V, I, R measurements

590B

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SI units/History

Resistivity measurements
Alessandro Giuseppe Antonio Anastasio Volta
1745-1827
Count (made by Napoleon 1810)
1881- Volt unit adopted internationally

First reliable source of electricity
Alternating plates of Zn and Cu separated by cardboard soaked in saltwater

Electrical action is proportional to the number of plates
Months after 1819 Hans Christian Ørsted's discovery of magnetic action of electrical current.

1820 Law of electromagnetism (Ampère's law) magnetic force between two electric currents.

First measurement technique for electricity
Needle galvanometer
1826 Ohm’s apparatus

Current measurement: magnetic needle

Voltage source: thermocouple (Seebeck 1821)
Steam heater
Ice cooler

**Georg Simon Ohm**
1789 - 1854

\[ I = \frac{V}{R} \]

Resistivity
\[ \rho = R \frac{A}{\ell} \]
Can be measured via
Magnetic action of electric current
Heat
Mass flow (electrolysis)
Light generation
Physiological action (Galvani, You can do anything with cats!)

Ampere main SI unit:
Definition based on force of interaction between parallel current
Replaced recently
Amount of deposited mass per unit time in electrolysis process

D'Arsonval galvanometer

Thompson (Kelvin) mirror galvanometer
Our common experience: resistance is the simplest quantity to measure

True, but only inside “comfort zone”

Digital Multi Meters - DMM

Use Ohm’s law

\[ \text{Resistor} \]

Apply known I (V) Measure V (I)
Calculate resistance

Implicit: Ohm’s law is valid for our measurement object, I-V curve is linear
May be far from true!
Implicit: our whole circuit is linear and no offsets!
Assumption: wire resistance is negligible

Typical characteristics
1 mV per last digit
1 μA per last digit
High input offset current
Low input impedance
Resistance 4-probe measurements (Kelvin probe measurement)

To minimize wire resistance effect for remote objects
To minimize the effect of contacts for resistivity measurements
Even allows slightly rectifying contacts
and “high” resistance contacts

FIGURE 3-15: Four-Wire Resistance Measurement

Current source in one circuit
Potential voltage measurement in ANOTHER circuit

Thanks Adam!
Resistance: bridge measurement

At balance \( I = 0 \)
\[ Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \]

Replaces I and V measurement by resistance compensation to obtain zero reading
No effect of circuit non-linearity,
In old days PRECISE DIGITAL measurement

Does not go well with modern electronics
Resistance 4-probe measurements
Consideration of noise sources

Wires generate spurious DC Voltages
- Thermoelectric (thermal gradients) 1/f noise
- Galvanic (oxidation) 1/f noise
- RF interference and rectification in contacts

\[ V_{\text{M}} = IR + V_{\text{offset}} \]

DC Delta method
Measure at I+
Measure at I- average
- Thermal EMF errors
  Most common source in low level Voltage measurements

  Each wire junction forms a thermocouple

- Galvanic potentials in contacts
  Gold plating to avoid oxidation

- Noise caused by magnetic fields

  Extremely important for AC measurements
  Wires vibrate and AC voltage generated

  Twisted pairs
  Fixed wires

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<table>
<thead>
<tr>
<th>Copper-to-</th>
<th>Approx. μV / °C</th>
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<tbody>
<tr>
<td>Copper</td>
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<tr>
<td>Gold</td>
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<td>Silver</td>
<td>0.5</td>
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<td>Brass</td>
<td>3</td>
</tr>
<tr>
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<td>5</td>
</tr>
<tr>
<td>Aluminum</td>
<td>5</td>
</tr>
<tr>
<td>Kovar or Alloy 42</td>
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<tr>
<td>Silicon</td>
<td>500</td>
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<tr>
<td>Copper-Oxide</td>
<td>1000</td>
</tr>
<tr>
<td>Cadmium-Tin Solder</td>
<td>0.2</td>
</tr>
<tr>
<td>Tin-Lead Solder</td>
<td>5</td>
</tr>
</tbody>
</table>

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**FIGURE 3-10: Minimizing Interference from Magnetic Fields**

- a. Source
- b. Source
Ground loops

Current source and voltmeter have grounded terminals

Grounds are not ideal and are under some potential

Devices grounded in different points acquire potential difference which contributes to the measured signal
Common mode rejection

Finite resistance between LO terminal and ground

If I am measuring voltage difference $V_{H} - V_{L}$, it will depend on $V_{L}$!

Noise caused by injected current

Same for capacitance charging
Mainly 60 Hz power line noise

N.B. The most important sources of error in 4-probe resistance measurements
Contact voltage drop is equivalent to common mode voltage
Typically Common Mode Rejection of DMM is $10^5$-$10^6$
Injected current bypasses the measured resistance  Injected current flows through the measured resistance

*Note: The measurement noise caused by injected current can be significantly reduced by setting the integration time of the DMM to 1 power line cycle (PLC) or greater.*
Loading errors due to input resistance

Important for high resistance measurements
Typical DC 10 GOhm

Loading errors due to input bias current
AC 1 MOhm
100 pF
Shielding

Electrostatic Faraday cage

Magnetic shielding

To prevent EMI

Unfortunately not very useful

In the lab, can not put electronics at low T
Shielding

General rule
Avoid grounds in measurement circuits
Ground shields
Johnston noise

**voltage**

\[ E = \sqrt{4kTRB} \text{ volts, rms} \]

**current**

\[ I = \frac{\sqrt{4kTRB}}{R} \text{ amperes, rms} \]

B bandwidth

**FIGURE 1-2: Theoretical Limits of Voltage Measurements**

- **Within theoretical limits**
- **Near theoretical limits**
- **Prohibited by noise**

Source Resistance

- 1Ω
- 1kΩ
- 1MΩ
- 1GΩ
- 1TΩ

Noise Voltage

- 1kV
- 1V
- 1mV
- 1μV
- 1nV
- 1pV

- 10^0
- 10^3
- 10^6
- 10^9
- 10^12

Time

- 0.1sec
- 10sec

300K
Going outside comfort zone: DC measurements

Special designs

High impedance source Electrometer
Used for $I<10\ \text{nA}$, $G>1\ \text{GΩ}$
Input impedance $\sim 100\ \text{TΩ}$
Input offset current $<3\text{fA}$
Capable of $R$ measurement up to $300\ \text{GΩ}$

Low impedance source- Nanovoltmeter
$<1\ \text{nV}$

Source-measure units for resistance measurements

Here DMM
is from Keithley,
not from Fluke!
Low resistances: AC may be a better choice

Lock-in resistance measurements
SR830
Built-in AC voltage generator

“Ohmic contacts” required for AC measurements

AC in differential mode
avoids offsets
Low frequency (below power grid, typically 10-20 Hz)

Problem: low input impedance,
Not good for high resistance sources

Problem: current source is not precise

Very popular simple and reasonably precise resistance measurement
Low resistances: AC may be a better choice

Resistance bridges
LR700, AVS47, SIM927 and LS370

Actually these are not bridges!
Do not use compensation
Ratiometric resistance measurement

- Low noise
- Low excitation power
- AC

Resistance measurements

AC to avoid offsets
Low frequency (below power grid, typically 10-20 Hz)

SIM927 Comparators measuring reference resistance voltage and in-phase component of sample resistance voltage
Ways to reduce noise:

**Reduce bandwidth**
- averaging (digital or analog)
- filtering

Very long term measurements are susceptible to other errors, Temperature drift

**Cool down the source**
300K to 3K
10 times noise decrease

Low temperature transformers and Preamplifiers in DR

**Source resistance**
Low resistance contacts
The Delta Method of Measuring Resistance

![Graph depicting voltage and time](image)

**Figure 2a:** The graph depicts an alternating, three-point delta method of measuring voltage with no thermoelectric voltage error.

![Graph depicting voltage and time](image)

**Figure 2b:** A linearly increasing temperature generates a changing thermoelectric voltage error, which is eliminated by the three-point delta method.

\[ V_1 = 2.5 \mu V \ ; \ V_2 = -2.5 \mu V \ ; \ V_3 = 2.5 \mu V \]

**3 point Delta Method**

\[ V_a = \text{negative-going step} = \frac{(V_1 - V_2)}{2} \]
\[ = 2.45 \mu V \]

\[ V_b = \text{positive-going step} = \frac{(V_3 - V_2)}{2} \]
\[ = 2.55 \mu V \]

\[ V_f = \text{final voltage reading} = \frac{(V_a + V_b)}{2} \]
\[ = \frac{1}{2}[(V_1 - V_2)/2 + (V_3 - V_2)/2] \]
\[ = 2.5 \mu V \]
Figure 3: A graph comparing the results of applying a two- and three-point delta method shows significant noise reduction using the three-point method.
Figure 6: Making differential conductance measurements using just two instruments that incorporate all of the instruments used in the AC technique.

Figure 7: The waveform used in the new technique is a linear staircase function that combines an alternating current with a staircase current.
High resistance measurements

FIGURE 2-33: Electrometer Ohmmeter for Measuring High Resistance

Special features: Guarded cables
Triaxial connectors

FIGURE 2-34a: Effects of Cable Resistance on High Resistance Measurements

FIGURE 2-34b: Equivalent Circuit of Figure 2-34a Showing Loading Effect of Cable Leakage Resistance $R_L$

FIGURE 2-34c: Guarding Cable Shield to Eliminate Leakage Resistance
High resistance measurements

Because of parasitic capacitance and high impedance, only DC measurements are necessary. It is important to make correct electrometer connections.

**FIGURE 2-35:** Settling Time is the Result of $R_S C_{SHUNT}$ Time Constant

\[ \tau = R_S C_{SHUNT} \]

**FIGURE 2-36:** Exponential Settling Time Caused by Time Constant of Shunt Capacitance and Source Resistance

![Diagram showing exponential settling time](image)
High resistance measurements

Guard ring technique:

- Volume resistance, $R_v$
- Surface leakage resistance, $R_s$

(a) Circuit that measures insulation volume resistance in parallel with surface leakage resistance

$$R_{meas} = R_s \parallel R_v = \frac{V}{I_s + I_v}$$

(b) Use of guard ring to measure only volume resistance

$$R_{meas} = R_v = \frac{V}{I_v}$$
Resistivity measurements: 4-probe

“Ohmic contacts” required for AC measurements

Concerns
Different I and V circuits
May be disconnected!

Potential contacts should be connected well to current path

Strict requirements on sample shape

Superconducting fluctuations and the Peierls instability in an organic solid
SSC 12, 1125 (1973)
Four probe measurement in bridge configuration

Four-Terminal Resistor

Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge

- $r_1$ causes no effect on the balance condition.
- The effects of $r_2$ and $r_3$ could be minimized, if $R_1 >> r_2$ and $R_a >> r_3$.
- The main error comes from $r_4$, even though this value is very small.
A METHOD OF MEASURING SPECIFIC RESISTIVITY AND HALL EFFECT OF DISCS OF ARBITRARY SHAPE

by L. J. van der PAUW

Summary
A method of measuring specific resistivity and Hall effect of flat samples of arbitrary shape is presented. The method is based upon a theorem which holds for a flat sample of arbitrary shape if the contacts are sufficiently small and located at the circumference of the sample. Furthermore, the sample must be singly connected, i.e., it should not have isolated holes.
Resistivity measurements: van der Pauw method

Very popular in semiconductor industry
Does not require sample of regular shape

Assumptions
1. Homogeneous sample
2. Isotropic sample
3. Two-dimensional, thickness is unimportant
4. Sample boundary sharply defined

Surface resistance

\[ \rho_{\square} = \frac{\pi}{\ln 2} \frac{R_{ab-cd} + R_{bc-da}}{2} f \]

Resistivity

\[ \rho = \frac{\pi}{\ln 2} W \frac{R_{ab-cd} + R_{bc-da}}{2} f \]
Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. Montgomery

Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07974

(Received 30 November 1970)

A rectangular prism with edges in principal crystal directions is prepared with electrodes on the corners of one face. Voltage–current ratios for opposite pairs of electrodes permit calculation of components of the resistivity tensor. The method can use small samples, and is best suited to materials describable by two or three tensor components. Examples are given of measurements of V$_2$O$_5$–Cr and oriented amorphous graphite.
Montgomery technique

\[(\rho_2/\rho_1)^{1/2} = (l_2/l_1) \times (l'_1/l'_2).\]

1. Van der Pauw resistivity measurements on samples of rectangular

\[R_1 = V_1/I_1 \quad \quad R_2 = V_2/I_2\]

2. Calculation of the anisotropy ratio for isotropic samples

3. Scaling anisotropic samples on isotropic by van der Pauw scaling transformation

**Fig. 3.** Resistance ratio versus sample dimension ratio. Solid line is for a thin sample; dashed line for a thick sample. Details of thickness dependence given in Table II.
Interference Factor

- Ohmic contact quality and size
- Sample uniformity and accurate thickness determination
- Photoconductive and photovoltaic effects
Reading

2. Lake Shore manual for LS370