

# X-ray diffraction: *the basics*

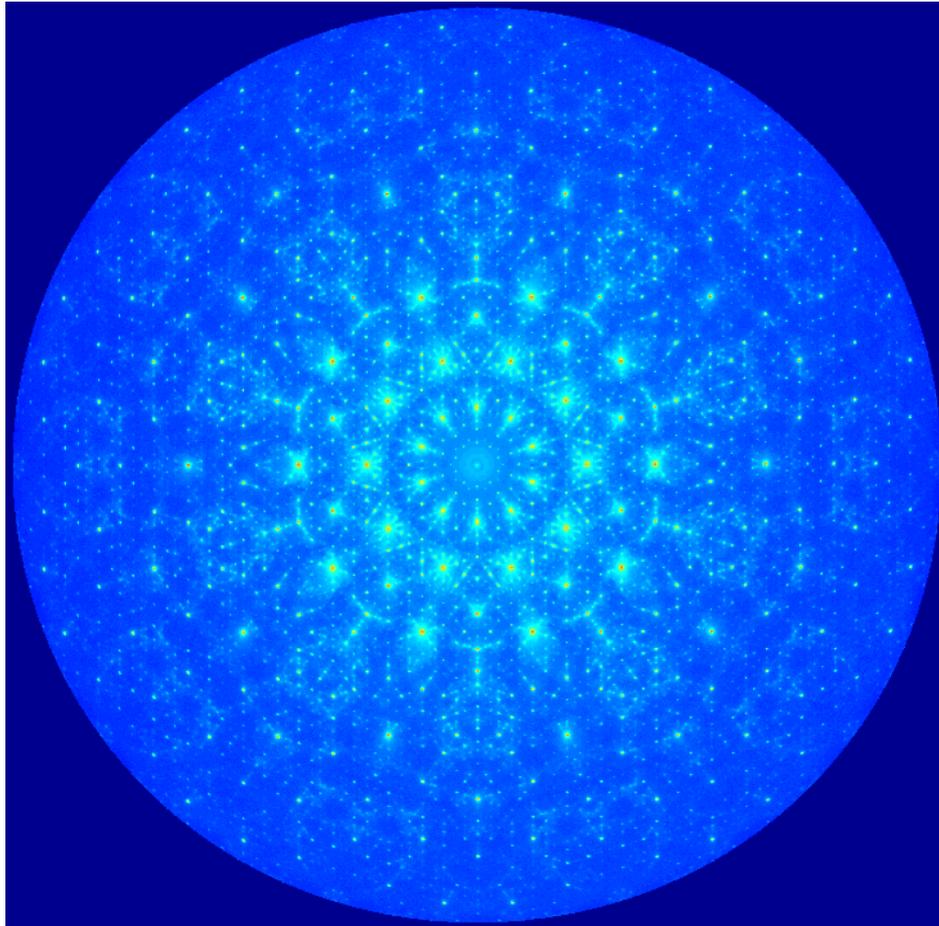


AMES LABORATORY

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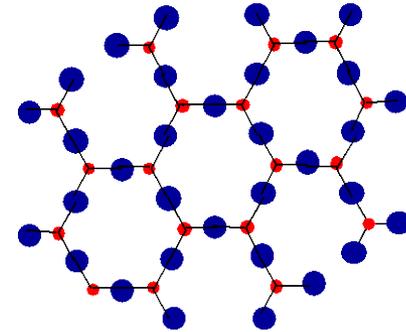
# What's wrong with this picture?



# What is the atomic scale structure?

## Crystalline materials...

- atoms pack in periodic, 3D arrays
- typical of: -metals  
-many ceramics  
-some polymers



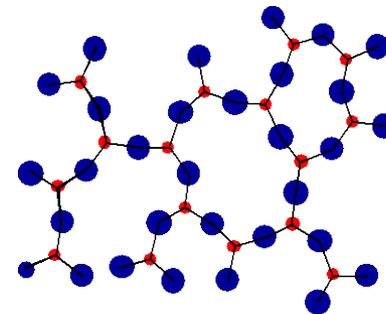
crystalline SiO<sub>2</sub>

Adapted from Fig. 3.18(a),  
*Callister 6e.*

## Noncrystalline materials...

- atoms have no “regular” packing
- occurs for: -complex structures  
-rapid cooling

• Si      • Oxygen



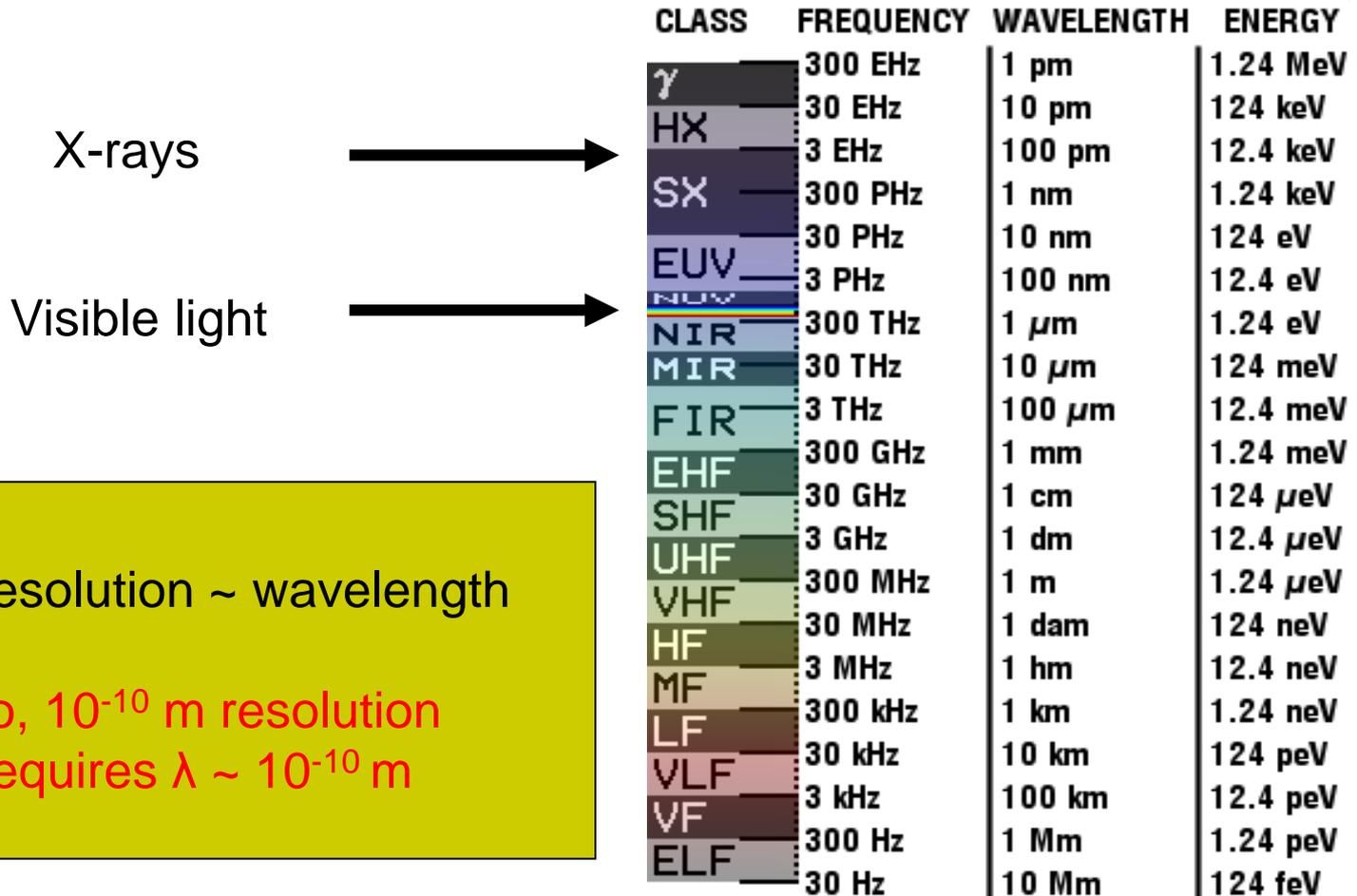
noncrystalline SiO<sub>2</sub>

Adapted from Fig. 3.18(b),  
*Callister 6e.*

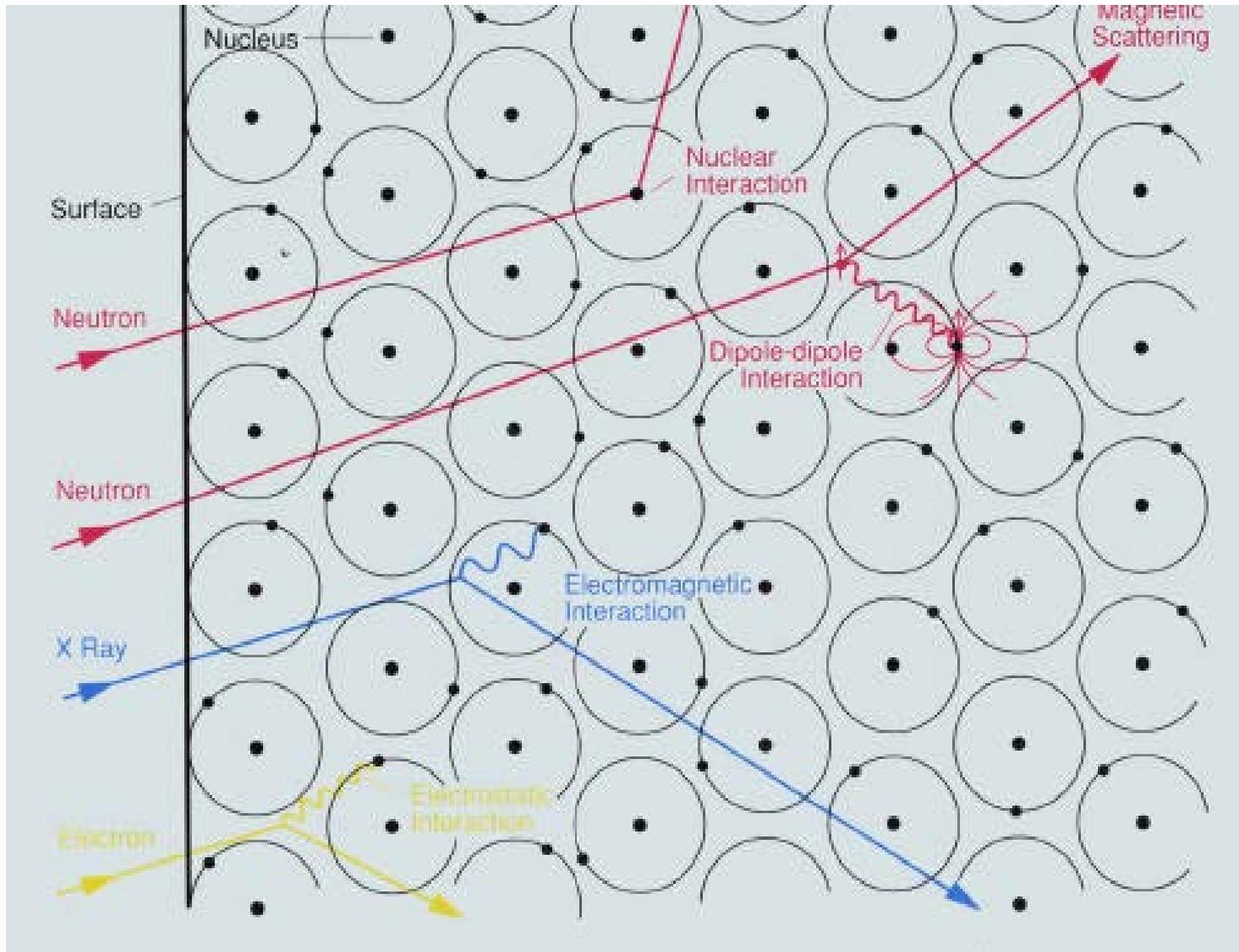
"Amorphous" = Noncrystalline

Distance between atoms ~ Å (10<sup>-10</sup> m)

# How do we use x-rays to study crystal structures?



# Different probes



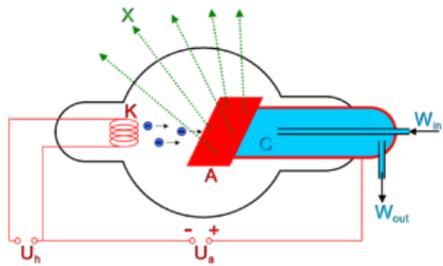
# Different probes

	NEUTRONS	X-RAYS	ELECTRONS
<b>Wavelength range</b>	0.4 - 10 Å	0.1 - 5 Å	0.04 - 0.2 Å
<b>Energy range</b>	0.001 - 0.5 eV	3000 - 100000 eV	6000 - 120000 eV
<b>Cross-section</b>	$10^{-25}$ barns	$10^{-25} Z^2$ barns	$\sim 10^{-22}$ barns
<b>Penetration depth</b>	$\sim$ cm	$\sim$ $\mu\text{m}$	$\sim$ nm
<b>Typical flux</b>	$10^{11} \text{ s}^{-1} \text{ m}^{-2}$	$10^{24} \text{ s}^{-1} \text{ m}^{-2}$	$10^{26} \text{ s}^{-1} \text{ m}^{-2}$
<b>Beam size</b>	mm-cm	$\mu\text{m}$ -mm	nm- $\mu\text{m}$
<b>Typical sample</b>	Any bulk sample	Small crystals, powders, surfaces	Surfaces, thin films, grains, gases
<b>Techniques</b>	<b>Diffraction</b> Inelastic scattering Reflectivity	Diffraction Photon absorption Photoemission Inelastic scattering	Microscopy Diffraction Emission spectroscopy EELS
<b>Phenomena</b>	<b>Magnetic/crystal structures</b> collective excitations (phonons, spin waves) electronic excitations (crystal-field, spin-orbit)	Crystal structures, electronic transitions (photoemission, absorption),	microstructure crystal structures electronic transitions

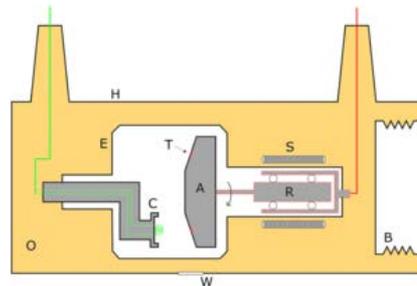
# Why x-rays??

- Easily accessible in the laboratory
- Penetration ~ microns to millimeters (tuned with energy)
  - Low-energies can be used to probe surface and near surface regions
  - High energies can probe the bulk of a sample
- High flux at National Facilities
  - Allows you to throw a lot of photons away → high energy and momentum resolution
  - Is highly polarized and highly collimated
  - Is continuously tunable across a wide range of energies (1 keV → 200 keV)
- Wavelength range ~ interatomic spacing (1-2 Å) → diffraction
  - interacts with electrons
  - Magnetic interactions are higher order

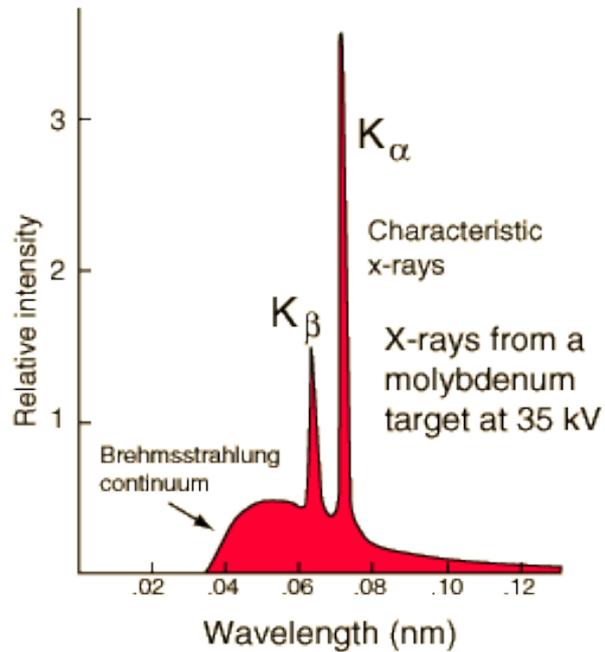
# Production of x-rays



X-ray tube

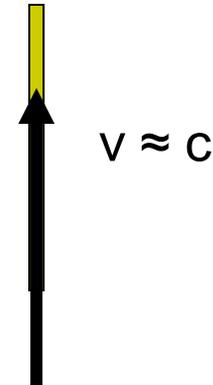
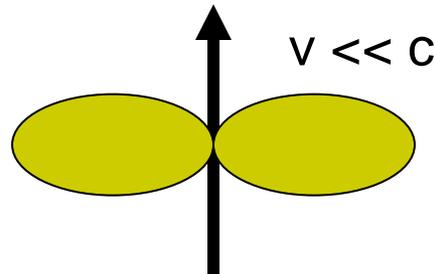
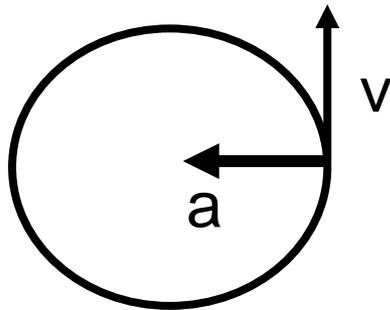


Rotating anode



# X-rays from a synchrotron source

- Static charge ---- Electric field
- Charge moving at constant  $v$  ---- Magnetic field
- Accelerating charge --- Electromagnetic radiation



Synchrotron radiation is

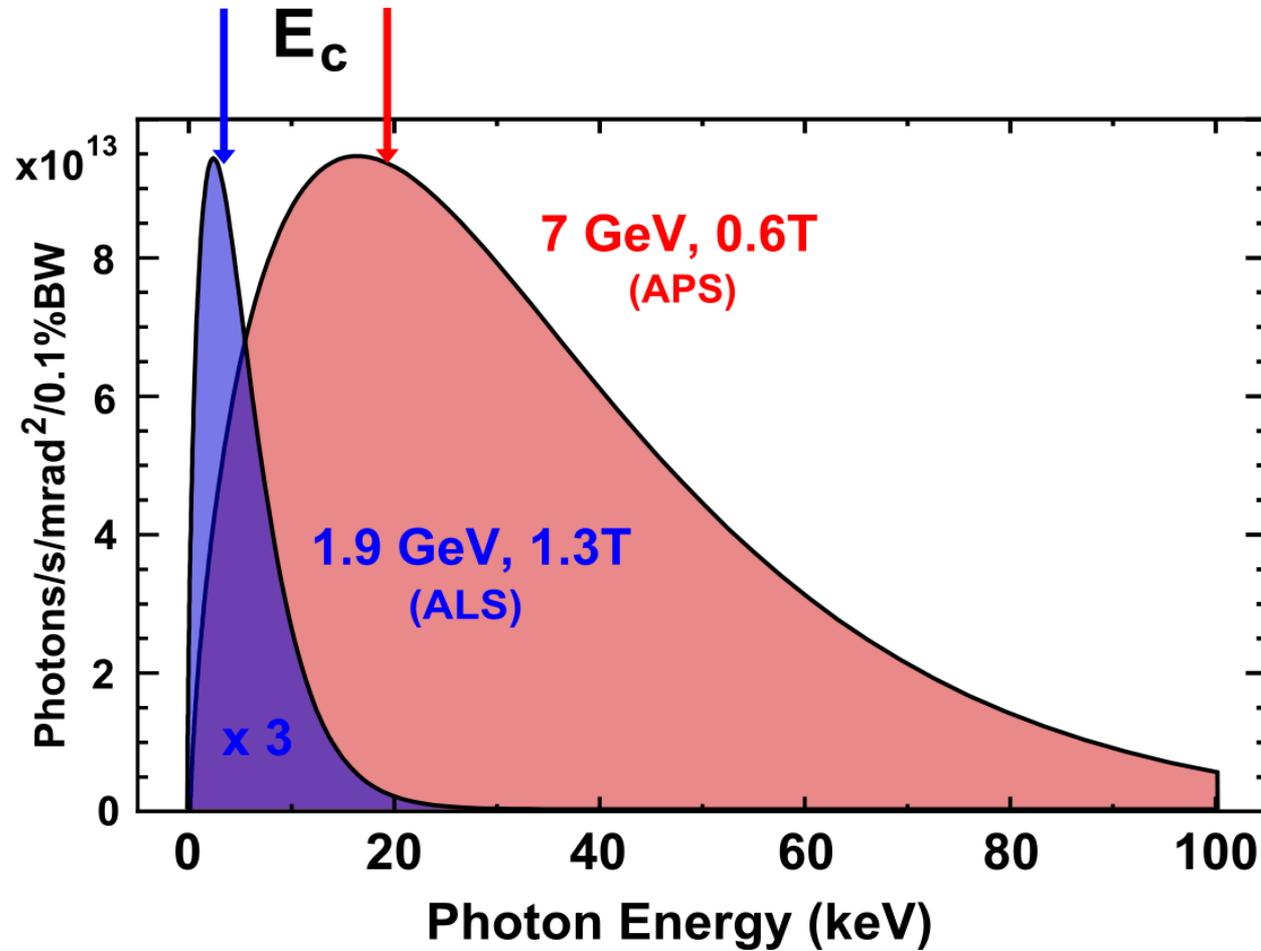
highly collimated

highly linearly polarized

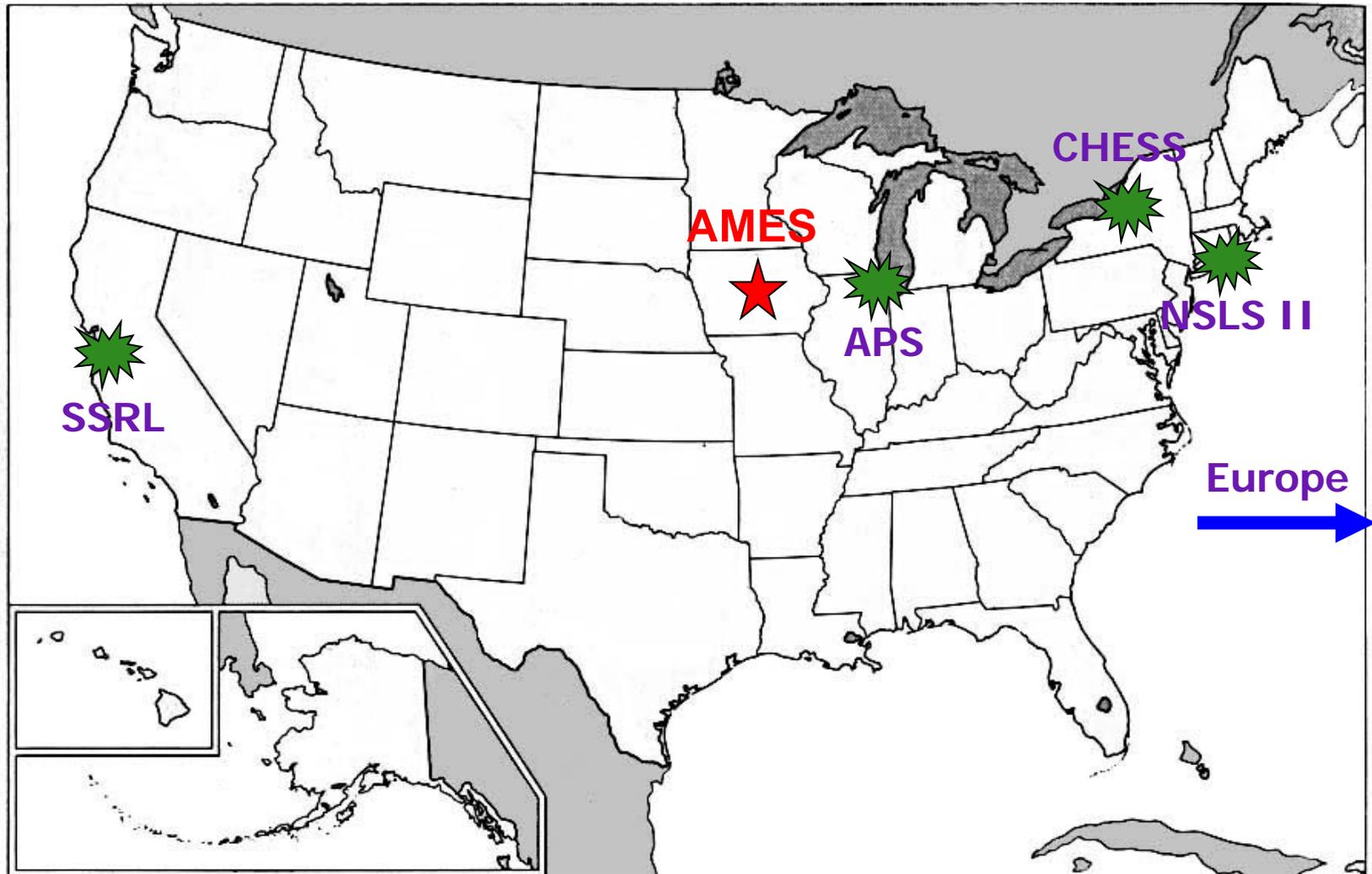
highly brilliant

continuous wavelength distribution (beyond Cu, Mo, etc..)

# Bending magnet radiation

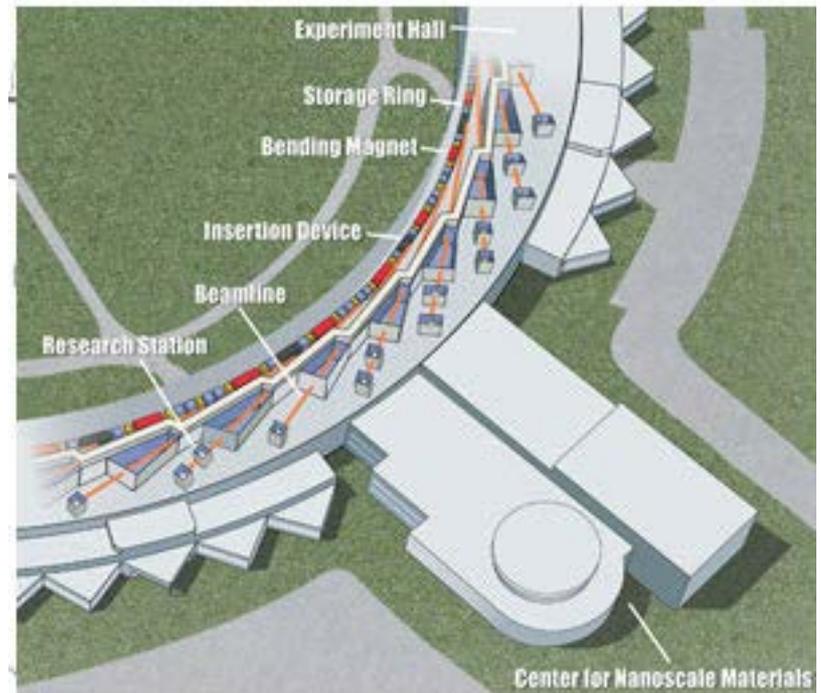


# Places to go

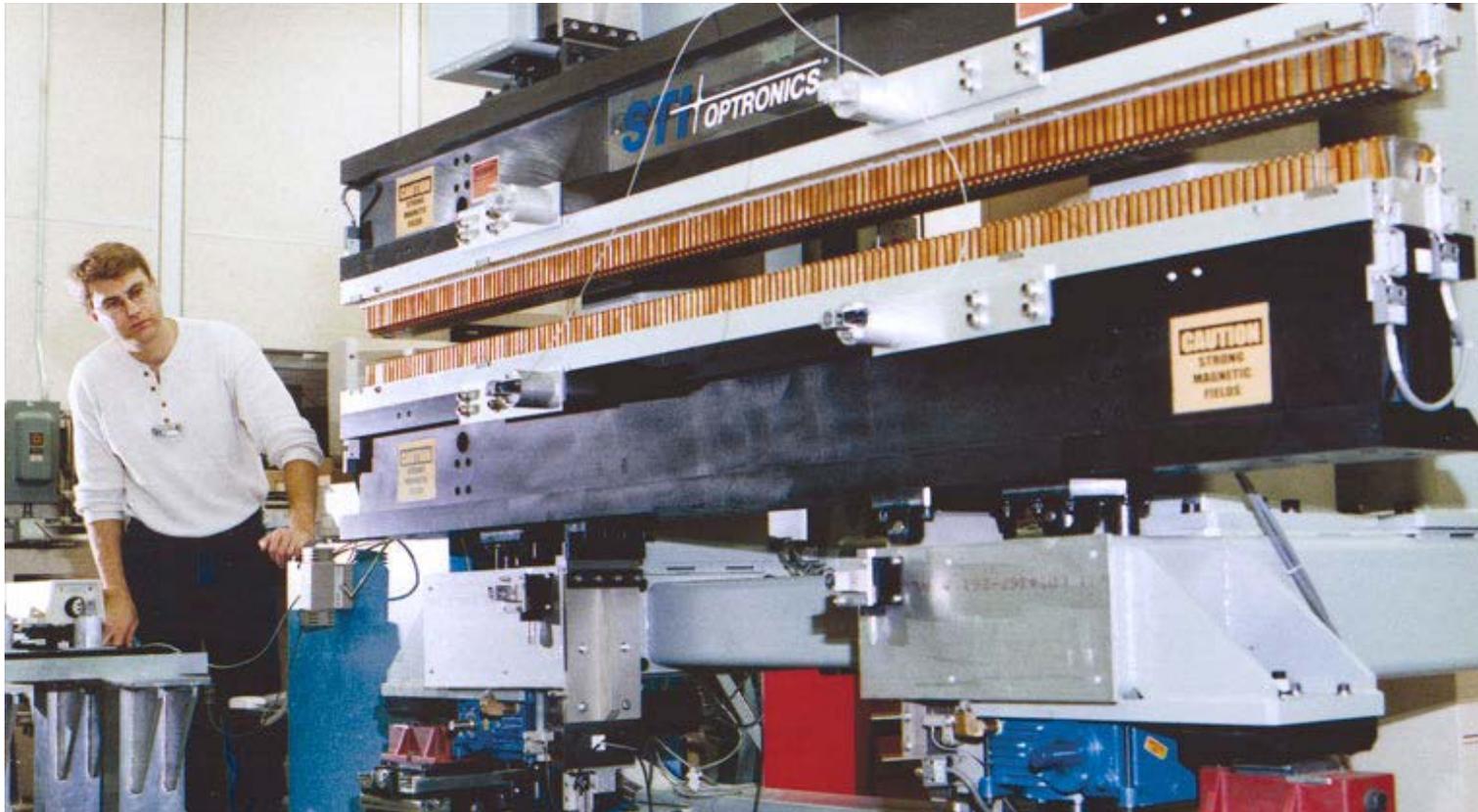


# Advanced Photon Source

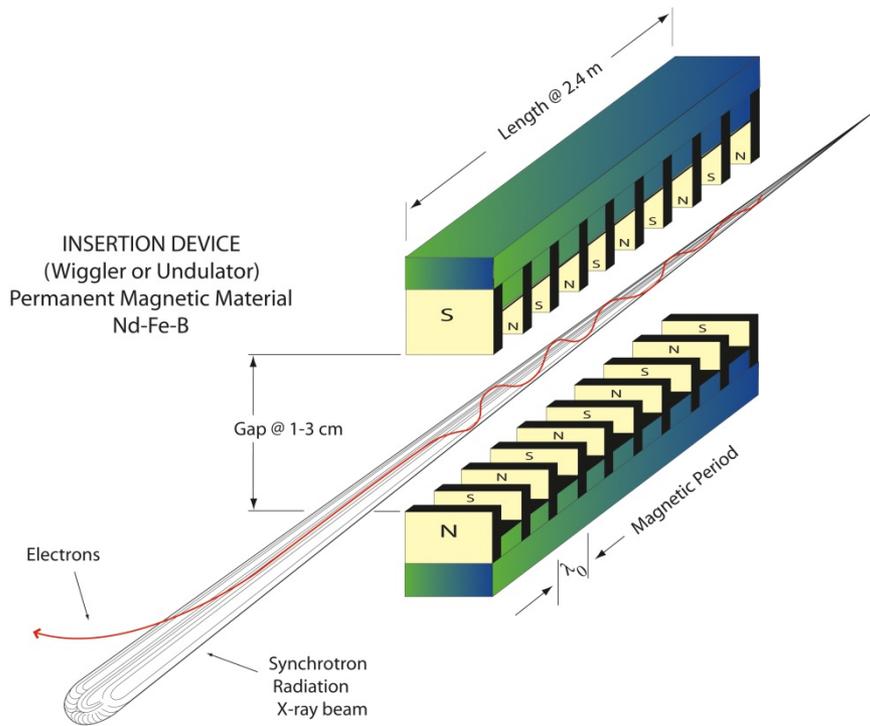
## Synchrotron



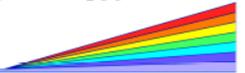
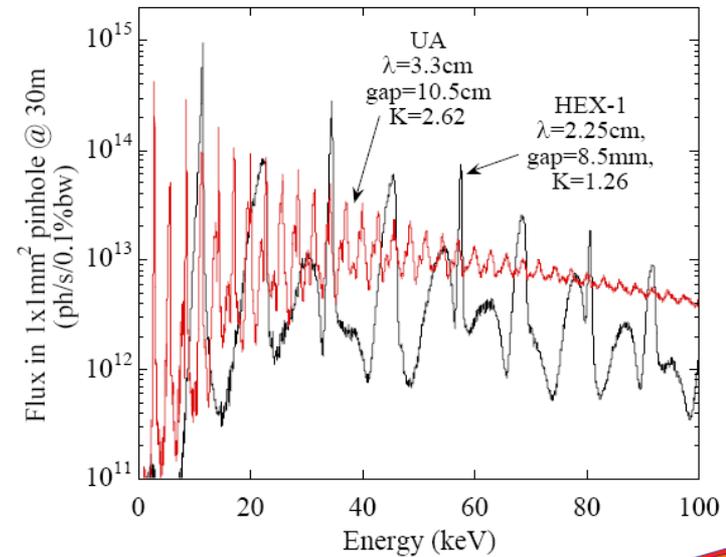




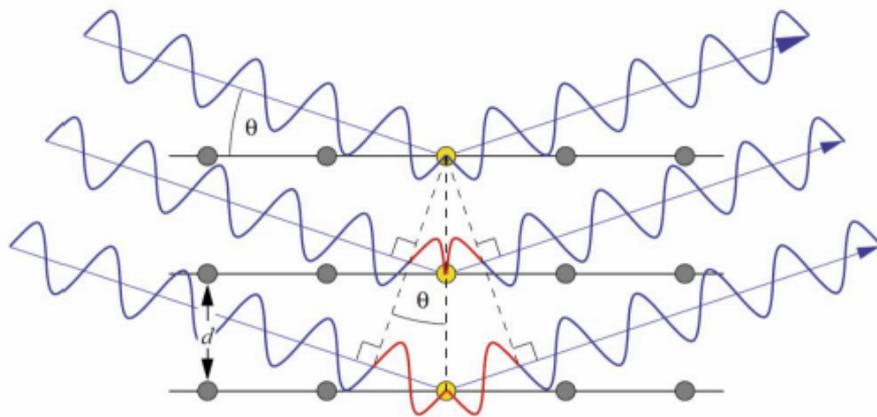
# Insertion Devices



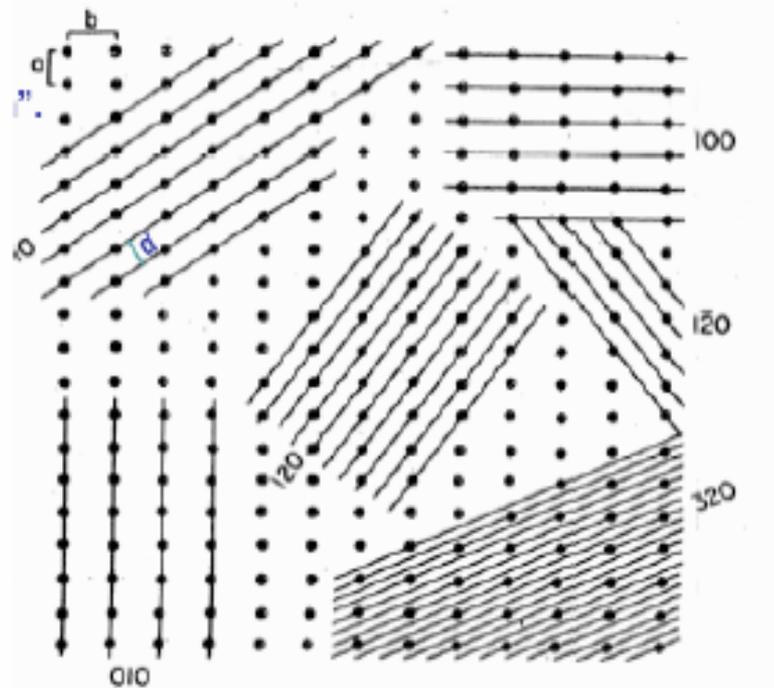
## High Energy Undulator Flux



# Bragg's Law –the view in real space



Constructive interference when  $n\lambda = 2d\sin\theta$

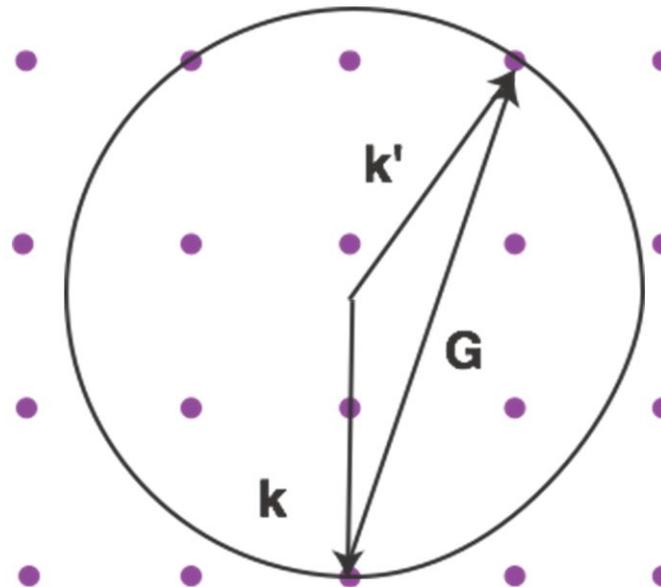


# But Physicists live in reciprocal space

## The Ewald construction

Laue condition  $\mathbf{K} = \mathbf{k}' - \mathbf{k} = \mathbf{G}$  if  $\mathbf{G}$  is a rec. lat. vec.

- Draw (cut through) the reciprocal lattice.
- Draw a  $\mathbf{k}$  vector corresponding to the incoming x-rays which ends in a reciprocal lattice point.
- Draw a circle around the origin of the  $\mathbf{k}$  vector.
- The Laue condition is fulfilled for all vectors  $\mathbf{k}'$  for which the circle hits a reciprocal lattice point.



# Diffraction peak intensities

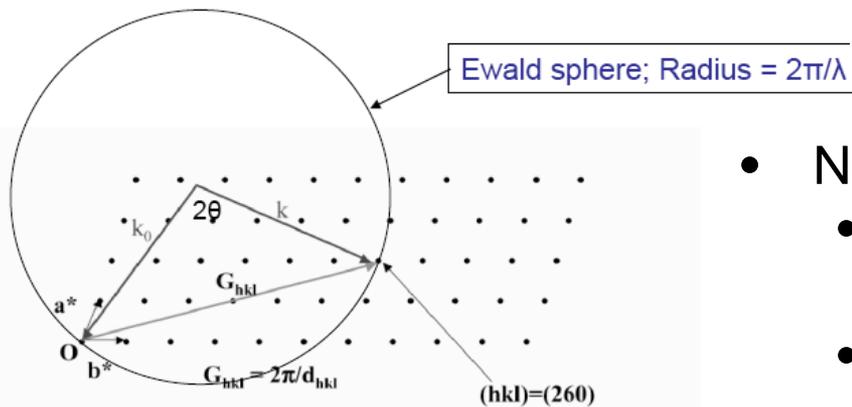
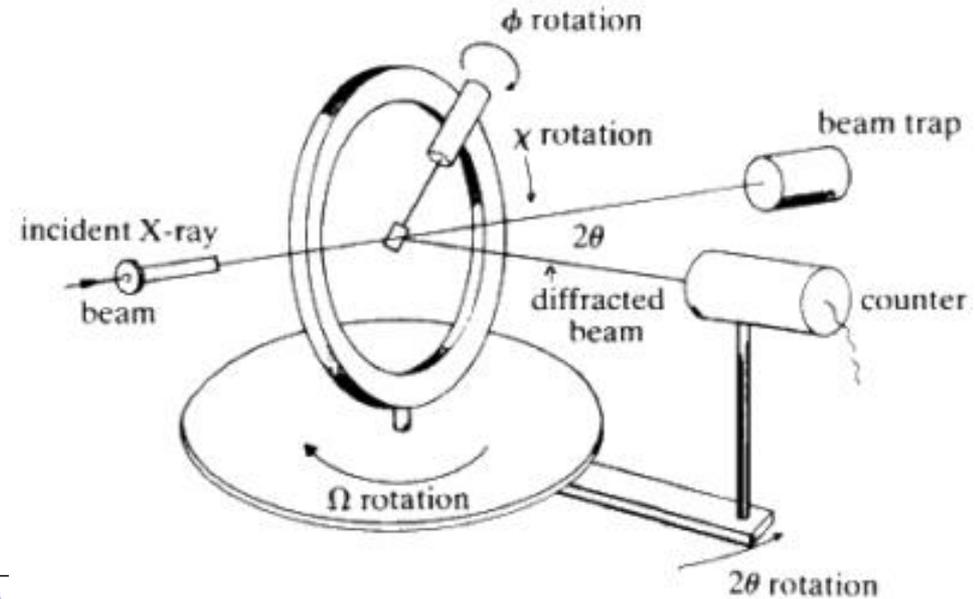
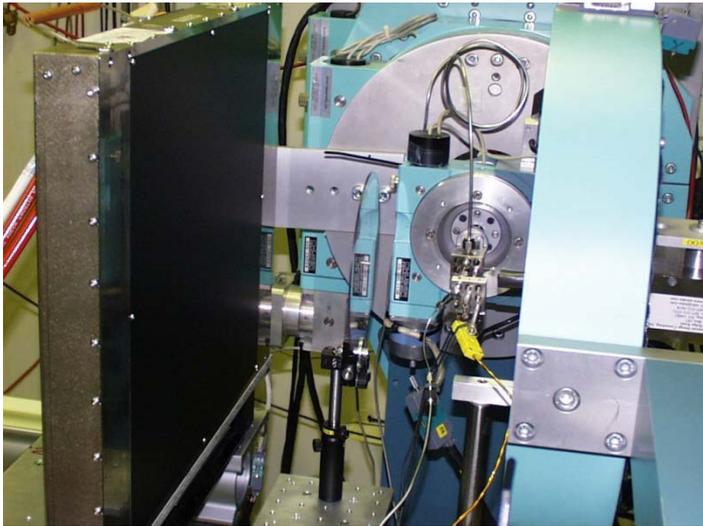
## Generally speaking:

Whereas the dimensions and symmetry of the unit cell determine peak positions, peak intensities are governed by how the atoms are arranged within the unit cell.

$$I_{hkl} \propto |F_{hkl}|^2$$
$$F_{hkl} = \sum_{j=1}^m N_j f_j \exp[2\pi i(hx_j + ky_j + lz_j)]$$

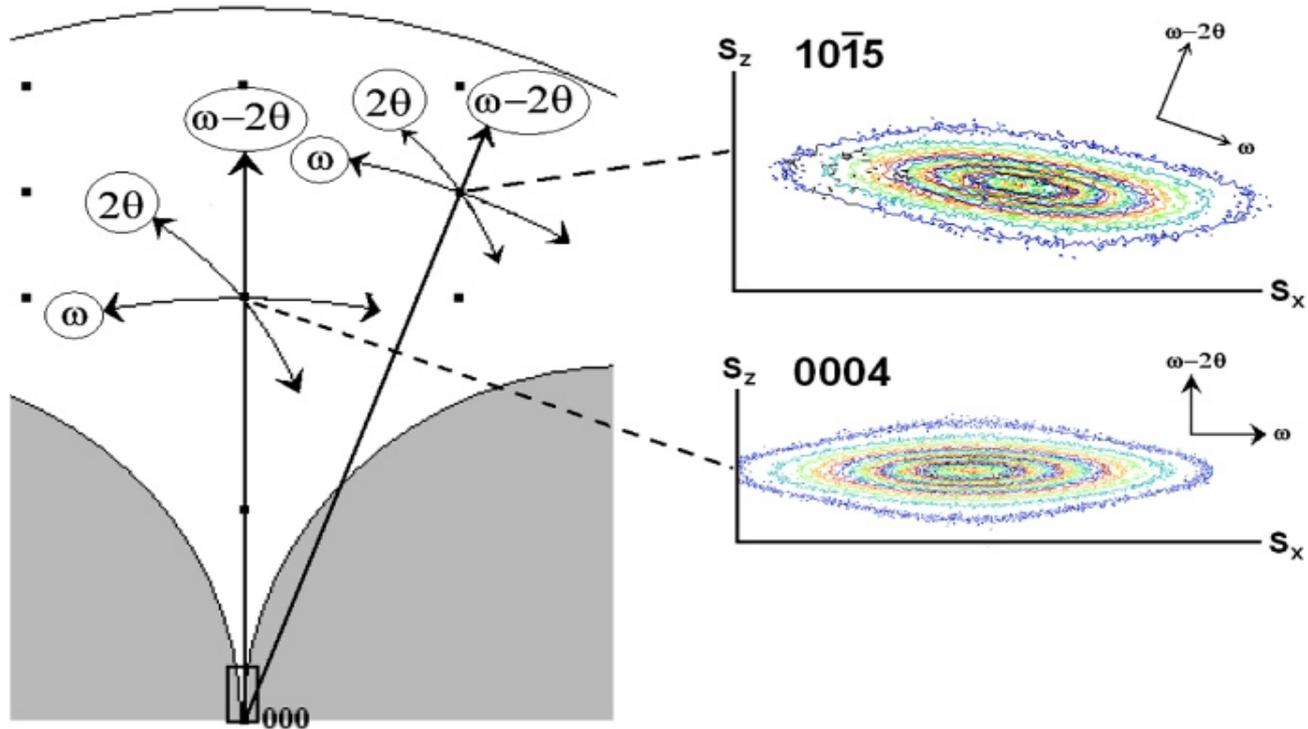
- where the atoms are on the atomic planes
  - this is expressed by the fractional coordinates  $x_j$   $y_j$   $z_j$
- what atoms are on the atomic planes
  - the scattering factor  $f_j$  quantifies the efficiency of X-ray scattering at any angle by the group of electrons in each atom
    - The scattering factor is equal to the number of electrons around the atom at  $0^\circ$   $\theta$ , the drops off as  $\theta$  increases
  - $N_j$  is the fraction of every equivalent position that is occupied by atom  $j$

# Single-crystal diffraction

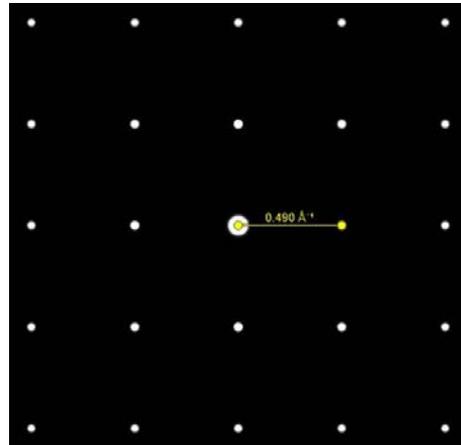
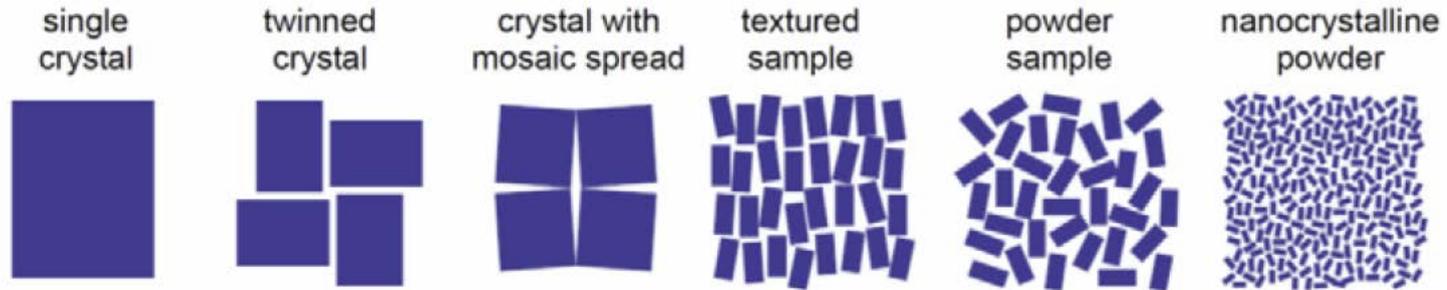


- Need to rotate through several angles
  - It's easy to get “lost in reciprocal space”
  - But you get a lot more detail!

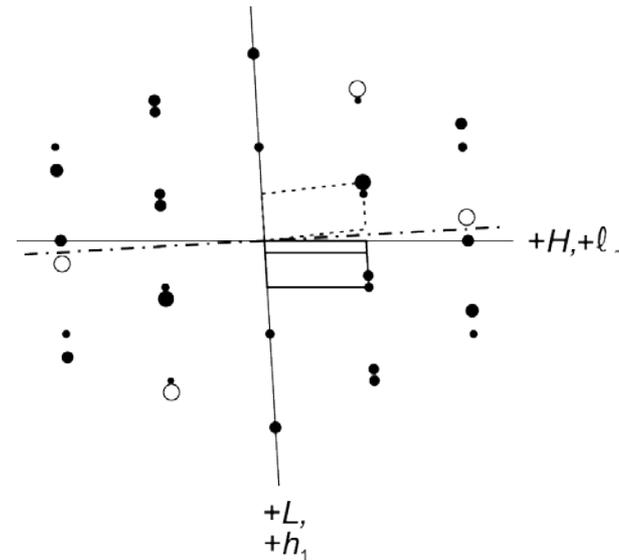
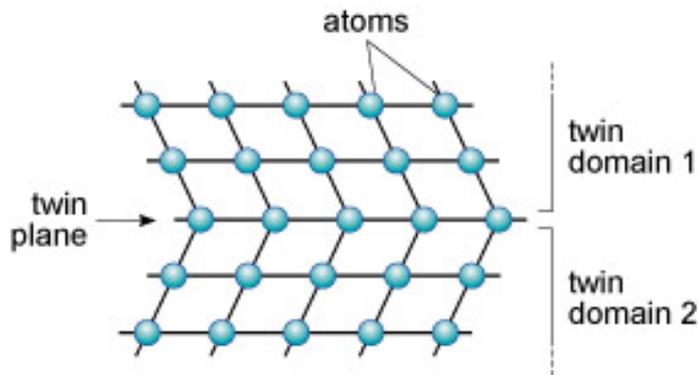
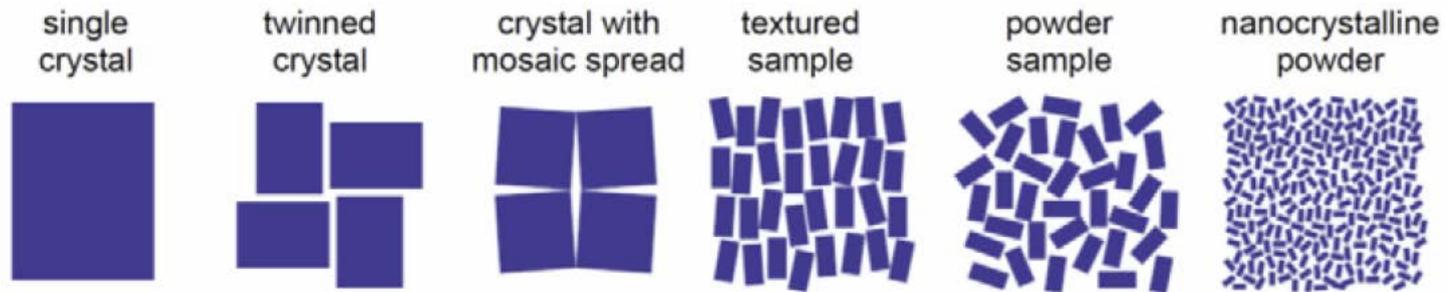
# Scanning reciprocal space



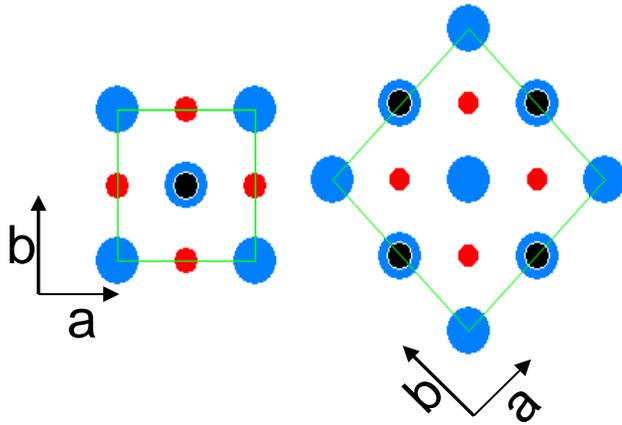
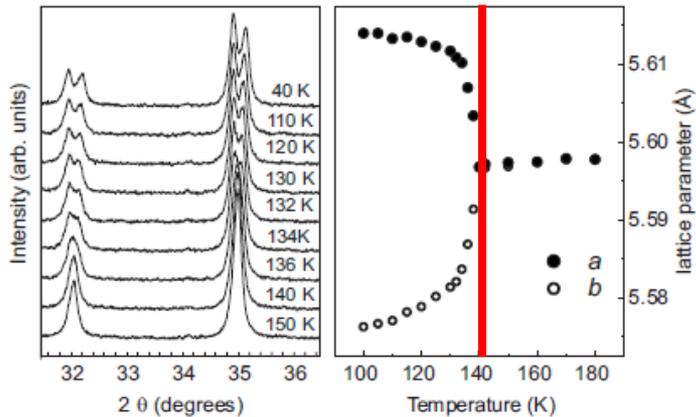
# Flavors of samples



# Twinned crystals

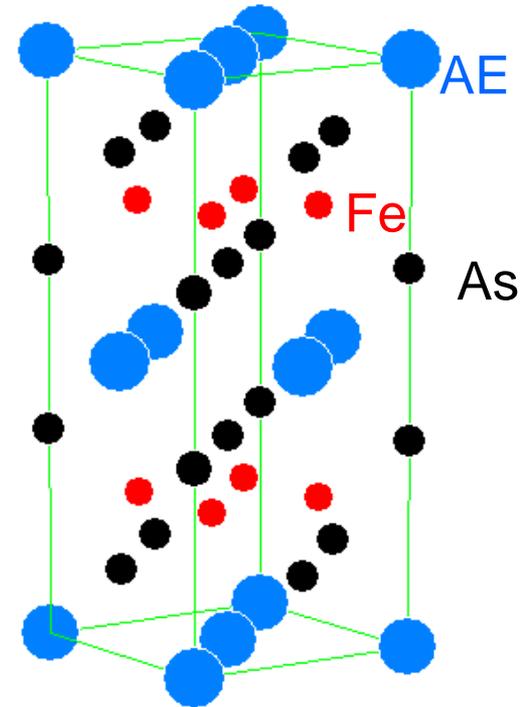
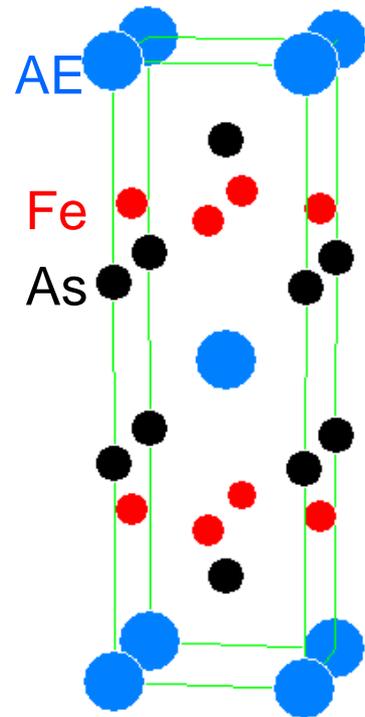


# Is this just a nuisance?



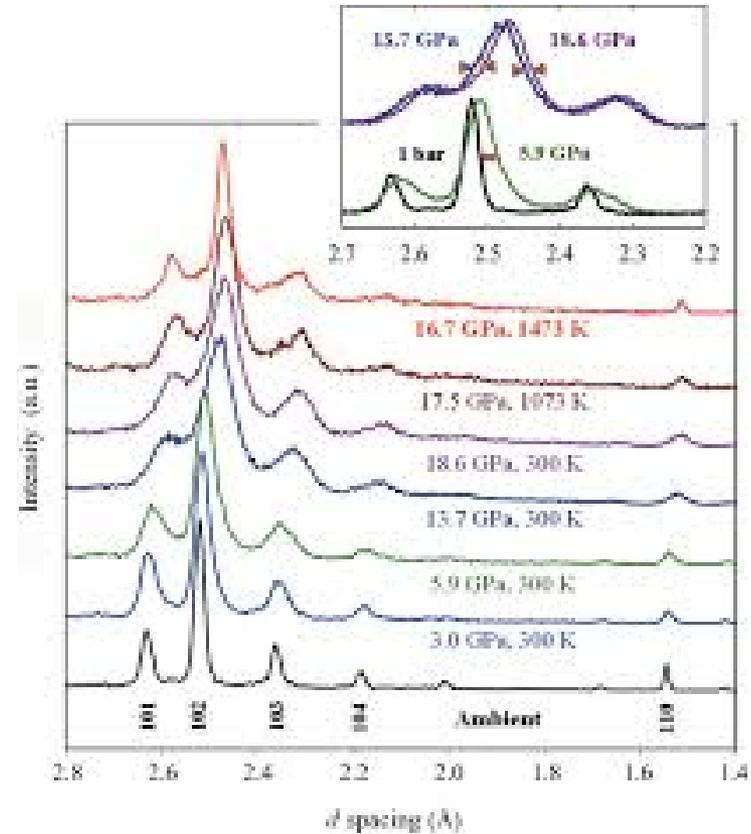
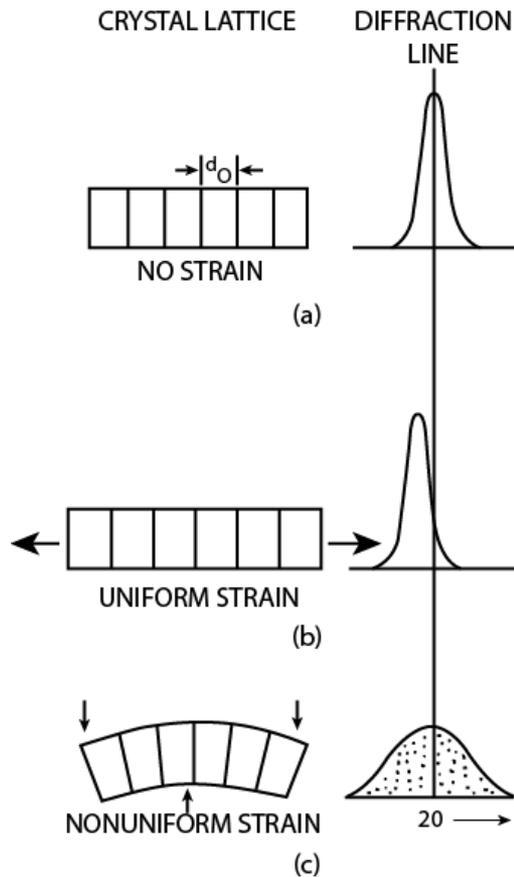
## Structural phase transition

$I4/mmm \rightarrow Fmmm$

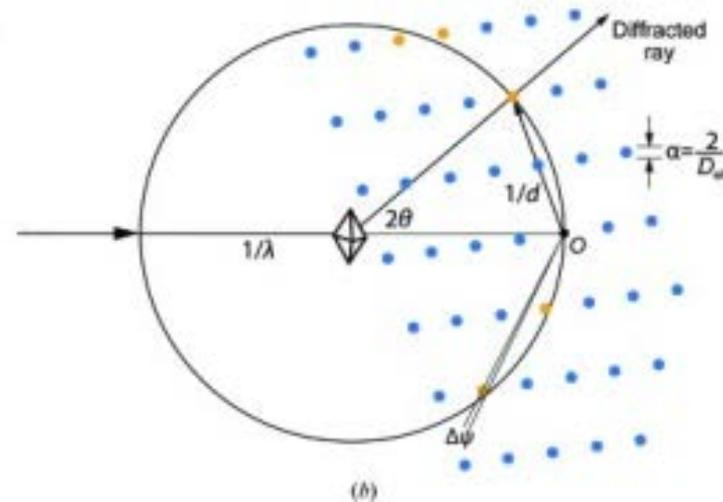
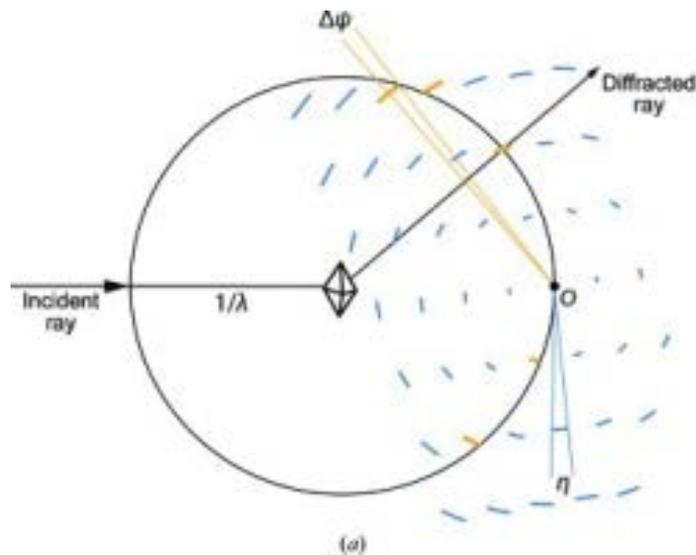
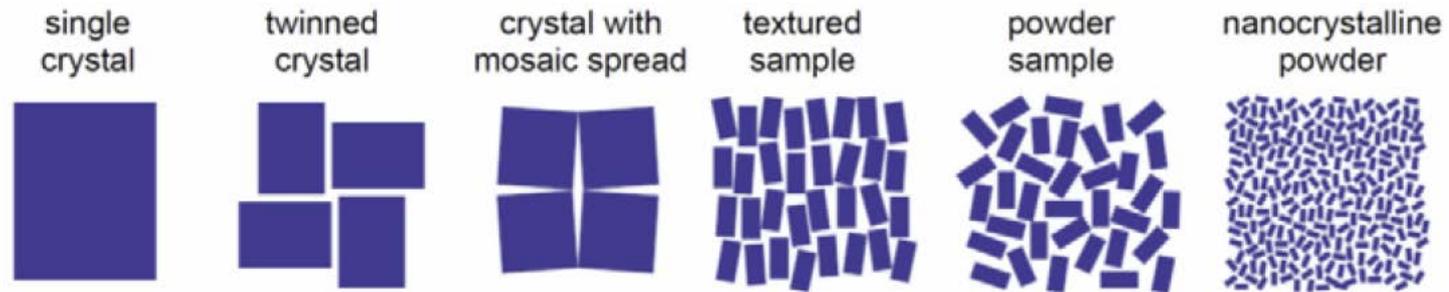


# Strained samples

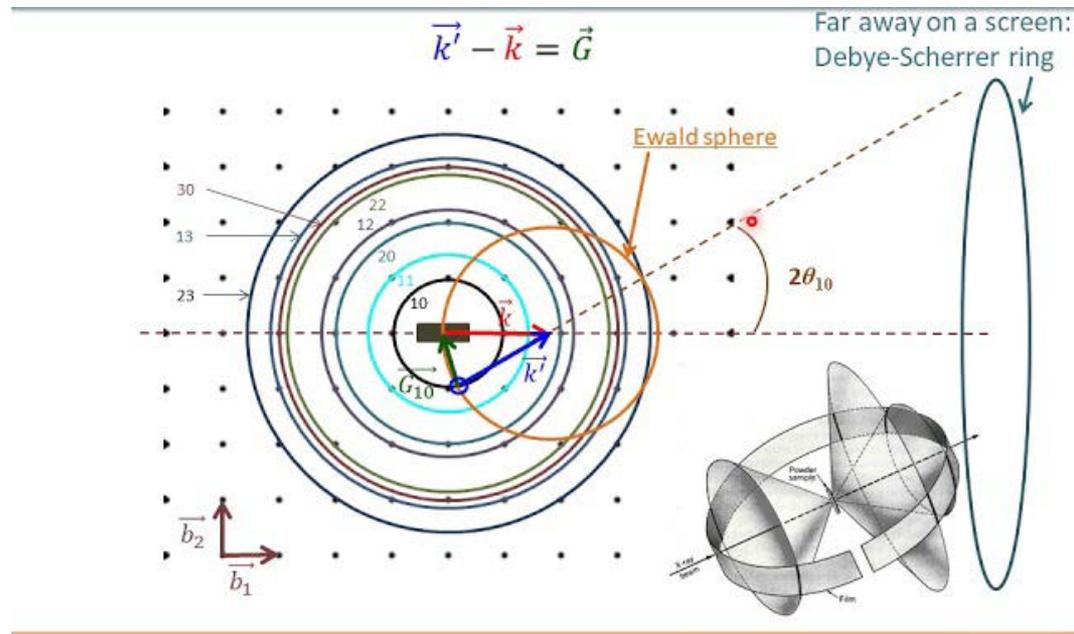
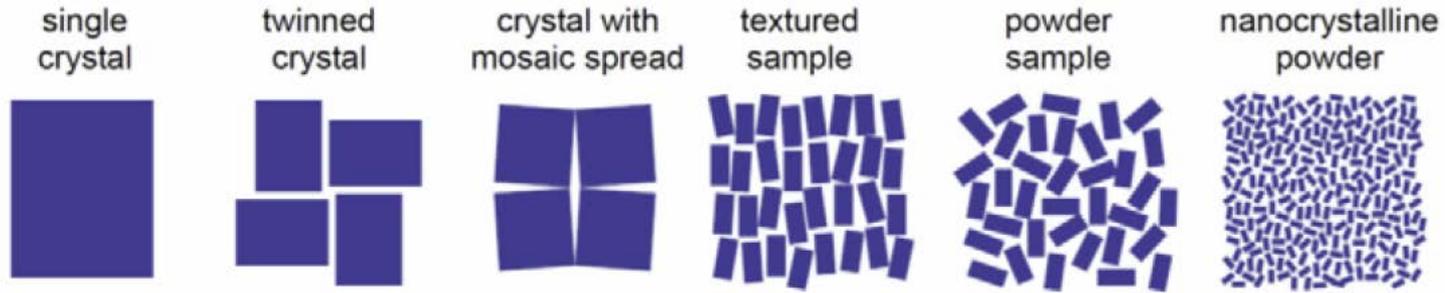
## Samples under pressure



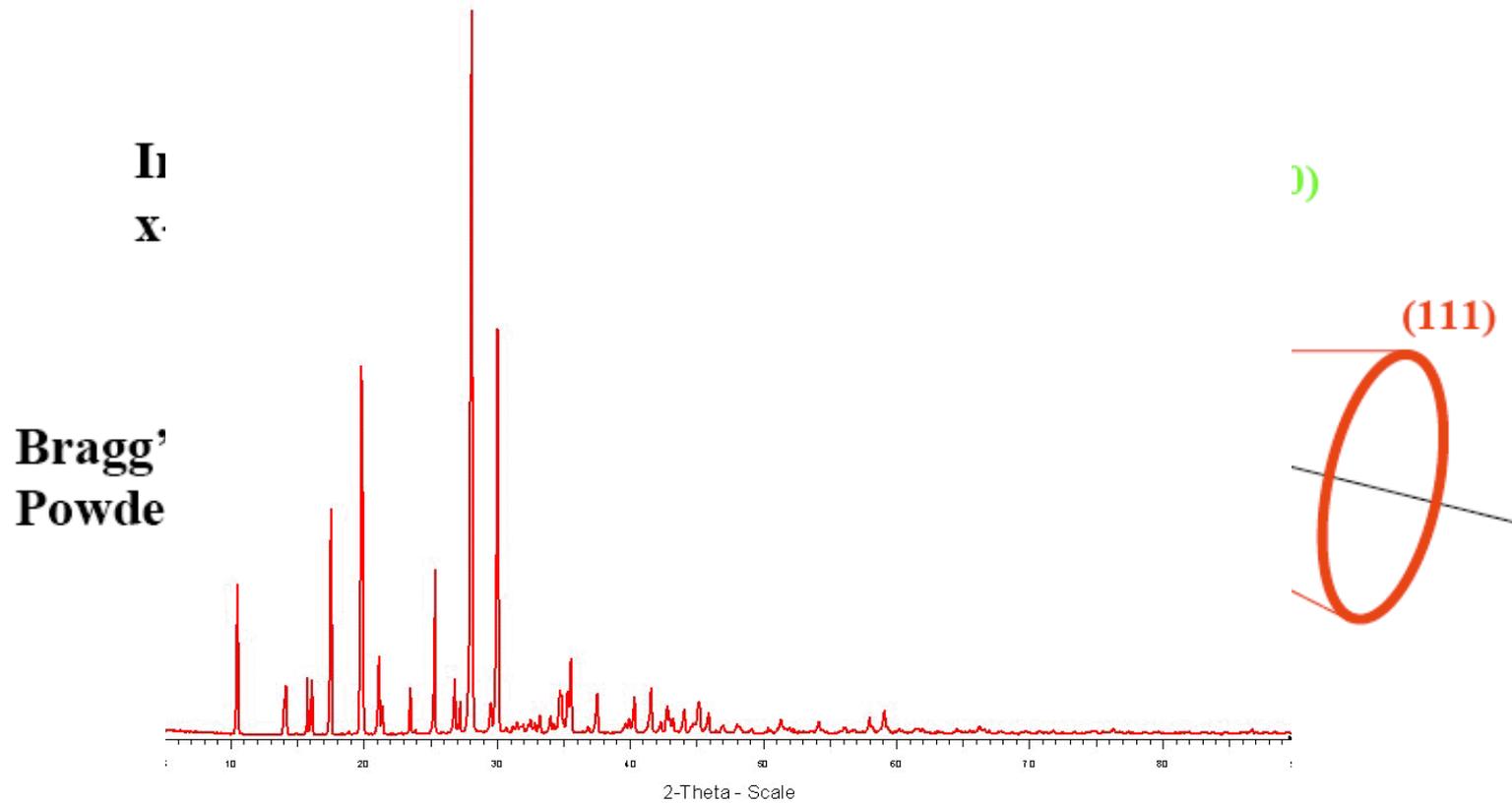
# Mosaic crystals



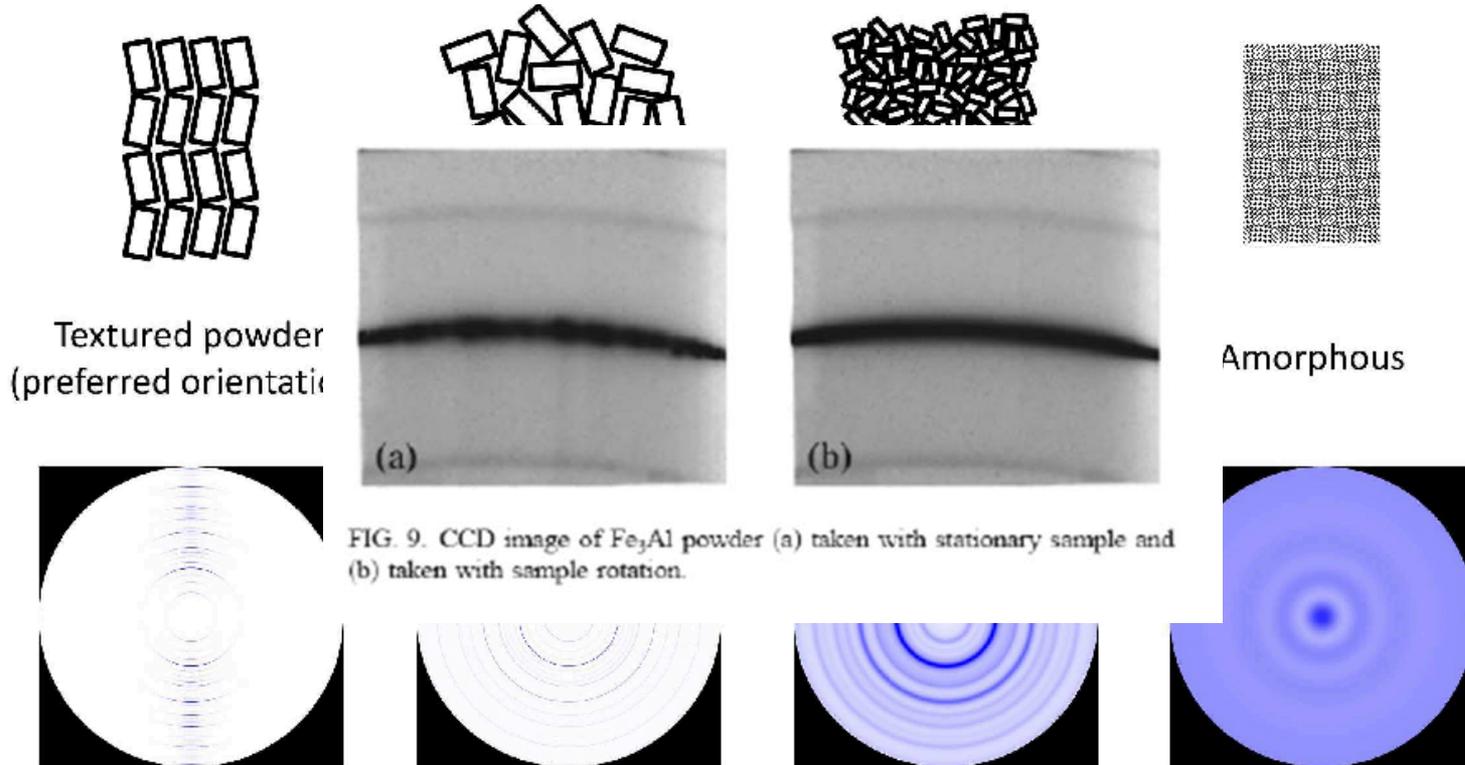
# Ideal powder samples



# Powder diffraction



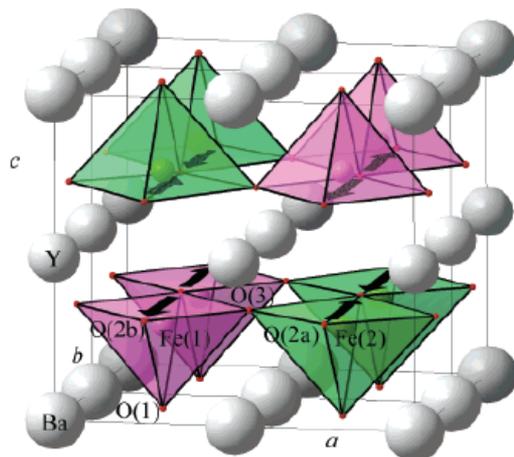
# Types of powder samples



# Fitting diffraction data

## ● Rietveld refinement

- Lattice constants
- space group
- Atom positions
- Site disorder/vacancies
- Thermal vibration amplitude
- Strain broadening
- R-factor



**Figure 3.** Crystal and magnetic structure of the class-I MV (charge-ordered)  $\text{YBaFe}_2\text{O}_5$  at 20 K. Magnetic unit cell ( $a \times 2b \times c$ ) is drawn.

**Table 4.**  $\text{YBaFe}_2\text{O}_5$  Structure Refinement Results from NPD Data

T (K):	20	280	300	320	340
$\lambda$ (Å)	1.5402	2.0783	2.0783	2.0783	2.0783
$R_{\text{wp}}$	0.0671	0.0789	0.0907	0.0783	0.0748
$\chi^2$	4.78	1.38	1.91	1.25	1.17
space group	$Pm\bar{m}a^a$	$Pm\bar{m}a^a$	$Pm\bar{m}a^a$	$Pm\bar{m}n^b$	$Pm\bar{m}n^b$
a (Å)	8.0251(1)	8.0162(2)	8.0141(2)	3.93329(7)	3.93181(7)
b (Å)	3.83834(6)	3.85238(7)	3.85511(9)	3.91342(7)	3.91717(7)
c (Å)	7.5312(1)	7.5541(2)	7.5577(2)	7.5652(1)	7.5683(1)
$a_3 - b_3$ (Å) <sup>c</sup>	0.1742	0.1557	0.1520	0.0199	0.0146
V (Å <sup>3</sup> )	231.983(8)	233.28(1)	233.49(1)	116.447(5)	116.563(5)
Fe(1) z	0.2542(4)	0.2568(9)	0.257(1)	0.2640(2)	0.2641(2)
Fe(2) z	0.2695(4)	0.2662(9)	0.265(1)		
O(1) z	0.003(1)	0.001(3)	0.000(3)	0	0
O(2a) z	0.3213(7)	0.324(2)	0.325(2)	0.3137(5)	0.3140(6)
O(2b) z	0.3132(7)	0.307(2)	0.308(2)		
O(3) x	0.0098(7)	0.012(1)	0.011(2)	0	0
O(3) z	0.3119(3)	0.3130(4)	0.3115(4)	0.3127(5)	0.3125(6)
Y $U_{\text{iso}}$ (Å <sup>2</sup> )	0.0063(4)	0.0123(9)	0.013(1)	0.0152(8)	0.0137(7)
Ba $U_{\text{iso}}$ (Å <sup>2</sup> )	0.0039(6)	0.014(1)	0.017(1)	0.017(1)	0.0144(9)
Fe $U_{\text{iso}}/U_{\text{eqv}}$ (Å <sup>2</sup> ) <sup>d,e</sup>	0.0039(3)	0.0124(6)	0.0139(8)	0.0132	0.0122
O(1) $U_{\text{eqv}}$ (Å <sup>2</sup> ) <sup>d</sup>	0.0080	0.0138	0.0159	0.0181	0.0136
O(2) $U_{\text{eqv}}$ (Å <sup>2</sup> ) <sup>d,f</sup>	0.0065	0.0123	0.0129	0.0186	0.0188
O(3) $U_{\text{eqv}}$ (Å <sup>2</sup> ) <sup>d</sup>	0.0078	0.0128	0.0141	0.0146	0.0138
Fe $M_y$ ( $\mu_B$ )	3.82(2)	3.41(3)	3.26(3)	2.88(2)	2.76(2)
Fe $M_z$ ( $\mu_B$ )	0	0	0	-0.17(8)	-0.20(8)
Fe $M_{\text{Total}}$ ( $\mu_B$ )	3.82(2)	3.41(3)	3.26(3)	2.89(2)	2.77(2)

<sup>a</sup> Wyckoff positions for space group  $Pm\bar{m}a$  (nuclear cell) are: Ba at 2a (0,0,0); Y at 2c (0,0,1/2); Fe(1) and O(1) at 2f (1/4,1/2,z); Fe(2) at 2f (3/4,1/2,z); O(2a) at 2e (3/4,0,z); O(2b) at 2e (1/4,0,z); O(3) at 4j (x,1/2,z).

<sup>b</sup> Wyckoff positions for space group  $Pm\bar{m}n$  (nuclear cell) are: Ba at 1a (0,0,0); Y at 1c (0,0,1/2); Fe at 2t (1/2,1/2,z); O(1) at 1f (1/2,1/2,0); O(2) at 2s (1/2,0,z); O(3) at 2r (0,1/2,z).

<sup>c</sup> Orthorhombic distortion; refers to the single-perovskite-type subcell. <sup>d</sup>  $U_{\text{eqv}}$  values are given for those atoms where anisotropic displacement parameters were used in the refinement. The  $U_{\text{eqv}}$  values are defined as one-third the trace of the diagonal matrix describing the shape of the thermal ellipsoid. A complete list of the anisotropic displacement parameters is given in the Supporting Information. <sup>e</sup> The displacement parameters for Fe(1) and Fe(2) were constrained to be equal. An isotropic displacement parameter was used in the charge-ordered state; anisotropic displacement parameters were used for the MV state. <sup>f</sup> The displacement parameters for O(2a) and O(2b) were constrained to be equal.

# More than “just diffraction”

- **X-ray resonant magnetic scattering<sup>\*\*\*</sup>**
  - Magnetic structures of neutron absorbing materials with very high resolution
- **Pump-probe experiments**
  - Dynamics on picosecond to millisecond timescales
- **Inelastic x-ray scattering**
  - Approaching neutron energy resolution
- **X-ray absorption Spectroscopy<sup>\*\*\*</sup>**
  - Studies of local atomic environments and chemistry
- **Circular magnetic x-ray dichroism<sup>\*\*\*</sup>**
  - Element specific magnetization measurements

# More than “just diffraction”

- **Small-angle scattering**
  - Studying mesoscopic-scale structures
- **X-ray radiography and tomography**
  - 3D views of structural features with submicron resolution
- **Surface scattering**
  - Studies of both solid and liquid surfaces
- **Coherent x-ray diffraction**
  - Probing coherence effects on the atomic scale