V, I, R measurements

590B

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September 23, 2009

SI units/History

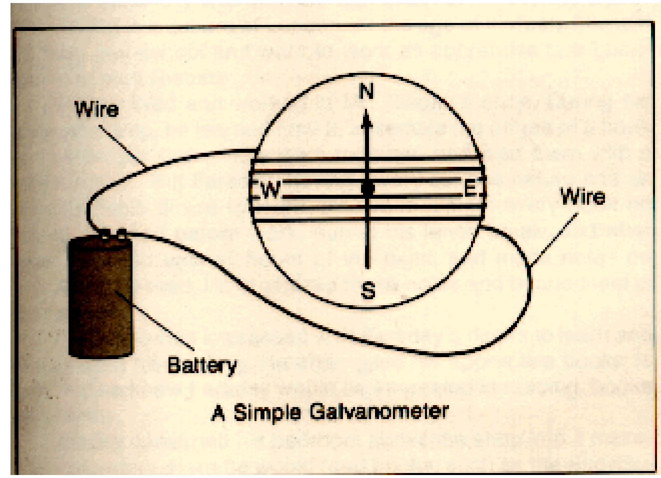
Resistivity measurements

First reliable source of electricity
Alternating
plates of Zn and Cu
separated by cardboard
soaked in saltwater

Electrical action is proportional to the number of plates

Alessandro Giuseppe Antonio Anastasio Volta
1745-1827
Count (made by Napoleon 1810)
1881- Volt unit adopted internationally

André-Marie Ampère
1775 - 1836



Months after 1819 Hans Christian Ørsted's discovery of magnetic action of electrical current

1820 Law of electromagnetism (Ampère's law) magnetic force between two electric currents.

First measurement technique for electricity
Needle galvanometer

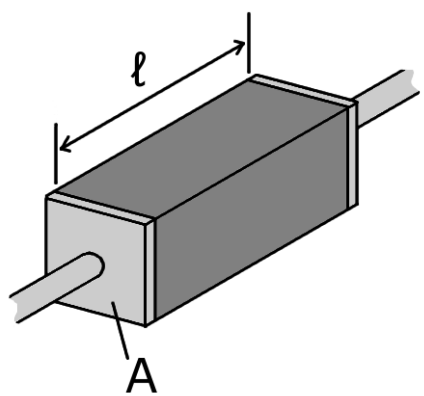
1826 Ohm's apparatus

Current measurement:
magnetic needle

Voltage source:
thermocouple (Seebeck 1821)
Steam heater
Ice cooler

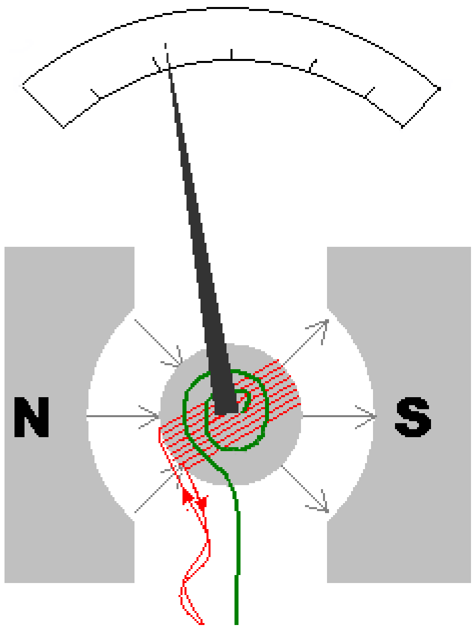
Georg Simon Ohm
1789 - 1854

I=V/R

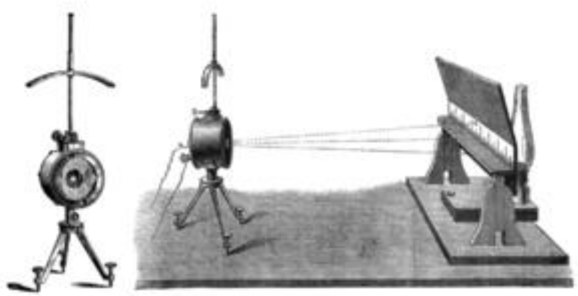


Resistivity

$$\rho = R\,\frac{A}{\ell}$$



D'Arsonval galvanometer



Thompson (Kelvin) mirror galvanometer

Can be measured via
Magnetic action of electric current
Heat
Mass flow (electrolysis)
Light generation
Physiological action (Galvani,
You can do anything with cats!)

Ampere main SI unit:

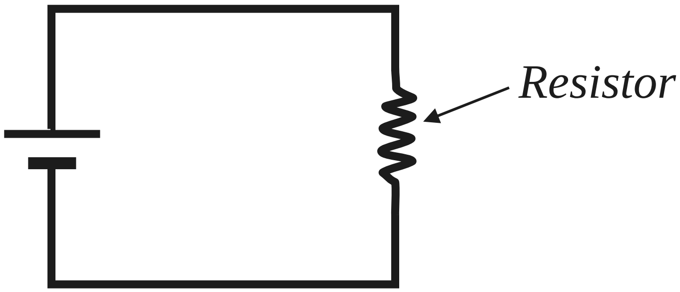
Definition based on force of
interaction between parallel current

Replaced recently
Amount of deposited mass per
unit time in electrolysis process

Our common experience: resistance is the simplest quantity to measure

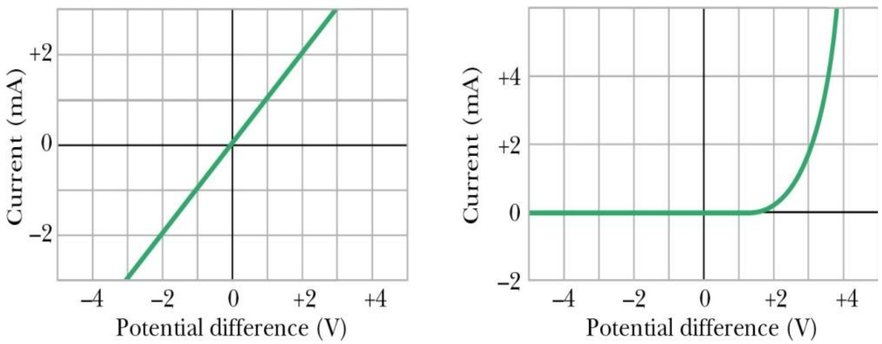
True, but only inside "comfort zone"

Use Ohm's law



Digital Multi Meters - DMM

Apply known I (V) Measure V (I)
Calculate resistance



(*b*) (*c*)

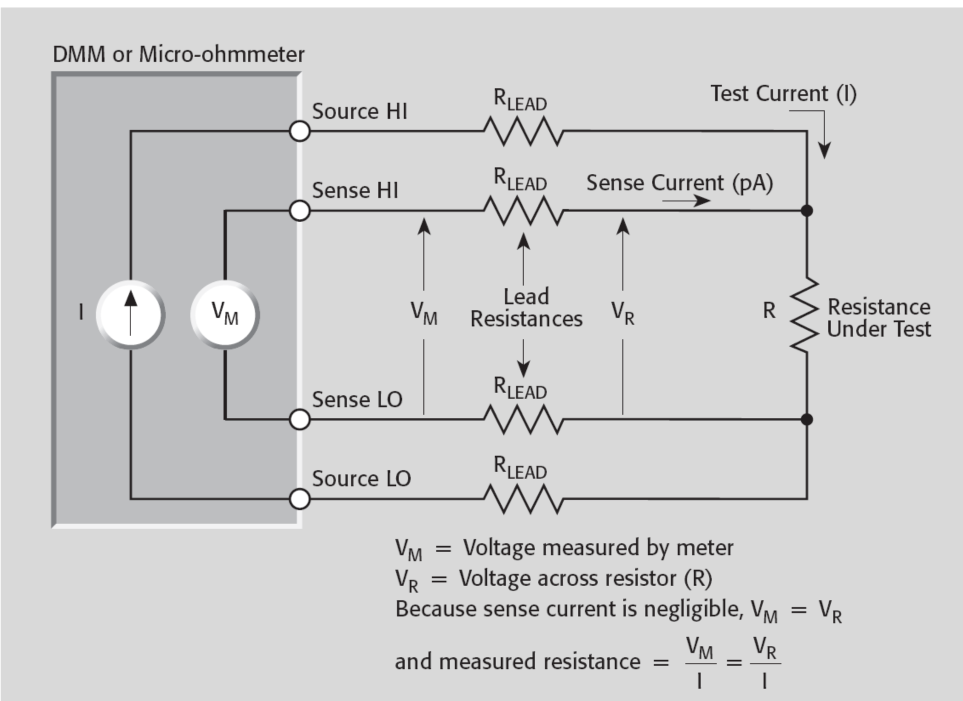
Implicit: Ohm's law is valid for our measurement object, I-V curve is linear
May be far from true!
Implicit: our whole circuit is linear and no offsets!
Assumption: wire resistance is negligible

Typical characteristics
1 mV per last digit
1 μA per last digit
High input offset current
Low input impedance

Resistance 4-probe measurements (Kelvin probe measurement)

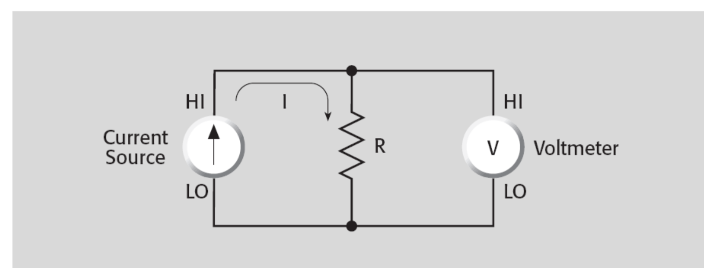
To minimize wire resistance effect for remote objects
To minimize the effect of contacts for resistivity measurements
Even allows slightly rectifying contacts
and "high" resistance contacts

FIGURE 3-15: Four-Wire Resistance Measurement



Current source in one circuit
Potential voltage measurement
in ANOTHER circuit

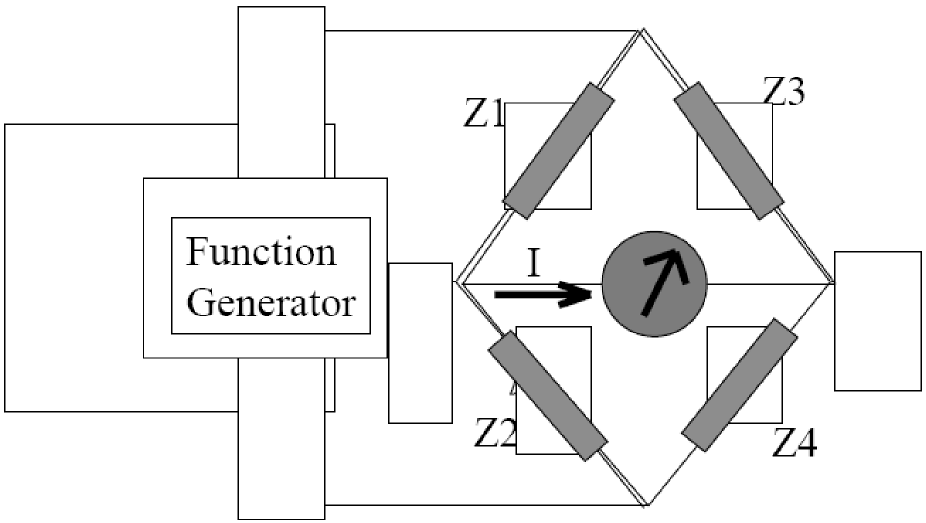
FIGURE 2-31: Constant-Current Method Using a Separate Current Source and Voltmeter



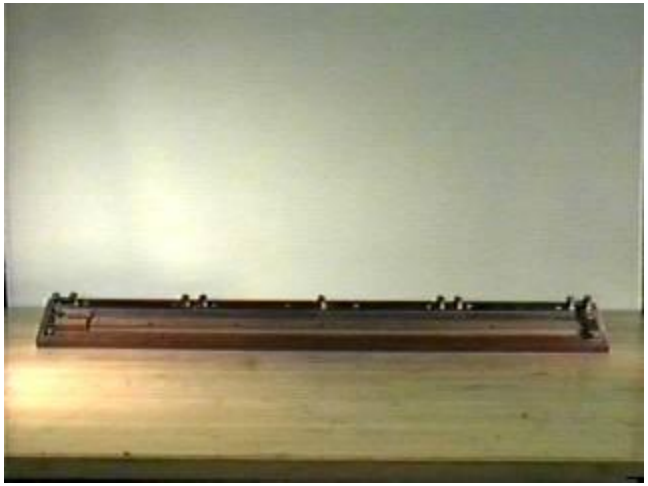
Thanks Adam!

Resistance: bridge measurement

$$\text{At balance } I = 0$$
$$Z1 \,.\, Z4 = Z2 \,.\, Z3$$



Slide Wire Wheatstone Bridge

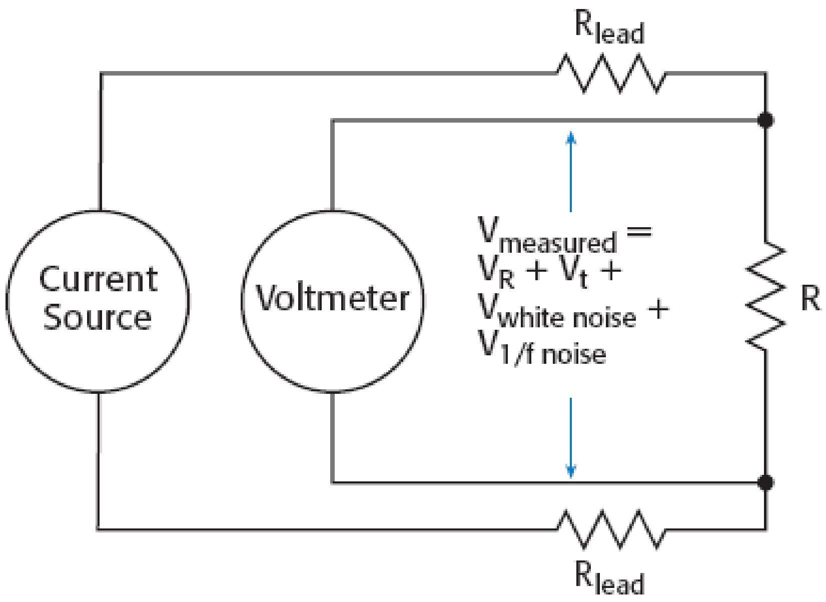


Resistance Decade Bridge

Replaces I and V measurement by resistance compensation to obtain zero reading
No effect of circuit non-linearity,
In old days PRECISE DIGITAL measurement

Does not go well with modern electronics

Resistance 4-probe measurements
Consideration of noise sources



Wires generate spurious
DC Voltages
-Thermoelectric
(thermal gradients) 1/f noise
-Galvanic (oxidation) 1/f noise
-RF interference
and rectification in contacts

$$V_M = IR + V_{offset}$$

DC Delta method
Measure at I+
Measure at I-
average

-Thermal EMF errors
Most common source in low level
Voltage measurements

Each wire junction forms
a thermocouple

Copper-to-	Approx. µV / °C
Copper	< 0.3
Gold	0.5
Silver	0.5
Brass	3
Beryllium Copper	5
Aluminum	5
Kovar or Alloy 42	40
Silicon	500
Copper-Oxide	1000
Cadmium-Tin Solder	0.2
Tin-Lead Solder	5

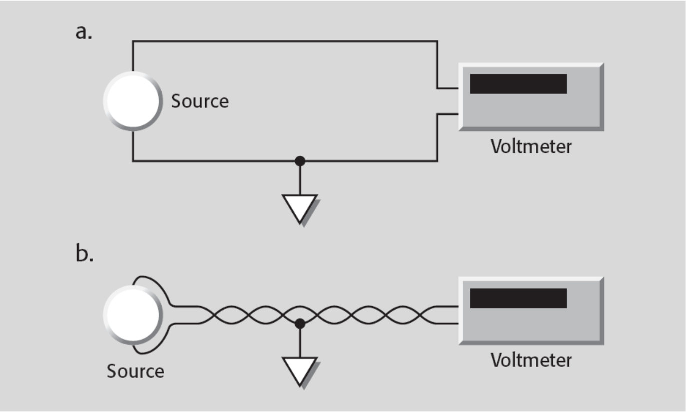
-Galvanic potentials in contacts
Gold plating to avoid oxidation

-Noise caused by magnetic fields

Extremely important for AC measurements
Wires vibrate and AC voltage generated

Twisted pairs
Fixed wires

FIGURE 3-10: Minimizing Interference from Magnetic Fields



Ground loops

Current source
and voltmeter
have grounded terminals

Grounds are not ideal and
are under some potential

Devices grounded
in different points acquire
potential difference which
contributes to the measured signal

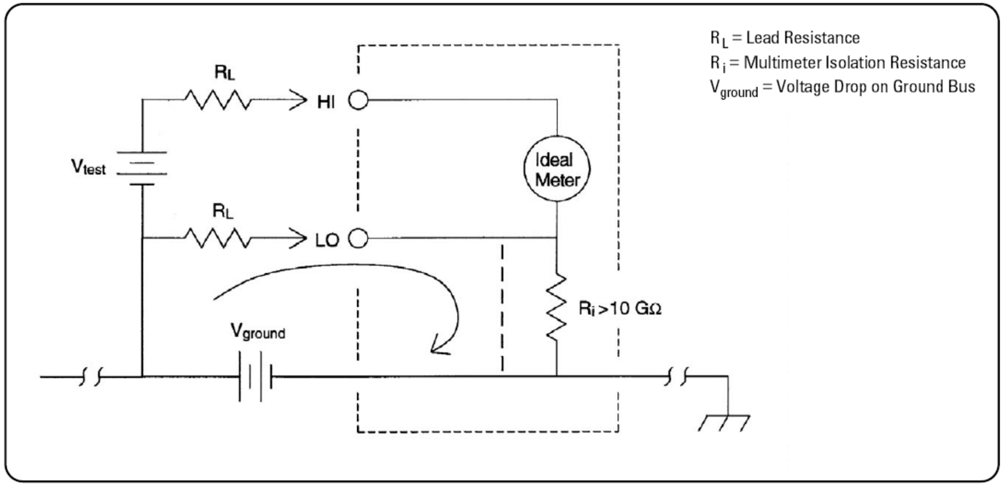


Figure 3.

Common mode rejection

Finite resistance
between LO terminal and ground

If I am measuring voltage difference V_H-V_L, it will depend on V_L!

Noise caused by injected current

Same for capacitance charging
Mainly 60 Hz power line noise

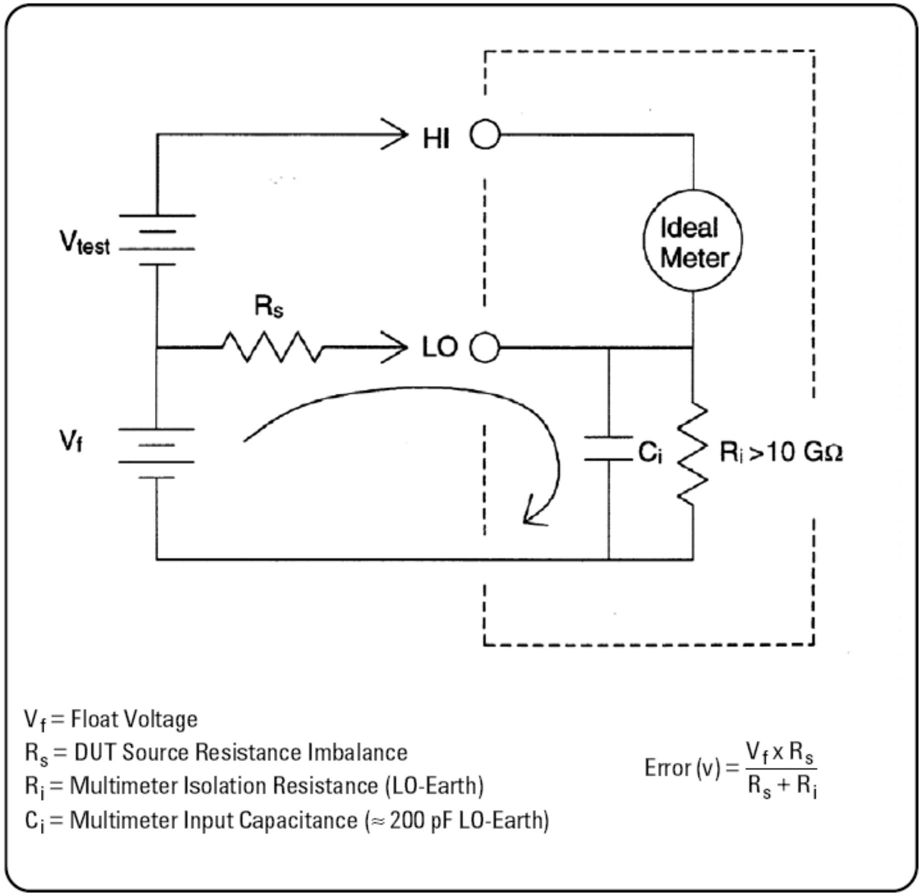


Figure 4.

N.B. The most important sources of error in 4-probe resistance measurements
Contact voltage drop is equivalent to common mode voltage
Typically Common Mode Rejection of DMM is 10^5-10^6

Good Bad

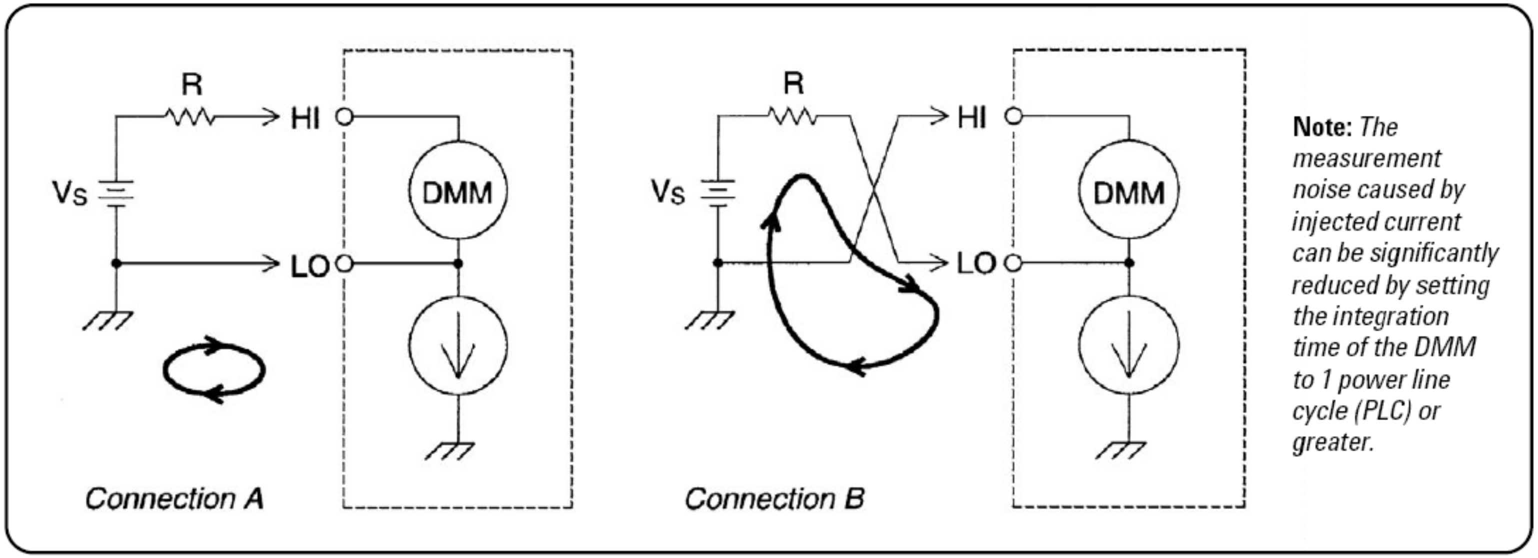


Figure 6.

Injected current bypasses the measured resistance

Injected current flows through the Measured resistance

Loading errors due to input resistance

Important for high resistance measurements
Typical
DC 10 GOhm

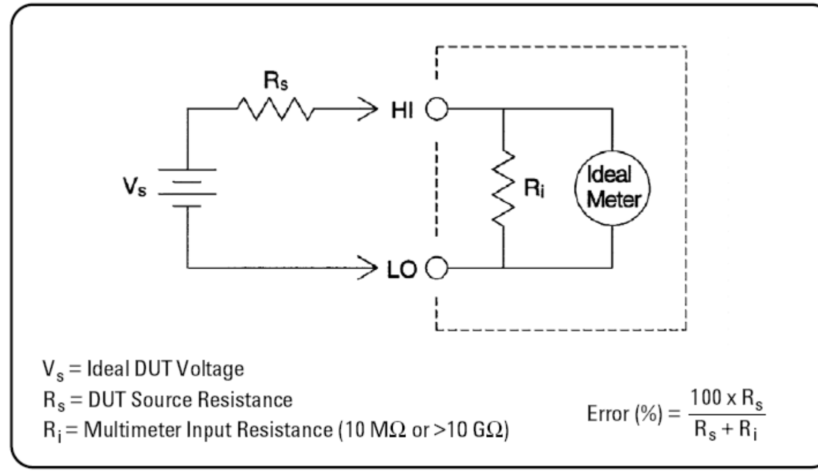


Figure 7.

Loading errors due to input bias current

AC 1 MOhm
100 pF

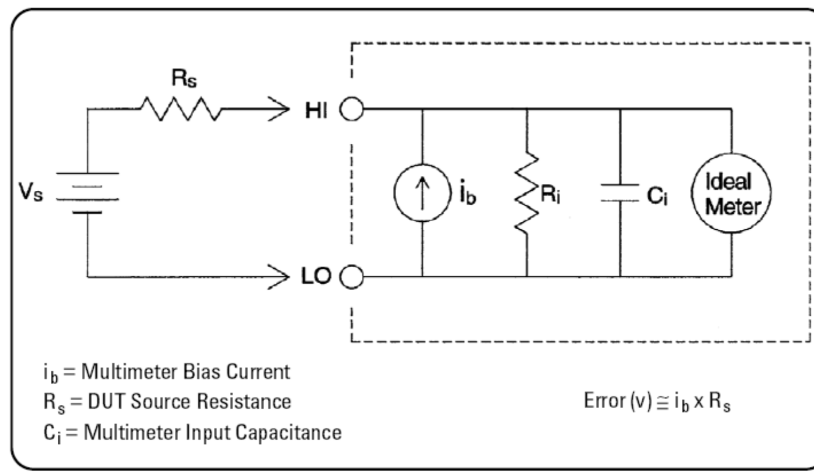
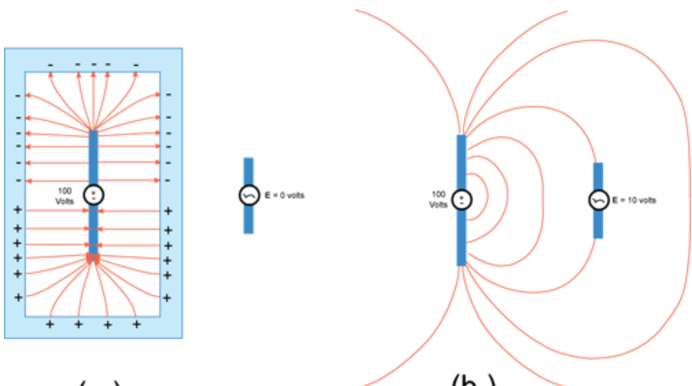


Figure 8.

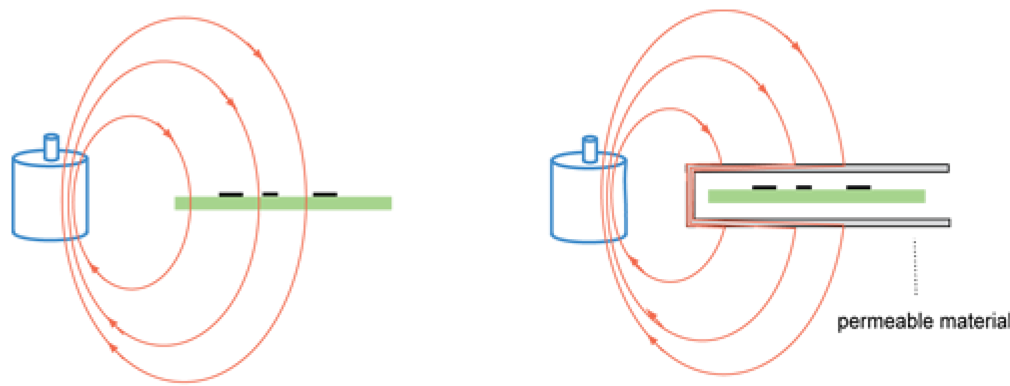
Shielding

Electrostatic Faraday cage

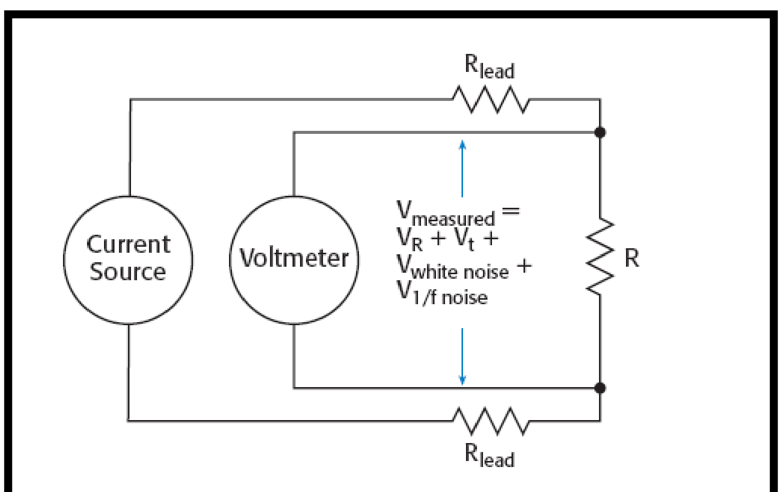


(a.) (b.)

Magnetic shielding



(a.) (b.)



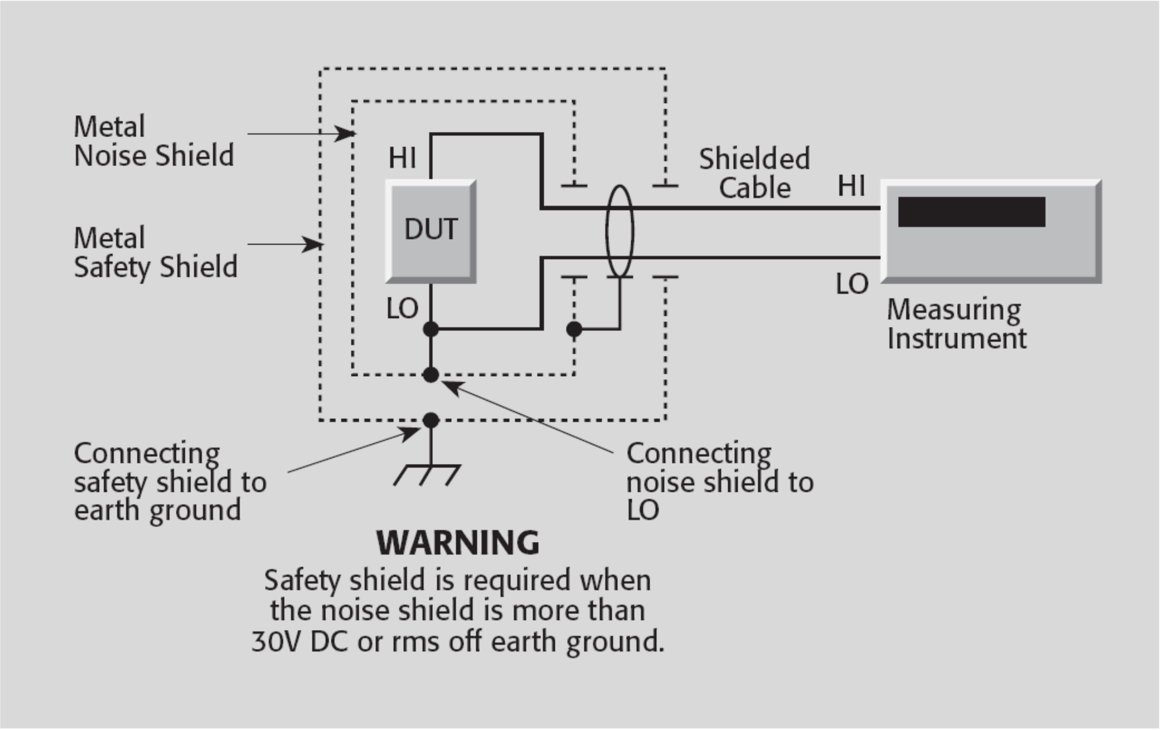
To prevent EMI

Unfortunately not very useful

In the lab, can not put electronics at low T

Shielding

FIGURE 3-6: Shielding to Attenuate RFI/EMI Interference



General rule

Avoid grounds in measurement circuits

Ground shields

Johnston noise

voltage

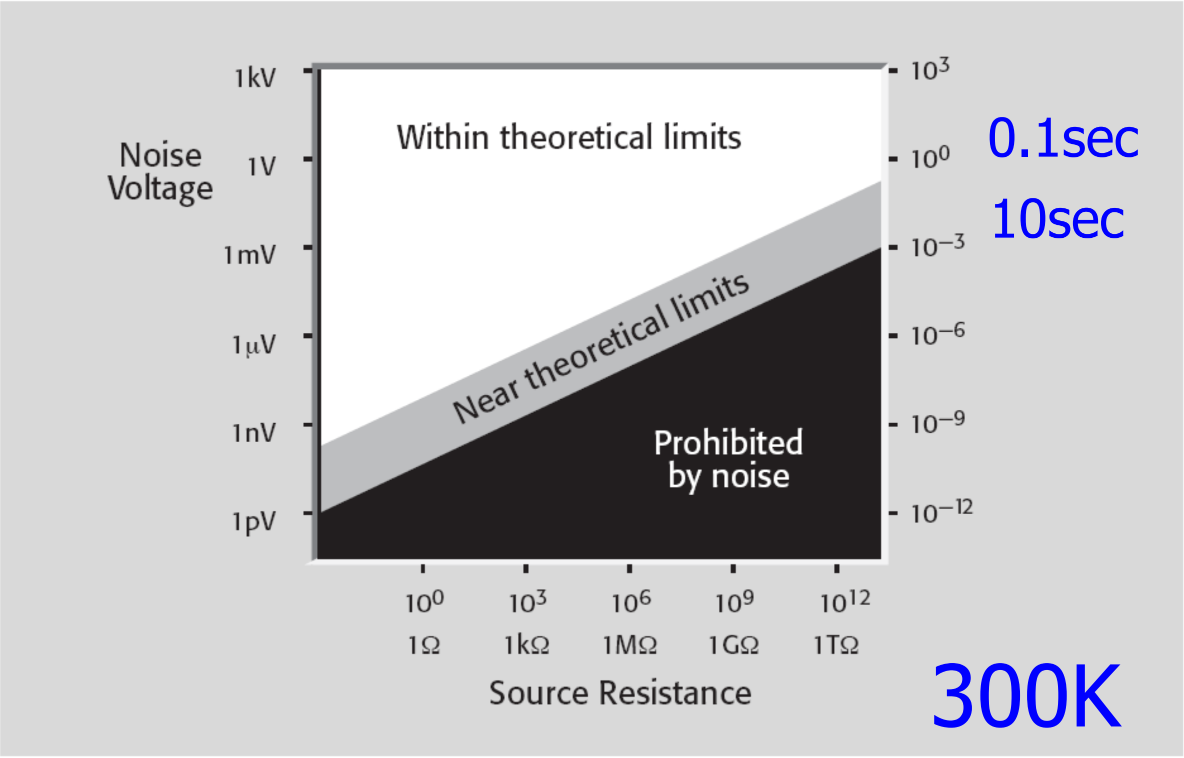
$$E = \sqrt{4kTRB} \text{ volts, rms}$$

current

$$I = \frac{\sqrt{4kTRB}}{R} \text{ amperes, rms}$$

B bandwidth

FIGURE 1-2: Theoretical Limits of Voltage Measurements



Going outside comfort zone: DC measurements

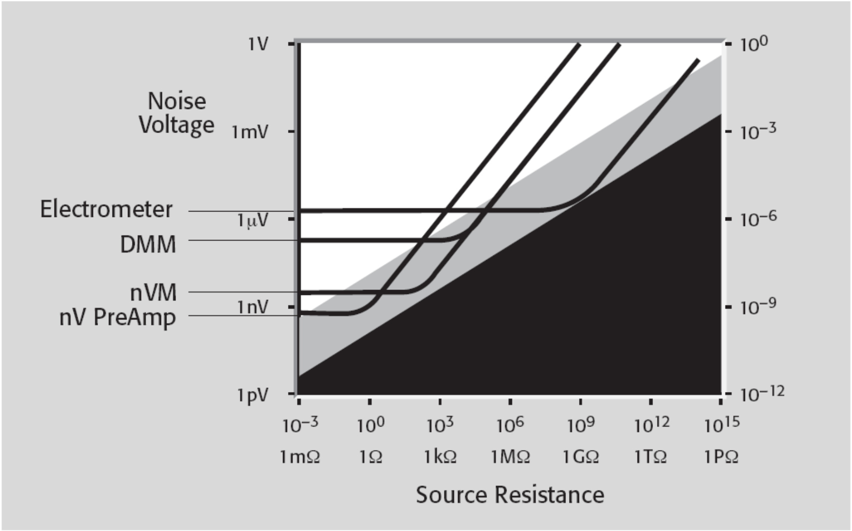
Special designs

High impedance source Electrometer
Used for I<10 nA, G>1 GΩ
Input impedance ~100 TΩ
Input offset current <3fA
Capable of R measurement up to
300 G Ω

Low impedance source- Nanovoltmeter
<1 nV

Source-measure units for
resistance measurements

FIGURE 1-3: Typical Digital Multimeter (DMM), Nanovoltmeter (nVM), Nanovolt Preamplifier (nV PreAmp), and Electrometer Limits of Measurement at Various Source Resistances



Here DMM
is from Keithley,
not from Fluke!

Low resistances: AC may be a better choice

Lock-in resistance measurements
SR830
Built-in AC voltage generator

"Ohmic contacts" required for AC measurements

AC in differential mode
avoids offsets
Low frequency (below power grid, typically 10-20 Hz)

Problem: low input impedance,
Not good for high resistance sources

Problem: current source is not precise

Very popular simple and reasonably
precise resistance measurement

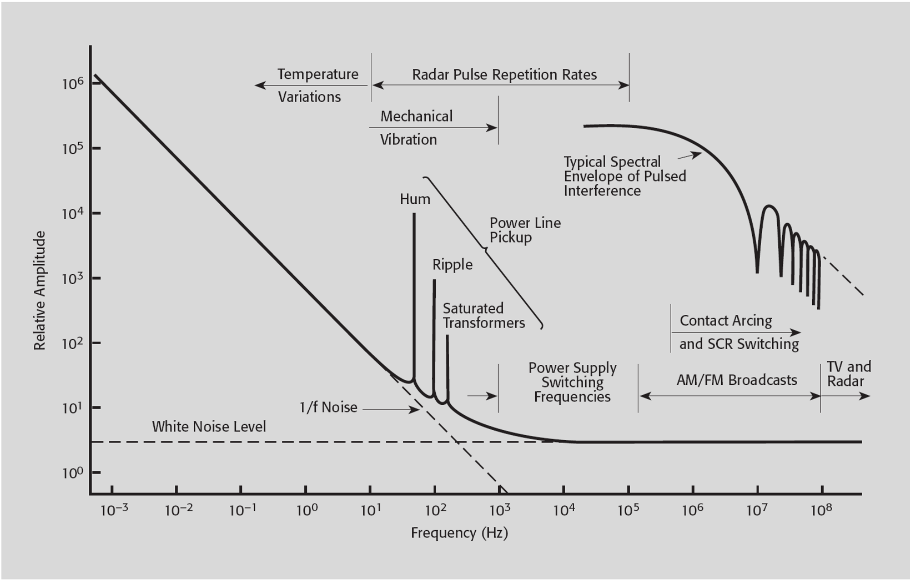


FIGURE 3-5: Voltage Noise Frequency Spectrum

Low resistances: AC may be a better choice

Resistance bridges
LR700, AVS47, SIM927 and LS370

Actually these are not bridges!
Do not use compensation
Ratiometric resistance measurement

Specialized for low-temperature precision resistance measurements

- **Low noise**
- **Low excitation power**
- **AC**

Resistance measurements

AC to avoid offsets
Low frequency (below power grid, typically 10-20 Hz)

SIM927 Comparators measuring reference resistance voltage and in-phase component of sample resistance voltage

Ways to reduce noise:

Reduce bandwidth
-averaging (digital or analog)
-filtering

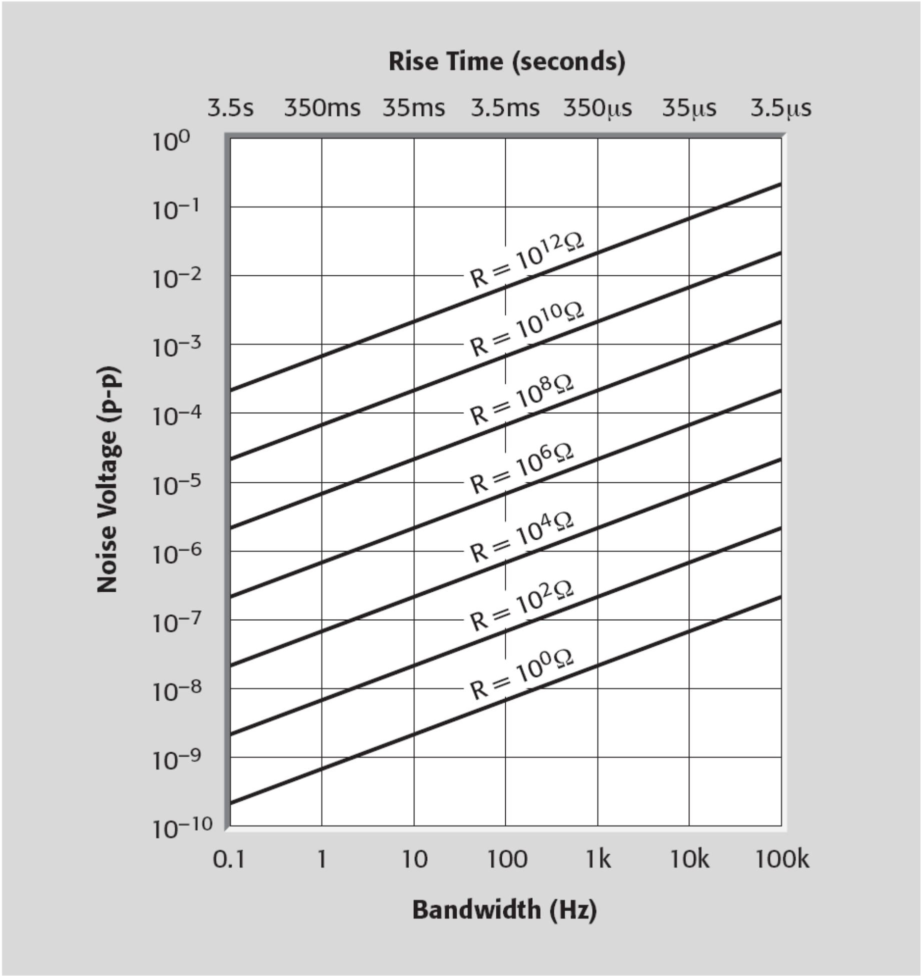
Very long term measurements are susceptible to other errors, Temperature drift

Cool down the source
300K to 3K
10 times noise decrease

Low temperature transformers and Preamplifiers in DR

Source resistance
Low resistance contacts

FIGURE 2-52: Noise Voltage vs. Bandwidth at Various Source Resistances



The Delta Method of Measuring Resistance

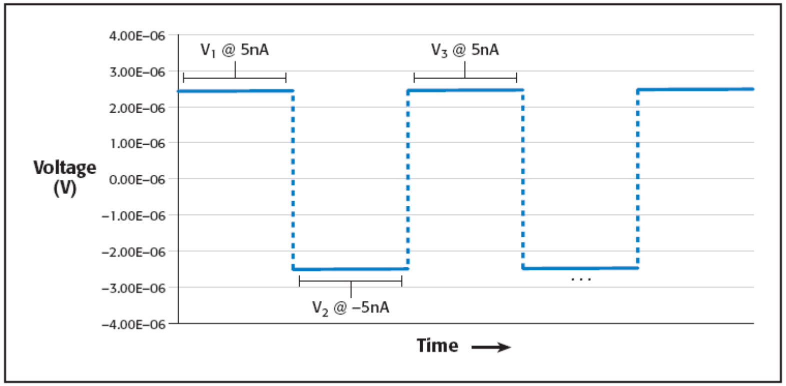


Figure 2a: The graph depicts an alternating, three-point delta method of measuring voltage with no thermoelectric voltage error.

$V_1 = 2.5\mu V$; $V_2 = -2.5\mu V$; $V_3 = 2.5\mu V$

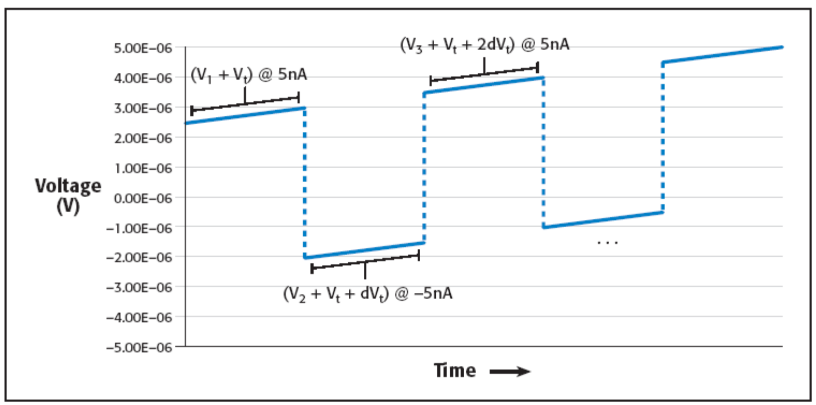


Figure 2b: A linearly increasing temperature generates a changing thermoelectric voltage error, which is eliminated by the three-point delta method.

3 point Delta Method

V_a = negative-going step = $(V_1 - V_2)/2$
= $2.45\ \mu V$

V_b = positive-going step = $(V_3 - V_2)/2$
= $2.55\ \mu V$

V_f = final voltage reading = $(V_a + V_b)/2$
= $\frac{1}{2}[(V_1 - V_2)/2 + (V_3 - V_2)/2]$
= $2.5\ \mu V$

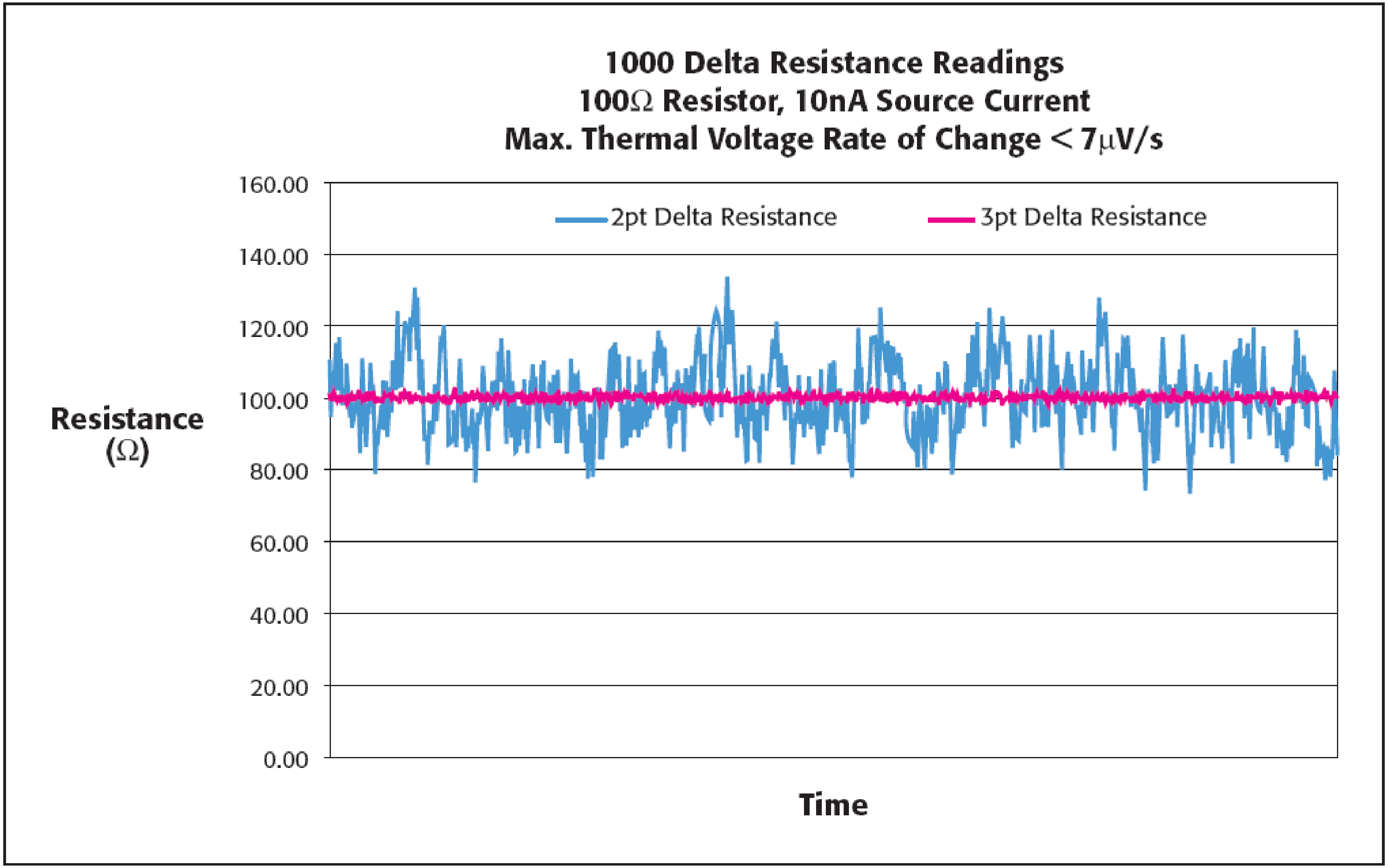


Figure 3: A graph comparing the results of applying a two- and three-point delta method shows significant noise reduction using the three-point method.

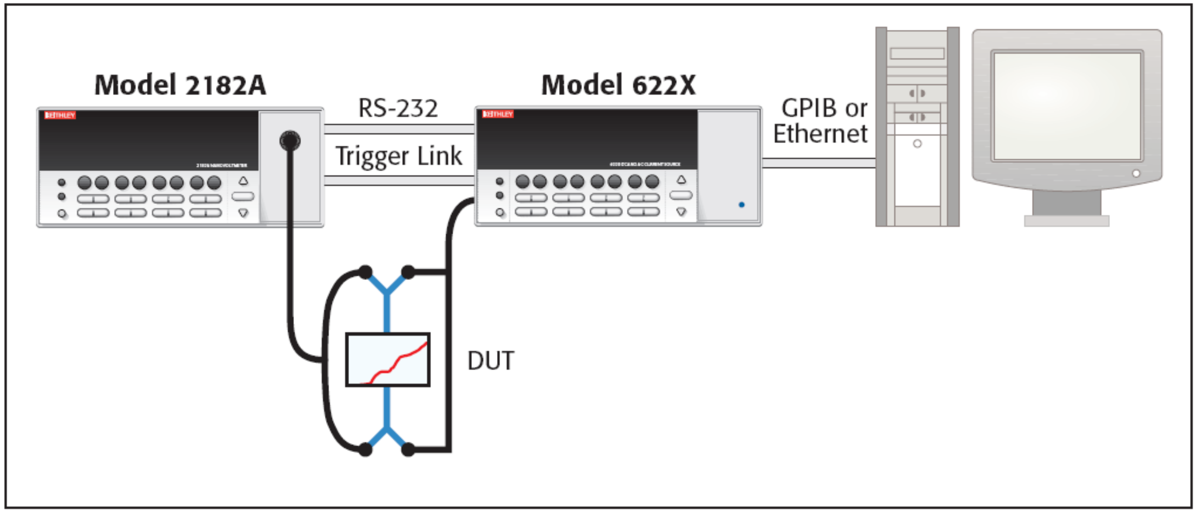


Figure 6: Making differential conductance measurements using just two instruments that incorporate all of the instruments used in the AC technique.

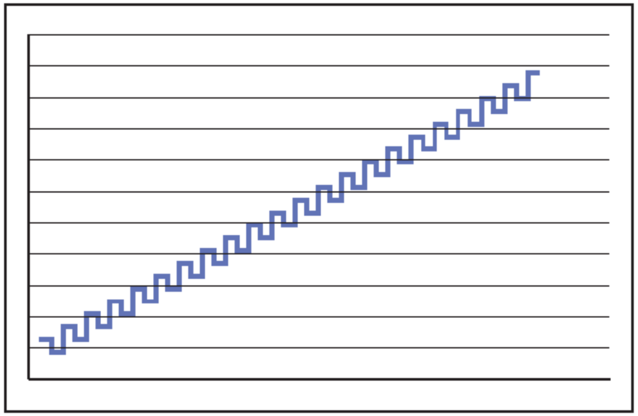
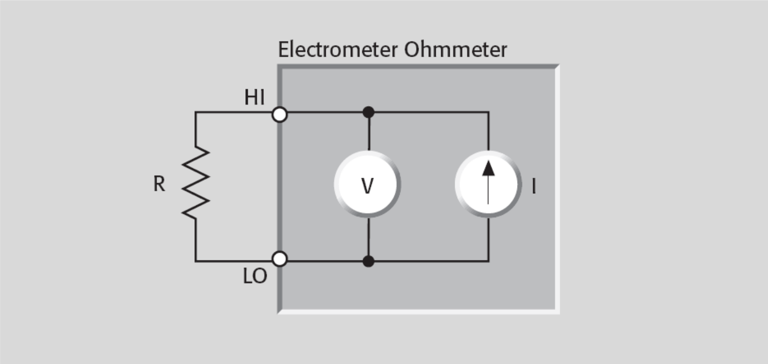


Figure 7: The waveform used in the new technique is a linear staircase function that combines an alternating current with a staircase current.

High resistance measurements

FIGURE 2-33: Electrometer Ohmmeter for Measuring High Resistance



Special features: Guarded cables
Triaxial connectors

FIGURE 2-34a: Effects of Cable Resistance on High Resistance Measurements

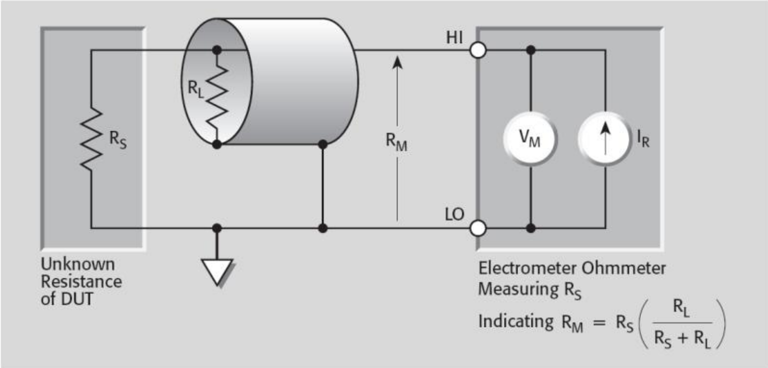


FIGURE 2-34b: Equivalent Circuit of Figure 2-34a Showing Loading Effect of Cable Leakage Resistance R_L.

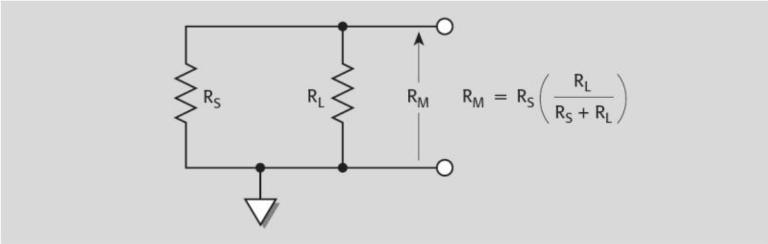
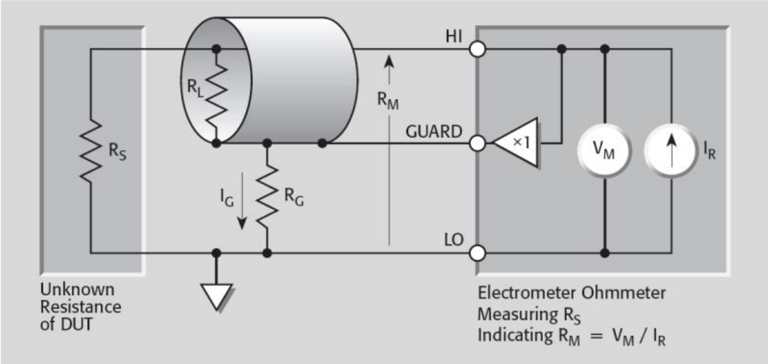


FIGURE 2-34c: Guarding Cable Shield to Eliminate Leakage Resistance



High resistance measurements

Because of parasitic capacitance
and high impedance
only DC measurements

Important to make
correct electrometer connections

FIGURE 2-40: Proper Connection

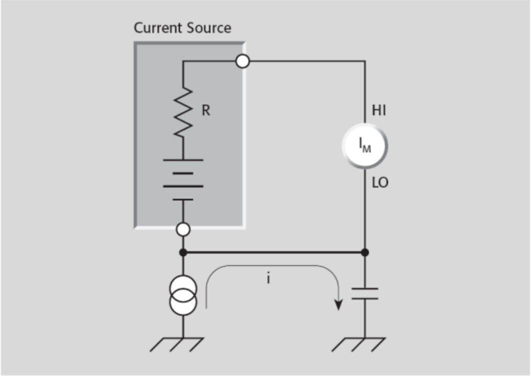


FIGURE 2-41: Improper Connection

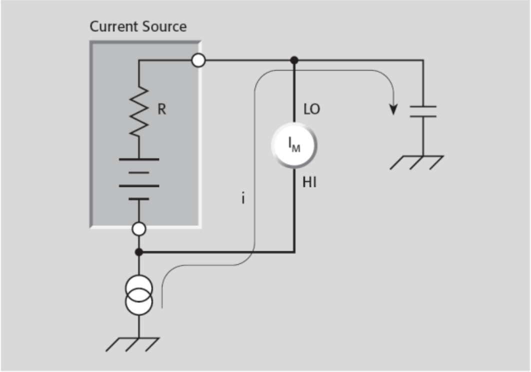


FIGURE 2-35: Settling Time is the Result of R_SC_{SHUNT} Time Constant

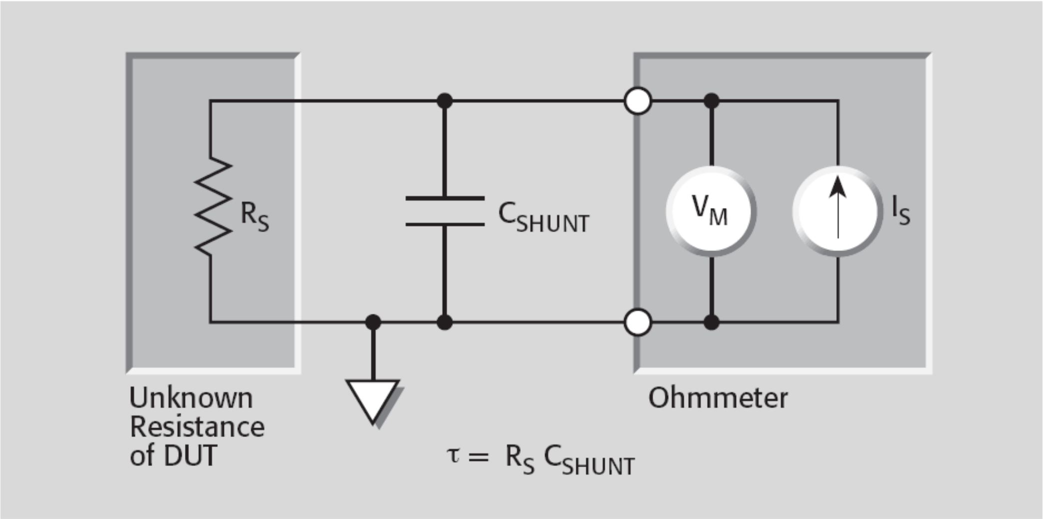
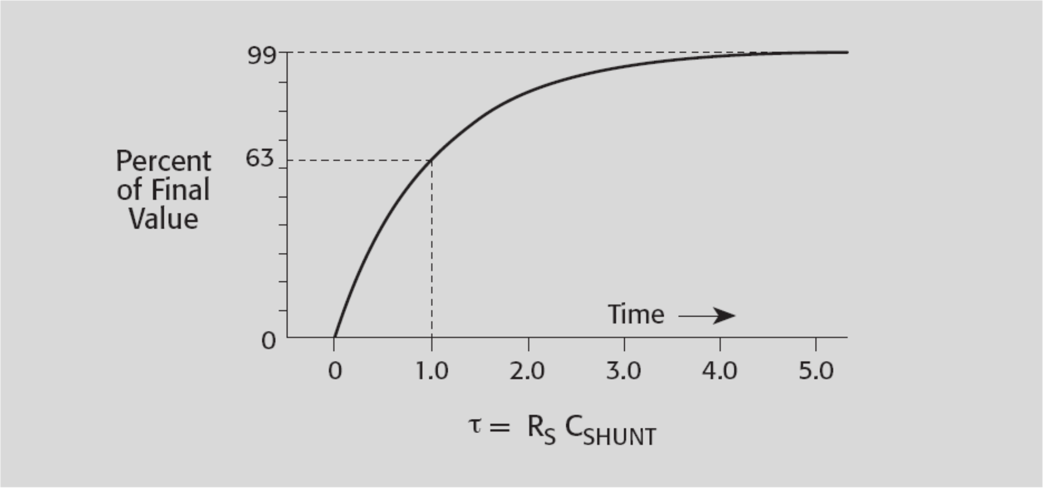


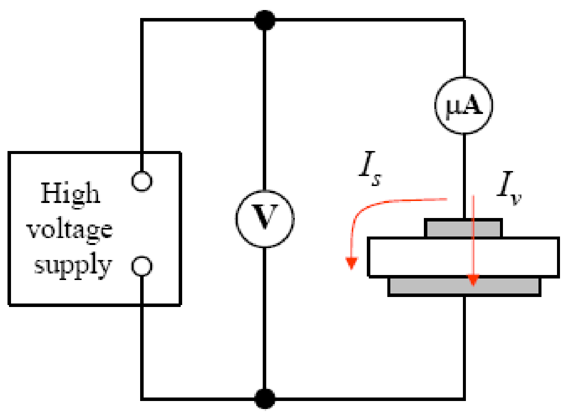
FIGURE 2-36: Exponential Settling Time Caused by Time Constant of Shunt Capacitance and Source Resistance



High resistance measurements

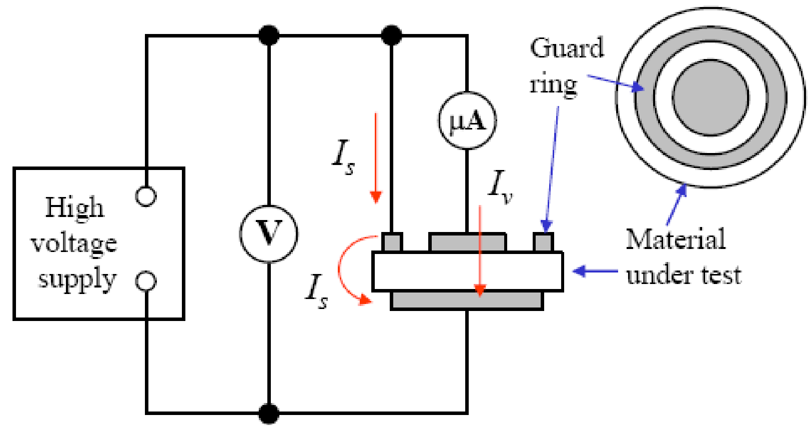
Guard ring technique:

- Volume resistance, R_V
- Surface leakage resistance, R_s



(a) Circuit that measures insulation volume resistance in parallel with surface leakage resistance

$$R_{meas} = R_s // R_v = \frac{V}{I_s + I_v}$$



(b) Use of guard ring to measure only volume resistance

$$R_{meas} = R_v = \frac{V}{I_v}$$

Resistivity measurements: 4-probe

"Ohmic contacts" required for AC measurements

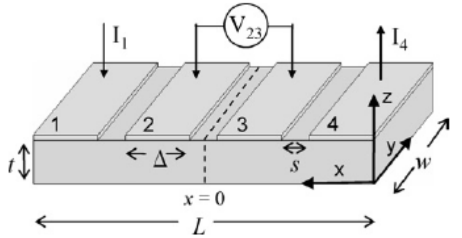
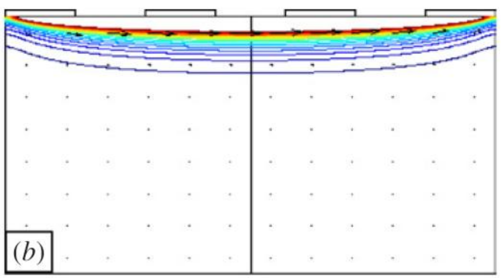
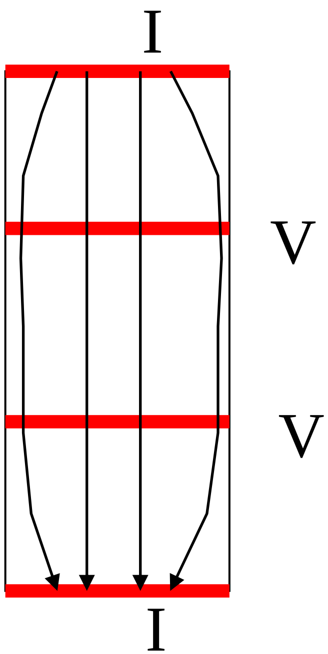


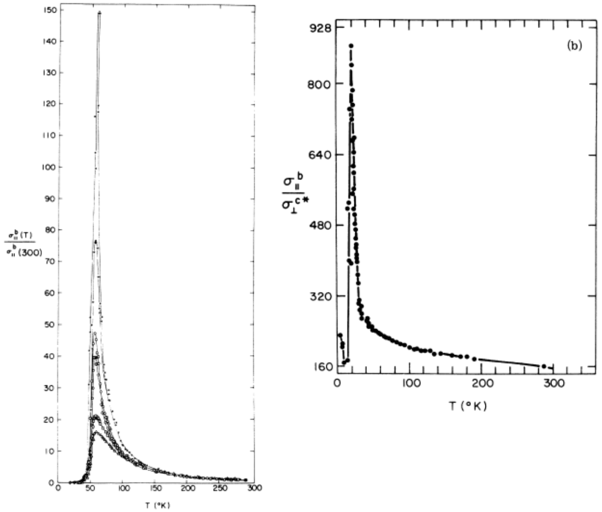
Figure 1. Schematic of the co-linear 4-probe configuration with electrodes that span the width of the specimen.



Strict requirements on sample shape



Probably most famous artefact



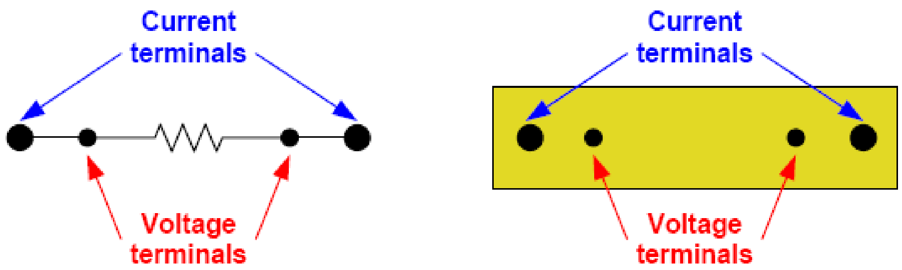
Superconducting fluctuations and the Peierls instability in an organic solid
SSC 12, 1125 (1973)

Concerns
Different I and V circuits
May be disconnected!

Potential contacts should be connected well to current path

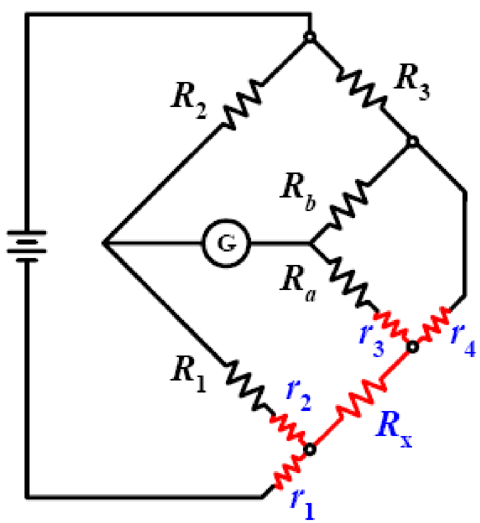
Four probe measurement in bridge configuration

Four-Terminal Resistor



Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge



- r_1 causes no effect on the balance condition.
- The effects of r_2 and r_3 could be minimized, if $R_1 >> r_2$ and $R_a >> r_3$.
- The main error comes from r_4, even though this value is very small.

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R 334 *Philips Res. Repts* **13**, *1-9, 1958*

A METHOD OF MEASURING SPECIFIC RESISTIVITY AND HALL EFFECT OF DISCS OF ARBITRARY SHAPE

by L. J. van der PAUW

537.723.1:53.081.7+538.632:083.9

Summary

A method of measuring specific resistivity and Hall effect of flat samples of arbitrary shape is presented. The method is based upon a theorem which holds for a flat sample of arbitrary shape if the contacts are sufficiently small and located at the circumference of the sample. Furthermore, the sample must be singly connected, i.e., it should not have isolated holes.

Resistivity measurements: van der Pauw method

Very popular in semiconductor industry
Does not require sample of regular shape

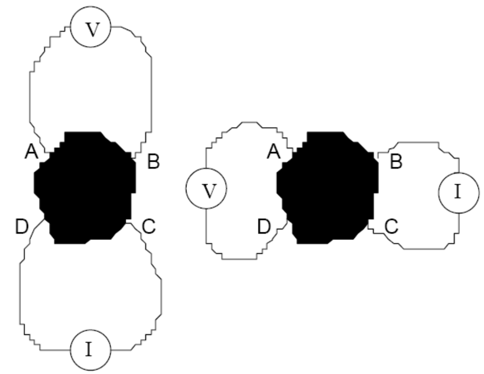
Assumptions
1. Homogeneous sample
2. Isotropic sample
3. Two-dimensional, thickness is unimportant
4. Sample boundary sharply defined

Surface resistance

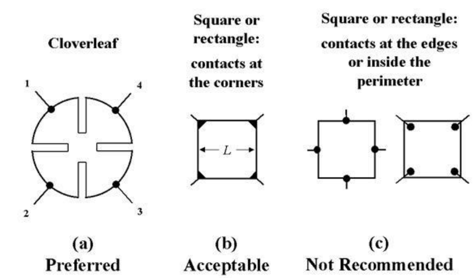
$$\rho_{\square} = \frac{\pi}{ln2} \frac{R_{ab\text{-}cd} + R_{bc\text{-}da}}{2} f$$

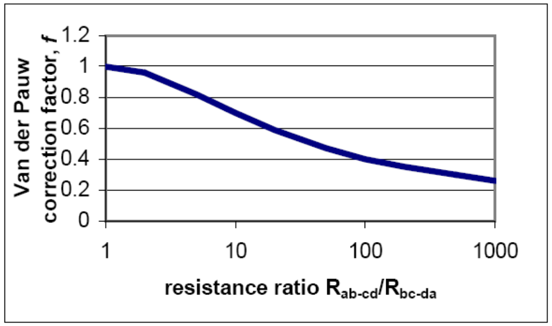
Resistivity

$$\rho = \frac{\pi}{ln2} W \frac{R_{ab\text{-}cd} + R_{bc\text{-}da}}{2} f$$



The contact arrangements for the two resistance measurements, $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$ used in the Van der Pauw resitivity measurement.





Correction factor based on the ratio of the two resistance measurements, $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$ used in the Van der Pauw resitivity measurement.

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Method for Measuring Electrical Resistivity of Anisotropic Materials

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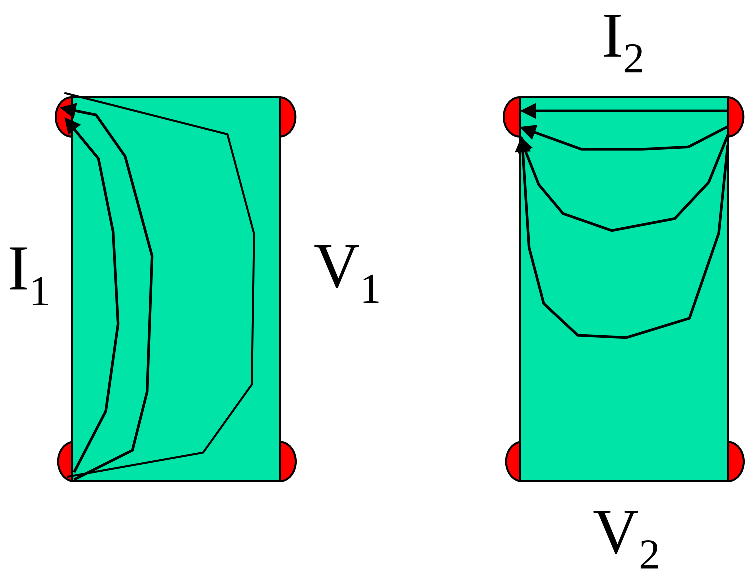
A rectangular prism with edges in principal crystal directions is prepared with electrodes on the corners of one face. Voltage–current ratios for opposite pairs of electrodes permit calculation of components of the resistivity tensor. The method can use small samples, and is best suited to materials describable by two or three tensor components. Examples are given of measurements of V_2O_3–Cr and oriented amorphous graphite.

Montgomery technique

$$(\rho_2/\rho_1)^{1/2} = (l_2/l_1) \times (l_1'/l_2').$$

1. Van der Pauw resistivity measurements on samples of rectangular



$R_1=V_1/I_1$ $R_2=V_2/I_2$

2. Calculation of the anisotropy ratio for isotropic samples

3. Scaling anisotropic samples on isotropic by van der Pauw scaling transformation

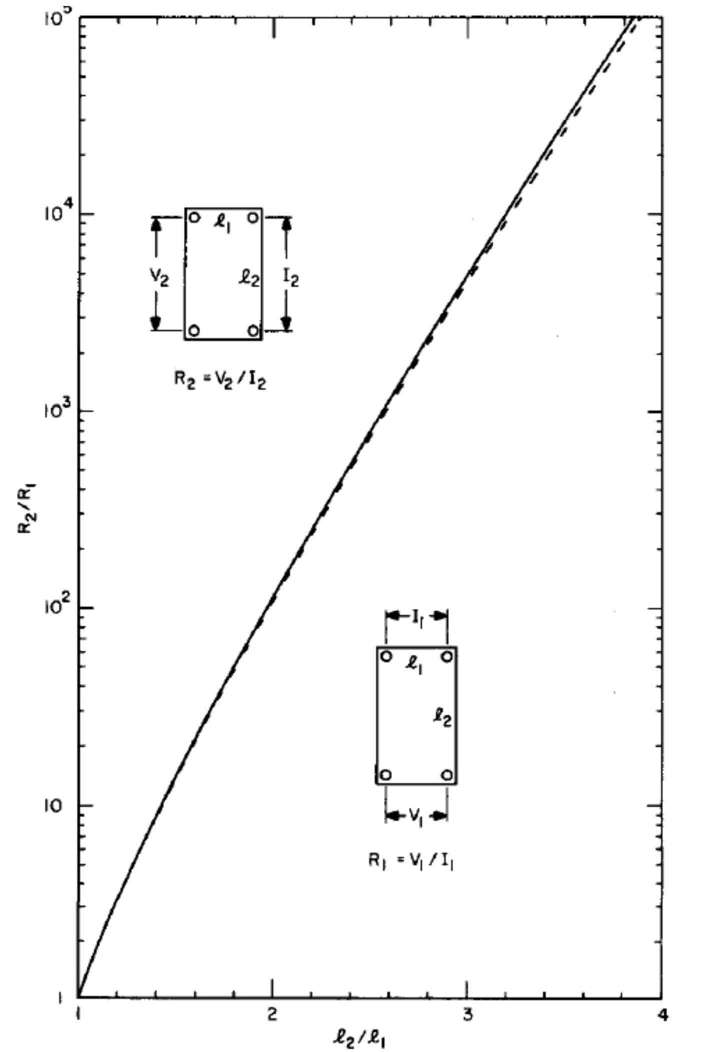


FIG. 3. Resistance ratio versus sample dimension ratio. Solid line is for a thin sample; dashed line for a thick sample. Details of thickness dependence given in Table II.

Interference Factor

- Ohmic contact quality and size
- Sample uniformity and accurate thickness determination
- Photoconductive and photovoltaic effects

Reading

1. Low Level Measurements Handbook, Keithley www.keithley.com
2. Lake Shore manual for LS370
3. J. M. Ziman, Principles of the Theory of Solids, 1964