Propagating Uncertainty

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We have learned how to find the mean value of a set of data, and its uncertainty. Now what if we use that mean value (and its uncertainty) in a calculation? How do we translate the uncertainty in the mean value to the uncertainty of the answer of a calculation that uses that mean value? For example, if I know average yield over time, and its associated uncertainty, how do I find the average dollar value of the crop and its associated uncertainty?

Suggested Procedure for Propagating Uncertainty

Follow these directions to propagate, or carry through, the uncertainty associated with a value used in a calculation in order to find the uncertainty in the answer to that calculation.

1. Determine the answer of the calculation using the mean value. Use the rules of significant figures in the calculation.

2. Determine the largest possible answer by adding or subtracting the uncertainty to the mean value. If the mean value is in a numerator, then adding the uncertainty will give the largest answer. If the mean value is in a denominator, then subtracting the uncertainty will give the largest answer. If other types of mathematical operations are required in the calculation, add or subtract the uncertainty to get the largest possible answer. Keep more decimal places than you will need in the final answer.

3. Determine the smallest possible answer by adding or subtracting the uncertainty to the mean value. Keep more decimal places than you will need in the final answer.

4. Subtract the smallest possible answer from the largest possible answer. Divide the result in half and round up the result so that the place of its least significant digit matches the place of the least significant digit of the answer of the calculation found using the mean value. This is the uncertainty in the answer.

5. Report the answer to the calculation as the answer plus or minus the uncertainty.

6. If there is more than one value in the expression for which you have an uncertainty, then use the same procedure described above to find the smallest answer to the expression and the largest answer to the expression.

This procedure is based upon *Gordon et al.* [1984] and *Schwartz* [1985].

Example

To convert corn yield, Y, from Mg ha^{-1} (megagrams per hectare), the units used in most parts of the world outside of the United States, to bu ac^{-1} (bushels per acre), the following expression is needed.

$$Y \text{ Mg ha}^{-1} \times \frac{10^3 \text{ kg}}{1 \text{ Mg}} \times \frac{1 \text{ kg wet}}{d \text{ kg}} \times \frac{1 \text{ bu}}{25.4 \text{ kg wet}} \times \frac{1 \text{ ha}}{2.471 \text{ ac}} \tag{1}$$

In this expression, d is the mass of 1 kg of corn grain after drying.

If $Y = 10.4 \pm 0.3$ Mg ha^{-1} and $d = 0.85 \pm 0.05$ kg, then using the mean values of both Y and d (and the rules of multiplication and division with significant figures) gives a result of 190 bu ac^{-1} since d has only two significant figures. To obtain the largest possible answer, we must use $Y = 10.4 + 0.3 = 10.7$ Mg ha^{-1} and $d = 0.85 - 0.05 = 0.80$ kg, and the result is 213.10 bu ac^{-1}. The smallest possible answer is when $Y = 10.4 - 0.3 = 10.1$ Mg ha^{-1} and $d = 0.85 + 0.05 = 0.90$ kg, which gives 178.80 bu ac^{-1}. $(213.10 - 178.80)/2 = 17.15$, which is an uncertainty of 20 bu ac^{-1} after rounding to the place of the least significant figure in the answer found using the mean values. The final answer with uncertainty is 190 ± 20 bu ac^{-1}.

References

Gordon, R., M. Pickering, and D. Bisson, Uncertainty analysis by the worst case method, *Journal of Chemical Education*, *61*(9), 780–781, doi:10.1021/ed061p780, 1984.

Schwartz, L. M., Propagation of significant figures, *Journal of Chemical Education*, *62*(8), 693–697, doi:10.1021/ed062p693, 1985.