Elastic X-ray Scattering
Vs
Elastic Neutron Scattering

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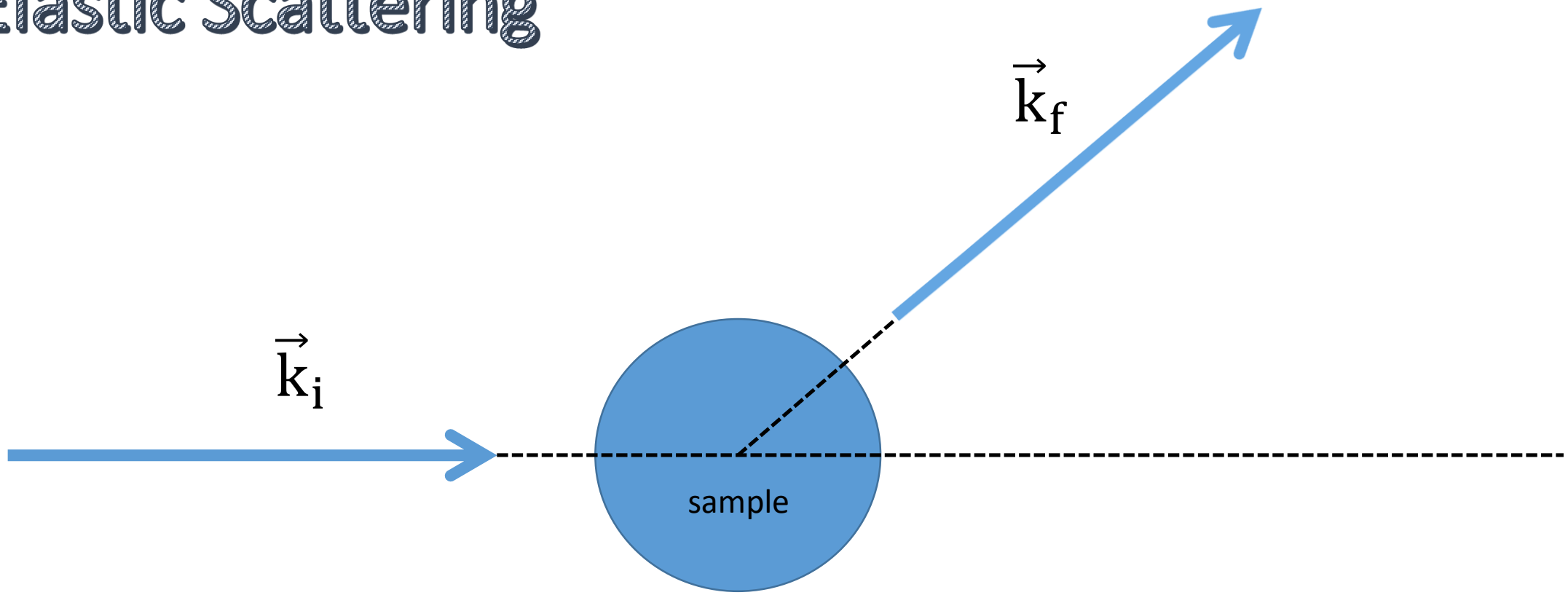
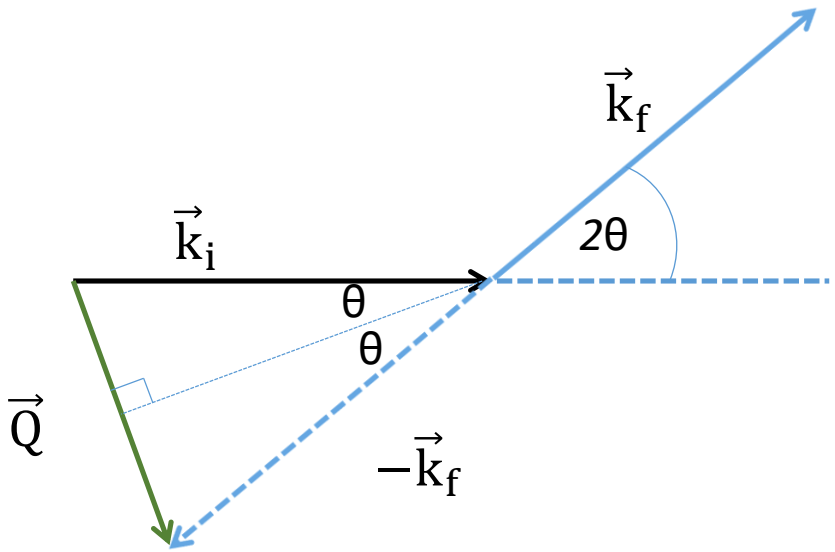
PH 590B

Elastic Scattering



In elastic scattering there is no energy exchange to the sample



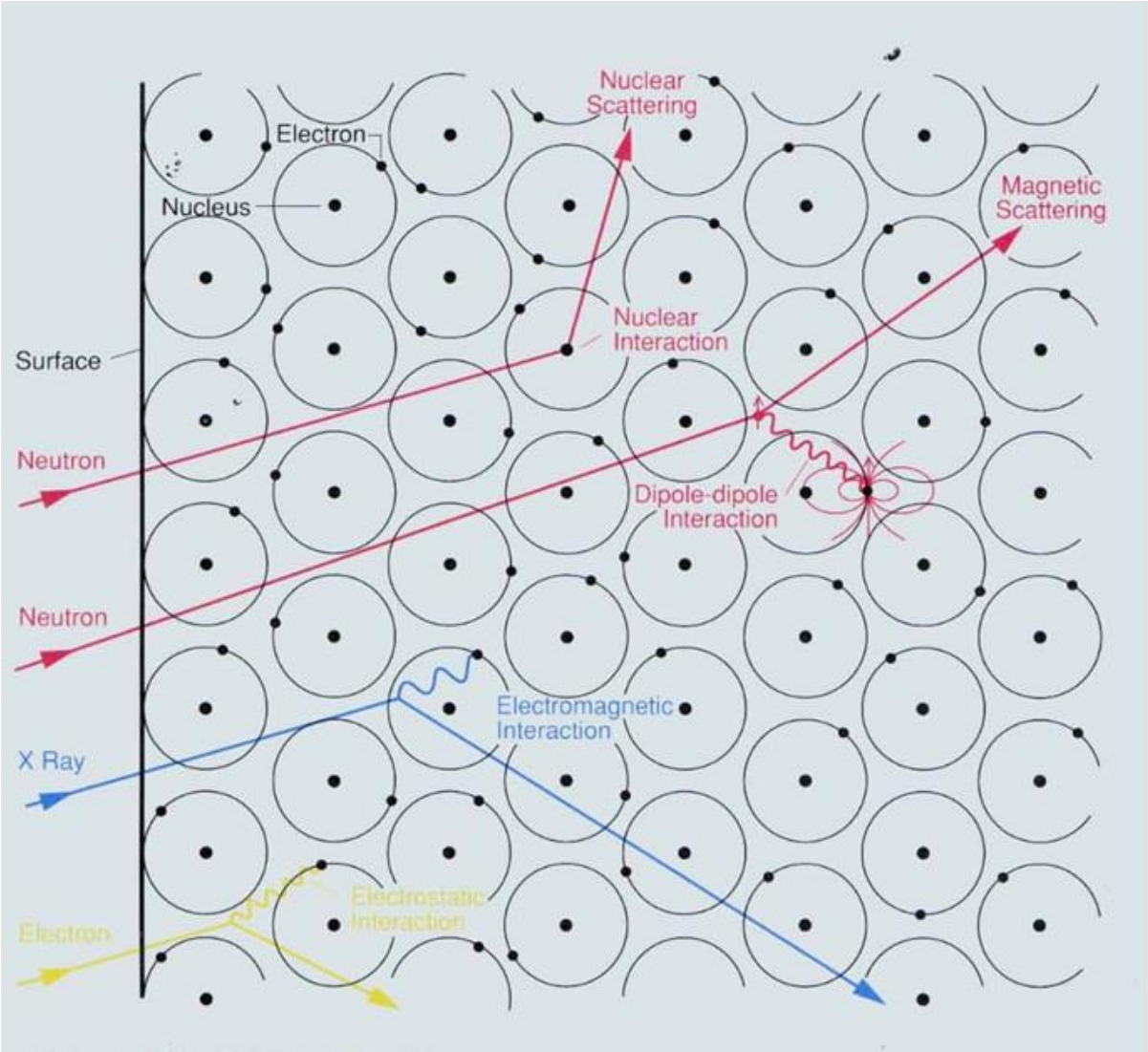
$$\vec{Q} = \vec{k}_i - \vec{k}_f$$

$$|\vec{k}_i| = |\vec{k}_f| = \frac{2\pi}{\lambda}$$

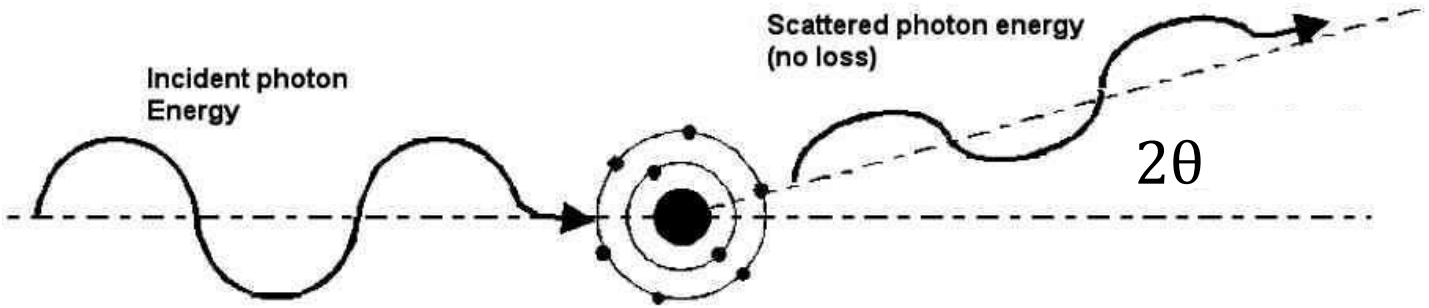
$$\frac{|\vec{Q}|}{2} = |\vec{k}_i| sin\theta = \frac{2\pi}{\lambda} sin\theta$$

$$\boxed{Q = \frac{4\pi\, sin\theta}{\lambda}}$$

Interaction with atoms



Elastic X-ray Scattering



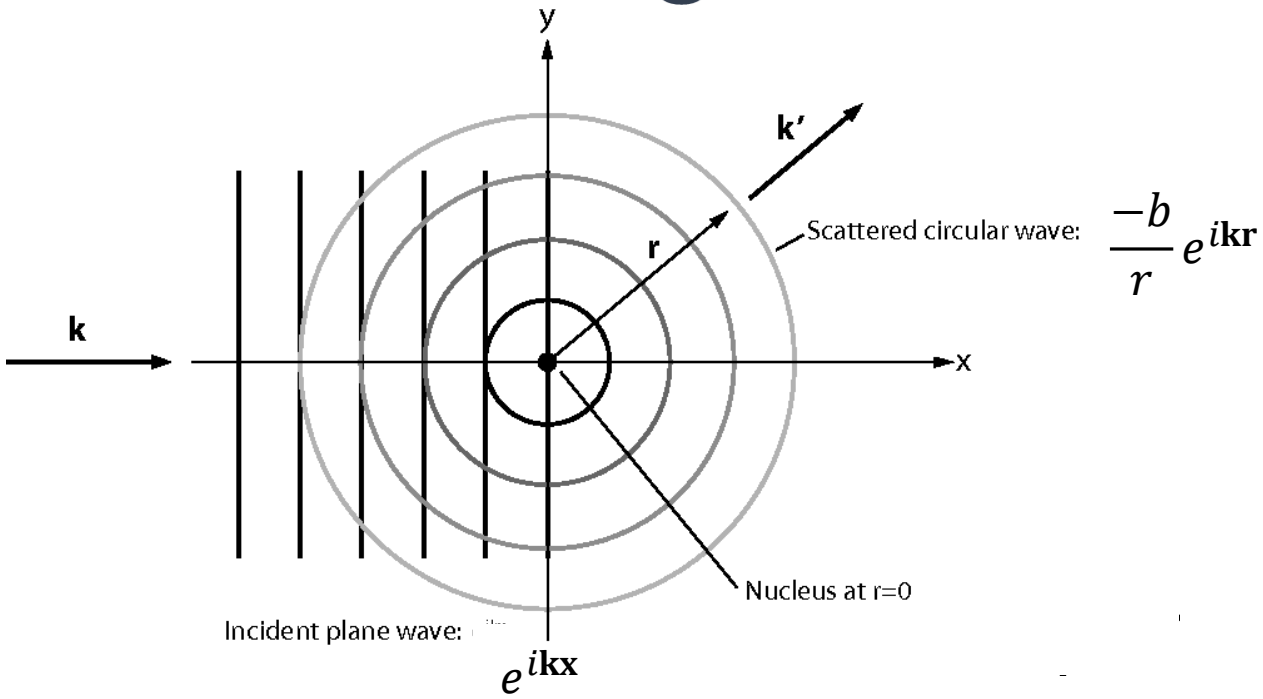
Elastic X- ray scattering is characterized by Thompson scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2 \frac{1 + cos^2 2\theta}{2}$$

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2 = \frac{8\pi}{3} {r_e}^2$$

$$r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.818\ fm$$

Elastic Neutron Scattering



$$\frac{d\sigma}{d\Omega} = b^2$$

$$\sigma = 4\pi b^2$$

Comparison

Elastic X-Ray Scattering	Elastic Neutrons Scattering
Both can be used to probe inside materials.	
Electromagnetic radiation interaction with electrons	Particle with spin ½, interaction with the nucleus
Give a description about electron density distribution	Gives the position of the nucleus
Atomic form Factor	No atomic form factor
Scattering length is proportional to Z	Scattering lengths are random
Do not interact good with light elements	Can identify light elements
Cannot distinguish isotope	Can identify isotopes
magnetic scattering relatively law	Magnetic scattering factor Can identify magnetic structure of a material
Not a good choice for sensitive sample characterization (such as biological samples)	Usually biological samples uses neutron scattering

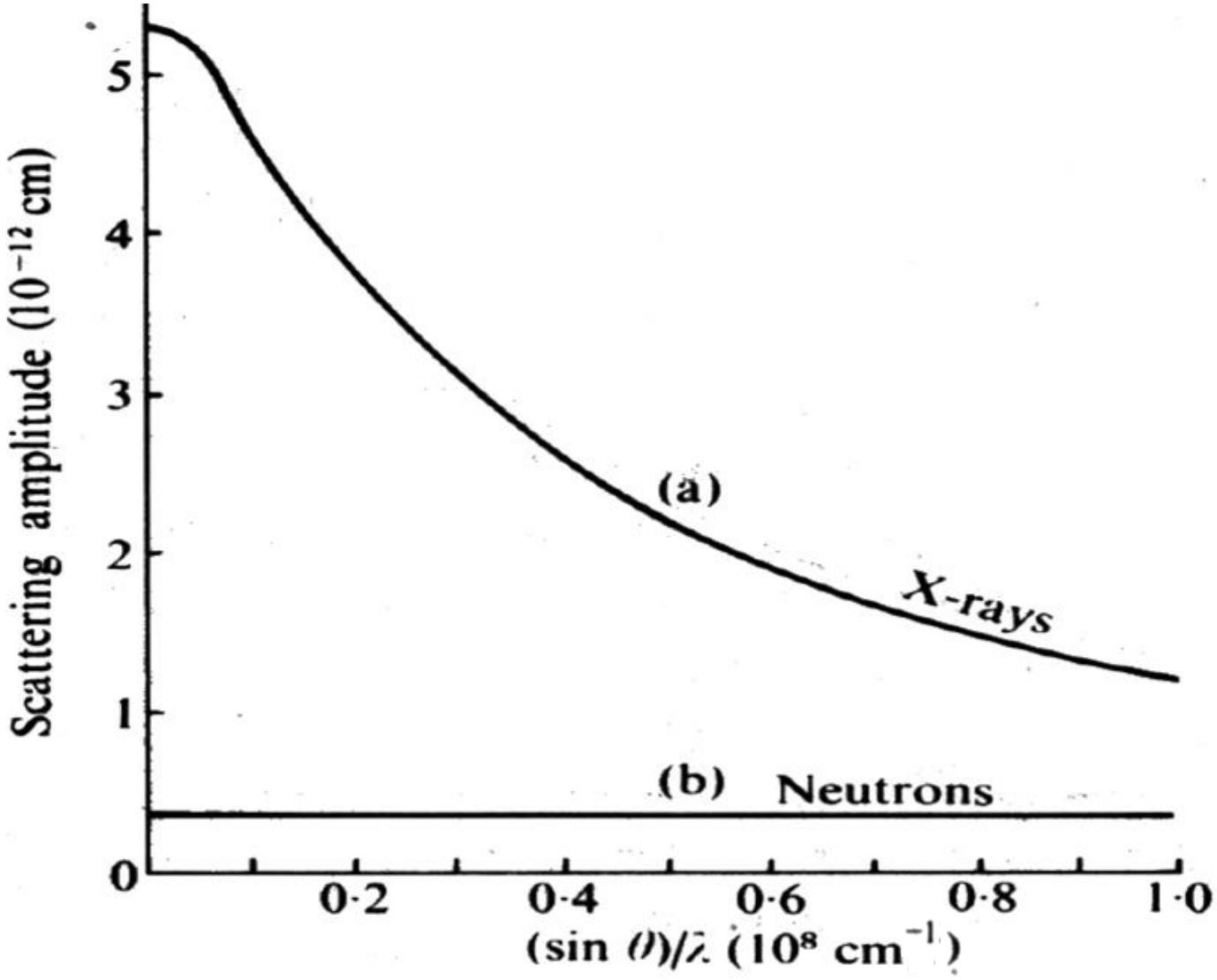
Atomic Form Factor

Measure of the scattering amplitude of a wave by an isolated atom.

$$f(\mathbf{Q}) = \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \mathrm{d}^3\mathbf{r},$$

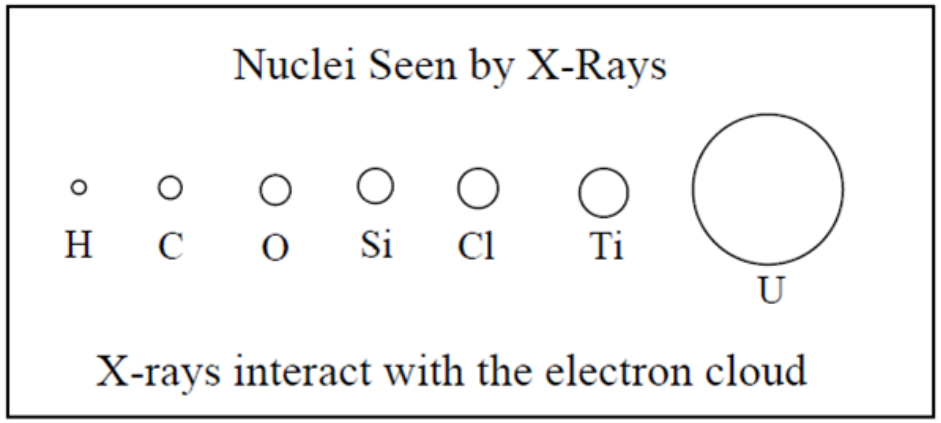
Proportional to Z.

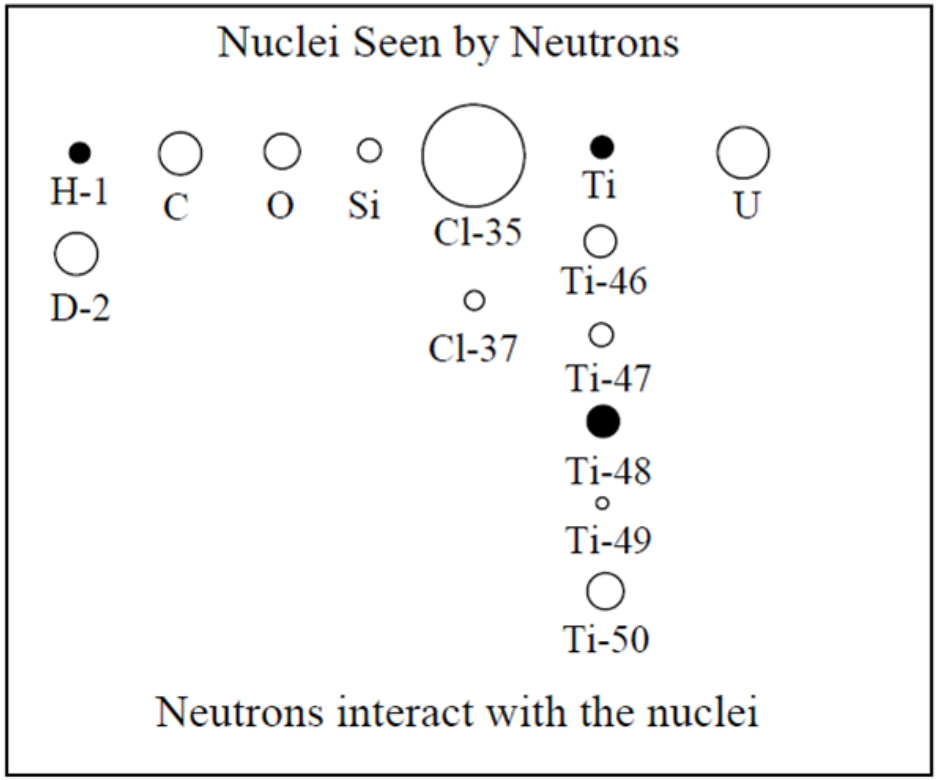
Mathematically speaking this is the Fourier transform of the electron density.



X-ray and neutron scattering amplitudes for a potassium atom.

Scattering Cross section





Element	H	D	U	Fe	Co	Ba	O	V	Ti	Zr
X-ray *f* (0°) (*Z*×2.818 fm)	1	1	92	26	27	56	8	23	22	40
Neutron *b* (fm)	-3.740	6.674	8.420	9.450	2.780	5.280	5.805	-0.3824	-3.438	5.280

Isotopes of a given element have different neutron scattering lengths (**H** and **D**)

Adjacent elements that would be difficult to distinguish in X ray scattering (**Fe** and **Co**)

“Heavy" elements do not scatter on average very much more than lighter ones (**D** and **U**)

Hard to identify elements with a large neutron absorption. (**Gd**, **Sm**, **Eu**, and **Dy**)

V is a weak scatter of neutron

Magnetic Sensitivity

Magnetic interaction with X rays in the order of 10^{-6}

Neutrons have ½ spin

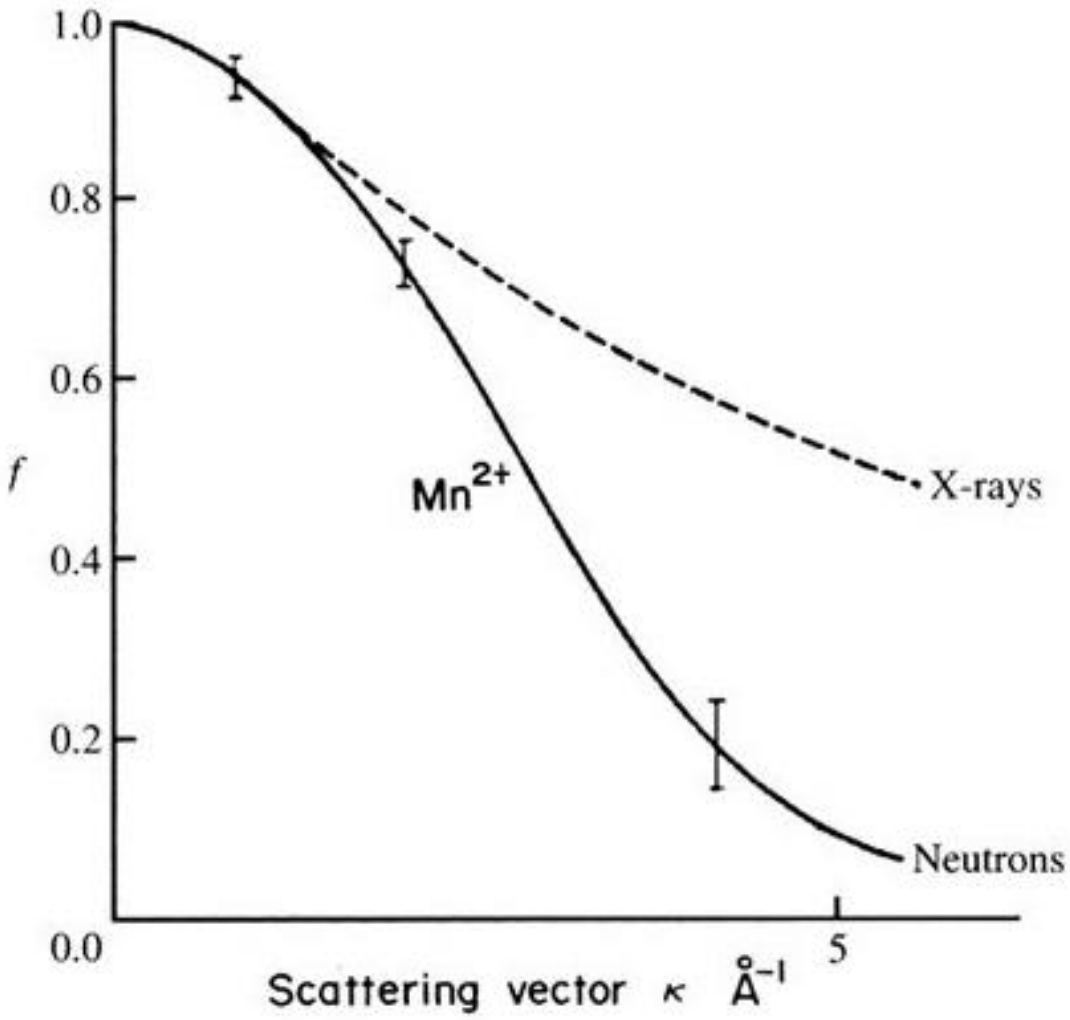
$$\mu_n = -1.913 \frac{eh}{2m_p}$$

$$\frac{\mu_n}{\mu_e} \sim \frac{m_e}{m_p} \sim \frac{1}{2000}$$

Neutron magnetic moment is very small compared an electron.

Even we can identify the magnetic structure of the materials using neutron scattering.

Magnetic Form Factor



Sensitive material

Neutrons and x-rays of the same wavelength will have extremely different energies.

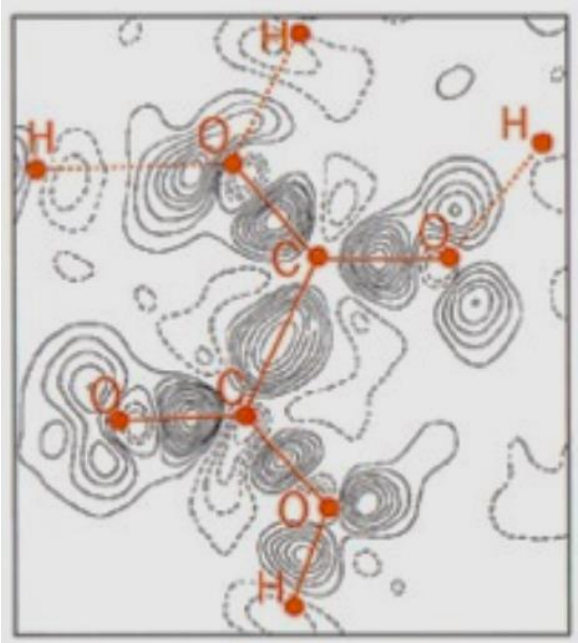
For example, a neutron with a De Broglie wavelength of 0.15nm will have an energy of 36.4 meV while an x-ray with 0.15nm wavelength has an energy of 8.2 keV.

The x-ray is more energetic than the neutron by a factor of 200,000 . This means that when it comes to investigating sensitive samples, such as biological samples, neutrons are preferable than x-rays.

X ray and Neutron diffraction maps

Sometimes, two methods are combined to cover different aspects of materials structures.

A neutron diffraction map can show the positions of the nuclei while X-ray diffraction map can give the distribution of the electrons.



References

[1] “Neutron Diffraction” by G.E Bacon

[2] “Elementary Scattering Theory” by D.S. Sivia

[3] L Finney, J. (1995). The complementary use of X-ray and neutron diffraction in the study of crystals. Acta Cryst.. B51. 10.1107/S0108768195002734.

[4] Lectures by Rogger Pynn

Thank you very much

Questions are welcome.