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Ground-force- or plate-displacement-based vibrator control?

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ABSTRACT

The feedback system of seismic vibrators minimizes the difference between the theoretical sweep and the seismic signal radiated into the earth. This requires the determination of the correct source signature, that is, the characteristic of the baseplate motion that is a true representation of the downgoing wave. The prevalent systems use ground-force-based vibrator control, which assumes that the amplitudes of the seismic waves are proportional to the force applied to the ground. Mathematical solution of the pressure boundary-value problem of linear elasticity is the theoretical underpinning of this method. However, the ground force has a significant disadvantage of not being a directly measurable parameter. It has to be inferred, with significant uncertainty, as a “weighted sum” of the accelerations of the baseplate and the reaction mass based on a simplified linear model of oscillating masses, springs, and dashpots. On the other hand, the baseplate displacement (acceleration) can itself serve as an alternative, model-independent source signature. To provide a theoretical justification for the displacement-based vibrator control, one has to show that the amplitudes of the outgoing waves are proportional to the plate displacement. This has to be done by solving a “mixed” boundary-value problem, in which displacements are specified under the plate and pressures everywhere else on the surface. Such a problem has historically been considered unsolvable, unlike the mathematically easier pressure-source radiation problem. However, the analysis of approximate approaches to the solution of the mixed boundary-value problem, as well as its exact solution for the static-displacement case, reveal equivalence between the source displacement and source force in being the proportionality factors controlling the amplitudes of the downgoing waves. This provides mathematical justification for the displacement being the correct source signature. The displacement-based feedback, or feedback based on a combination of the weighted-sum and displacement, may be a more accurate method.

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1. Introduction

The idea of the vibroseis method is to radiate a theoretically-prescribed frequency-modulated (sweep) signal, which provides the best identification of the returning seismic reflections, into the earth. Although creating a theoretical sweep at the output of an electronic generator is not technically difficult, this signal is then transformed, typically through a complex hydraulic-supply and servo-valve assembly, into the oscillations of the radiating baseplate of a seismic vibrator. The hydraulic assembly has its own complex transfer characteristics; in addition, the baseplate rests on a granular, uneven surface of the ground, underlain by a near-surface layer with complex mechanical properties. These combined two factors

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cause significant, often nonlinear and unpredictable distortions of the original idealized sweep; as a result, the true motion of the ground is never identical to the prescribed theoretical shape. Since the cross-correlation and deconvolution procedures, used in seismic processing to convert the recorded vibrograms into impulse seismograms showing individual reflections, still assume the radiation of a theoretical sweep, the violation of this assumption leads to the distortion of seismic images.

Realization of the fact that the true “source signature” (the signal realistically radiated into the earth) is not the same as the theoretical sweep is not new. Seismic vibrators are designed with sophisticated feedback (vibrator-control) systems that attempt to minimize the difference between the theoretical sweep (the system’s input) and the true motion of the baseplate (the system’s output). The question then arises what characteristic of the baseplate motion should be taken as the representation of the seismic signal entering the earth, to be used in the feedback control. This is the problem of the “source-signature” choice. Although different solutions have been proposed, the now prevalent approach is to use the “ground force” (the force exerted by the vibrator’s plate on the ground) as the correct source signature.

The ground-force feedback was originally proposed by Castanet and Lavergne [1] but was not widely implemented in practice until it was re-introduced by Sallas [2]. Sallas’s motivation was in the theoretical problem solved by Miller and Pursey [3,4], in which seismic radiation was calculated from a circular disk vibrating normally on the surface of an elastic half-space. The disk applied a sinusoidal force with constant amplitude to the ground. Miller and Pursey’s approximate theoretical particle displacement in the downgoing P -wave in the far field (e.g., Ref. [4], Eq. (1)) was directly proportional to the force applied to the ground. Sallas [2] therefore concluded that the force was responsible for the formation of the far-field P -wave radiation and was thus theoretically justified to be used as the source signature in vibrator’s phase control.

Production seismic vibrators are typically equipped with accelerometers installed on the reaction mass and the top cross of the assembly rigidly connected to the baseplate, but the force applied to the ground is not directly measured. To circumvent this difficulty, Castanet and Lavergne [1] and Sallas [2] used a simplified lumped-parameter model of the vibrator, consisting of a linear oscillating system of masses, springs, and dashpots. The model allowed a representation of the force F_g applied to the ground as a “weighted sum” of the accelerations of the reaction mass and the baseplate,

$$F_g = M_r \ddot{z}_1 + M_b \ddot{z}_2, \quad (1)$$

where M_r and M_b are the reaction mass and the mass of the baseplate, z_1 and z_2 are their displacements, respectively, and \ddot{z}_1 and \ddot{z}_2 are the accelerations. The advantage of using Eq. (1) is that it expresses the ground force through the quantities that are routinely measured; the weighted sum can then be used, as a proxy for the force, in feedback control.

Whether or not the force expressed by Eq. (1) is an adequate representation of the actual ground force has been a subject of debate. Two sources of its discrepancy with the true force are possible. First, Eq. (1) follows from an idealized linear-oscillator model of a complex coupled system of the mechanical vibrator and the ground; there is little doubt that the model oversimplifies the vibrator’s real dynamic behavior (e.g., [5]). Second, the model assumes a perfectly rigid baseplate. Flexural vibrations always exist in a real baseplate, causing variations in the phase and amplitude of the acceleration over the plate surface [6]. The value of the weighted sum will therefore depend on the position of a reference accelerometer, resulting in arbitrary deviations of the weighted sum from the true force value. Such departures have, for example, been documented by van der Veen et al. [Ref. [7], Fig. 7], Wei [Ref. [8], Fig. 4], and Wei [Ref. [9], Figs. 4–6], who measured the true force by load cells (stress sensors) installed under the plate. The deviations typically occur at the higher end of the frequency band used in the sweeps, typically above 80–100 Hz [6; Ref. [8], Fig. 4; Ref. [9], Fig. 10]. Lebedev and Beresnev [6] concluded theoretically that stiffening the baseplate, in combination with selecting an optimum location of the reference accelerometer, may significantly reduce the flexural vibrations. The data of Wei [Ref. [8], Fig. 4] and Wei [Ref. [9], Figs. 7–8, 10] support this conclusion.

While the effect of the baseplate flexure can in theory be reduced, the effect of simplified model assumptions cannot. With the ground force being a not directly determinable quantity, and the weighted sum, as its substitute, containing significant uncertainties in its relationship to the true force, there is motivation for revisiting the issue of the appropriateness of the ground force as the reliable source signature. Would it not be more accurate to replace it with a quantity that is directly measured?

The system characteristics that are directly measured are the accelerations (displacements) of the baseplate and of the reaction mass. The displacement of the plate represents the motion applied to the earth surface as a boundary condition. It seems to be natural to use this displacement as the source signature, avoiding the uncertainty of assuming any particular, imperfect models of the vibrator’s dynamic behavior.

Sallas [2] used the Miller and Pursey solution as the theoretical underpinning for using the ground force as the source signature. This solution specifically states that the seismic-wave displacement is proportional to the force applied to the ground. Analogous solutions, showing the proportionality of the wave displacement to the displacement applied to the ground, which would equally serve as the justification for the displacement-based control method, do not appear to exist. In general, there is no apparent physical reason why the source displacement cannot be equivalent to the force in controlling the seismic waveforms as proportionality coefficients. The unavailability of such theoretical solutions thus cannot serve as a reason for not using the baseplate displacement as the source signature, unless there is clear fundamental difference between how the force and displacement in the source scale the radiated waveforms. Revisiting this issue and the underlying theories is the subject of this paper.

Sallas [Ref. [2], Figs. 8, 9, 11] compared the auto-correlation of the theoretical pilot and the pilot's cross-correlation with the acceleration signals observed in a borehole, for the cases of a vibrator controlled using the baseplate displacement and the weighted sum. The evidence shown in support of the weighted-sum based control is hardly conclusive: the differences in the quality of the respective cross-correlation wavelets are subtle to the eye. This is further emphasized by the numerical simulations of the radiation from a vibrating plate on a half-space conducted by Baeten et al. [10]. For example, the authors demonstrate that simply changing the values of the P - and S -wave velocities in the half-space improves the cross-correlation between the far-field displacement and the baseplate velocity dramatically, to the extent that the modulus of the cross-correlation is almost indistinguishable from the autocorrelation of the sweep (cf. Figs. 10(d) and 11). Evidently the baseplate signal in this situation would be an accurate source signature.

In the following, we address the reasons for the lack of “displacement-source” radiation solutions, as opposed to the “force-source” solutions. We begin with the review of the types of boundary conditions used in the linear theories of radiation from a vibrating plate, for both force sources and displacement sources, highlighting the principal differences. We show that the differences are purely mathematical and do not explicitly preclude using the source displacement, as opposed to force, as the equivalent coefficient in the expression for the far-field radiation. We propose that the displacement-based vibrator control may be a more accurate method.

2. Types of boundary conditions in linear theories of surface radiators

The linear radiation problems are based on the solution of the equations of motion of linear elasticity. The general solutions are known; obtaining a particular result is a matter of resolving the arbitrary constants, appearing in these solutions, to match the specific boundary conditions for the radiator of interest.

Two types of boundary conditions for the problems of mechanical radiation from piston sources vibrating on the surface of a half-space typically occur [10,11]. The first type is the stress condition, corresponding to the problem of a “force” (or “pressure”) source. For the vertical vibrator, this condition prescribes the normal stress σ_{zz} to be constant (a sinusoidal factor will always be assumed) under the vibrating plate and zero outside, and the tangential stress σ_{zr} to be zero everywhere:

$$\sigma_{zz} = \sigma_0 (r \leq a), \quad \sigma_{zz} = 0 (r > a), \quad \sigma_{zr} = 0 \quad (\text{for all } r), \quad (2)$$

where a is the radius of the plate and r is the radial distance in the cylindrical coordinate system. This boundary-value problem was solved by Miller and Pursey [3,4]. The knowledge of stress everywhere on the surface allows explicit determination of all arbitrary constants, making an analytical solution possible.

The second type of boundary condition is the “mixed” form, in which both stresses and displacements on the surface are prescribed [11]. It corresponds to the problem of a “displacement” source. For a vertical vibrator, this means a constant vertical displacement d under the plate, zero normal stress outside, and zero tangential stress everywhere:

$$w = d (r \leq a), \quad \sigma_{zz} = 0 (r > a), \quad \sigma_{zr} = 0 \quad (\text{for all } r), \quad (3)$$

where w is the vertical displacement. An assumption of frictionless contact is typically made, which postulates no shear stress under the vibrator. Note that, in this problem, the vertical displacement is unknown outside the plate, while the vertical stress is unknown under the plate; as a result, neither of them is prescribed over the entire surface. This makes the mixed boundary-value problem fundamentally different mathematically from the stress problem. Due to the lack of knowledge of a continuous distribution of either the vertical stress or displacement over the surface, there are no mathematical means to explicitly resolve the arbitrary constants that would automatically satisfy the boundary conditions everywhere. The problem becomes mathematically unsolvable in explicit terms; the solution can only be formulated implicitly as a system of dual integral equations. Such implicit solutions of the mixed boundary-value problem have been presented by Bycroft [12], Awojobi and Grootenhuis [13], Robertson [14], and Baeten et al. [10]. Depending on the formulation, either the unknown arbitrary functions [Ref. [12], Eqs. (66)–(68); Ref. [14], Eqs. (3.3)–(3.4) or (3.7)] or unknown surface stresses [Ref. [13], Eq. (22)] appear under the integral sign. This explains why even approximate analytical solutions for the mixed boundary-value problem, unlike the stress problem of Miller and Pursey [3,4], have not been known. The control of the seismic radiation by the displacement at the source thus cannot be directly demonstrated.

3. “Quasistatic” approximation in the mixed boundary-value problem

3.1. The quasistatic solution of Bycroft [12]

Bycroft [12] proposed an approximate approach to the mixed boundary-value problem, avoiding the solution of dual integral equations based on the quasistatic assumption. The approach is to assume smallness of the disk's size compared to the radiated wavelength, in which case the instantaneous stress distribution under the vibrating plate can be considered to be approximately the same as under the plate with a respective static displacement (the longer the wavelength, the better the approximation). On the other hand, the exact distribution of stress under a static load is known (the classic Boussinesq solution). The assumption of smallness of source dimensions relative to the wavelength is common in seismic exploration, which makes Bycroft's approach reasonable to apply (note that Miller and Pursey make the same assumption in deriving

their analytical expressions for the wave shapes in the far field). Applying Boussinesq's stress distribution, Bycroft resolved the arbitrary constants [Ref. [12], Eqs. (72) and (65)], which allows calculation of the radiated fields from a vertical vibrator anywhere in the half-space using the general integral solutions of the equations of motion [Ref. [12], Eqs. (61)–(62)]. The latter equations, with the arbitrary constants in them thus determined, become equivalent to the stress-source solutions of Miller and Pursey [Ref. [3], Eqs. (72)–(73)], from which Miller and Pursey obtained their asymptotic analytic expressions for the compressional waves [Ref. [3], Eq. (116); Ref. [4], Eq. (1)], relating the waves to the force applied to the ground.

Unlike Miller and Pursey, though, Bycroft did not analyze the asymptotic behavior of the general solutions in the far field and did not obtain the respective body-wave shapes. Bycroft's work thus does not offer a theoretical justification for the control of the far-field radiation by the source displacement, although such an asymptotic analysis can potentially be performed.

3.2. An approximate quasistatic solution for the downgoing wave

3.2.1. An approximate "model" solution

A reasonable substitute for such an analysis, useful in the context of this discussion, can be obtained as follows.

The exact solution of the equations of motion of linear elasticity for the vertical component of the radiated field, for the axisymmetric case, is

$$w = \left(\frac{A\sqrt{x^2-h^2}e^{-\sqrt{x^2-h^2}z}}{h^2} - \frac{Cx^2e^{-\sqrt{x^2-k^2}z}}{k^2} \right) J_0(xr)e^{i\omega t}, \quad (4)$$

where z is the vertical coordinate, h and k are the compressional and shear wavenumbers, respectively, ω is the angular frequency, $J_0(xr)$ is the Bessel function of the first kind, and A , C , and x are arbitrary constants [Ref. [12], Eq. (51)] (in Bycroft's notation). The normal stress is

$$\sigma_{zz} = \mu \left[\frac{A(k^2-2x^2)e^{-\sqrt{x^2-h^2}z}}{h^2} + \frac{2Cx^2\sqrt{x^2-k^2}e^{-\sqrt{x^2-k^2}z}}{k^2} \right] J_0(xr)e^{i\omega t}, \quad (5)$$

where μ is the shear modulus [Ref. [12], Eq. (52)]. The mixed boundary conditions are given by Eq. (3). From the zero condition on the shear stress, Bycroft [Ref. [12], Eq. (65)] derived the relationship between the constants A and C :

$$C = -\frac{2k^2A\sqrt{x^2-h^2}}{h^2(k^2-2x^2)}. \quad (6)$$

Using this relation, the vertical displacement (Eq. (4)) and normal stress (Eq. (5)) at the surface ($z=0$) are rewritten (omitting the oscillation factors) as

$$w(r,0) = \frac{A\sqrt{x^2-h^2}}{h^2} \left(1 + \frac{2x^2}{k^2-2x^2} \right) J_0(xr), \quad (7)$$

$$\sigma_{zz}(r,0) = \frac{\mu A}{h^2} (k^2-2x^2) \left[1 - \frac{4x^2\sqrt{(x^2-h^2)(x^2-k^2)}}{(k^2-2x^2)^2} \right] J_0(xr) \quad (8)$$

We now assume, following the quasistatic approximation, that the normal stress at the boundary under the vibrator is that of the static-displacement case, which represents the Boussinesq stress distribution [Ref. [12], Eq. (47)]:

$$\sigma_{zz}^{\text{static}}(r,0) = \frac{4\mu d(h^2/k^2-1)}{\pi\sqrt{a^2-r^2}}, \quad r \leq a. \quad (9)$$

Note that the Boussinesq stress under a constant-displacement condition experiences a singularity at the edge of the plate ($r=a$). In reality, such a singularity is prevented by a local plastic yielding of the material along the edge [Ref. [15], p. 408]. Equating Eqs. (8) and (9) resolves the constant A through x :

$$A = \frac{4h^2(h^2-k^2)d}{\pi k^2(k^2-2x^2) \left[1 - \frac{4x^2\sqrt{(x^2-h^2)(x^2-k^2)}}{(k^2-2x^2)^2} \right] \sqrt{(a^2-r^2)} J_0(xr)}, \quad r \leq a. \quad (10)$$

We can now use the displacement boundary condition to determine the remaining undefined constant x . Substituting Eq. (10) into (7) and equating the result to d leads to an algebraic equation for x :

$$4(h^2-k^2)\sqrt{x^2-h^2} = \pi\sqrt{a^2-r^2} \left[(k^2-2x^2)^2 - 4x^2\sqrt{(x^2-h^2)(x^2-k^2)} \right], \quad r \leq a. \quad (11)$$

3.2.2. Analysis of the solution

Let us analyze the radiation solution thus determined for a simple case of $r=0$ (the downgoing wave) and a particular elastic scenario corresponding to the Poisson hypothesis, $k^2=3h^2$ [Ref. [16], p. 12]. Under these conditions, Eq. (11) simplifies to

$$-8\sqrt{x^2/h^2-1} = \pi(l/\lambda_p) \left[(2x^2/h^2-3)^2 - 4x^2/h^2 \sqrt{(x^2/h^2-1)(x^2/h^2-3)} \right], \quad (12)$$

where l is the circumference of the plate and λ_p is the P -wavelength. We will subsequently denote the root of this equation as $x_1 \equiv x^2/h^2$. The root resolves the constant A through Eq. (10), which then determines the constant C via Eq. (6). Substituting them into Eq. (4) leads to the solution for the vertical component of the downgoing wave (the oscillation factor has been restored):

$$w = -\frac{d}{3} \left[(2x_1-3)e^{-h\sqrt{x_1-1}z} - 2x_1 e^{-h\sqrt{x_1-3}z} \right] e^{i\omega t} \quad (13)$$

This solution is not yet complete as one should still determine the value of the root x_1 of Eq. (12). Our analysis has been performed in the assumption of smallness of the vibrating-plate's size relative to the wavelength. For a particular value of $l/\lambda_p=0.001$, the only real root is numerically found as $x_1 \approx 405,289$. This root is unphysical, as its substitution into Eq. (13) leads to an unrealistically strong attenuation of the downgoing wave. We were also unable to find any complex roots of the equation. However, an inspection of Eq. (12) suggests that it is very closely satisfied by $x_1 = x^2/h^2 \approx 1$. Indeed, at $x_1 \approx 1$, Eq. (12) reduces to $\pi(l/\lambda_p) \approx 0$, which approaches zero as the source size becomes sufficiently small. This is consistent with the approximations already made. For $x_1 \approx 1$, we then arrive at the final approximate form of the solution (13):

$$w \approx \frac{d}{3} \left[e^{i\omega t} + 2e^{i(\omega t - \sqrt{2}hz)} \right] \quad (14)$$

We now need to extract the physical meaning from this solution, in light of the approximations made, which has an unusual form. However, each term carries its particular meaning following from the way the problem was set up. The first term in the brackets describes the vibration of the entire half-space as a whole. This is a mathematical consequence of the quasistatic approximation: we in fact used the stress distribution (Eq. (9)) under the plate that is exact for the static load, assuming it equals the stress under the vibrating plate at any given moment, appropriately scaled by an instantaneous displacement. The second term in the brackets describes a downward-propagating compressional wave, albeit with the velocity of $1/\sqrt{2}$ of the P -wave velocity. This wave term is a desired consequence of the fact that the dynamic problem was solved. The lower than expected propagation velocity reflects the approximate character of the solution: we used an approximate root of Eq. (12). Symmetry dictates that there is no S -wave motion at $r=0$. Every term in Eq. (14), therefore, has its explanation. Finally, there is no geometric spreading in this solution, which also can be understood, because we have only satisfied the boundary conditions “partially” – under the plate but not over the entire surface.

However, using this approximate “model” approach, we have obtained a useful and expected result: the displacement in the downgoing wave is proportional to the displacement d in the source. The source displacement merely appears as a coefficient of proportionality at the radiated waveform, just as the source force does in the Miller–Purseley solution. It is reasonable to expect that, if a closed-form solution were available for the dynamic mixed-boundary-value problem, it would exhibit the same simple dependence.

4. Exact static solution of the mixed boundary-value problem

The approximate dynamic solution obtained provides the first piece of evidence that it is reasonable to expect the proportionality of the far-field wave displacement to the displacement applied to the ground.

The mixed boundary-value problem has an exact analytical solution for the true static case, which we can also examine for supporting evidence. This is the solution of the Boussinesq problem of the static load. The distribution of displacements inside the half-space, resulting from the application of the boundary conditions of Eq. (3), has been determined by Sneddon [17]. For example, the vertical displacement is

$$w = \frac{2d}{\pi} \left[J_0^0 + \frac{\lambda + \mu}{(\lambda + 2\mu)a} z J_1^0 \right], \quad (15)$$

where λ is the Lamé elastic coefficient and (in Sneddon's notation) J_0^0 and J_1^0 are coefficients depending only on the coordinates r and z [Ref. [17], Eq. (9)]. One thus again finds the confirmation of the anticipated conclusion that the displacement in the interior of the half-space is proportional to the displacement applied to the ground.

In case of the use of plate displacement for control, it should be kept in mind that one of the reasons why the switch to the force-based control took place in the past is the very low plate-displacement output at low frequencies, due to the vibrator-ground system's transfer function [Ref. [6], Fig. 3; Ref. [19]]. A practical solution would be the use of a combination of the weighted sum and the baseplate displacement at the low-frequency end of sweep signals, and of the displacement only at higher frequencies.

5. Conclusions

In order to achieve the closest possible similarity between the radiated signal and the theoretical sweep, various methods of phase control of a seismic vibrator's baseplate have been proposed. Historically, the phaselock of the sweep signal to the motion of the baseplate (the plate-displacement-based vibrator control) was the first to be implemented [2,5,18]. Lerwill [19] proposed a different approach, in which the theoretical sweep was phaselocked to the acceleration of the reaction mass. Sallas and Weber [20] and Sallas [2] showed that the acceleration of the reaction mass did not correctly represent the force applied to the ground and argued for an alternative way of estimating the ground force via the weighted-sum approximation (Eq. (1)). The weighted-sum-based feedback in vibrator control has become the prevalent approach in practical exploration.

There are two principal types of boundary-value problems for the surface-source radiation problems of linear elasticity: the pressure and the displacement types. The pressure problem specifies stresses over the entire surface of the half-space, which makes an explicit mathematical solution possible. This problem corresponds to the force source in seismic exploration. The displacement problem, known as the case of the mixed boundary conditions, prescribes a displacement boundary condition under the source and a normal-stress boundary condition elsewhere, while neither normal stress nor displacement are known everywhere on the surface. Due to the lack of knowledge of the full distribution of either variable along the surface, there are no explicit mathematical means to obtain a solution: the solutions appear implicitly as unsolvable integral equations. This is the fundamental mathematical difference between the two formulations, explaining why asymptotic analytical solutions historically have been obtained for the pressure source but not the displacement source.

However, despite the mathematical differences, there is no reason to believe that there is any fundamental asymmetry between the force and displacement at the source controlling the amplitudes of radiated waves. Although direct mathematical solutions of the mixed boundary-value problem are not possible, an approximation reasonably accurate in seismic applications is to assume the instantaneous stress under the vibrating plate as that under a static load. This “low-frequency” approximation is valid for seismic sources small relative to the radiated wavelength. An approximate treatment of this quasistatic situation has led to a solution that shows the expected proportionality of the radiated waveforms to the displacement applied by the source. The same proportionality is shown by the exact solutions of the mixed boundary-value problem known for purely static loads.

The control of the downgoing wave by the source displacement thus appears in all plausible solutions of the mixed problem, making the source displacement equivalent to the source force as the valid source signature. This justifies the usage of the source displacement, as opposed to the ground force, as an alternative control signal to phaselock to the theoretical sweep in vibrator's feedback.

The plate-displacement-based vibrator control has a significant advantage of the displacement being a directly measurable parameter, unlike the ground force that has to be inferred, in form of the weighted sum, based on simplified model assumptions, with a significant and unknown uncertainty. Even in the presence of flexural vibrations, the displacement-based control may still achieve a greater similarity between the far-field wave shape and the theoretical sweep. In this case, flexure will remain the only source of uncertainty, whereas the weighted-sum-based control will have two: both the flexure and the reliance on model assumptions.

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References

- [1] A. Castanet, M. Lavergne, Vibrator controlling system, U. S. Patent No. 3,208,550, 1965.
- [2] J.J. Sallas, Seismic vibrator control and the downgoing *P*-wave, *Geophysics* 49 (1984) 732–740.
- [3] G.F. Miller, H. Pursey, The field and radiation impedance of mechanical radiators on the free surface of a semi-infinite isotropic solid, *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences* 223 (1954) 521–541.
- [4] G.F. Miller, H. Pursey, On the partition of energy between elastic waves in a semi-infinite solid, *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences* 233 (1955) 55–69.
- [5] J.E. Martin, I.G. Jack, The behaviour of a seismic vibrator using different phase control methods and drive levels, *First Break* 8 (1990) 404–414.
- [6] A. Lebedev, I. Beresnev, Radiation from flexural vibrations of the baseplate and their effect on the accuracy of traveltimes measurements, *Geophysical Prospecting* 53 (2005) 543–555.
- [7] M. van der Veen, J. Brouwer, K. Helbig, Weighted sum method for calculating ground force: an evaluation by using a portable vibrator system, *Geophysical Prospecting* 47 (1999) 251–267.
- [8] Z. Wei, Design of a *P*-wave seismic vibrator with advanced performance, *GeoArabia* 13 (2008) 123–136.
- [9] Z. Wei, How good is the weighted-sum estimate of the vibrator ground force? *The Leading Edge* 28 (2009) 960–965.
- [10] G.J.M. Baeten, J.T. Fokkema, A.M. Ziolkowski, Seismic vibrator modeling, *Geophysical Prospecting* 36 (1988) 22–65.
- [11] G.M.L. Gladwell, The calculation of mechanical impedances relating to an indenter vibrating on the surface of a semi-infinite elastic body, *Journal of Sound and Vibration* 8 (1968) 215–228.
- [12] G.N. Bycroft, Forced vibrations of a rigid circular plate on a semi-infinite elastic space and on an elastic stratum, *Philosophical Transactions of the Royal Society of London, Series A, Mathematical and Physical Sciences* 248 (1956) 327–368.

- [13] A.O. Awojobi, P. Grootenhuis, Vibration of rigid bodies on semi-infinite elastic media, *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences* 287 (1965) 27–63.
- [14] I.A. Robertson, Forced vertical vibration of a rigid circular disc on a semi-infinite elastic solid, *Mathematical Proceedings of the Cambridge Philosophical Society* 62 (1966) 547–553.
- [15] S.P. Timoshenko, J.N. Goodier, *Theory of Elasticity*, McGraw-Hill, New York, 1970.
- [16] H. Lamb, On the propagation of tremors over the surface of an elastic solid, *Philosophical Transactions of the Royal Society of London, Series A, Mathematical and Physical Sciences* 203 (1904) 1–42.
- [17] I.N. Sneddon, Boussinesq's problem for a flat-ended cylinder, *Mathematical Proceedings of the Cambridge Philosophical Society* 42 (1946) 29–39.
- [18] Z. Wei, T.F. Phillips, M.A. Hall, Fundamental discussions on seismic vibrators, *Geophysics* 75 (2010) W13–W25.
- [19] W.E. Lerwill, The amplitude and phase response of a seismic vibrator, *Geophysical Prospecting* 29 (1981) 503–528.
- [20] J.J. Sallas, R.M. Weber, Comments on "The amplitude and phase response of a seismic vibrator" by W. E. Lerwill, *Geophysical Prospecting* 30 (1982) 935–938.