Elastic X-ray Scattering
Vs
Elastic Neutron Scattering

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In elastic scattering there is no energy exchange to the sample.
\[ \vec{Q} = \vec{k}_i - \vec{k}_f \]

\[ |\vec{k}_i| = |\vec{k}_f| = \frac{2\pi}{\lambda} \]

\[ \frac{|\vec{Q}|}{2} = |\vec{k}_i| \sin \theta = \frac{2\pi}{\lambda} \sin \theta \]

\[ Q = \frac{4\pi \sin \theta}{\lambda} \]
Interaction with atoms
Elastic X-ray scattering is characterized by Thompson scattering

\[
\frac{d\sigma}{d\Omega} = \left( \frac{e^2}{4\pi\varepsilon_0 mc^2} \right)^2 \frac{1 + \cos^2 2\theta}{2}
\]

\[
\sigma = \frac{8\pi}{3} \left( \frac{e^2}{4\pi\varepsilon_0 mc^2} \right)^2 = \frac{8\pi}{3} r_e^2
\]

\[
r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.818 \text{ fm}
\]
Elastic Neutron Scattering

\[
\frac{d\sigma}{d\Omega} = b^2
\]

\[
\sigma = 4\pi b^2
\]
## Comparison

<table>
<thead>
<tr>
<th>Elastic X-Ray Scattering</th>
<th>Elastic Neutrons Scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both can be used to probe inside materials.</td>
<td></td>
</tr>
<tr>
<td>Electromagnetic radiation interaction with electrons</td>
<td>Particle with spin $\frac{1}{2}$, interaction with the nucleus</td>
</tr>
<tr>
<td>Give a description about electron density distribution</td>
<td>Gives the position of the nucleus</td>
</tr>
<tr>
<td>Atomic form Factor</td>
<td>No atomic form factor</td>
</tr>
<tr>
<td>Scattering length is proportional to $Z$</td>
<td>Scattering lengths are random</td>
</tr>
<tr>
<td>Do not interact good with light elements</td>
<td>Can identify light elements</td>
</tr>
<tr>
<td>Cannot distinguish isotope</td>
<td>Can identify isotopes</td>
</tr>
<tr>
<td>Magnetic scattering relatively law</td>
<td>Magnetic scattering factor Can identify magnetic structure of a material</td>
</tr>
<tr>
<td>Not a good choice for sensitive sample characterization (such as biological samples)</td>
<td>Usually biological samples uses neutron scattering</td>
</tr>
</tbody>
</table>
Atomic Form Factor

Measure of the scattering amplitude of a wave by an isolated atom.

\[ f(Q) = \int \rho(r) e^{iQ \cdot r} \, d^3r, \]

Proportional to \ Z.

Mathematically speaking this is the Fourier transform of the electron density.
X-ray and neutron scattering amplitudes for a potassium atom.
Scattering Cross section

Nuclei Seen by X-Rays

X-rays interact with the electron cloud

Nuclei Seen by Neutrons

Neutrons interact with the nuclei
<table>
<thead>
<tr>
<th>Element</th>
<th>H</th>
<th>D</th>
<th>U</th>
<th>Fe</th>
<th>Co</th>
<th>Ba</th>
<th>O</th>
<th>V</th>
<th>Ti</th>
<th>Zr</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-ray f (0°) (Z×2.818 fm)</td>
<td>1</td>
<td>1</td>
<td>92</td>
<td>26</td>
<td>27</td>
<td>56</td>
<td>8</td>
<td>23</td>
<td>22</td>
<td>40</td>
</tr>
<tr>
<td>Neutron b (fm)</td>
<td>-3.740</td>
<td>6.674</td>
<td>8.420</td>
<td>9.450</td>
<td>2.780</td>
<td>5.280</td>
<td>5.805</td>
<td>-0.3824</td>
<td>-3.438</td>
<td>5.280</td>
</tr>
</tbody>
</table>

Isotopes of a given element have different neutron scattering lengths (H and D).

Adjacent elements that would be difficult to distinguish in X-ray scattering (Fe and Co).

"Heavy" elements do not scatter on average very much more than lighter ones (D and U).

Hard to identify elements with a large neutron absorption. (Gd, Sm, Eu, and Dy).

V is a weak scatter of neutron
Magnetic Sensitivity

Magnetic interaction with X rays in the order of $10^{-6}$

Neutrons have $\frac{1}{2}$ spin

$$\mu_n = -1.913 \frac{eh}{2m_p}$$

$$\frac{\mu_n}{\mu_e} \sim \frac{m_e}{m_p} \sim \frac{1}{2000}$$

Neutron magnetic moment is very small compared an electron.

Even we can identify the magnetic structure of the materials using neutron scattering.
Magnetic Form Factor
Neutrons and x-rays of the same wavelength will have extremely different energies.

For example, a neutron with a De Broglie wavelength of 0.15nm will have an energy of 36.4 meV while an x-ray with 0.15nm wavelength has an energy of 8.2 keV.

The x-ray is more energetic than the neutron by a factor of 200,000. This means that when it comes to investigating sensitive samples, such as biological samples, neutrons are preferable than x-rays.
Sometimes, two methods are combined to cover different aspects of materials structures.

A neutron diffraction map can show the positions of the nuclei while X-ray diffraction map can give the distribution of the electrons.
References

[1] “Neutron Diffraction” by G.E Bacon
[4] Lectures by Roger Pynn
Thank you very much

Questions are welcome.