Resistivity measurements

590B

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SI units/History

Resistivity measurements
Alessandro Giuseppe Antonio Anastasio Volta
1745-1827
Count (made by Napoleon 1810)
1881- Volt unit adopted internationally

First reliable source of electricity
Alternating plates of Zn and Cu separated by cardboard soaked in saltwater

Electrical action is proportional to the number of plates
Months after 1819 Hans Christian Ørsted's discovery of magnetic action of electrical current

1820 Law of electromagnetism (Ampère's law) magnetic force between two electric currents.

First measurement technique for electricity
Needle galvanometer
Georg Simon Ohm
1789 - 1854

1826 Ohm’s apparatus

Current measurement:
magnetic needle

Voltage source:
thermocouple (Seebeck 1821)
Steam heater
Ice cooler

\[ I = \frac{V}{R} \]

Resistivity
\[ \rho = R \frac{A}{\ell} \]
Can be measured via
Magnetic action of electric current
Heat
Mass flow (electrolysis)
Light generation
Physiological action (Galvani,
You can do anything with cats!)

**D'Arsonval galvanometer**

Ampere main SI unit:

Definition based on force of interaction between parallel current

Replaced recently
Amount of deposited mass per unit time in electrolysis process

**Thompson (Kelvin) mirror galvanometer**
Our common experience: resistance is the simplest quantity to measure
True, but only inside “comfort zone”

Digital Multi Meters - DMM

Use Ohm’s law

Apply known I (V) Measure V (I)
Calculate resistance

Implicit: Ohm’s law is valid for our measurement object, I-V curve is linear
May be far from true!
Implicit: our whole circuit is linear and no offsets!
Assumption: wire resistance is negligible

Typical characteristics
1 mV per last digit
1 μA per last digit
High input offset current
Low input impedance
Resistance 4-probe measurements (Kelvin probe measurement)

To minimize wire resistance effect for remote objects
To minimize the effect of contacts for resistivity measurements
Even allows slightly rectifying contacts and “high” resistance contacts

**Warning!**
Current source in one circuit
Potential voltage measurement in ANOTHER circuit

**FIGURE 3-15: Four-Wire Resistance Measurement**

**FIGURE 2-31: Constant-Current Method Using a Separate Current Source and Voltmeter**
Resistance: bridge measurement

At balance $I = 0$

$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$

Replaces $I$ and $V$ measurement by resistance compensation to obtain zero reading.

No effect of circuit non-linearity,

In old days PRECISE DIGITAL measurement

Does not go well with modern electronics

But the name and reputation remained!
Four probe measurement in bridge configuration

Four-Terminal Resistor

Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge

- $r_1$ causes no effect on the balance condition.
- The effects of $r_2$ and $r_3$ could be minimized, if $R_1 \gg r_2$ and $R_a \gg r_3$.
- The main error comes from $r_4$, even though this value is very small.
Resistance 4-probe measurements
Consideration of noise sources

Wires generate spurious DC Voltages
- Thermoelectric (thermal gradients) 1/f noise
- Galvanic (oxidation) 1/f noise
- RF interference and rectification in contacts

\[ V_{\text{M}} = IR + V_{\text{offset}} \]

DC Delta method
Measure at I+
Measure at I-
average
- Thermal EMF errors
  Most common source in low level voltage measurements

  Each wire junction forms a thermocouple

- Galvanic potentials in contacts
  Gold plating to avoid oxidation

- Noise caused by magnetic fields

Extremely important for AC measurements
Wires vibrate and AC voltage generated

Twisted pairs
Fixed wires

### Copper-to-

<table>
<thead>
<tr>
<th></th>
<th>Approx. μV / °C</th>
</tr>
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<tbody>
<tr>
<td>Copper</td>
<td>&lt; 0.3</td>
</tr>
<tr>
<td>Gold</td>
<td>0.5</td>
</tr>
<tr>
<td>Silver</td>
<td>0.5</td>
</tr>
<tr>
<td>Brass</td>
<td>3</td>
</tr>
<tr>
<td>Beryllium Copper</td>
<td>5</td>
</tr>
<tr>
<td>Aluminum</td>
<td>5</td>
</tr>
<tr>
<td>Kovar or Alloy 42</td>
<td>40</td>
</tr>
<tr>
<td>Silicon</td>
<td>500</td>
</tr>
<tr>
<td>Copper-Oxide</td>
<td>1000</td>
</tr>
<tr>
<td>Cadmium-Tin Solder</td>
<td>0.2</td>
</tr>
<tr>
<td>Tin-Lead Solder</td>
<td>5</td>
</tr>
</tbody>
</table>

**FIGURE 2-10: Minimizing Interference from Magnetic Fields**

- Diagram a: Source connected to voltmeter
- Diagram b: Source connected to voltmeter through twisted pair of wires
Capacitive coupling

\[ i = C_{\text{stray}} \frac{dV}{dt} = \omega C_{\text{stray}} V_{\text{noise}} \]

Cures for capacitive noise coupling include:

1) Removing or turning off the noise source.

2) Keeping the noise source far from the experiment (reducing \( C_{\text{stray}} \)). Do not bring the signal cables close to the noise source.

3) Designing the experiment to measure voltages with low impedance (noise current generates very little voltage).

4) Installing capacitive shielding by placing both the experiment and detector in a metal box.

\[ f=60 \text{ Hz}, \ V=110 \text{ V} \ S=1 \text{ cm}^2 \ d=10 \text{ cm} \ I=400 \text{ pA}=0.4 \text{ nA} \]
Cures for inductively coupled noise include:

1) Removing or turning off the interfering noise source.

2) Reduce the area of the pick-up loop by using twisted pairs or coaxial cables, or even twisting the 2 coaxial cables used in differential connections.

3) Using magnetic shielding to prevent the magnetic field from crossing the area of the experiment.

4) Measuring currents, not voltages, from high impedance detectors.
Ground loops

Current source and voltmeter have grounded terminals

Grounds are not ideal and are under some potential

Devices grounded in different points acquire potential difference which contributes to the measured signal

Cures for ground loop problems include:

1) Grounding everything to the same physical point.

2) Using a heavy ground bus to reduce the resistance of ground connections.

3) Removing sources of large ground currents from the ground bus used for small signals.
Common mode rejection

Finite resistance between LO terminal and ground

If I am measuring voltage difference $V_H - V_L$, it will depend on $V_L$!

Noise caused by injected current

Same for capacitance charging
Mainly 60 Hz power line noise

N.B. The most important sources of error in 4-probe resistance measurements
Contact voltage drop is equivalent to common mode voltage
Typically Common Mode Rejection of DMM is $10^5$-$10^6$
Good

Injected current bypasses the measured resistance

Bad

Injected current flows through the Measured resistance

Note: The measurement noise caused by injected current can be significantly reduced by setting the integration time of the DMM to 1 power line cycle (PLC) or greater.
Loading errors due to input resistance

Important for high resistance measurements
Typical DC 10 ГОм

Loading errors due to input bias current
AC 1 МОм
100 пФ
Shielding

Electrostatic Faraday cage

Magnetic shielding

Unfortunately not very useful
In the lab, can not put electronics at low T

To prevent EMI
General rule
Avoid grounds in measurement circuits
Ground shields

**FIGURE 3-6: Shielding to Attenuate RFI/EMI Interference**

*WARNING*
Safety shield is required when the noise shield is more than 30V DC or rms off earth ground.
Johnson noise

voltage

\[ E = \sqrt{4kTRB} \text{ volts, rms} \]

current

\[ I = \frac{\sqrt{4kTRB}}{R} \text{ amperes, rms} \]

B bandwidth

FIGURE 1-2: Theoretical Limits of Voltage Measurements

Within theoretical limits

Near theoretical limits

Prohibited by noise

0.1sec
10sec

300K
Going outside comfort zone: DC measurements

Special designs

High impedance source Electrometer
Used for I<10 nA, G>1 GΩ
Input impedance ~100 TΩ
Input offset current <3fA
Capable of R measurement up to 300 GΩ

Low impedance source- Nanovoltmeter
<1 nV

Source-measure units for resistance measurements

Here DMM is from Keithley, not from Fluke!
AC in differential mode avoids offsets
Low frequency (below power grid, typically 10-20 Hz)

Problem: low input impedance,
Not good for high resistance sources

Problem: current source is not precise

Very popular simple and reasonably precise resistance measurement

“Ohmic contacts” required for AC measurements

Lock-in resistance measurements
SR830
Built-in AC voltage generator

Low resistances: AC may be a better choice
Low resistances: AC may be a better choice

Resistance bridges
LR700, AVS47, SIM927 and LS370

Actually these are not bridges!
Do not use compensation
Ratiometric resistance measurement

- Low noise
- Low excitation power
- AC
Resistance measurements

AC to avoid offsets
Low frequency (below power grid, typically 10-20 Hz)

SIM927 Comparators measuring reference resistance voltage and in-phase component of sample resistance voltage

Specialized for low-temperature precision resistance measurements
Ways to reduce noise:

**Reduce bandwidth**
- averaging (digital or analog)
- filtering

Very long term measurements are susceptible to other errors, Temperature drift

**Cool down the source**
300K to 3K
10 times noise decrease

Low temperature transformers and Preamplifiers in DR

**Source resistance**
Low resistance contacts
The Delta Method of Measuring Resistance

\[ V_1 = 2.5 \mu V \; ; \; V_2 = -2.5 \mu V \; ; \; V_3 = 2.5 \mu V \]

3 point Delta Method

\[ V_a = \text{negative-going step} = (V_1 - V_2)/2 \]
\[ = 2.45 \mu V \]

\[ V_b = \text{positive-going step} = (V_3 - V_2)/2 \]
\[ = 2.55 \mu V \]

\[ V_f = \text{final voltage reading} = (V_a + V_b)/2 \]
\[ = \frac{1}{2}[(V_1 - V_2)/2 + (V_3 - V_2)/2] \]
\[ = 2.5 \mu V \]
Figure 3: A graph comparing the results of applying a two- and three-point delta method shows significant noise reduction using the three-point method.
Figure 6: Making differential conductance measurements using just two instruments that incorporate all of the instruments used in the AC technique.

Figure 7: The waveform used in the new technique is a linear staircase function that combines an alternating current with a staircase current.
High resistance measurements

Special features: Guarded cables
Triaxial connectors
High resistance measurements

Because of parasitic capacitance and high impedance, only DC measurements are appropriate. It is important to make correct electrometer connections.

**FIGURE 2-35:** Settling Time is the Result of $R_S C_{SHUNT}$ Time Constant

**FIGURE 2-36:** Exponential Settling Time Caused by Time Constant of Shunt Capacitance and Source Resistance
High resistance measurements

Guard ring technique:

- Volume resistance, $R_v$
- Surface leakage resistance, $R_s$

(a) Circuit that measures insulation volume resistance in parallel with surface leakage resistance

$$R_{\text{meas}} = R_s \parallel R_v = \frac{V}{I_s + I_v}$$

(b) Use of guard ring to measure only volume resistance

$$R_{\text{meas}} = R_v = \frac{V}{I_v}$$
Resistivity measurements: 4-probe

“Ohmic contacts” required for AC measurements

Strict requirements on sample shape

Figure 1. Schematic of the co-linear 4-probe configuration with electrodes that span the width of the specimen.

Concerns
Different I and V circuits
May be disconnected!

Potential contacts should be connected well to current path

Important: contact is not necessary
homogeneous funny pattern of currents
Resistivity 4-probe measurements: sample geometry

Problem with anisotropic samples
Actual dimensions $l_1, l_2$

Effective sample dimensions
$\frac{l*_{1}}{l*_{2}}=\sqrt{\frac{\rho_2}{\rho_1}} \frac{l_1}{l_2}$

For example $\rho_d/\rho_a$ cuprates organics $10^3$-$10^4$
typical sample 1mm size 0.1 mm thick
Effectively $\frac{l*_{a}}{l*_{c}} \sim 0.1$
Very difficult correct in-plane resistivity measurements
Superconducting fluctuations and the Peierls instability in an organic solid

SSC 12, 1125 (1973)
A METHOD OF MEASURING SPECIFIC RESISTIVITY AND HALL EFFECT OF DISCS OF ARBITRARY SHAPE

by L. J. van der PAUW

Summary
A method of measuring specific resistivity and Hall effect of flat samples of arbitrary shape is presented. The method is based upon a theorem which holds for a flat sample of arbitrary shape if the contacts are sufficiently small and located at the circumference of the sample. Furthermore, the sample must be singly connected, i.e., it should not have isolated holes.
Resistivity measurements: van der Pauw method

Very popular in semiconductor industry
Does not require sample of regular shape

Assumptions
1. Homogeneous sample
2. Isotropic sample
3. Two-dimensional, thickness is unimportant
4. Sample boundary sharply defined

Surface resistance

$$\rho = \frac{\pi}{ln2} \frac{R_{ab-cd} + R_{bc-da}}{2} f$$

Resistivity

$$\rho = \frac{\pi}{ln2} W \frac{R_{ab-cd} + R_{bc-da}}{2} f$$

Correction factor based on the ratio of the two resistance measurements, $R_{ab-cd}$ and $R_{bc-da}$ used in the Van der Pauw resistivity measurement.
Method for Measuring Electrical Resistivity of Anisotropic Materials

H. C. Montgomery
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07974
(Received 30 November 1970)

A rectangular prism with edges in principal crystal directions is prepared with electrodes on the corners of one face. Voltage–current ratios for opposite pairs of electrodes permit calculation of components of the resistivity tensor. The method can use small samples, and is best suited to materials describable by two or three tensor components. Examples are given of measurements of $\text{V}_2\text{O}_3$–Cr and oriented amorphous graphite.
Montgomery technique

1. Van der Pauw resistivity measurements on samples of rectangular

\[
R_1 = \frac{V_1}{I_1} \quad R_2 = \frac{V_2}{I_2}
\]

2. Calculation of the anisotropy ratio for isotropic samples

\[
\left(\frac{\rho_2}{\rho_1}\right)^{1/2} = \left(\frac{l_2}{l_1}\right) \times \left(\frac{l_1'}{l_2'}\right)
\]

3. Scaling anisotropic samples on isotropic by van der Pauw scaling transformation

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**Fig. 3.** Resistance ratio versus sample dimension ratio. Solid line is for a thin sample; dashed line for a thick sample. Details of thickness dependence given in Table II.
Interference Factor

- Ohmic contact quality and size
- Sample uniformity and accurate thickness determination
- Photoconductive and photovoltaic effects
Reading

2. Lake Shore manual for LS370
3. SR-830 manual [www.thinksrs.com/.../PDFs/Manuals/SR830m](http://www.thinksrs.com/.../PDFs/Manuals/SR830m)