Short Notes

What We Can and Cannot Learn about Earthquake Sources from the Spectra of Seismic Waves

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Abstract Earthquake sources are commonly viewed as shear dislocations. This imposes distinct limitations on what source parameters can be realistically determined from radiated shear-wave spectra. First, the slip velocity on the fault is the real parameter that controls the strength of the high-frequency radiation; it can be directly determined from acceleration spectra by fitting their high-frequency level. Second, the relationship between corner frequency of the spectrum and the radius of the source is fundamentally unclear. As a result, the source dimensions cannot be accurately determined from the spectra; such an estimate would be as accurate as any other informed guess. Third, the stress drop only serves as a proxy for the source radius in the relationship between the radius and the corner frequency; it thus cannot be reliably determined from the spectra. The quantity usually obtained from the spectra and referred to as the stress drop is a poorly defined parameter that may bear little relevance to the actual stresses acting on faults. This parameter has little meaning unless converted to the maximum slip velocity, which is the only quantity that can be accurately determined from the spectra. The typical value of stress drop of 100 bars, established from the spectra of California events, may imply that the typical slip velocities have been on the order of 0.5 m/sec, although it is more accurate to determine slip velocities directly from the spectra.

Introduction

The classic theory of seismic radiation from an earthquake source is based on the model of shear dislocation. This model is commonly used to determine important source parameters, such as the source radius or the tectonic shear stress that caused the rupture. Clearly, the nature of the model dictates some limitations on what source characteristics can be realistically determined from the observed spectra. To distinctly outline these limitations, I revisit the issue by providing a summary of the relationships commonly used (e.g., Brune, 1970, 1971; Savage, 1972; Tumarkin and Archuleta, 1994) that highlights the key points.

Seismic Spectra and Slip Velocity

The displacement on the dislocation that leads to a ω^2 spectrum of shear-wave radiation is

$$u(t) = u(\infty)\left[1 - \left(1 + \frac{t}{\tau}\right)e^{-t/\tau}\right], \qquad (1)$$

where τ is the parameter governing the speed of the rise in dislocation displacement to its final value of $u(\infty)$ (e.g., Beresnev and Atkinson, 1997). The acceleration spectrum in the far field is then given by the ω^2 function

$$a(\omega) \sim \frac{M_0\omega^2}{1 + \left(\dfrac{\omega}{\omega_c}\right)^2},$$

where $\sim$ indicates proportionality, M_0 is the seismic moment, ω is the angular frequency, and $\omega_c \equiv 1/\tau$ is the corner frequency of the spectrum. The quantity ω_c is thus also related to the speed of dislocation rise.

At high frequencies ($\omega > \omega_c$), the acceleration spectrum is constant:

$$a_{hf}(\omega) \sim M_0\omega_c^2. \qquad (2)$$

Thus a_{hf} is also controlled by the speed of the dislocation rise.

By taking the time derivative of (1), we can determine the slip velocity (v), which has its maximum at $t = \tau$:

$$v_{\max} = \frac{u(\infty)}{e\tau},$$

whence, using the earlier definition of corner frequency,

$$\omega_c = e\,\frac{v_{\max}}{u(\infty)}. \tag{3}$$

We thus conclude that the corner frequency is controlled by the maximum slip velocity, and that from equations (2) and (3),

$$a_{hf}\,(\omega) \sim M_0 v_{\max}^2. \tag{4}$$

It follows that slip velocity is the source parameter that is directly obtainable from the high-frequency spectrum.

Radius of the Source

Can we obtain any other information on the source based on the seismic spectra? We could try to link the corner frequency to source dimensions. The source rise time (T) can be defined as the time it takes the dislocation to reach a certain fraction of its final displacement. For example, we could take this fraction as a half of total displacement. Then, from (1),

$$\frac{u(T)}{u(\infty)} = \left[1 - \left(1 + \frac{T}{\tau}\right)e^{-T/\tau}\right] = 0.5 \tag{5}$$

If we introduce the dimensionless rise time $T/\tau \equiv z$, equation (5) can be viewed as an algebraic equation for z, from which we have $z \approx 1.68$. Thus, the corner frequency can be approximately linked to the rise time as

$$\omega_c = \frac{1.68}{T}. \tag{6}$$

Equation (6) does not yet provide the relationship we need, since it remains to relate the rise time to the source dimension. We can do it reasonably well by assuming that each dislocation on the fault rises as long as the rupture propagates across the fault. This assumption links the rise time to the fault linear dimension (L):

$$T = \frac{L}{V_R}, \tag{7}$$

where V_R is the rupture-propagation velocity. Substituting (7) into (6) gives

$$\omega_c = \frac{1.68 V_R}{L}, \tag{8}$$

which is the relationship sought. We notice, however, that instead of providing a means of determining fault dimensions from corner frequency, this relationship cautions us that such a solution is, in fact, fundamentally unclear. First, we have the coefficient of 1.68 in relationship (8), which arises from a rather arbitrary definition of the rise time as the time over which the fault reaches its half-displacement (why half?). Second, it involves the rupture velocity, V_R, which is an unknown quantity but can be related (albeit with significant uncertainty) to the shear-wave propagation velocity, for example, $V_R \approx 0.8\ V_S$ (again, why 0.8?). Third, equation (8) is based on the stated hypothesis that, locally, the dislocation grows as long as the rupture propagates along the fault; however, the local rise time (T) could also be hypothesized to be shorter than the total rupture-propagation time (L/V_R) (e.g., Heaton, 1990). If we use all these assumptions, then

$$\omega_c \approx \frac{1.68 \times 0.8 \times V_S}{L} = 0.67\,\frac{V_S}{R}, \tag{9}$$

where R is the radius of the source.

Brune (1970, 1971) proposed the classic formula

$$\omega_c = 2.34\,\frac{V_S}{R}$$

with a different coefficient, which has been used extensively in observational seismology. However, in view of the previous reasoning, any particular value of this coefficient has no rigorous meaning because of the ambiguities involved in deriving the relationship (9). The use of relationships of this kind to determine the radius of the source from the spectra would therefore be as accurate as any other informed guess.

Stress

It is common to express corner frequency in terms of the change in tectonic stress, which provides the basis for the determination of stress drop from the seismic spectra. However, the approach taken is still based on equation (9), in which R is simply substituted by the moment and stress. The resulting relationship is thus subject to the same reservations.

The seismic moment is defined as

$$M_0 = \mu u(\infty) A, \tag{10}$$

where μ is the shear modulus and A is the area of the rupture. By rewriting the area as $A \approx L^2$, we determine L from (10):

$$L = \left[\frac{M_0}{\mu u(\infty)}\right]^{1/2}. \tag{11}$$

The derivation then uses the definition of stress drop ($\Delta\sigma$), which is simply the quantity proportional to the fault displacement normalized by fault dimension:

$$\Delta\sigma \equiv \mu \frac{u(\infty)}{L}. \tag{12}$$

Using (12) in (11) gives

$$L = \left(\frac{M_0}{\Delta\sigma}\right)^{1/3} \text{ or } R = \frac{1}{2}\left(\frac{M_0}{\Delta\sigma}\right)^{1/3}. \tag{13}$$

Finally, substituting (13) into (9) gives

$$\omega_c \approx 0.67 \times 2 \times V_S \left(\frac{\Delta\sigma}{M_0}\right)^{1/3}. \tag{14}$$

Equation (14) gives an impression that the stress drop can be determined from the earthquake spectrum, even though this equation has been simply obtained as a proxy for the (uncertain) relationship (9) using the definition (12). It thus carries no new information about the source and can even be misleading.

Indeed, instead of recognizing slip velocity as the real physical parameter (other than the moment) governing the high-frequency level of the spectrum, the stress drop is introduced as a controlling factor, creating confusion. It is clear even from simple reasoning that stress drop cannot be responsible for the strength of the high-frequency radiation: for example, the same drop in stress can lead to either very strong or no high-frequency radiation, depending on whether it occurred over a 1 second or 1 year interval. Clearly, the rate at which the stress changes is a key factor, which again emphasizes the significance of slip velocity (bringing us back to equation 4).

Do published estimates of the stress drop, obtained from shear-wave spectra, bear any relationship to the real stresses acting on tectonic faults? To answer this question, we should keep in mind that the quantity determined from the spectra through equation (14) (or its analogs) is that originally introduced by equation (12). The question is thus reformulated as that of determining the physical meaning of $\Delta\sigma$ in (12).

It is common to assume that $\Delta\sigma$ in equation (12), which resembles Hooke's law, is closely related to the actual stress change during an earthquake. In looking for the theoretical basis for this assumption, we might go back to the original work of Eshelby (1957) that is often referred to (e.g., Kanamori and Anderson, 1975) as the source for this conclusion. Eshelby (1957) considered the elastic deformation of a thin ellipsoidal cavity ($\mu = 0$) in a matrix subjected to homogeneous simple shear. Eshelby found an expression for the relative displacement of the faces of the cavity (Δu_1) (equation 5.7 in Eshelby's paper). The maximum displacement occurs at the center of the cavity:

$$\Delta u_1 = \frac{2bS}{\mu\eta}, \tag{15}$$

where b is the radius of the cavity and S is the ambient shear stress (I keep the author's original notation and assume spheroidal cavity shape). The coefficient η is given by equation 5.3 of Eshelby (1957):

$$\eta = \frac{\pi(2 - \sigma)}{4(1 - \sigma)}, \tag{16}$$

where σ is Poisson's ratio. Combining (15) and (16), we have

$$\Delta u_1 = \frac{8bS}{\pi\mu}\frac{1 - \sigma}{2 - \sigma}.$$

S then can be recovered from Δu_1 by

$$S = \frac{\pi}{8}\frac{2 - \sigma}{1 - \sigma}\mu\frac{\Delta u_1}{b}.$$

Poisson's ratio is $\sigma = \dfrac{\lambda}{2(\lambda + \mu)}$, where λ is Lamé constant. Then, for $\lambda = \mu$ (a common simplifying assumption), $\sigma = \frac{1}{4}$. We thus have

$$S = \frac{7\pi}{24}\mu\frac{\Delta u_1}{b} = \frac{7\pi}{12}\mu\frac{\Delta u_1}{L} \tag{17}$$

($L = 2b$), which has the form of equation (12). Equation (17), which we derived from the work of Eshelby (1957), is the absolute value of driving shear stress reconstructed from the maximum relative displacement of the faces of the cavity, for $\lambda = \mu$. Note that the deformation of the cavity was assumed totally elastic; that is, the cavity was assumed to return to its original shape once the ambient stress had been removed (no breakage assumed). Clearly, this is an inadequate model for the earthquake source, which involves a break in material continuity and cannot be considered to deform elastically. We conclude that the values of stress determined using equation (17), if the displacement discontinuity across the fault is substituted for Δu_1, may have little relevance to the actual stress that caused the fault to break. The calculated quantity will probably have no other meaning than simply quantifying the fault displacement normalized by its length. Equating it to the stress that acted on the fault plane before breakage would be incorrect.

In introducing $\Delta\sigma$ into a popular source-radiation model, Boore (1983) was in fact cautious about attributing this parameter any direct physical meaning, other than simply that of a parameter controlling the strength of the high-frequency radiation. Some of the subsequent papers have been equally frank in acknowledging the lack of physical meaning behind $\Delta\sigma$ determined from the spectra (Atkinson, 1993; Atkinson and Boore, 1995; Atkinson and Beresnev, 1997). Nevertheless, determining $\Delta\sigma$ from relationships such as (14) and equating it to the real stress in the crust became widespread practice.

In spite of the limited practical value of $\Delta\sigma$, the fact cannot be disregarded that a large number of determinations of this parameter from earthquake spectra have been published. We could try to make use of these data by back-calculating from them the slip velocity, as the source parameter that has a simple and clear meaning. We equate expressions (3) and (8) for the corner frequency:

$$e\,\frac{v_{\max}}{u(\infty)} = \frac{1.68 V_{\mathrm{R}}}{L}. \tag{18}$$

Using the definition of stress drop (12), the relationship $V_{\mathrm{R}} \approx 0.8\ V_{\mathrm{S}}$, and equation $\mu = V_{\mathrm{s}}^{2}\rho$, where ρ is the density, we rewrite (18) as

$$v_{\max} = \frac{1.68 \times 0.8}{\mathrm{e}} \frac{\Delta\sigma}{V_{\mathrm{S}}\rho}. \tag{19}$$

For example, Brune (1970) determined from seismic spectra that the stress drop for California earthquakes was typically on the order of 100 bars. This value has become classic and is quoted in many textbooks. Using this value in (19) and assuming typical crustal density and shear-wave velocity ($\rho = 2700$ kg/m^3 and $V_{\mathrm{S}} = 3600$ m/sec), we obtain $v_{\max} \approx 0.5$ m/sec. This is the probable typical velocity of slip on rupturing faults calculated from the spectra of seismic waves. However, we again note the ambiguous character of equation (19) that contains uncertain coefficients. The best way of obtaining the slip velocity from seismic data would be to directly fit the observed spectra with relationship (4), which would allow us to avoid the use of the ambiguously defined coefficients that enter (19).

Conclusions

I come to the following conclusions:

1. The maximum velocity of slip on rupturing faults ($v_{\max}$) is the real physical parameter controlling the high-frequency level of the radiated shear-wave spectra. It can be determined directly from the observed high-frequency levels of acceleration spectra.
2. There is no exact relationship between the source radius and the corner frequency of the spectrum. The accuracy of the existing approximate relationships is as good as that of any other reasonable guess.
3. In the popular source models, the stress drop serves as a proxy for the slip velocity and can be back-calculated into $v_{\max}$. The stress drop itself is a poorly defined source parameter and, in its classic definition, does not quantify the actual stress change during an earthquake. Its values derived from the seismic spectra have little physical meaning unless converted back into the slip velocity. For example, the typical values of stress drop of around 100 bars derived for California events may imply that the typical slip velocity has been around 0.5 m/sec, although the equation that establishes this link involves significant ambiguity.

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