the sensor at $T = 1.46$ K.
7.2.1 Magnetic susceptibility thermometry

An ideal paramagnetic material is one in which there are no interactions between the magnetic dipoles. At low magnetic field strength, the magnetic susceptibility of an ideal paramagnetic material follows Curie’s law,

\[
\chi = \frac{M}{B} = \frac{C}{T},
\]

where \( B \) is the field acting on the dipoles and \( M \) the magnetisation (see Chapter 6). The Curie constant \( C \) may be calculated from first principles, so measurement of the ideal susceptibility yields absolute temperature directly. Unfortunately, at a particular value of applied field, as the temperature falls, the magnetisation increases and a point is reached where the magnetisation tends to saturate. To extend the measurement range below this point, the applied magnetic field may be reduced. The minimum value of the applied field which may be used is that which is comparable to the internal fields. This determines the minimum temperature that may be measured using paramagnetic susceptibility. The salt cerium magnesium nitrate (CMN), used for cooling by adiabatic demagnetisation, is used in paramagnetic thermometry between about 2 mK and 1 K because it has relatively low internal fields. Using dilute magnetic media in which the magnetic atoms are well separated (for example, CLMN with about 5 per cent cerium to lanthanum) reduces the problem of internal fields to some extent, but the susceptibility is smaller and, consequently, harder to measure accurately.

Figure 7.12 (top) shows schematically a simple arrangement for magnetic thermometry. The paramagnetic sample is placed within two coils, which make a transformer, and the mutual inductance between them is measured using A.C. techniques. The mutual inductance is given in terms of the

![Diagram of CMN magnetic susceptibility thermometer system](image-url)
NMR thermometer

To measure temperatures much below a millikelvin, a nuclear paramagnetic material must be used because its internal field is much smaller. However, the method used to measure the static magnetic susceptibility of electronic paramagnetic materials lacks sufficient sensitivity for use with nuclear materials. The nuclear moment is a thousand times less than the electronic moment, and the sensitivity of the above method is proportional to $\mu^2$, i.e. a million times less. For this reason, susceptibility is determined using the more sensitive and selective technique of nuclear magnetic resonance (NMR) (see Figure 7.13). If a nuclear paramagnetic sample is placed in a magnetic field then the nuclear dipole precesses about the field at a characteristic frequency, $f_0 = \gamma B_0/2\pi$ where $\gamma$ is the magnetogyric ratio, just as a mechanical gyroscope precesses about the earth's gravitational field. Typically, $f_0$ is in the radiofrequency range $f_0 = 1 - 40$ MHz in fields of order 1 T. If the sample is placed in a coil oriented at right angles to the applied field (Figure 7.13a), and the coil is energised by a strong pulse of radiofrequency current with a frequency equal to the precessional frequency, then the magnetisation, $\mathbf{M}$, is tipped away from the direction of the applied magnetic field towards the axis of the coil as shown in Figure 7.13c. If the pulse strength and duration are chosen correctly, the magnetisation is tipped into the axis of the coil, i.e. at right angles to the applied field. Such a pulse
12.6 Noise Thermometry

The conduction electrons in a metal perform random thermal movements (Brownian motion), which result in statistical voltage fluctuations of a resistive element. Therefore all resistive elements of an electronic circuit are noise sources. This noise is statistical, therefore we cannot make statements about its value at a fixed time, and the mean value of the noise voltage vanishes. However, we can calculate an effective time averaged mean square noise voltage

\[ U_{\text{rms}} = \sqrt{\langle u^2 \rangle_t} \quad \text{(12.10)} \]

This noise voltage was investigated in 1928 by J.B. Johnson (Johnson noise) and H. Nyquist (Nyquist theorem). Nyquist arrived at the following equation for the component of the time averaged noise voltage within the frequency band from \( \nu \) to \( \nu + d\nu \) of a resistor with value \( R \) at temperature \( T \):

\[ \langle u^2(\nu) \rangle_t = 4k_B TRd\nu \quad \text{(12.11)} \]

If we measure within a frequency band of width \( \Delta\nu \) we obtain

\[ \langle u^2 \rangle = \int_{\nu}^{\nu+\Delta\nu} 4k_B TRd\nu = 4k_B TR\Delta\nu \quad \text{(12.12)} \]
Secondary thermometers

• Resistance
  – Metallic (positive) – highly reproducible, but not too sensitive at low Ts
  – Semiconducting (negative) – very sensitive, but require calibration (impurity-dependent $R(T)$)

• diodes – easy to read (voltage $\sim 1$ V) – not as accurate and reproducible as semiconductor or metallic sensors

• thermocouples – difficult to read (uV range) – wide T-range, convenience (passive sensor)

• Capacitance – smallest H-field error. Not reproducible (need second sensor) – control sensors
Important practical characteristics

- accuracy is determined by
  - sensitivity
  - resolution
- sensitivity to a magnetic field
- reproducibility
- uniqueness of calibration
- easy of use
- cost
- packaging
Fig. 5.5 Comparison of dimensionless sensitivity of various commercial cryogenic thermometers (given as an absolute value, since some of the sensitivities are negative) (from Lake Shore Cryotronics 2002 and product literature). The dimensionless sensitivity is defined as the relative change in a sensor's output $dO/O$ from a given relative change in temperature $dT/T$; that is, $[(dO/O)/(dT/T)]$, or equivalently, $[d \ln O / d \ln T]$. The higher a curve's position on the plot, the better:

- Au–Fe thermocouple: Thermocouple (K-P Chromel vs. Au–0.07%Fe referenced to 0 K)
- Capacitor: Capacitance thermometer
- Carbon–glass: Carbon–glass resistor
- Cernox™: Zirconium–oxynitride resistor
- GaAlAs diode: GaAlAs diode operating at 10 μA
- Ge: Germanium resistor
- Pt: Platinum resistor
- Rh–Fe: Rhodium–iron resistor
- Ru–O: Ruthenium–oxide resistor
- Si diode: Silicon diode operating at 10 μA
temperature resolution

Fig. 5.6 Normalized temperature resolution of commercial cryogenic thermometers, $\Delta T/T$ (from Lake Shore Cryotronics 2002 and product literature). $\Delta T$ is the smallest temperature change that can be resolved under the typical operating conditions listed below; the lower the curve's position on the plot, the better. The data assume a 5-1/2 digit voltage readout with a 0.1 µV resolution or a 5-digit capacitance readout with a 0.1 pF resolution.

- Au–Fe thermocouple: Thermocouple (KP Chromel vs. Au–0.07%Fe referenced to 0 K).
- Capacitor: Capacitance thermometer: 5 V at 5 kHz.
- Carbon–glass: Carbon–glass resistor: 2 mV or 0.1 µA minimum operating current.
- Cernox™: Zirconium–oxynitride resistor: 2 mV or 0.1 µA minimum operating current.
- GaAlAs diode: GaAlAs diode operating at 10 µA.
- Ge: Germanium resistor: 2 mV or 0.3 µA minimum operating current.
- Pt: Platinum resistor: 100 µA operating current.
- Rh–Fe: Rhodium–iron resistor: 300 µA operating current.
- Ru–O: Ruthenium–oxide resistor: 10 µA operating current.
- Si diode: Silicon diode operating at 10 µA.
Fig. 5.7 Temperature error, \( (T_{\text{app}} - T_{\text{act}})/T_{\text{act}} \) (%), (absolute value) of common cryogenic thermometers arising from a magnetic field of 2.5 T. For most sensors, the relative magnetic-field errors diminish as temperature increases. Diode sensors are generally the most affected by magnetic field. Capacitance thermometers are the least affected, but they have poor reproducibility. Cernox\textsuperscript{TM} and carbon–glass sensors are the best thermometers for magnetic-field applications over the entire cryogenic range from 1 to 300 K, and platinum resistors are also excellent above liquid-nitrogen temperature. (GaAlAs diode data are for 2.0 T rather than 2.5 T.) (Compiled from data supplied by Lake Shore Cryotronics 1995 and 1991, product literature, and Rubin et al. 1986.)
**Figure 7.6** Thermocouple suitable for measuring low temperatures. 1: gold (0.3 percent iron); 2: Chromel
In 1821, the German–Estonian physicist Thomas Johann Seebeck discovered that when any conductor (such as a metal) is subjected to a thermal gradient, it will generate a voltage. This is now known as the thermoelectric effect or Seebeck effect. Any attempt to measure this voltage necessarily involves connecting another conductor to the "hot" end. This additional conductor will then also experience the temperature gradient, and develop a voltage of its own which will oppose the original. Fortunately, the magnitude of the effect depends on the metal in use. Using a dissimilar metal to complete the circuit creates a circuit in which the two legs generate different voltages, leaving a small difference in voltage available for measurement. That difference increases with temperature, and can typically be between 1 and 70 microvolts per degree Celsius (µV/°C) for the modern range of available metal combinations. Certain combinations have become popular as industry standards, driven by cost, availability, convenience, melting point, chemical properties, stability, and output. This coupling of two metals gives the thermocouple its name.

Thermocouples measure the temperature difference between two points, not absolute temperature. In traditional applications, one of the junctions—the cold junction—was maintained at a known (reference) temperature, while the other end was attached to a probe.

Having available a known temperature cold junction, while useful for laboratory calibrations, is simply not convenient for most directly connected indicating and control instruments. They incorporate into their circuits an artificial cold junction using some other thermally sensitive device, such as a thermistor or diode, to measure the temperature of the input connections at the instrument, with special care being taken to minimize any temperature gradient between terminals. Hence, the voltage from a known cold junction can be simulated, and the appropriate correction applied. This is known as cold junction compensation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Temperature range °C (continuous)</th>
<th>Temperature range °C (short term)</th>
<th>Tolerance class one (°C)</th>
<th>Tolerance class two (°C)</th>
<th>IEC Color code</th>
<th>BS Color code</th>
<th>ANSI Color code</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>0 to +1100</td>
<td>-180 to +390</td>
<td>±1.5 between -40 °C and 375 °C</td>
<td>±2.5 between -40 °C and 330 °C</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 375 °C and 1000 °C</td>
<td>±0.007±T between 333 °C and 1200 °C</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>J</td>
<td>0 to +760</td>
<td>-180 to +800</td>
<td>±1.5 between -40 °C and 375 °C</td>
<td>±2.5 between -40 °C and 330 °C</td>
<td>Black</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 375 °C and 750 °C</td>
<td>±0.007±T between 333 °C and 750 °C</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td>N</td>
<td>0 to +1100</td>
<td>-270 to +390</td>
<td>±1.5 between -40 °C and 375 °C</td>
<td>±2.5 between -40 °C and 330 °C</td>
<td>Purple</td>
<td>Purple</td>
<td>Purple</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 375 °C and 1000 °C</td>
<td>±0.007±T between 333 °C and 1500 °C</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
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<tr>
<td>R</td>
<td>0 to +1600</td>
<td>-50 to +1700</td>
<td>±1.5 between 0 °C and 1100 °C</td>
<td>±1.5 between 0 °C and 600 °C</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 1100 °C and 1600 °C</td>
<td>±0.002±T between 600 °C and 1600 °C</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>S</td>
<td>0 to 1600</td>
<td>-50 to +1759</td>
<td>±1.5 between 0 °C and 1100 °C</td>
<td>±1.5 between 0 °C and 600 °C</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 1100 °C and 1600 °C</td>
<td>±0.002±T between 600 °C and 1600 °C</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>B</td>
<td>+200 to +1100</td>
<td>U to +1150</td>
<td>Not Available</td>
<td>±0.005±T between 160 °C and 1100 °C</td>
<td>Black</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No standard use copper wire</td>
<td>Black</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>T</td>
<td>-105 to +300</td>
<td>-250 to +409</td>
<td>±1.5 between -40 °C and 125 °C</td>
<td>±1.5 between -40 °C and 330 °C</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 125 °C and 350 °C</td>
<td>±0.007±T between 133 °C and 350 °C</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td>E</td>
<td>0 to +600</td>
<td>-40 to +900</td>
<td>±1.5 between -40 °C and 375 °C</td>
<td>±2.5 between -40 °C and 330 °C</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>±0.004±T between 375 °C and 800 °C</td>
<td>±0.007±T between 333 °C and 900 °C</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td>Chrome/Au/Fe</td>
<td>-272 to +300</td>
<td>n/a</td>
<td>Reproducibility ±2% of the voltage, each sensor needs individual calibration.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Cryogenic Temperature Sensors

## Sensor Overview

<table>
<thead>
<tr>
<th></th>
<th>Temperature Range</th>
<th>Standard Curve</th>
<th>Below 1 K</th>
<th>Can be used in radiation</th>
<th>Performance in magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diodes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon</td>
<td>1.4 K to 500 K</td>
<td>×</td>
<td></td>
<td></td>
<td>Fair above 60 K</td>
</tr>
<tr>
<td>GaAlAs</td>
<td>1.4 K to 500 K</td>
<td></td>
<td></td>
<td></td>
<td>Fair</td>
</tr>
<tr>
<td><strong>Positive Temperature Coefficient RTDs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Platinum</td>
<td>14 K to 873 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Fair above 30 K</td>
</tr>
<tr>
<td>Rhodium-Iron</td>
<td>0.65 K to 500 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Fair above 77 K</td>
</tr>
<tr>
<td><strong>Negative Temperature Coefficient RTDs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cernox™</td>
<td>0.10 K to 325 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Excellent above 1 K</td>
</tr>
<tr>
<td>Cernox™ HT</td>
<td>0.10 K to 420 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Excellent above 1 K</td>
</tr>
<tr>
<td>Germanium</td>
<td>0.05 K to 100 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Not recommended</td>
</tr>
<tr>
<td>Carbon-Glass</td>
<td>1.4 K to 325 K</td>
<td>×</td>
<td></td>
<td></td>
<td>Good</td>
</tr>
<tr>
<td>Ruthenium oxide*</td>
<td>0.01 K to 40 K</td>
<td>×</td>
<td>×</td>
<td></td>
<td>Good below 1 K</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermocouples</td>
<td>1.2 K to 1543 K</td>
<td>×</td>
<td></td>
<td></td>
<td>Fair</td>
</tr>
<tr>
<td>Capacitance</td>
<td>1.4 K to 290 K</td>
<td></td>
<td></td>
<td></td>
<td>Excellent</td>
</tr>
</tbody>
</table>
# Silicone Diodes

## DT-670 Silicon Diodes

### DT-670-SD Features
- Best accuracy across the widest useful temperature range — 1.4 K to 500 K — of any silicon diode in the industry
- Tightest tolerances for applications from 30 K to 500 K of any silicon diode to date
- Rugged, reliable Lake Shore SD package designed to withstand repeated thermal cycling and minimize sensor self-heating
- Conformance to standard Curve 10 temperature response curve

## DT-400 Series Silicon Diodes

### DT-470-SD Features
- Monotonic temperature response from 1.4 K to 500 K
- Conformance to standard Curve 10 temperature response curve
- Useful above 60 K in magnetic fields up to 5 T
- The rugged, reliable Lake Shore SD package designed to withstand repeated thermal cycling and minimize sensor self-heating
- Variety of packaging options
GaAlAs diodes

**Description**

The TG-120 gallium-aluminum-arsenide (GaAlAs) diode cryogenic temperature sensors are particularly well suited for low to moderate magnetic field applications at low temperatures. The GaAlAs sensing element exhibits high sensitivity (dV/dT) at low temperatures. Voltage-temperature characteristics are monotonic over the sensor's useful range from 1.4 K to 500 K (see data plots below).

Gallium-aluminum-arsenide diodes are direct band-gap, single junction devices that produce small output variances in the presence of magnetic fields. Consequently, their low magnetic field dependence makes them ideally suited for applications in moderate magnetic fields up to five tesla.

**TG-120-SD Features**

- Monotonic temperature response from 1.4 K to 500 K
- Excellent sensitivity (dV/dT) at temperatures below 50 K
- Relatively low magnetic field-induced errors
- Rugged, reliable Lake Shore SD package designed to withstand repeated thermal cycling and minimize sensor self-heating
- Variety of packaging options

**TG-120-P Features**

- Temperature range: 1.4 K to 325 K
- Reproducibility at 4.2 K: ±10 mK
Resistance thermometers

Resistance thermometers, also called resistance temperature detectors (RTDs), are temperature sensors that exploit the predictable change in electrical resistance of some materials with changing temperature.
Fig. 3.7 The electrical resistance $R(T)$ of some typical thermometers. A-B denotes Allen-Bradley carbon resistor. Speer is a carbon resistor. CG is carbon-in-glass. CX 1050 is a Cernox and RX 202A is a ruthenium oxide from LakeShore. Ge 100 and Ge 1000 are Cryocal germanium thermometers.
Relative Resistance Vs Temperature of Typical RTDs and Thermistors

- Thermistor
- Nickel
- Balco
- Platinum

Relative Resistance \( \frac{R(T)}{R(0\, ^\circ C)} \) vs Temperature

- Degrees Celsius: -100 to 700
- Degrees Fahrenheit: -148 to 1292
Pt resistance thermometers

**PT-100 Series Platinum RTDs**

**Features**

- Temperature range: 14 K to 873 K (model dependent)
- Conforms to IEC 751 standards down to 70 K
- High reproducibility: ≤5 mK at 77 K
- Low magnetic field dependence above 40 K
- Excellent for use in ionizing radiation
- SoftCal™ calibration available

**Description**

PT-100 platinum resistance thermometers (PRTs) are an excellent choice for use as cryogenic temperature sensing and control elements in the range from 30 K to 873 K (-243 °C to 600 °C). Over this temperature span, PRTs offer high repeatability and nearly constant sensitivity (dR/dT). Platinum resistors are also useful as control elements in magnetic field environments where errors approaching one degree can be tolerated. PRTs are interchangeable above 70 K. The use of controlled-purity platinum assures uniformity from one device to another.

PRTs experience rapidly decreasing sensitivity below approximately 30 K. They should be calibrated in order to achieve maximum accuracy for use below 100 K. The plot illustrates platinum sensor performance to the IEC 751 curve.

**Packaging Options** AL, AM

- Click here for Adding Length to Sensor Leads

**Matching**

If your application requires more than one platinum resistor, up to five platinum resistors can be matched to one another to within ±0.1 K at liquid nitrogen temperature with the purchase of only one calibration.
Rhodium-Iron RTDs

**Description**

Rhodium-iron cryogenic temperature sensors offer a positive temperature coefficient, monotonic response over a wide temperature range, and high resistance to ionizing radiation.

**RF-100**

The Lake Shore thin film rhodium-iron temperature sensor offers significant advantages over comparable wire-wound resistance sensors. The thin film sensors feature a smaller package size, which makes them useful in a broader range of experimental mounting schemes, and they are available at a much lower cost. Additionally, they have proven to be very stable over repeated thermal cycling and under extended exposure to ionizing radiation. Furthermore, the thermal time constant of these film rhodium-iron cryogenic temperature sensors (bare chip) is on the order of milliseconds, while the thermal time constant of wire-wound resistors is on the order of seconds.

**RF-800**

The RF-800 rhodium-iron resistance sensor features monotonically decreasing resistivity from 500 K to 0.65 K, although sensitivity (dR/dT) falls off in the region of 30 K. From 100 K to 273 K the resistance changes linearly with temperature to within 1 K. RF-800-4 sensors also exhibit monotonic response at higher temperatures, hence their adaptability for use over the broad range from 1.4 K to 500 K.

---

The RF-100U and RF-100T sensors have been discontinued and there is only a limited quantity available. Please refer to [http://www.lakeshore.com/cold/obsa.html](http://www.lakeshore.com/cold/obsa.html) for remaining quantities. Alternative sensors are: Cerinox for measuring temperatures from 100 mK to 420 K or Platinum for measuring temperatures from 14 K to 873 K.
Figure 7.8 Resistance—temperature characteristics for a typical Rhodium–iron resistance thermometer [Courtesy of Oxford Instruments UK Ltd.]
negative T coefficient: Cernox

**Description**

Cernox™ thin film resistance cryogenic temperature sensors offer significant advantages over comparable bulk or thick film resistance sensors. The smaller package size of these thin film sensors makes them useful in a broader range of experimental mounting schemes, and they are also available in a chip form. They are easily mounted in packages designed for excellent heat transfer, yielding a characteristic thermal response time much faster than possible with bulk devices requiring strain-free mounting. Additionally, they have been proven very stable over repeated thermal cycling and under extended exposure to ionizing radiation.

**Packaging Options**

- AA, BC, BG, BO, BR, CD, CO, CU, ET, ER, MR, SD

  - Click here for Adding Length to Sensor Leads

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**CX-1010 – the Ideal Replacement for Germanium RTDs**

The CX-1010 is the first Cernox™ designed to operate down to 100 mK, making it an ideal replacement for Germanium RTDs. Unlike Germanium, all Cernox models have the added advantage of being able to be used to room temperature. In addition, Cernox is offered in the incredibly robust Lake Shore SD package, giving researchers more flexibility in sensor mounting.

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**Features**

- Low magnetic field-induced errors
- Temperature range of 100 mK to 420 K (model dependent)
- High sensitivity at low temperatures and good sensitivity over a broad range
- Excellent resistance to ionizing radiation
- Bare die cryogenic temperature sensor with fast characteristic thermal response times: 1.9 ms at 4.2 K, 50 ms at 77 K
- Broad selection of models to meet your thermometry needs
- Excellent stability
- Variety of packaging options

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**The Lake Shore SD Package – The Most Rugged, Versatile Package in the Industry**

The SD package, with direct sensor-to-sapphire base mounting, hermetic seal, and soldered copper leads, provides the industry’s most rugged, versatile cryogenic temperature sensors with the best sample to chip connection. Designed so heat coming down the leads bypasses the chip, it can survive several thousand hours at 420 K (depending on model) and is compatible with most ultra-high vacuum applications. It can be indium soldered to samples without sensor calibration shift.
Germanium resistance

Description

The GR-200 Germanium Resistance Temperature Sensor is recognized as a "Secondary Standard Thermometer" and has been employed in the measurement of temperature from 0.05 K to 30 K for more than 30 years.

GR-200 sensors have a useful temperature range of about two orders of magnitude. The exact range depends upon the doping of the germanium element. Cryogenic temperature sensors with ranges from below 0.05 K to 100 K are available. Between 100 K and 300 K, $dR/dT$ changes sign and $dR/dT$ above 100 K is very small for all models. Sensor resistance varies from several ohms at its upper useful temperature to several tens of kilohms at its lower temperature.

Features

- Recognized as a "Secondary Standard Thermometer"
- High sensitivity provides submillikelvin control at 4.2 K and below
- Excellent reproducibility better than ±0.5 mK at 4.2 K
- Various models for use from 0.05 K to 100 K
- Excellent resistance to ionizing radiation

Because device sensitivity increases rapidly with decreasing temperature, a high degree of resolution is achieved at lower temperatures, making these resistors very useful for submillikelvin control at 4.2 K and below.

The GR-200 sensors have excellent stability, and ±0.5 mK reproducibility at 4.2 K. The germanium resistor is usually the best choice for high-accuracy work below 30 K. Use in a magnetic field is not recommended.
Figure 7.9 Typical resistance–temperature characteristics for a selection of germanium thermometer elements. The thermometers are characterised by their resistance at 4.2 K. That is, 50, 100, 250, 500, 1000 and 1500 Ω for curves 1–6 respectively. Also shown is the maximum recommended measurement current [Courtesy of Oxford Instruments UK Ltd.]
Carbon-Glass

**Description**

Carbon-Glass RTDs (CGRs) have the longest history of use of any cryogenic temperature sensor suitable for high magnetic fields and wide range temperature sensing. These resistance cryogenic temperature sensors are highly reproducible and can be used from 1.4 K to 100 K and in magnetic fields up to 20 tesla. Their extremely high sensitivity at liquid helium temperatures makes them very useful for submillikelvin control below 10 K. CGR sensors are monotonic in resistance temperature characteristic between 1.4 K and 325 K, but their reduced sensitivity (<0.01 W/K) above 100 K limits their usage at higher temperatures.

**Features**

- Low magnetic field induced errors
- For use in magnetic fields up to 20 tesla
- Reproducible in the 1.4 K to 100 K range
- Monotonic R vs. T and dR/dT vs. T response curves
- High sensitivity provides submillikelvin control at 4.2 K and below
- Usable sensitivity over the broad range of 1.4 K to 325 K
- Good resistance to ionizing radiation at low temperatures

**Typical Carbon-Glass Resistance Values**

**Typical Carbon-Glass Sensitivity Values**

**Typical Carbon-Glass Dimensionless Sensitivity Values**
Ruthenium oxide

Description

Ruthenium oxide temperature sensors are thick-film resistors used in applications involving magnetic fields. These composite sensors consist of bismuth ruthenate, ruthenium oxides, binders, and other compounds that allow them to obtain the necessary temperature and resistance characteristics. Each Lake Shore Kox™ model adheres to a single resistance versus temperature curve.

Features

RX-102A Features
- Standard curve interchangeable
- Good radiation resistance
- Useful down to 50 mK
- Low magnetic field-induced errors

RX-102B Features
- Useful down to 10 mK; calibrations down to 20 mK available
- Monotonic from 10 mK to 300 K

RX-202A Features
- Standard curve interchangeable
- Good radiation resistance
- Monotonic from 50 mK to 300 K
- 4K improvement in magnetic field-induced errors over other ruthenium oxides

RX-103A Features
- Standard curve interchangeable
- Good radiation resistance
- Best choice for interchangeability from 1.4 K to 40 K
- Low magnetic field-induced errors

RX-102A
The RX-102A (1000 Ω at room temperature) is useful down to 50 mK and has better interchangeability than the RX-202A as well as low magnetic field induced errors below 1 K.

RX-102B-CB
The RX-102B-CB (1000 Ω at room temperature) is useful down to 10 mK (calibrations available down to 20 mK) and monotonic from 10 mK to 300 K. The unique package design maximizes thermal connection and minimizes heat capacity at ultra low temperatures. The RX-102B-CB is not interchangeable to a standard curve and not recommended for use in magnetic fields.

RX-202A
The RX-202A (2000 Ω at room temperature) has a 4K improvement in magnetic field-induced errors over other commercially available ruthenium oxide temperature sensors with similar resistances and sensitivities. Most ruthenium oxide sensors have a maximum useful temperature limit well below room temperature, where the sensitivity changes from negative to positive. The RX-202A however, is designed to have a monotonic response from 0.85 K up to 300 K.

RX-103A
The RX-103A (10,000 Ω at room temperature) has a unique resistance and temperature response curve combined with low magnetic field-induced errors, and is the best choice for interchangeability from 1.4 K to 40 K.

Click here for Adding Length to Sensor Leads
capacitance

Description

CS capacitance sensors are ideally suited for use as cryogenic temperature control sensors in strong magnetic fields because they exhibit virtually no magnetic field dependence. Displacement current is not affected by magnetic fields. Consequently, temperature control fluctuations are kept to a minimum when sweeping magnetic field or when changing field values under constant temperature operation.

Features

- Virtually no magnetic field-induced errors
- Capable of mK control stability in the presence of strong magnetic fields
- Monotonic in C versus T to nearly room temperature

Because small variations in the capacitance/temperature curves occur upon thermal cycling, calibrations must be transferred to the capacitor from another sensor after cooling for the best accuracy. It is recommended that temperature in zero field be measured with another temperature sensor and that the capacitance sensor be employed as a control element only.
practical thermometry

• Zero-field
  – Platinum 77-300 K
  – Cernox (zirconium-axynitride) 0.5-77 K
  – Ruthenium oxide (20 mK - 0.5 K)
  – (simplicity – silicon diode (1.5 – 300 K)

• In a magnetic field
  – Cernox (various models – check the specific characteristics)
    0.3-325 K
  – Platinum 77-300 K
  – Capacitance 1- 290 K
Fig. 5.12 Bolt-on copper bobbin (a), and copper block (b), for thermally anchoring a thermometer and its leads (adapted from Lake Shore Cryotronics 2002). For the bobbin technique, the thermometer leads are wrapped around the bobbin and thermally attached to it with varnish or epoxy. For the copper block technique, a beryllium-oxide chip with electric-terminal metallization on top can be used to thermally anchor the thermometer leads. To enhance the thermal contact between the bobbin/block and sample holder, a thin layer of thermally conductive grease is applied to the bottom of the bobbin/block before it is bolted in place.
Varnish, such as IMI 7031 varnish (formerly GE 7031), can be painted directly onto the copper sample holder to provide an electrically insulating layer. Application is easier if the varnish is first partially thinned with ethanol. Normal varnish curing time (12–24 h) can be shortened by using a heat lamp. If the varnish needs to be removed later, the same thinner can be used. This method is well suited to large or long samples, since large areas can be readily painted.

Cigarette paper can add some robustness to varnish (the paper is available commercially). This method is appropriate for situations where the soft varnish layer might be scratched. Thoroughly soak the paper with thinned varnish to adhere the thin paper to the copper sample holder as well as to provide good thermal conduction through the paper.

Mylar™- or Kapton™-based tape is perhaps the easiest to install. Simply stick it to the copper sample holder. “Yellow” Mylar™ tape (3M #56) is very thin, and its adhesion to copper actually improves with thermal cycling (application notes and supplier information are given in the table of sticky stuff, Appendix A3.10). Thin, adhesive-coated Kapton™ tape also works well. Both types of tape are more robust than a varnish layer alone.

Thin pieces of sapphire or silicon wafer can be greased to the sample holder. This method works best for electrically isolating small samples that do not require large sample holders.
thermometer location

Fig. 5.13  Ideal arrangement for thermometer location.
temperature control

- on-off controllers
- proportional
- proportional-integral-derivative
When selecting a commercial temperature controller, it pays to check the following:

- **Resolution and absolute accuracy**: Is it adequate to meet your temperature-measurement needs?
- **Input modules**: Are they available for the particular sensors you want to use?
- **Sensor excitation level**: Can it be set low enough to avoid self-heating for a particular sensor and temperature range?
- **ac sensor excitation**: Is it available to avoid the effects of slowly drifting thermoelectric voltages?
- **Interpolation schemes**: What types are used to fit the calibration data? (See suggestion list in Sec. 5.1.6.) Can new calibrations be entered?
- **Preloaded calibrations**: Are they available for nonprecision work with interchangeable thermometers?
- **Heater power**: Is it adequate and a genuine analogue output (not pulsed, which causes electromagnetic interference for precise work)?
- **Computer interface**: Is it available? Is it electrically isolated from the cryogenic outputs to minimize interference?
1 - Line Input Assembly
2 - Heater Fuse
3 - Heater Output
4 - Option Slots
5 - Data Card
6 - IEEE-488 Interface
7 - Serial (RS-232C) I/O
8 - Digital I/O
9 - Relays
10 - Analog Outputs
11 - Standard Sensor Inputs
Lake Shore offers a complete line of cryogenic accessories for sensor installation and general-purpose cryogenic use.

**Cryogenic Wire and Cable**

Used to minimize heat leak into the sensor and cryogenic system, cryogenic wire has a much lower thermal conductivity (and higher electrical resistivity) than copper wire.

The most common type of cryogenic wire is phosphor bronze. This wire is available in one-, two-, and four-lead configurations. Four-lead configurations are available as Quad-twist™ (two twisted pairs) or Quad-lead™ (ribbon). Wire gauge is 32 or 36 AWG, with polyimide or polyvinyl formal (Formvar®) used to insulate the wires.

Other common cryogenic wires and coaxial cables include manganin, nichrome heater wire, and HD-30 heavy duty copper wire. For high-frequency signals, Lake Shore provides various coaxial cables: ultra miniature coaxial cables and semi-rigid coaxial with a stainless steel center conductor.

**Solder**

The most common electrical connections are solder joints. Solder can also be used to install various sensors to improve thermal heat sinking. Common solders are indium solder and 90/10 Pb/Sn. Indium solder is used for various applications including sensor installation to provide excellent thermal contact with the sample. 90/10 Pb/Sn solder is used for applications requiring a higher temperature (liquidus point of 575 K and solidus point 458 K). Oсталой® 158 solder is used as a seal for demountable vacuum cans and electric feedthroughs in cryogenic systems.

**Epoxy, Thermal Grease, and Varnish**

Thermal greases and epoxies are used to install and fasten sensors, while providing thermal contact and/or electrical insulation, with the sample. Epoxy can be used for mechanical attachment and joints.

The most common varnish for cryogenic installations is VGE-7031 varnish. It has good chemical resistance, bonds to a variety of materials, and has a fast tack time. Stydast® 2850FT is composed of a black epoxy resin, and has a thermal expansion coefficient that is matched to copper. A silver-filled, low-temperature conducting epoxy provides excellent strength, along with electrical and thermal conductivity.

Thermal grease, Apiezon N and Apiezon H, is suitable for enhancing thermal contact, especially for sensors inserted into cavities. Apiezon N is for low temperature applications, while H is for high temperature.

**Miscellaneous Cryogenic Accessories**

Lake Shore also supplies heat sink bobbins, a beryllium oxide heat sink chip, and a four-lead resistance sample holder. Cartridge heaters and vacuum feed through products are also available.
Table 5.1 Properties of selected cryogenic thermometers (−150 to 500 K).
A more complete listing of cryogenic thermometers and their properties is given in the addendum at the end of this chapter (Sec. 5.5) and in Appendix A5.2.

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Temperature Range</th>
<th>Accuracy (± value)</th>
<th>Reproducibility (± value)</th>
<th>Long-Term Calibration Drift</th>
<th>Interchangeability</th>
<th>Magnetic Field Use</th>
<th>Best Use</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platinum resistance thermometer</td>
<td>77.800 K</td>
<td>With impurity</td>
<td>10 mK from 77 K to 305 K</td>
<td>±10 mK/yr at 77 K to 237 K</td>
<td>Yes</td>
<td>Recommended</td>
<td>Measurements above 77 K</td>
<td>High without calibration</td>
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<tr>
<td></td>
<td></td>
<td>correction:</td>
<td></td>
<td></td>
<td></td>
<td>above 70 K,</td>
<td>Excellent reproducibility</td>
<td>High with individual calibration</td>
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<td></td>
<td>20–77 K</td>
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<td>interchangeability,</td>
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<td>low magnetic field error</td>
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<td>(Appendix A5.5b)</td>
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<td>Many shapes and sizes</td>
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<td>One of the best sensors</td>
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<td>for use in magnetic fields</td>
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<td>Good sensitivity</td>
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<td>over a wide temperature</td>
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<td>Fast response time as chip</td>
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<tr>
<td>Zirconium-oxytitanate</td>
<td>0.3–325 K</td>
<td>Must be individually calibrated</td>
<td>3 mK at 4.2 K</td>
<td>±25 mK/yr at 1 K to 100 K</td>
<td>No</td>
<td>Recommended</td>
<td>Low chemical and electrical</td>
<td>High with individual calibration</td>
</tr>
<tr>
<td>resistance thermometer (Cernox™)</td>
<td></td>
<td>5 mK at 4.2 K</td>
<td>0.05% of reading at 100 K</td>
<td></td>
<td></td>
<td>Lowest error</td>
<td>Standard correction</td>
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<td></td>
<td></td>
<td>&lt; 0.1% at &gt; 10 K</td>
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<td>Appendix A5.6</td>
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<tr>
<td>Germanium resistance thermometer</td>
<td>0.05–100 K</td>
<td>Must be individually calibrated</td>
<td>0.5 mK at 4.2 K</td>
<td>±1 mK/yr at 4.2 K to 77 K</td>
<td>No</td>
<td>Not recommended</td>
<td>Secondary-standard thermometer</td>
<td>High with individual calibration</td>
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<tr>
<td></td>
<td></td>
<td>With individual calibration:</td>
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<td>Excellent reproducibility</td>
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<td></td>
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<td>5 mK at &lt; 10 K</td>
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<td>0.07% at &gt; 10 K</td>
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<tr>
<td>Silicon diode thermometer</td>
<td>1.4–450 K</td>
<td>Without calibration:</td>
<td>5 mK at 4.2 K</td>
<td>±10 mK/yr at 4.2 K to 77 K</td>
<td>Yes</td>
<td>Not recommended</td>
<td>Relatively inexpensive,</td>
<td>Medium for low accuracy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 K at &lt; 100 K,</td>
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<td></td>
<td></td>
<td>easily measured output</td>
<td>High with individual calibration</td>
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<tr>
<td></td>
<td></td>
<td>1 K at 100 K to 300 K</td>
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<td></td>
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<td>Small size</td>
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<td></td>
<td></td>
<td>20 mK at 1.4 K–10 K</td>
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<td></td>
<td></td>
<td>15 mK at 300 K</td>
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</tbody>
</table>


b Reproducibility: the change in apparent temperature when the sensor is subjected to repeated thermal cycling from room temperature.

c Interchangeability: the ability to substitute one sensor for another with little change in calibration.