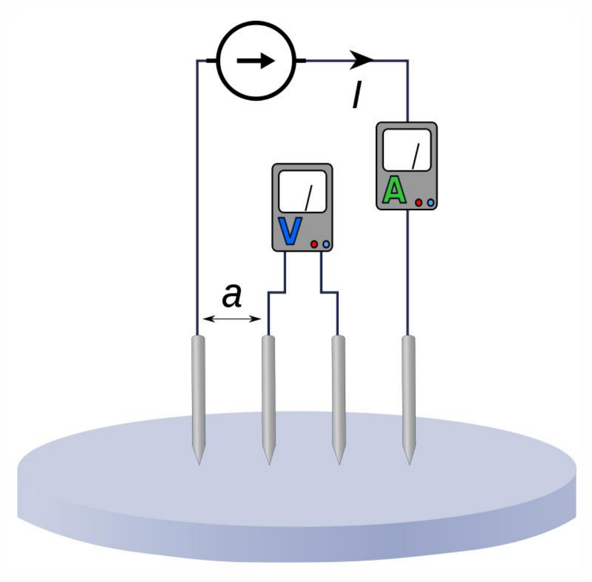
Resistivity

$$\rho = R\,\frac{A}{\ell}$$

Compare and Contrast electrical resistivity measurements in linear 4 probe and Montgomery



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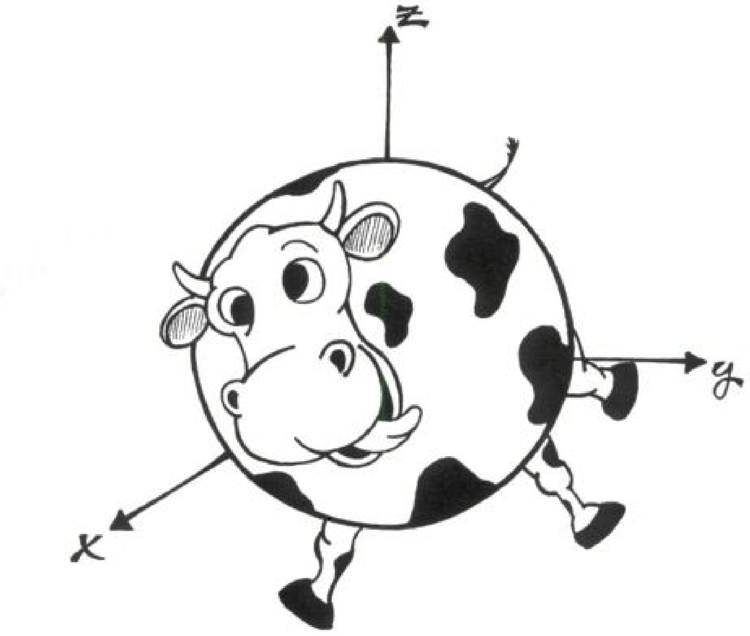
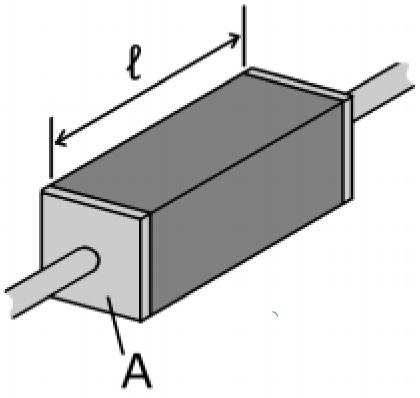


What is Electrical Resistivity

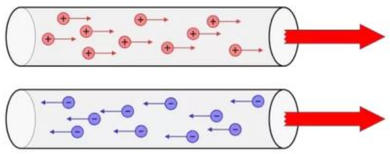
Electrical resistivity **F**undamental property of a material that quantifies how strongly the material opposes the flow of electric current. A low resistivity indicates a material that readily allows the flow of electric current

https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

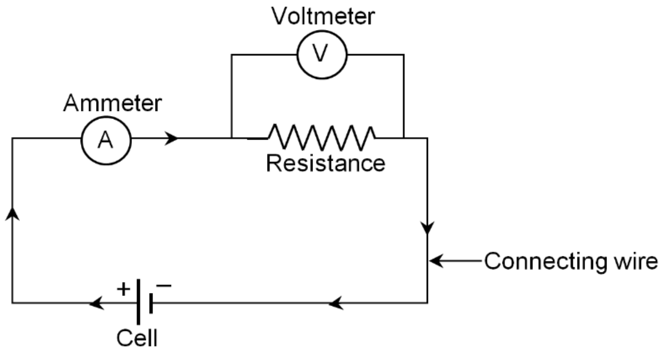


Resistivity

$$\rho = R\,\frac{A}{\ell}$$



I=V/R

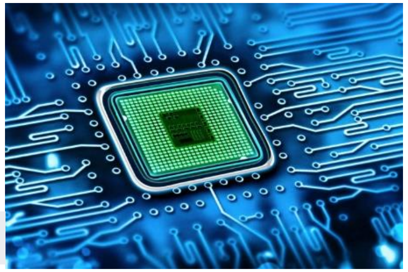


Georg Simon Ohm
1789 - 1854

Alternative expression

$E = \rho J$ *E= Electric Field , J = Current Density*

Motivation for finding electrical resistivity



To know the material ---- Metal, Semiconductor, Superconductor, Insulator

Resistivity gives us idea about other physical quantities of interest – Mobility of electron and hole in semiconductor (Grain size , mid-gap states)

(Condensed Matter Physics) Electrical Transport measurement

Phase transitions in conducting materials are observable as a sharp feature or change in slope of the temperature dependence of the resistivity.

Presence of anisotropies in electrical transport can reflect the presence of broken rotational symmetries.

Measurement of these anisotropies can contribute to understanding the nature and origin of associated phase transitions, particularly those that are driven by interactions at the Fermi-level

In the case of an electronic nematic phase transition close to the critical temperature the resistivity anisotropy is proportional to the nematic order parameter.

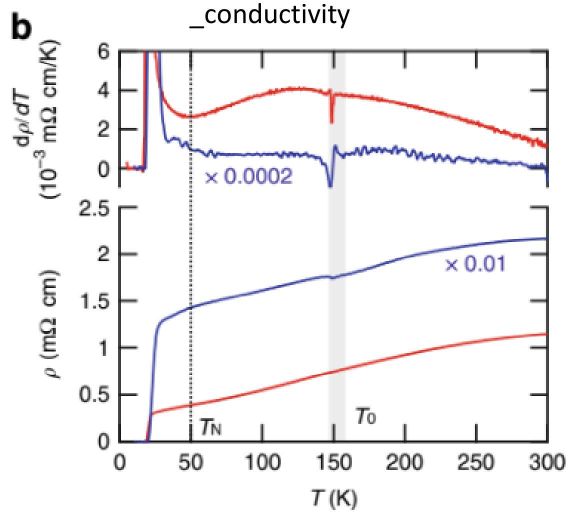
Determination of the resistivity anisotropy of orthorhombic materials via transverse resistivity measurements

P. Walmsley[1,2,a] and I. R. Fisher[1,2]

	ρ (Ω·m) at 20 °C
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Annealed copper[b]	1.72×10^{-8}
Gold[c]	2.44×10^{-8}
GaAs	1.00×10^{-3} to 1×10^{8}
Germanium[i]	4.6×10^{-1}
Fused quartz	7.5×10^{17}
PET	1×10^{21}
Teflon	1×10^{23} to 1×10^{25}

https://www.morningoutlook.com/image-processor-market-global-insights-and-trends-2018/

https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity



NATURE COMMUNICATIONS|8: 2167 |DOI: 10.1038/s41467-017-02327-0

Problem !!!!!! Contact Resistance

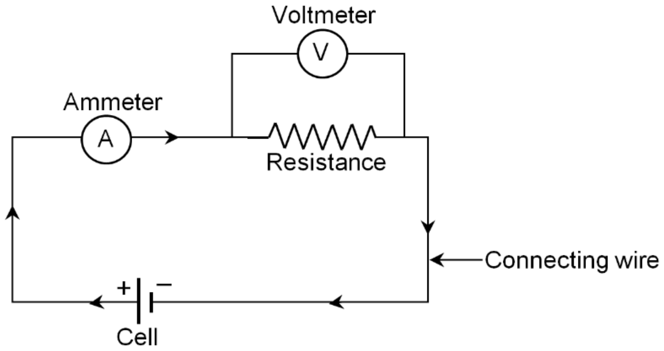
Resistance due to interface of electrical leads and connections (Parasitic resistance)

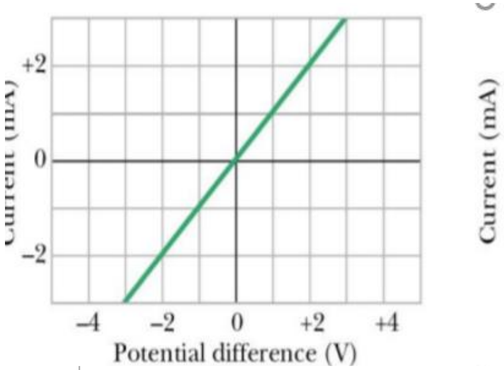
Due to oxidation of metal on the surface of the metal used

Ways to avoid contact resistance

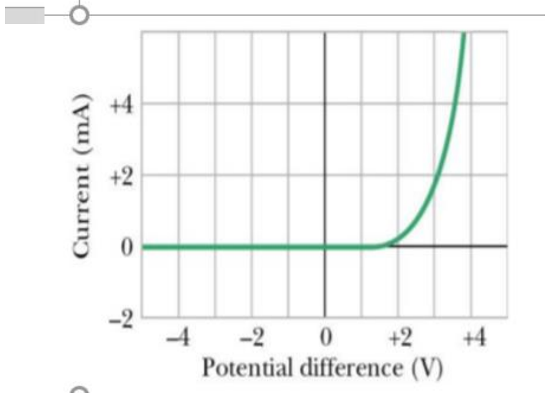
Metal and Metal --- soldering, welding

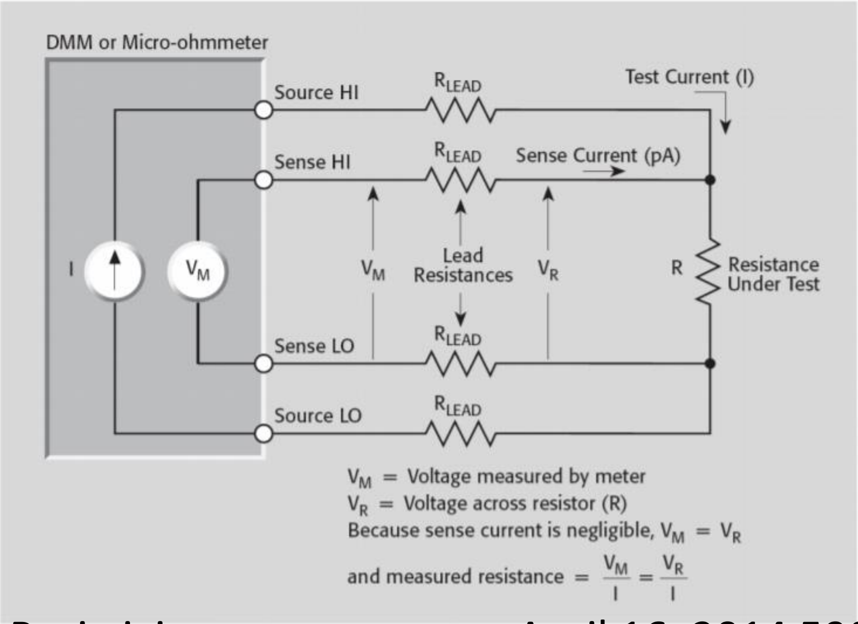
Semiconductor ------Metal in contact with highly doped polycrystalline leading to quantum tunneling





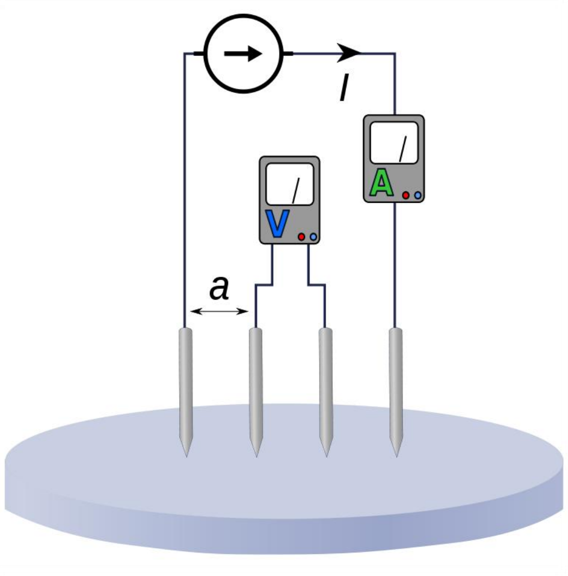
leading



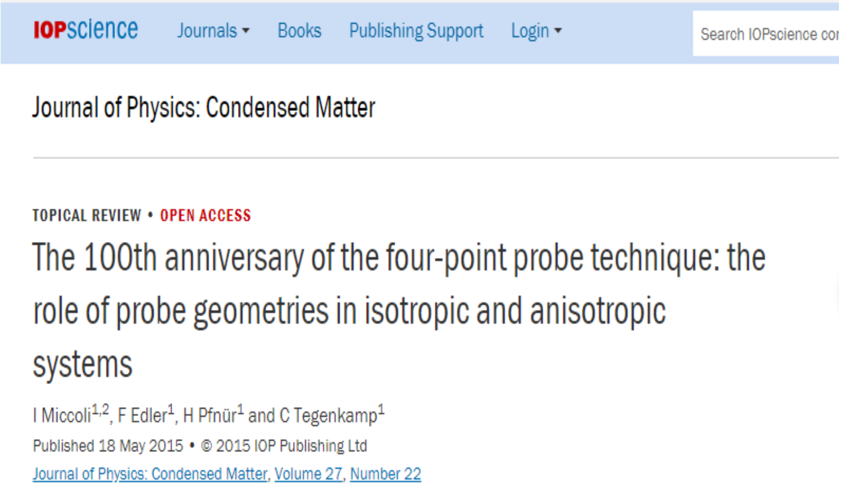


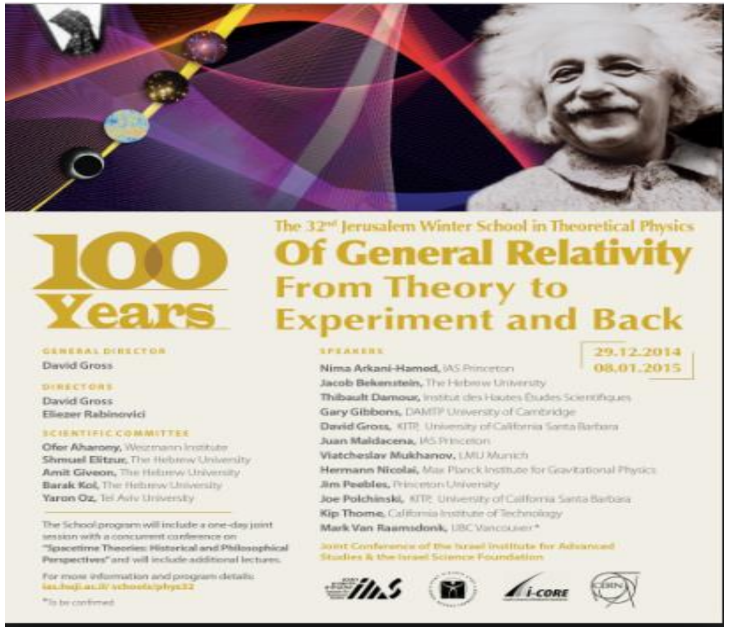
Resistivity measurements April 16, 2014 590B Makariy A. Tanatar

History

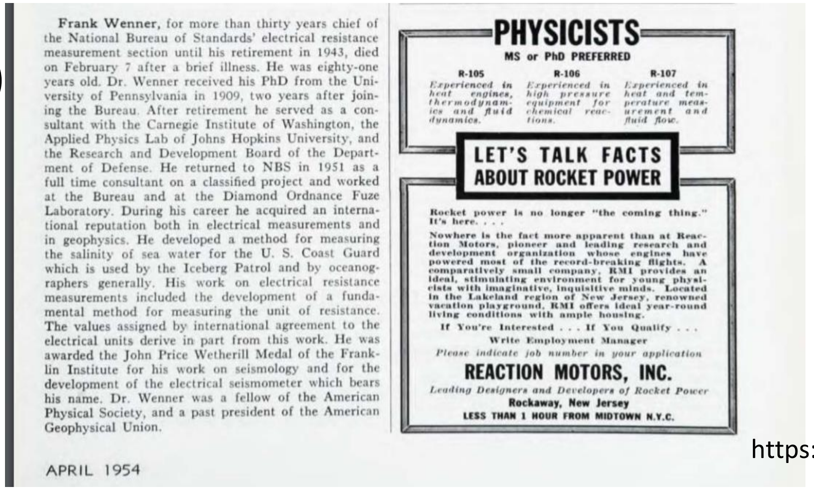


https://en.wikipedia.org/wiki/File:Wenner_electrode_array.svg



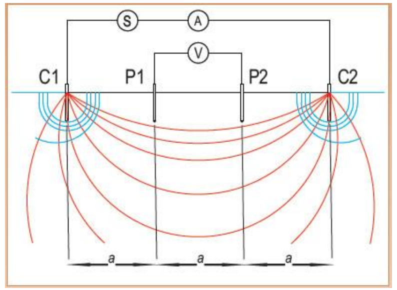


American Physicist Frank Wenner (1873-1954) (American Bureau of Standards) invented Wenner array to measure resistivity of soil



Frank Wenner, for more than thirty years chief of the National Bureau of Standards' electrical resistance measurement section until his retirement in 1943, died on February 7 after a brief illness. He was eighty-one years old. Dr. Wenner received his PhD from the University of Pennsylvania in 1909, two years after joining the Bureau. After retirement he served as a consultant with the Carnegie Institute of Washington, the Applied Physics Lab of Johns Hopkins University, and the Research and Development Board of the Department of Defense. He returned to NBS in 1951 as a full time consultant on a classified project and worked at the Bureau and at the Diamond Ordnance Fuze Laboratory. During his career he acquired an international reputation both in electrical measurements and in geophysics. He developed a method for measuring the salinity of sea water for the U. S. Coast Guard which is used by the Iceberg Patrol and by oceanographers generally. His work on electrical resistance measurements included the development of a fundamental method for measuring the unit of resistance. The values assigned by international agreement to the electrical units derive in part from this work. He was awarded the John Price Wetherill Medal of the Franklin Institute for his work on seismology and for the development of the electrical seismometer which bears his name. Dr. Wenner was a fellow of the American Physical Society, and a past president of the American Geophysical Union.

APRIL 1954



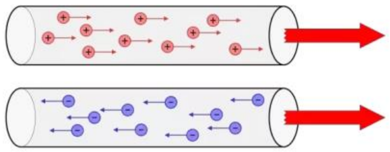
https://www.geophysical.biz/soil-resistivity.htm

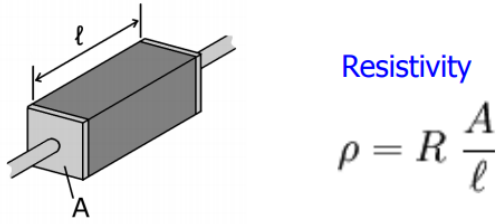
Geometry of sample : Pandora's Box

- Finite shape and size

  Two length scales -------Spacing between probes and Dimension of sample

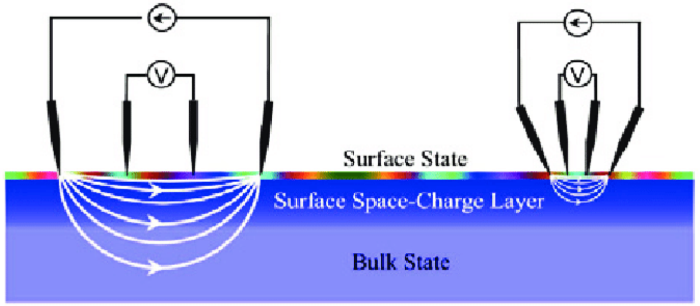
  Two Special case
  Semi infinite 3D sample
  Infinite 2D sheets





It's all about how current distribute in sample

We have opened Pandora's box.



Surface Review and Letters 10(6):963 · December

Special Case:Semi –infinite 3D and infinite 2D sheets

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2. Four-probe methods for isotropic semi-infinite 3D bulks and infinite 2D sheets

For the ideal case of a 3D semi-infinite material with the four electrodes equally spaced and aligned along a straight line (a 4P in-line array, see figure 1(b)), the material resistivity is given by [6]

$$\rho_{3D}^{\text{line}} = 2\pi s \frac{V}{I}, \qquad (2.1)$$

where V is the measured voltage drop between the two inner probes, I is the current flowing through the outer pair of probes and s is the probe spacing between the two probes. Equation (2.1) can be easily derived considering that the current $+I$, injected by first electrode in figure 1(a), spreads spherically into a homogeneous and isotropic material. Therefore, at a distance r_1 from this electrode, the current density $J = I/2\pi r_1^2$ and the associated electric field, i.e. the negative gradient of the potential, can be expressed as

$$E(r_1) = \rho J = \frac{\rho I}{2\pi r_1^2} = -\frac{\mathrm{d}V}{\mathrm{d}r}. \qquad (2.2)$$

By integrating both sides of (2.2), the potential at a point P reads

$$\int_0^V \mathrm{d}V = -\frac{\rho I}{2\pi}\int_0^{r_1} \frac{\mathrm{d}r}{r^2} \Rightarrow V(P) = \frac{I\rho}{2\pi r_1}. \qquad (2.3)$$

For the scenario shown in figure 1(a), the voltage drop is then given by the potential difference measured between the two probes, i.e.

$$V(P) = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \qquad (2.4)$$

This concept can be easily extended to 4P geometries where the problem of contact resistances (see above) is usually avoided. According to figure 1(b), the concept presented above can be generalized and the voltage drop between the two inner probes of a 4P in-line array is

$$V = V_2 - V_3 = \frac{I\rho}{2\pi}\left[\left(\frac{1}{s_1} - \frac{1}{s_2}\right) - \left(\frac{1}{s_3} - \frac{1}{s_4}\right)\right], \qquad (2.5)$$

which, for the special case of an equally spaced 4P probe geometry (with $s_1 = s_4 = s$ and $s_2 = s_3 = 2s$), is equivalent to (2.1).

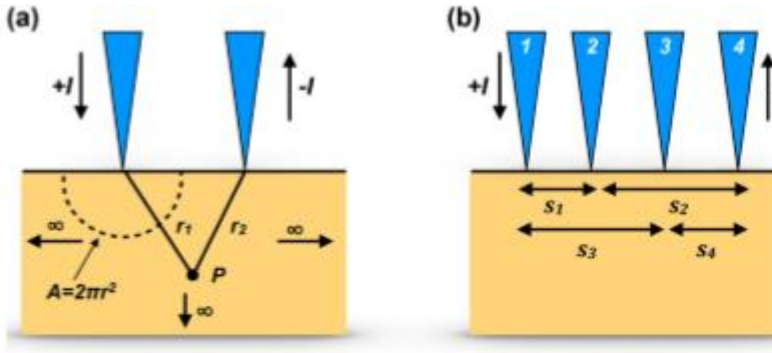
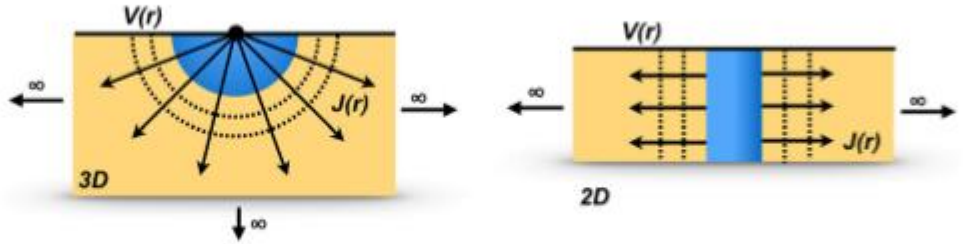
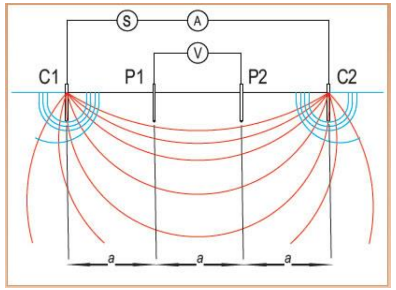


Figure 1. Schematics of (a) a two-point probe and (b) a collinear 4P probe array with equidista





https://www.geophysical.biz/soil-resistivity.htm

Figure 2. Voltage $V(r)$ and current density $J(r)$ profiles for a semi-infinite 3D material and infinite 2D sheet.

Contd………

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- For thickness t << S (probe spacing)

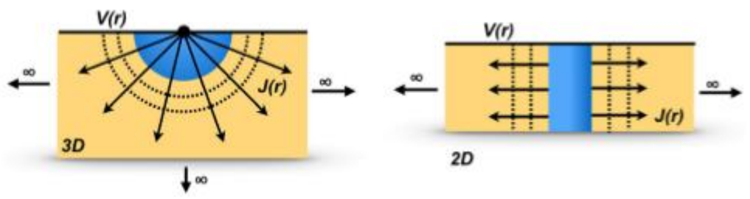


Figure 2. Voltage $V(r)$ and current density $J(r)$ profiles for a semi-infinite 3D material and infinite 2D sheet.

$$E(r) = \rho J = \frac{\rho I}{2\pi r t} = -\frac{\mathrm{d}V}{\mathrm{d}r}.$$

Repeating the same steps as for (2.3)–(2.5), a loga
dependency is obtained for the voltage drop between
inner probes:

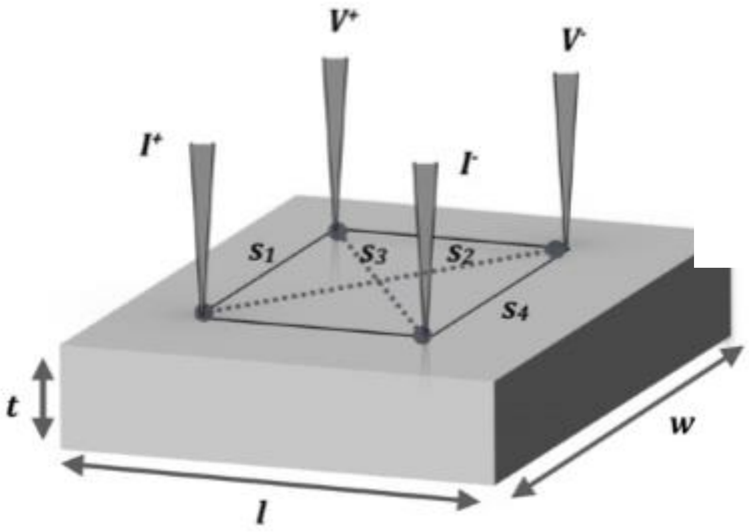
$$V = V_2 - V_3 = \frac{I\rho}{2\pi t}\ln\left(\frac{s_2 s_3}{s_1 s_4}\right).$$

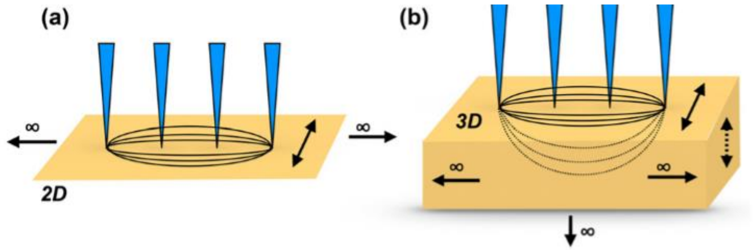
In the case of an equally spaced in-line 4P geometry t
resistivity is given by

$$\rho_{2D}^{\text{line}} = \frac{\pi t}{\ln 2}\frac{V}{I},$$

i.e. the resistance is not dependent on the probe distanc
directly underlines the 2D character of the specimen.
of a homogenous and finitely thick sample the resisti
be assumed to be constant, thus the bulk resistivity is often
replaced by the so-called sheet resistance R_{sh} defined as

$$R_{sh} = \frac{\rho}{t}\ (\Omega)\,. \qquad (2.9)$$





Sample shape	4P in-line
3D bulk[a]	$2\pi s \frac{V}{I}$
2D sheet[b]	$\frac{\pi}{\ln 2}\frac{V}{I}$
1D wire[a]	$\frac{\sum}{s}\frac{V}{I}$

Correction Factor for finite thickness

Resistivity ρ = F1.F2.F3 (V/I)

Correction factor

a) Thickness of sample (F1)

b) Position of probe from edge of sample (F2)

c) Lateral dimension of the sample (F3)

It's all about how current distribute in sample

$$\rho = R^{\text{line}}_{\text{sh}-2\text{D}} \cdot t \cdot F_1\left(\frac{t}{s}\right) = \left[\frac{\pi}{\ln 2}\frac{V}{I}\right] \cdot t \cdot F_1\left(\frac{t}{s}\right)$$

$$F_1 = \frac{\ln 2}{\ln\{[\sinh(t/s)]/[\sinh(t/2s)]\}}.$$

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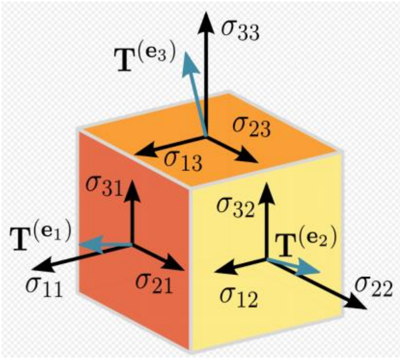
Anisotropy in crystal structure
Electrical resistivity : Scalar or Tensor

Alternative expression

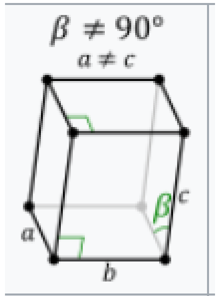
$E = \rho J$ *E= Electric Field , J = Current Density*

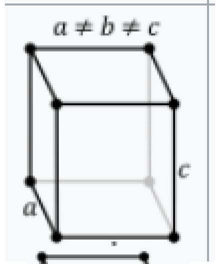
//en.wikipedia.org/wiki/Crystal_structure

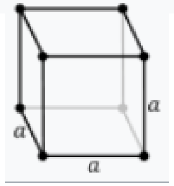
$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho y_x & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{xx} & \rho_{xx} \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}$$



https://en.wikipedia.org/wiki/Tensor







Symmetric tensor by Reciprocity theorem

Six independent components

Further reduces down to one two or three or more depending on crystal symmetry

Cubic ---1 , Trigonal and Tetragonal -2 ……. Orthorhombic -3 Monoclinic ----6

Why not only three ? Diagonalize symmetric matrix and find 3 eigenvectors(principal axis)

Yes. For specific orientation only 3 independent components is sufficient . But its impossible to know the required orientation apriori from crystal structure alone.

Montgomery Method: Exploring unknown with known

In 1970 Montgomery proposed a **GRAPHICAL** method for specifying the resistivity of anisotropic materials cut in the form of a parallelepiped with the three orthogonal edges l_1' l_2' l_3' sides

Consists of basically two steps.

Step 1: Wasscher mapping : Mapping isotropic parallelepiped on an anisotropic parallelepiped

Step 2: Relating the bulk resistivity of a rectangular isotropic prism to voltage- current measurements (Taking account of geometry(size and shape) of sample and probe

H. C. Montgomery
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07974
(Received 30 November 1970)

A rectangular prism with edges in principal crystal directions is prepared with electrodes on the corners of one face. Voltage–current ratios for opposite pairs of electrodes permit calculation of components of the resistivity tensor. The method can use small samples, and is best suited to materials describable by two or three tensor components. Examples are given of measurements of V_2O_3–Cr and oriented amorphous graphite.

Electrical resistivity, defined in terms of the vector electric field intensity and the vector current density, is represented by a second-rank tensor. Considerations of detailed balance indicate that this tensor is symmetrical, requiring not more than six parameters. In a great many materials of practical interest, considerations of crystal symmetry further reduce the required number of parameters to one, two, or three.

Various methods have been described for measuring the components of the resistivity tensor for anisotropic conductors.[1–3] Some of these involve cutting out several samples in different orientations or placing electrodes in several configurations on a sample. We have used a method which has advantages in certain applications in that it requires only a single sample with fixed electrodes to measure two components of the resistivity tensor and the sample can be quite small. Very satisfactory measurements were made with a V_2O_3–Cr sample about 1-mm square and 0.5-mm thick. The method is especially useful in measuring resistivity as a rapidly varying function of temperature since two tensor components can be measured on a single sample in rapid succession. The principal directions of the resistivity tensor must be known when the sample is prepared.

A sample is cut in the form of a rectangular prism, one of whose faces has edges in the directions of the two resistivity components to be measured. Small electrodes are attached at the four corners of this face. The ratio of voltage between two adjacent electrodes to the current between the other two electrodes is measured, then a similar measurement is made with all connections rotated by 90°. From these measurements and the dimensions of the sample, two components of the resistivity tensor can be calculated. If a third component is required, a different face of the same sample or a separate sample with its principal face in a different orientation may be used. A large class of anisotropic materials can be characterized by

tropic solid of any shape to an equivalent isotropic solid, based on work of van der Pauw,[2] and (2) relating the bulk resistivity of a rectangular isotropic prism to voltage–current measurements of the sort described

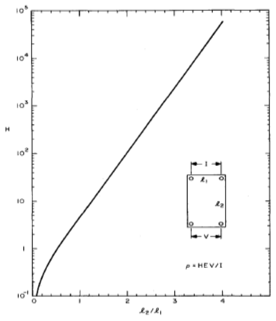


Fig. 1. Function H relating resistivity to sample dimensions and voltage–current ratio.

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Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. Montgomery

Montgomery Method Step 1: Wasscher mapping

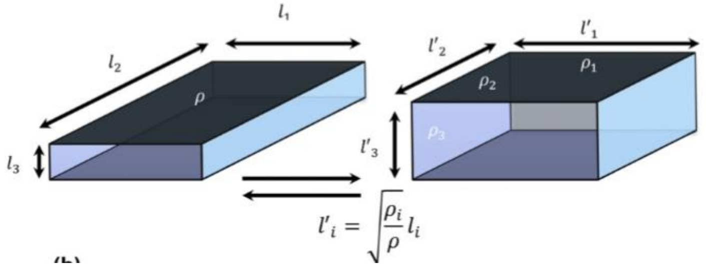
Let l_1 l_2 l_3 and l'_1 l'_2 l'_3 are sides of isotropic and anisotropic material

ρ_1 ,$\rho_{2\ and}$ ρ_3 $resitivity\ of\ anisotropic\ material$ and $\rho = resistivity\ of\ isotropic\ material$

Mapping $\rho^3 = \rho_1\rho_2\rho_3$ $and\ l_i = l'_i(\rho_{i/}\rho\ )^{1/2}$

Mapping preserves preserve voltage and current relationship,

i.e. they do not affect the resistance R



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Anisotropic solid can be mapped to an equivalent isotropic solid. Equivalent means that when identical currents flow into corresponding contact areas on the two solids the potential differences between corresponding points are identical.

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Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. Montgomery

Montgomery Method Step 2:Relating bulk resistivity with voltage and current

Relating the bulk resistivity of a rectangular isotropic prism to voltage- current measurements Taking account of geometry(size and shape) of sample and probe.

$\rho = H\left(\frac{l2}{l1}\right) E\ R$ where E is the effective thickness

H is the correction factor for finite size (lateral dimension)

- Effective thickness equal to real thickness for $\frac{l_3}{(l_1 l_2)^{1/2}} < 1$

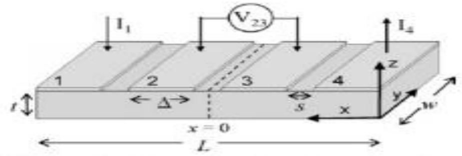
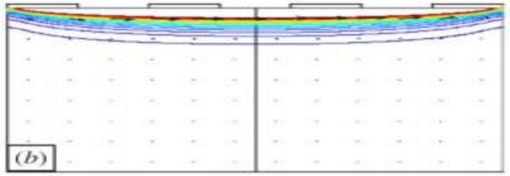


Figure 1. Schematic of the co-linear 4-probe configuration with electrodes that span the width of the specimen.



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Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. MONTGOMERY

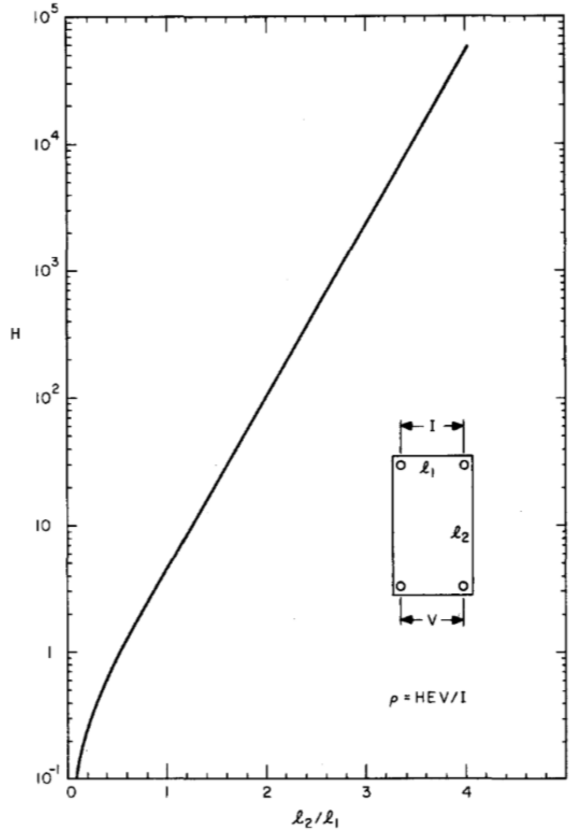

FIG. 1. Function H relating resistivity to sample dimensions and voltage–current ratio.

Montgomery Method Step 2:Relating bulk resistivity with voltage and current

- H and E are independent parameters

For $\frac{l_3}{(l_1 l_2)^{1/2}} < 1$

E/$(l_1 l_2)^{1/2}$ is fairly independent of l_2/l_1

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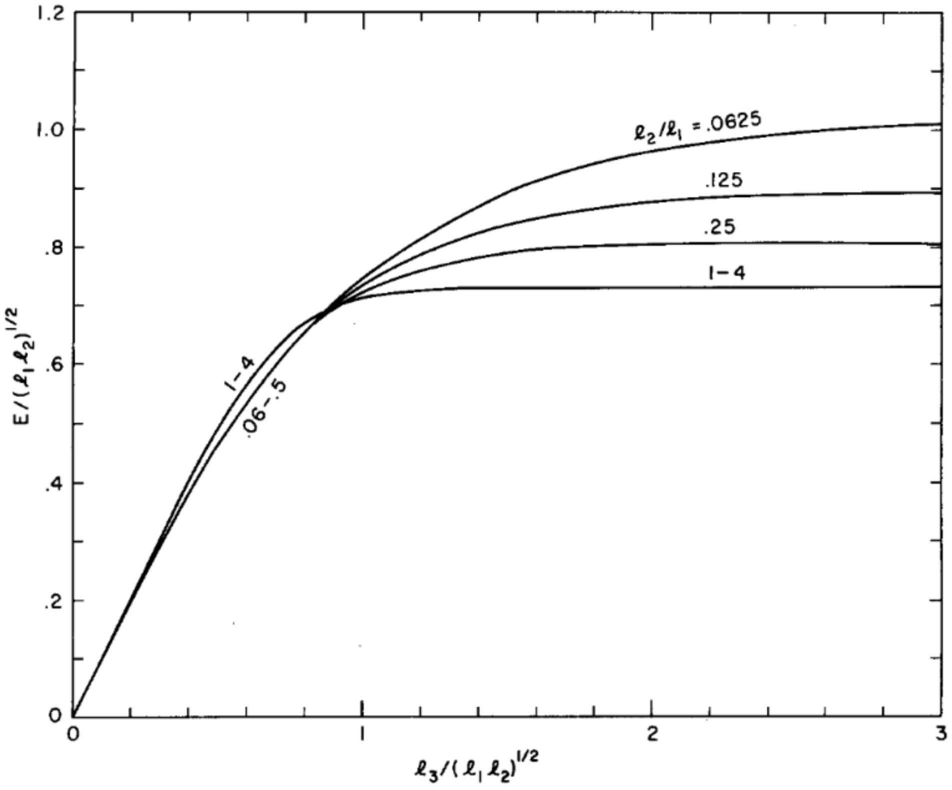
Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. MONTGOMERY



Not end yet but its getting there….Procedure……

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Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. MONTGOMERY

Step 1: Van der Pauw resistivity measurements

$R_1 = V_1/I_1$ V_1 voltage between the two contacts on an edge $l_1{}'$

I_1 the current between the opposite two contacts $l_2{}'$

Current and Voltage connections are rotated by 90 degree to get R_2.

R_2/R_1 is independent of thickness

From $l_i = l_i'(\rho_{i}/\rho)^{1/2}$ *we get* $(\rho_2/\rho_1)^{½} = (l_2/l_1)(l_1{}'/l_2{}')$ ……….i

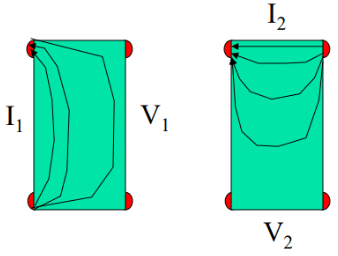
Using $\rho = H\left(\frac{l2}{l1}\right) E\ R$ and $l_i = l_i'(\rho_{i}/\rho)^{1/2}$ and assuming $\frac{l_3}{(l_1 l_2)^{1/2}} < 1$

$(\rho_2\rho_1)^{½} = H l_3' R_1$ ……………………….ii

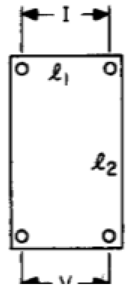
Solving (i) and (ii) we get ρ_1 *and* ρ_2

We use Graph to find value of H and (l_2/l_1) ----------GRAPHICAL METHOD

Second sample face normal to first one is used to find ρ_3



$R_1 = V_1/I_1$ $R_2 = V_2/I_2$



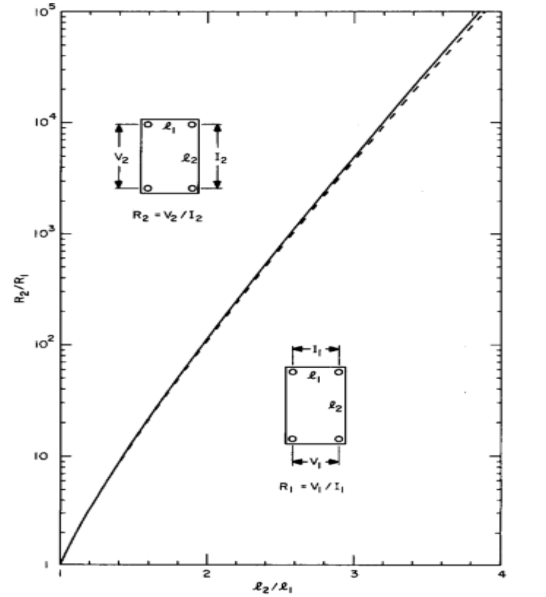


FIG. 3. Resistance ratio versus sample dimension ratio. Solid line is for a thin sample; dashed line for a thick sample. Details of thickness dependence given in Table II.

If sample thickness is not small and all three resistivity are different

We assume any value of ρ_3 and make multiple measurement on two perpendicular face until we get self consistent solution.

Why to take all this trouble ? Why not Linear Four Probe

We can measure resistivity in various direction

with single sample , same sets of electrodes in quick

succession.

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Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. Montgomery

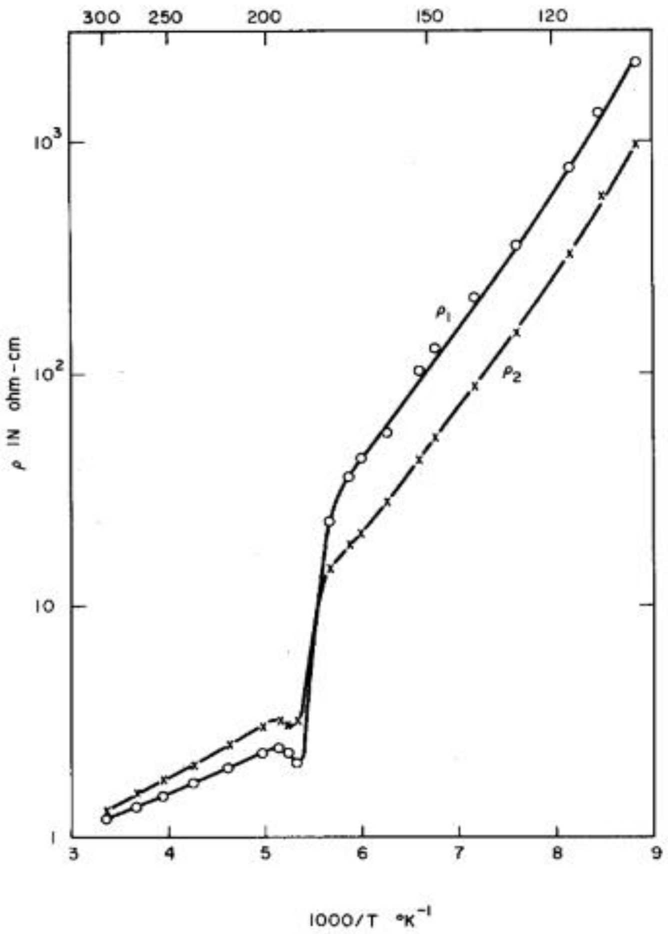

Fig. 4. Resistivity components for a V_2O_3–Cr sample at various temperatures; ρ_2 is parallel to the c axis.

No Free lunch

Montgomery method also has many limitations.

- Too many assumptions (e.g. Validity of Wasscher mapping)
- For the Montgomery method to be valid the sample must be square or rectangular in the plane and of constant thickness.
- Highly sensitive to current path (Sample homogeneity)
- Measured voltage have non linear relationship with sample thickness.

Comparative study

$$\rho = R^{\mathrm{line}}_{\mathrm{sh-2D}} \cdot t \cdot F_1\left(\frac{t}{s}\right) = \left[\frac{\pi}{\ln 2}\frac{V}{I}\right] \cdot t \cdot F_1\left(\frac{t}{s}\right)$$

$$F_1 = \frac{\ln 2}{\ln\{[\sinh(t/s)]/[\sinh(t/2s)]\}}$$

	Linear Four Probe	Montgomery
Crystal Symmetry	Isotropic Material ρ	Anisotropic Material ρ_1 , ρ_2 and ρ_3
Sensitivity	Less susceptible to sample thickness and homogeneity compared to Montgomery	Highly sensitive to sample thickness. Doesn't account for probe position . Based on validity assumptions
Availability of Sample	Many samples	Few samples
Sample Geometry	Thin films or very bulky sample Correction Factor are well defined for finite thickness	Good for parallelepiped but can have various thickness
Time and Simultaneity	Simultaneous measurements of ρ_1 , ρ_2 and ρ_3 $is\ not$ possible	****Simultaneous measurement of ρ_1 , ρ_2 and ρ_3 is possible *** **Ubiquitous signatures of nematic quantum criticality in optimally doped Fe-based superconductors.** Kuo HH[1], Chu JH[2], Palmstrom JC[2], Kivelson SA[3], Fisher IR Possible to measure in quick

THANK YOU

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