Magnetic field

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Physics 590B
What is magnetic field?

• The definitions are still debated
• \( H \) – magnetic field strength
• \( B \) – magnetic induction (or flux density)

• Some insight: Maxwell equations

\[
\nabla \times H = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial B}{\partial t}
\]

\[
\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}
\]
magnetic field due to current

cgs
H (Oe)
B (G)

SI
H (A/m)
B (T)
Magnetic moment

Magnetic moment of a closed loop carrying current $I$:

$$M_i = \frac{I}{2\pi} \oint_{C} \mathbf{r} \times d\mathbf{l} = ISn$$

Magnetic field on the axis of a loop of radius $R$ at a distance $z$ is:

$$H_z = \frac{2M_i}{(R^2 + z^2)^{3/2}}$$

Total magnetic moment:

$$\mathbf{M} = \sum \mathbf{M}_i \quad \text{(superposition principle)}$$
what about MPMS?

it measures a total magnetic moment in cgs (emu)

1 emu is:
- M of a 1 m² loop carrying a 1 mA current
- M of a loop of radius 1.78 cm carrying a 1 A current
- Typical permanent magnet (1 mm³) ~ 1 emu

- M of a neutron star ~ $10^{30}$ emu
- The Earth’s magnetic moment ~ $8 \times 10^{25}$ emu
- An electron spin: $\mu_B \sim 10^{-20}$ emu
- Proton and neutron: $\mu_N \sim 10^{-23}$ emu

- One Abrikosov vortex (0.1 mm long) ~ $10^{-10}$ emu
- Change in M due to d-wave gap < $10^{-10}$ emu/K
- Hard superconductors ~ 0.1 emu
magnetic moment in a magnetic field


torque: \[ \tau = \mu \times B \]

Energy: \[ W = -\mu B = -\mu B \cos(\theta) \]

Force: \[ F = -\text{grad}(W) = \text{grad}(\mu B) \]

for example, for \[ B = \begin{bmatrix} B_x(x), 0, 0 \end{bmatrix}, \mu = (\mu_x, \mu_y, 0) \]

\[ \mu B = \mu_x B_x \quad \text{and} \quad F = \mu_x \frac{dB_x}{dx} \]

in inhomogeneous magnetic field

\[ \mu_x > 0 \]

\[ \mu_x < 0 \]

\[ F \] changes sign however torque aligns along the field
origin of magnetism (except for currents...)

\[ \mathbf{M}_{\text{ion}} = \gamma \hbar \mathbf{J} = -g \mu_B \mathbf{J} \]

\[ \hbar \mathbf{J} = \hbar \mathbf{L} + \hbar \mathbf{S} \]

\[ g \text{- gyromagnetic ratio} \]
\[ g \text{- Landé factor} \]

\[ \mu_B = \frac{e \hbar}{2mc} \approx 9.2741 \times 10^{-21} \text{ erg} \frac{G}{\text{Bohr magneton}} \]

\[ g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \]

free electron:

\[ g = 2.0023 \approx 2.00 \]

Magnetic moment:

\[ \mathbf{M}_e \approx \mu_B \]

\( J=S=1/2 \)
Practical definitions

per unit volume

\[ 4\pi M = B - H \]

bound (molecular, spin etc) currents
free currents (moving charge)

What is the problem with this definition?
1. Assumes uniform H (ellipsoids only)
2. Assumes uniform B (homogeneous system)

\[ \chi = \frac{M}{H} \ - \ \text{dimensionless!} \quad \chi_{SI} = 4\pi \chi_{cgs} \]

some other quantities are used:

\[ \chi_m = \frac{\chi}{\rho} \left[ \frac{cc}{cc \cdot g} = g^{-1} \right] \]

\[ \chi_{mol} = \chi_m M_m \left[ \frac{g}{g \cdot mol} = mol^{-1} \right] \]
Let’s see how well it works

a simple classical paramagnet

\[ M = M_s L(x), \quad x = \frac{p \mu_B H}{k_B T} \]

\[ \frac{\mu_B}{k_B} = 0.671 \begin{bmatrix} K \\ T \end{bmatrix} \]

\[ \frac{\mu_B}{k_B} \]

\[ 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \]

\[ -0.2 \quad -0.1 \quad 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \]

\[ x=\mu H/k_B T \]

\[ -60 \quad -40 \quad -20 \quad 0 \quad 20 \quad 40 \quad 60 \]

\[ x=\mu H/k_B T \]
Spatial inhomogeneity

$M$ is not a single-valued function of $H$

demagnetization energy, magnetostatic energy, dipolar or fringe (stray) fields energy

$\sim M^2V$

ferromagnetic domains

domain walls energy

$\sigma \sim 1 \frac{erg}{cm^2}$

gain – energy of field outside
loose – energy of the domain wall
Magnetic hysteresis
type-II superconductor

Meissner State

Partial Penetration

Trapped Flux
Using Faraday law of induction

\[ V = -\frac{1}{c} \frac{d\Phi}{dt} \]
How to measure magnetic field?

Miniature Hall-probe Array