Thermal conductivity:
-why?
-how?
-what can we get?

590B

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• Experimental constraints
• Experiment design
• Dilution refrigerator (DF)
• Heat conduction in superconductors
Thermal conductivity: Old problem

\[ j = \sigma E \]

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\[ Q = -\kappa \nabla T \]

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\[ \kappa = \frac{d Q}{S (T_+ - T_-)} \]
Conduction of heat: Old problem

150 years of scientific exploration

Wiedemann-Franz law
Good electrical conductors are good thermal conductors
For metal $\kappa \sim \sigma$

Common experience
Dense crystalline solids are better heat conductors than porous materials

Big tables on thermal conductivity of various materials

Technically important number
Examples
Thermal conductivity and heat capacity are closely related.

**Phonons:** for $T < 0.1 \ \Theta_D \ C_g \sim T^3$

**Electrons:** $C_e \sim T$

Low temperatures $\lambda = \text{const}$

Phonons $\kappa_g \sim T^3$

Electrons $\kappa_e \sim T$
Thermal conductivity: Old means well understood!

Lorenz formulation of the Wiedemann-Franz law

\[ L = \frac{\kappa}{T \sigma} \]

\( L \) approximately \( T \)-independent Lorenz number

Sommerfeld in 1927 calculated Lorenz number for the particles obeying Fermi-Dirac statistics

\[ L_0 = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2 \]

Perfect QUANTITATIVE agreement in \( T \to 0 \) limit

Why do we bother to study?
Quantum states of matter are characterized by different statistics hence different $L_0$

Examples:

**superconductivity**  
Superconducting condensate carries charge without entropy  
superconductors are poor heat conductors $L_0 \equiv 0$

**Quantum Hall effect**: charge transport without entropy  
new statistics for the quasi-particles

So, because …

**Low temperature thermal conductivity is**  
a semi-quantitative tool for studying quantum states of matter
How? Experimental constraints

Unlike electrical conduction, heat does not need mobile charge carriers and is not confined to electrical wires. Heat flows everywhere!

We want to study.

Heat conduction:
Heat goes through a static material (medium).

We want to exclude:

Heat convection:
Heat goes through a moving medium or is carried away by a moving medium (fluid, gas).
Vacuum \(\sim 10^{-6}\) Torr may be not good enough for poorly conducting samples! M.Y.Choi, P. M. Chaikin, R.L. Greene PRB34, 7727 (86)

Heat radiation:
Heat travels through space with or without a medium.
Thermal radiation: Stefan-Boltzmann law

Total energy radiated per unit surface area of a black body in unit time, $J^*$, is directly proportional to the fourth power of the black body absolute temperature

$$J^* = \sigma T^4$$

$\sigma = 5.67 \times 10^{-12} \text{W/cm}^2\text{K}^{-4}$

Room temperature, $J^* = 5.67 \times (300)^4 = 46 \text{ mW/cm}^2$

1 K, $J^* = 5.67 \times \text{picoW/cm}^2$

Is this a lot?
### Thermal conductivity: radiation

**Room temperature**

<table>
<thead>
<tr>
<th>Substance</th>
<th>$\kappa$ [mW/cm K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>4200</td>
</tr>
<tr>
<td>Copper</td>
<td>3800</td>
</tr>
<tr>
<td>Steel</td>
<td>400</td>
</tr>
<tr>
<td>Water</td>
<td>20</td>
</tr>
<tr>
<td>Glass</td>
<td>8.4</td>
</tr>
<tr>
<td>Wood</td>
<td>1</td>
</tr>
<tr>
<td>Wool</td>
<td>0.4</td>
</tr>
<tr>
<td>Polyuretane</td>
<td>0.24</td>
</tr>
<tr>
<td>Air</td>
<td>0.23</td>
</tr>
</tbody>
</table>

**Room temperature, $J^* = 46$ mW/cm$^2$**

For $\Delta T = 1$K and 1 cm$^2$ area, $\Delta J^* = 0.5$ mW/cm$^2$

But this can be not sample but the apparatus surface!

**BIG problem for poor conductors!**

Degrees of freedom to play
Sample geometry
Quasi-isothermal measurements
Thermal shielding geometry
Heat exchange through radiation is not important at 10 K and below.

Cryogenic pumping makes perfect vacuum

Dilution refrigerator sets friendly environment for thermal conductivity measurements*

GENERAL PROBLEMS FOR ALL MEASUREMENTS AT LOW TEMPERATURES

- RF heating
- Vibration
- Kapitza resistance*

Extremely important for thermal measurements

*Cheese is free only in the mouse trap!
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Atomic mass (m_a/u)</th>
<th>Natural abundance (atom %)</th>
<th>Nuclear spin (I)</th>
<th>Magnetic moment (μ/μ_N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>^3He</td>
<td>3.016 029 309 7(9)</td>
<td>0.000137 (3)</td>
<td>1/2</td>
<td>-2.127624</td>
</tr>
<tr>
<td>^4He</td>
<td>4.002 603 2497(10)</td>
<td>99.999863 (3)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Physical properties**

<table>
<thead>
<tr>
<th></th>
<th>Helium-3</th>
<th>Helium-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling (1atm)</td>
<td>3.19 K</td>
<td>4.23 K</td>
</tr>
<tr>
<td>Critical point</td>
<td>3.35 K</td>
<td>5.19 K</td>
</tr>
<tr>
<td>Density of liquid</td>
<td>0.059 g/ml</td>
<td>0.12473 g/ml</td>
</tr>
<tr>
<td>(at boiling point, 1atm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latent heat of vaporization</td>
<td>0.026 kJ/mol</td>
<td>0.0829 kJ/mol</td>
</tr>
</tbody>
</table>
Cooling power of evaporating cryogenic liquid

\[ Q = n \Delta H = nL, \]

Q cooling power
n rate of evaporation, molecules/time
\( \Delta H \) enthalpy of evaporation
L latent heat of evaporation

For a pump with constant volume rate V

\[ Q = VP(T)L \]

L approximately constant

\[ \text{Latent heat } L \ [\text{J/mole}] \]

\[ \text{Temperature } [\text{K}] \]
Cooling power proportional to vapor pressure
$Q \sim P(T) \sim \exp(-1/T)$

Exponentially small at low $T$
We can get by pumping on
He-4 $T \sim 1 K$
He-3 $T \sim 0.26 K$

Evaporative cooling is used in
1K pot
He-3 cryostat
He-3 refrigerator

Typical features:
Sample in vacuum
(rarely sample in He-3 liquid)
One shot mode of operation
Hold time 10-60 hours

Operation sequence:
Release He-3 from cryopump
Condense by heat exchange with
1K pot
Cool condensate to 1.5K
Start cryopumping to reach
base temperature

He-3 is stored in a sealed space to avoid loss
He-3 pump is called Sorb, uses cryopumping
No solid phase due to:
weak van der Waals inter-atomic interactions, $E_{pot}$ is low
Large quantum mechanical zero-point energy $E_0 = \hbar^2/8ma^2$
due to small mass, $E_{kin}$ is high
Bose-Einstein condensate instead of a solid

Quantum liquids, ratio $\lambda = E_{kin}/E_{pot}$ He4 $\lambda = 2.64$ He3 $\lambda = 3.05$
Amplitude of vibrations about 1/3 of interatomic space
He-3 nucleus has spin 1/2, Fermion

Additional spin entropy

Bose-Einstein condensate of pairs, several superfluid phases

Special feature: Below $T_F$ spins in the liquid phase are spatially indistinguishable. Therefore they start obeying Fermi statistics and are more ordered than in the paramagnetic solid phase!
Mixture of He3 and He-4

Phase separation of the mixture into He3 rich and He3 poor phases, but not pure He3 and He4

Pure quantum effect classical liquids should separate into pure components to obey 3rd law of thermodynamics, $S=0$

In case of He3-He4 mixture, $S=0$ can be for finite concentration because of the Fermi statistics for He3 and Bose statistics for He4

Phase separation starts below $T=0.867$ K (max at $X=0.675$)
Cooling power:
Power law decrease
Instead of exponential decrease

Enthalpy of mixing uses the difference
In specific heat of two phases

\[ \Delta H = \int \Delta C dt \]
\[ Q \sim x \Delta H \sim T^2 \]
Evaporative cooling

Dilution

Still evaporates He3 from mixture

Mixing chamber

Phase separation line
Dilution Refrigerator: more details

- Concentration phase
- Dilute phase
- Phase boundary
- Condenser line
- Main flow impedance
- Still heat exchanger
- Secondary flow impedance
- Almost pure He3
- >90% He3 vapor
- To He3 pump
-几乎纯的He3
- Still 0.7K
- Dilute phase
- Heat exchangers
- Dilute phase
- Heat flow
- Concentration phase
- Dilute phase
- Mixing chamber 0.01K
- 1K pot
- Concentric
- Sintered
- Gravity
- Temperature
- Phase boundary
Kapitza resistance

A discontinuity in temperature across the interface of two materials through which heat current is flowing acoustic impedance mismatch at a boundary of two substances phonons have probability to be reflected

Kapitza resistance, $\sim T^3$
Important effect as $T$ tends to 0
1K vs 10 mK
6 orders of magnitude change!

$$\hat{Q} = \kappa_K \Delta T$$

$\kappa_K$ - Kapitza conductance

Good Thermal contact at low temperatures needs conduction electrons
Kapitza resistance

Mismatch of the excitation spectra
\( R_K \) dissimilar > \( R_K \) similar

Pobell’s recommendation

- Metal-metal
- Polished surfaces
- Strong mechanical force
- Gold-plating
- Welding

Soldering
- Silver based allows
- Few solders are not SC!

Insulators
- GE
- Stycast 1266
Dilution refrigerator: heat exchanging
Need big surface area contacts!

Concentric heat exchanger
High temperatures

Welded Cu-Ni foil
Sintered submicron silver powder
Close to mixing chamber
Dilution refrigerator: Experiment cooling

Do not rely on insulating contacts!

Vibration

RF heating
Dilution refrigerator: gas handling at room temperature

Key elements:
- He3-He4 Gas storage “Dump”
- Vacuum pump for 1K pot
- Vacuum pump for He3 circulation
- Roots (booster) pump for Still line pumping
- Cold traps for mixture cleaning

Very demanding to vacuum leaks
To avoid loss of mixture, all operation goes at $P<P_{\text{atm}}$
Leaks in, not out!
Dilution refrigerator: gas handling system

Front view

Back view

He3  He4
Kapitza resistance in the bulk: electron-phonon decoupling

Phonons vs electrons
Bosons vs Fermions

Contact Sample

\[ R_{el-ph}^{-1} \sim K_{el-ph} T^n, \quad n=4-5. \]

FIG. 1. A simple picture of the experimental configuration for thermal conductivity measurements. The thermal resistance to phonon and electron heat currents that occurs before the current enters the sample are represented by \( R_{ph(c)} \) and \( R_{el(c)} \), respectively. The thermal resistance to phonon and electron heat flow through the sample are represented by \( R_{ph} \) and \( R_{el} \), respectively. The electron-phonon heat transfer rate is associated with \( R_{el-ph} \). Temperatures at different positions have been labeled.

M. Smith et al. PRB71, 014506 (2005)
How? Thermalization

Electrical connection
Thermal insulation

4 probe
electrical resistivity
thermal conductivity
Thermopower
But never the same
Sample wires!

Test results
How? Sample preparation and mounting

Figure 4.1: CeCoIn$_5$ sample with In-soldered leads.
Something about superconductivity

BCS theory of superconductivity
- conventional
- unconventional

Thermal conductivity of superconductors
- $T$ zero field
- $J_\varphi \theta$ anisotropy
- $X$ impurities
- $H$ vortex state
- $H_{\Phi \theta}$ anisotropy
- $H_{c2}$
Metallic state

fermionic quasiparticles

Pauli exclusion principle

finite attraction

Cooper pair – boson
All occupy same quantum state

BCS coherent groundstate
- pairwise correlated
NO one-electron picture

\( \Psi(r) \sim \Delta(r) \)

Superconducting state

composite boson
“Cooper pair”
\( k_1 = -k_2 \)

energy break pair

\( 2\Delta \)
**BCS model – ‘conventional’ superconductivity**

- **energy gap** $\Delta$, $\Psi(r)$
- **Isotropic s-wave**
- **spin singlet**
- **phonon** mediated coupling

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**Experimental Manifestation – fully gapped superconductor**

- For low T expect Activated behaviour
  - e.g. $C_e \sim \exp \left( -\frac{\Delta}{k_BT} \right)$

- Minimal effect of impurities unless magnetic
Heat conduction conventional SC: electrons

Electronic excitations freeze out, Condensate carries no entropy $\kappa \equiv 0$

SC=thermal insulator

BRT theory
J. Bardeen, G. Rickayzen, L. Tewordt (1959)

Low $T$, $\sim \rho_0$ regime ($\sigma_e \sim \text{const}$)

$$\kappa_e = (nt_{tr}T / 2m) \int_{\Delta/T}^{\infty} x^2 ch^{-2} (x / 2)dx$$

$$\kappa_{es} \propto \left( \frac{\Delta}{k_B T} \right)^2 \exp \left( -\frac{\Delta}{k_B T} \right)$$

$T \ll T_c$

+phonons!

Fig. 3. Ratio of superconducting to normal thermal conductivity for aluminum.
C. B. Satterthwaite, Phys. Rev. 125 873 (1962)
The physics of heat conduction: metals, insulators, SC

Metal

Insulator

SC

In experiment we study thermal metal-insulator crossover

Phonon scattering on conduction electrons dies
The physics of heat conduction: superconductors $T \ll T_c$

\[ \frac{\kappa_{es}}{\kappa_n} \propto \left( \frac{\Delta}{k_B T} \right)^2 \exp \left( - \frac{\Delta}{k_B T} \right) \]

Phonon peak

Electrons $\sim T^\alpha \exp(\Delta/T_c) \sim 0$

Phonons $\sim T^\alpha$

\[ \alpha \sim 3 \]
Heat conduction conventional SC: magnetic impurities

With pairbreaking
Gap in DOS is filled

Fig. 6. The normalized density of states $N/N_0$ plotted as a function of the reduced quasiparticle energy for several values of the reduced inverse collision time $\Gamma/\Delta$ (from Skalski et al., Ref. 7). The corresponding values of the gap $\Omega_g$ are indicated.

Fig. 2. Measured tunnel conductance $g(V)$ of the Pb-Gd system as a function of energy $V$ for various Gd concentrations $c$. Curves (a) and (b) were taken at 1.1$^\circ$K; curves (c) and (d) at 0.4$^\circ$K. The transition temperature of the pure superconductor is denoted by $T_{C_p}$. [The finite values of $g(V)$ near $V=0$ are unreliable; see text.]

Magnetic pair-breaking smears the SC gap edges
Finite density of states in the SC gap

Wolf PR137, 557 (1965)
Heat conduction conventional SC: magnetic impurities "Gado"

V. Ambegaokar, A. Griffin
PR137, 1151 (1965)

Localized states?

R. L. Cappelletti, D. K. Finnemore
PR188, 123 (1969)

Fig. 3. The ratio of the electronic thermal conductivity in the superconducting and normal states \((K_s/K_n)\) vs \(t = T/T_c\) for various paramagnetic impurity concentrations. \(T_c\) is the transition temperature for the relevant impurity concentration, the latter being expressed in terms of \(n_{cr}\), the concentration required to completely destroy superconductivity. The curve for nonmagnetic impurities alone is denoted by \(n_i = 0.00 n_{cr}\). The \(n_i = 0.70 n_{cr}\) curve is almost identical to the 0.85 curve for \(t\geq 0.7\) and is not shown explicitly in this region. These results are based on (1.1) or (3.11).

Fig. 6. Comparison of pure Th with the Th-Gd alloys. At high temperatures the alloy data lie below pure Th, and at low temperatures they lie above the pure-Th data.
Heat conduction conventional SC: magnetic impurities

**Pb:Gd**

![Graph showing heat conduction data for Pb:Gd films.](image)

**FIG. 4.** Experimental and theoretical (Ref. 2) reduced transition temperature as a function of the reduced impurity concentration for the Pb-Gd films. The indicated impurity concentrations are those determined by chemical analysis of the ingots from which the films were made.

B.J. Mrstic, D.M. Grinsberg, PRB7, 4844 (1973)

**FIG. 6.** Reduced thermal conductivity as a function of the reduced temperature for the two Pb-Gd films. The dashed lines show the theoretical values for a weak-coupling superconductor, and the solid lines show the values obtained when the theory is modified by strong-coupling corrections for a reduced gap appropriate for pure lead. The zero has been shifted up by 0.2 for the higher-concentration alloy to avoid overlap. The indicated impurity concentrations are those determined from the measured transition temperatures.
Heat conduction conventional SC: magnetic impurities

Fig. 4. Reduced thermal conductivity $K_r/K_e$ vs. $T/T_c$ for a LaAl$_2$ single crystal in three different annealing stages, characterized by the resistance ratio $n$. Solid line: modified BRT curve.

Fig. 5. $K_r$ reduced by $K_e(T_c)/T$ as a function of $T/T_c$ for the (La$_1-x$Gd$_x$)Al$_2$ single crystal with 0.23 a/o Gd before (full triangles) and after annealing (open triangles). Dashed curves: fit results corresponding to Equation 5 with the parameters: $y_0 = 0.9$, $r_1/t_g = 0.6$, $t_1 = 0.0$ ($r = 33$) and 0.6 ($r = 155$).

CeAl$_2$:Gd

$T_c/T_{c0}$

Suggestion:
Localized-delocalized character of the quasi-particles
Need to include into consideration
distance of impurity level to the band-gap

**Thermal conductivity of Conventional SC: Vortex State**

\[ H > H_{c1} \text{ Vortex state} \]

- **Transport:** overlap of the bound states

- **Phonons, } T \neq 0\]

\[ 1/\kappa_g \sim H \sim \gamma_S / \gamma_N \]

**Vortex scattering**

\[ \kappa / \kappa_N \sim H / H_{c2} \]
‘Unconventional’ superconductivity

The symmetry of the SC state is lower than the symmetry of the N state.

Gap is not constant at nodes the SC gap vanishes.
‘Unconventional’ superconductivity: residual linear term

$T \ll T_c$

Clean limit
linear increase of density of states with $E$

Standard techniques
penetration depth
heat capacity
NMR relaxation rate

$\kappa^2 \propto T^\beta$
$C \propto T^\gamma$
$T_1^{-1} \propto T^\delta$

No information on location of the nodes on FS

Unconventional SC = bad thermal metal

$\gamma$ impurity bandwidth

$\frac{\kappa}{T}$

phonons $\sim T^3$
electrons $\sim T$

0 $T^2$
C. B. Satterthwaite, Phys. Rev. 125 873 (1962)

**Heat conduction SC zero field: electrons**

**Full gap: “Conventional”**

![Graph showing the ratio of superconducting to normal thermal conductivity for aluminum.]

**Nodal: “Unconventional”**

![Graphs of $\kappa_a/\kappa$ and $\kappa_s/\kappa_n$ for CeIrIn$_5$ a-axis.]

Fig. 3. Ratio of superconducting to normal thermal conductivity for aluminum.

$2\varepsilon_0(0) = 3.0kT_c$

$2\varepsilon_0(0) = 3.25kT_c$

$2\varepsilon_0(0) = 3.52kT_c$

$\rho(T_c)/\rho(0) \sim 1.5$

H. Shakeripour, unpublished
Heat conduction SC zero field: effect of inelastic scattering

$\frac{\kappa_a}{T} \approx 120$ (N)

$\frac{\kappa_a}{T} \approx 60$ (SC)

$\frac{\kappa_s}{\kappa_N} \approx 1.0$

$T/T_c$

$\rho(T_c)/\rho(0) \approx 1.5$

$\rho(T_c)/\rho(0) \approx 20$ pure

$\sim 5\%$

Similar effect in the cuprates
The physics of heat conduction: Inelastic scattering

BRT Metal

\[ \frac{\kappa^e}{T} = L_0 \sigma^e \]

\[ \kappa^g = \frac{1}{3} C^g v_s \lambda^g_e \]

\[ \lambda^g_e \sim 1/C^e \sim 1/T \]

\[ \kappa^g \sim T^2 \]

Strong Inelastic + nodal

BRT SC

Phonon scattering on conduction electrons dies

In experiment we study thermal metal-insulator crossover
‘Unconventional’ superconductivity: residual linear term

**Tl2201**

Residual linear term = Line nodes in the SC gap

Why $\kappa/T$ is T-linear?
Electronic contributions +
Phonons are scattered by QP
### ‘Unconventional’ superconductivity: anisotropy

**Allowed representations in D\(_{4h}\) symmetry**

**TABLE I:** Even-parity (spin-singlet) pair states in a tetragonal crystal with point group \(D_{4h}\) [22]. (\(V\) = vertical line node, \(H\) = horizontal line node.)

<table>
<thead>
<tr>
<th>Representation ()</th>
<th>Gap ()</th>
<th>Basis function ()</th>
<th>Nodes ()</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_{1g})</td>
<td>s-wave</td>
<td>(1, (x^2 + y^2), z^2)</td>
<td>none</td>
</tr>
<tr>
<td>(A_{2g})</td>
<td>g-wave</td>
<td>(xy(x^2 + y^2))</td>
<td>(V)</td>
</tr>
<tr>
<td>(B_{1g})</td>
<td>(d_{x^2-y^2})</td>
<td>(x^2 + y^2)</td>
<td>(V)</td>
</tr>
<tr>
<td>(B_{2g})</td>
<td>(d_{xy})</td>
<td>(xy)</td>
<td>(V)</td>
</tr>
<tr>
<td>(E_g) (1, 0)</td>
<td>-</td>
<td>(xz)</td>
<td>(V+H)</td>
</tr>
<tr>
<td>(E_g) (1, 1)</td>
<td>-</td>
<td>((x + y)z)</td>
<td>(V+H)</td>
</tr>
<tr>
<td>(E_g) (1, (i))</td>
<td>hybrid-I</td>
<td>((x + iy)z)</td>
<td>(H+points)</td>
</tr>
</tbody>
</table>

**d\(_{x^2-y^2}\)**

**hybrid**
CeIrIn$_5$ anisotropic superconductivity

Quasiparticle heat conduction: $J // a$ vs $J // c$

→ pronounced $a$-$c$ anisotropy of nodal structure

CeIrIn$_5$ anisotropic superconductivity

Quasiparticle heat conduction: $J // a$ vs $J // c$

Vertical line nodes: NO

Hybrid-I gap: OK

$\rightarrow$ $E_g (1,i)$ state

Thermal conductivity: effect of impurities

Clean limit:
Universal conductivity

\[
\frac{\kappa_{00}}{T} = \text{const}(\Gamma)
\]

Thermal conductivity of Unconventional SC: Vortex State

\( H > H_{c1} \) Vortex state

Transport: bulk itinerant states

\( \kappa: \sqrt{H} \) INCREASE WITH FIELD \( \sim H_{c1} \)

Phonons, \( T \neq 0 \)

\( 1/\kappa_g \sim \sqrt{H} \sim \gamma_S/\gamma_N \)

QP scattering

Close relation between the structure of vortex and \( \kappa \)
Thermal conductivity of Unconventional SC: H dependence

**Tl2201**

- $T_c = 76$ K
- $11.5$ T
- $0$ T

Graph showing the thermal conductivity $\kappa$ versus temperature $T$ for $J\parallel a$ in Tl2201, CeIrIn5, and Nb. The thermal conductivity ratio $\kappa_S/\kappa_N$ is also shown for various compounds including d-wave and s-wave scenarios.
Thermal conductivity with in-plane rotation of H

YBCO

Fourfold symmetry

Nb

No fourfold symmetry

Twofold symmetry

The difference between the effective DOS for QPs traveling parallel to the vortices and for those moving in the perpendicular direction

H. Aubin et al. PRL 78, 2624 (97)  
R. Ocano and P. Esquinazi, cond-mat/0207072
Doppler shift of the quasiparticle spectrum

G.E. Volovik, JETP Lett. 58, 469 (1993)

\[ d \frac{x^2 - y^2}{2} \text{ case} \]

1. \[ H \]
2. \[ p \cdot v_s > \Delta \]

2 nodes contribute
DOS small

4 nodes contribute
DOS large

I. Vekhter \textit{et al.} PRB 59, R9023 (99)
H. Won and K. Maki, cond-mat/0004105

4-fold oscillation of DOS

\[ \kappa \sim \sqrt{H} \]

\[ \kappa / \kappa_N \]

H \[ \sim \sqrt{H} \]

4-fold oscillation of QP scattering time
CeCoIn$_5$  \hspace{2cm} \kappa-(ET)$_2$Cu(NCS)$_2$  \hspace{2cm} Sr$_2$RuO$_4$

YNi$_2$B$_2$C  \hspace{2cm} PrOs$_4$Sb$_{12}$
Thermal conductivity: phase transition at $H_{c2}$

Sr$_2$RuO$_4$

Fig. 1. Temperature dependence of $C_v/T$ in magnetic fields precisely parallel to the [100] direction.

Fig. 2. Phase diagram of Sr$_2$RuO$_4$ for $H \parallel [100]$, based on specific heat. $H_{c2}$ and $H_s$ are the upper critical field and the critical field for the second superconducting transition. $H_{c2}$ is the critical field for normalization shown in Fig. 4(b). The inset shows an enlargement of Fig. 1 and the definition of $H_{c2}$ and $H_s$.

Fig. 3. Transformation of the field dependence of $C_v/T$ near $H_{c2}$ at 0.10 K on each field angle $\theta$. Except for $0.0^\circ$, each trace has an offset.

Fig. 4. (a) Transformation of the field dependence of $\kappa/T$ at 0.32 K on each field angle $\theta$. (b) The same dependence normalized by $H_{c2}$, treated as a fitting parameter.
# Thermal conductivity of SC: summary

<table>
<thead>
<tr>
<th>Isotropic SC</th>
<th>Nodal (Unconventional)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$\kappa/T = 0$ in $T=0$ limit</td>
</tr>
<tr>
<td>$X$</td>
<td>no effect</td>
</tr>
<tr>
<td>$H$</td>
<td>activated</td>
</tr>
<tr>
<td>$H\phi\theta$</td>
<td>2-fold</td>
</tr>
<tr>
<td>$J\phi\theta$</td>
<td>reflects band structure</td>
</tr>
</tbody>
</table>
Thermal conductivity of SC: comparison with other experiments

Petrovic et al.
Advantages of thermal conductivity in the SC state

\[ \kappa_e = C_e l_e V_F \]  
Thermal conductivity is closely linked with specific heat

Bulk, insensitive to superconducting filaments
No localized contributions (Schottky anomaly)
Insensitive to the transformations in the vortex lattice.
Line nodes (from temperature dependence in low temperature limit)
Position of nodes on the Fermi surface
(dependence on direction of magnetic field and of the heat flow).
Characterization of the upper critical field,
allows discrimination of 1st and 2nd order transitions.

**Thermal conductivity**
- Good at \( T \to 0 \)
- Bad at \( T_c \)

**Specific heat**
- Good at \( T_c \)
- Bad at \( T \to 0 \)
FIG. 1. Thermal conductivity vs temperature in the $ab$ plane of single-crystalline $\text{Mg}_{1-y}\text{Al}_y\text{B}_2$ ($y=0.02$ and 0.07) in zero magnetic field (open symbols) and $H\parallel c=50$ kOe (solid symbols). The arrows indicate the critical temperatures in zero magnetic field.
FIG. 3. (a) Thermal conductivity and heat capacity of NbSe$_2$ normalized to the normal state value vs $H/H_{c2}$. The heat capacity was measured in two different ways: (i) at $T = 2.4$ K on the same crystals as used in this study [10], and (ii) extrapolated to $T \rightarrow 0$ from various temperature sweeps on different crystals [18]. (b) Equivalent data for MgB$_2$ single crystals [19,20]. (c) Equivalent data for V$_3$Si, with a theoretical curve for $\kappa/T$ [9]. The specific heat is measured at $T = 3.5$ K [21] and extrapolated to $T = 0$ [22]. The straight line is a linear fit. The thermal conductivity is seen to follow the specific heat very closely for both NbSe$_2$ and the multiband superconductor MgB$_2$. It does not, however, for the conventional $s$-wave superconductor V$_3$Si.
Heat conduction SC: multiband

NbSe$_2$
### Something about quantum criticality

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### Summary & Conclusions
Lorenz ratio \( \kappa / \sigma T \)

\[ T \to 0 : L = L_0 \]

Wiedemann-Franz law

\[ T \approx T_D \]

Vanadium

Temperature dependence of Lorenz ratio

at \( T \approx T_D \) phonon scattering becomes quasi-elastic

characteristic energy scale

“Horizontal” scattering processes

Heat scattered
Charge scattered

\( k_s \)

“Vertical” scattering processes

Heat scattered
Charge is not scattered

\( k_i \)

\[ L = L_0 \]

\[ L < L_0 \]

Jung et al., PRB 15, 659 (1977)
CeRhIn$_5$: localized f-electron AF

FIG. 2. The principal dHvA frequencies in Ce$_x$La$_{1-x}$RhIn$_5$ plotted versus $x$, (a) for $B$ applied along [001] and (b) for $B$ applied along [100].

Localized f-electrons
CeMIn$_5$: Magnetic contribution to resistivity

Residual resistivity $\rho_0$

LaRhIn$_5$ etc. <0.01 $\mu\Omega$.cm

CeRhIn$_5$ 0.02 $\mu\Omega$.cm

CeCoIn$_5$ 0.1-0.2 $\mu\Omega$.cm
Importance of purity

Heat transport

- **Electronic thermal conductivity**

\[ \kappa = \kappa_{\text{electrons}} + \kappa_{\text{phonons}} \]

\[ \rho_0 = 40 \text{ n}\Omega \text{ cm} \]

[Graph showing \( \frac{\kappa}{T} \) vs. \( T^2 \) with linear trend for phonons and \( T \) for electrons]

[Graph showing \( \kappa \) vs. \( T \) with peak at \( T_N \)]
CeRhIn$_5$ heat & charge transport

Scattering mechanism

Scattering $\sim$ spin disorder

characteristic temperature $T_{sf}$

J. Paglione et al., PRL 94, 216602 (2005)
CeRhIn$_5$ heat & charge transport

Lorenz ratio $\kappa / \sigma T$

$T \to 0 : L = L_0$

Wiedemann-Franz law

J. Paglione et al., PRL 94, 216602 (2005)

A new probe
- $T \to 0 :$ test of WF law
- High $T : T_{SF}$
Thermal resistivity

\[ \Delta \rho = A(H)T^2 \]
\[ \Delta (L_0T/\kappa) = B(H)T^2 \]

FL temperature \( T_{FL} \)

same critical exponent in heat and charge transport

J. Paglione et al., PRL97, 106606 (2006)
Difference in resistivities: thermal – electrical

characteristic temperature $T_{SF}$
$H = H_c$ : electrical resistivity
What can be the cause of the violation?

Thermopower contribution to $\kappa$

$$\tilde{\kappa} = \kappa - T \sigma S^2$$

a. $S/T \sim \ln(1/T)$ Paul & Kotliar PRB 64, 184414 (2001)

b. $S/T \sim \gamma$ (FL, FM QCP)
   $S/T \ll \gamma$ (AFM QCP)
   Miyake & Kohno JPSJ 74 254 (2005)

c. $S \not\Rightarrow 0$ Podolsky et al PRB 75 014520 (2007)

d. $S \not\Rightarrow 0$ Khodel et al unpub.

In metals $S \rightarrow 0$ when $T \rightarrow 0$ correction unimportant
What if not?
For heat current along c, $S \to 0$ when $T \to 0$ with constant slope. Very unusual for QCP!
Reading:
General
J. M. Ziman Principles of the theory of solids

Experimental details
F. Pobell Matter and Methods at Low temperatures
A. C. Anderson, Instrumentation at temperatures below 1K,
B. RSI 51, 1603 (1980)

Review articles on physics of thermal and thermoelectric phenomena
Summary of transport measurements

Resistivity:
Good: direct information about conduction electrons
Bad: no quantitative theoretical description
Important info: charge gap (carrier density) and entropy (disorder, scattering)

Seebeck effect:
Good: charge carrier sign, density of states
Bad: no quantitative theoretical description, phonon drag
Important info: entropy per charge carrier

Hall effect:
Good: carrier charge, density and mobility (in combination with resistivity)
Good: analysis of multi-carrier transport
Bad: magnetic scattering
Important info: carrier density
Magnetoresistance
Good: can distinguish multiple carrier case from single carrier case
Good: good theoretical description (Kohler rule)
Important info:
carrier density in multiple carrier conductivity
Magnetic scattering

Nernst effect:
Good: understanding multi-carrier situation
Bad: difficult to measure
Important info:
Multiband conductivity
exotic states of matter

Thermal conductivity:
Good: well understood theoretically
Bad: phonon contribution
Important info:
Charge scattering mechanism
characterization of unusual states of matter