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A model for nonlinear seismic waves in a medium with instability

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Experimental investigations show that the earth's crust is elastically nonlinear and contains the sources of accumulated elastic energy, so that it can be considered as a nonlinear active medium. The effects of stimulation of narrow- and broad-band seismic emission and the existence of dominant frequencies for which the medium is transparent have been reported. A model evolutionary equation which takes into account nonlinearity, instability, dissipation of energy and dispersion is proposed and solved numerically. The temporal evolution of chaotic signals results in the formation of quasi-sinusoidal steady-state wave trains. The dependence of their amplitude on the values of coefficients of nonlinearity and dispersion is found. The mathematical model proposed can be used to describe the observed seismic phenomena.

1. Introduction

Seismic experiments carried out in recent years showed the existence of the effects which cannot be described by the linear elasticity concept generally accepted in seismology. There is experimental evidence that the geophysical medium has significant nonlinear elasticity manifested even in the propagation of small-amplitude seismic signals [1–3] (henceforth we will imply by the term of geophysical medium the rocks forming the earth's crust). Dynamical instability and stimulated emission of noise were observed in the propagation of long sinusoidal seismic signals [4–6]. It was noted that the real media can be transparent for the signals having some particular frequencies and opaque for the other frequencies. Such "preferred" frequencies were coined the dominant ones [7].

The results outlined above show that the real earth media are nonlinear and they contain the internal sources of accumulated elastic energy which may be triggered by the external excitation. Such media are usually called the active ones [8]. No adequate theory exists which describes all these phenomena using a single approach. In this investigation we propose an evolution equation which gives similar effects, and we analyze it numerically.

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2. Stimulation of narrow-band noise emission

A particular experiment showing that the narrow-band seismic noise may be excited by a long periodic elastic wave propagating in the earth's crust was carried out by Kuznetsov et al. [4].

The authors observed the temporal evolution of a spectrum of quasi-monochromatic vibratory signal having frequency of 12 Hz, at a distance of 200 m from the source. Fig. 1 shows the plots of spectral density taken at different instants of time. The spectra of ambient microseismic noise were measured before the onset of vibrations. Then vibrations started, which is indicated by the appearance of spikes at 12 and 24 Hz. It is seen that the vibrations cause the appearance of new components in the seismic spectra which did not exist before the beginning of vibro-action (e.g. 2.8, 5.6, 6.5 Hz), and the simultaneous increase

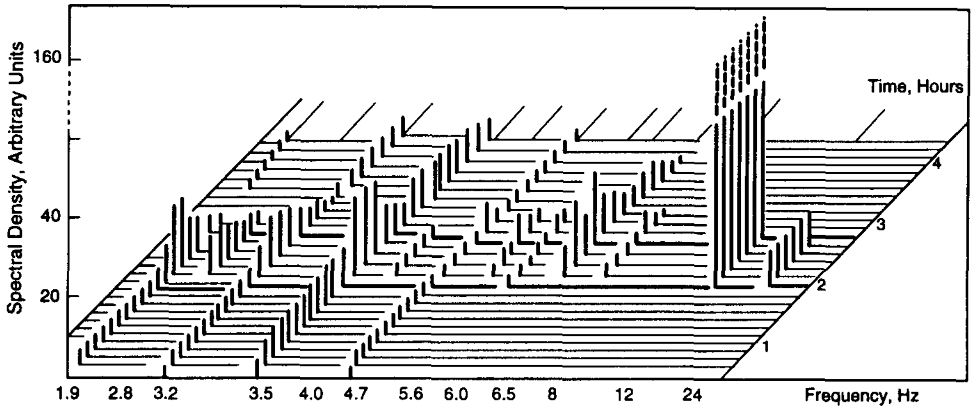

Fig. 1. Temporal evolution of the spectral components of the seismic wave field during the vibro-actions with frequency 12 Hz [4].

of the amplitudes of some components which were present before. This phenomenon was attributed to the seismic emission induced by vibro-action. Fig. 1 shows that the seismic emission appears in a narrow range of frequencies.

3. A model of nonlinear active medium

The mathematical description of the real geophysical medium required the use of generalized continuum theories.

Nikolaevskiy [9] proposed a theory of viscoelastic media with internal oscillators which takes into account nonlinearity, dissipation of energy, instability and dispersion. The amplitude variation of the signals propagating in such a medium is given by an evolutionary equation of the form

$$\frac{\partial v}{\partial t} + nv\frac{\partial v}{\partial x} + \alpha\frac{\partial^2 v}{\partial x^2} + \beta\frac{\partial^3 v}{\partial x^3} + \tau\frac{\partial^4 v}{\partial x^4} + \delta\frac{\partial^5 v}{\partial x^5} + \varepsilon\frac{\partial^6 v}{\partial x^6} = 0, \qquad (1)$$

where v is the particle velocity, x is the moving space coordinate, n is the dimensionless coefficient of nonlinearity, and $\alpha, \beta, \tau, \delta, \varepsilon$ are constants.

The system described by eq. (1) has a characteristic to amplify the waves having wavenumbers k belonging to certain interval and to suppress all waves beyond it, i.e., the interval of "dominant" frequencies exists.

To make it sure, let us consider the solution of a linear version of eq. (1) (without nonlinear term) in the form $v = \exp(ikx + \sigma t)$, where k is a wavenumber and σ is the attentuation (amplification) coefficient. Substituting it into eq. (1) we obtain a linear dispersion relation:

$$\sigma = (\alpha k^2 - \tau k^4 + \varepsilon k^6) + \mathrm{i}(\beta k^3 - \delta k^5) \equiv \mathrm{Re}(\sigma) + \mathrm{i}\,\mathrm{Im}(\sigma). \qquad (2)$$

The first term in the right-hand side of (2) describes the amplification (damping) of the waves having wavenumber k. Fig. 2 shows the graph of function $\mathrm{Re}[\sigma(k)]$. The interval of

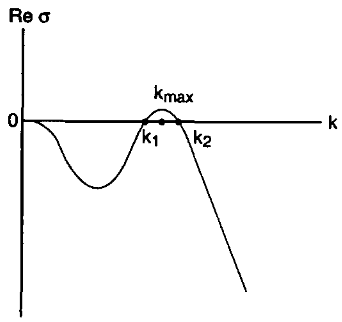

Fig. 2. Frequency dependence of the attentuation (amplification) coefficient.

wavenumbers exists where $\mathrm{Re}(\sigma)$ is positive, i.e., the amplitude of such waves will increase (unboundedly in the linear case); the other waves will decay rapidly. We will call it the interval of dominant frequencies. Its width and the height of the "hump" in fig. 2 depend on a choice of the coefficients α, τ, ε. Note that all of these coefficients should be negative.

The frequencies $k_{1,2}$ of zero-crossings are given by the expression

$$k_{1,2} = \sqrt{\frac{\tau \pm \sqrt{\tau^2 - 4\alpha\varepsilon}}{2\varepsilon}} . \tag{3}$$

A fourth-order equation of the type (1) with $\alpha, \tau \geq 0$ was previously analyzed by Kawahara [10]. However, only the presence of a sixth-order term results in the appearance of the interval of dominant frequencies. The system described by eq. (1) behaves in a significantly different way compared with a fourth-order equation.

The presence of dominant frequencies in the system means in particular the following. Let us assume an initial condition in the form of white noise. Evolving in time the system will automatically select the dominant frequencies from the initially random spectrum and will amplify them. This means that within some finite period of time the noisy initial condition will evolve into quasi-periodic solution having a dominant frequency. The width of the interval of dominant frequencies should be small enough, which is determined by the values of coefficients α, τ, ε.

In the linear case the amplitude of the solution will grow unboundedly. However, we can surmise that nonlinearity plays a stabilizing role leading to the amplitude saturation and a formation of steady-state solutions. We check it numerically.

4. Numerical results

To study the evolution of noisy initial conditions in frame of eq. (1) we solve it numerically. To solve the fourth-order evolutionary equation, Kawahara [10] applied a finite-difference method in space and the Runge–Kutta–Gill method in time. Our experience showed that such a scheme does not work well when applied to the sixth-order equation. It may be explained by the fact that the high-order derivative term value is proportional to k^n, where n is the order of derivative. This factor considerably increases the error in the numerical evaluation of the derivative. This causes much stronger instabilities in the case of a sixth-order equation than in the fourth-order one.

In our calculations we used the finite-difference approximation of the second order of accuracy for spatial derivatives and the Adams predictor–corrector scheme of fourth order for stepping in time. The characteristic time scale of the problem may be chosen as $1/\mathrm{Re}(\sigma)$. Our scheme proved to be stable up to the time about $7/\mathrm{Re}(\sigma)$. That is why we could not advance in the time domain farther beyond this limit. It remains an open question as to what scheme should be used to improve the numerical stability.

Fig. 3 shows the results of the calculation of the evolution of chaotic disturbance using eq. (1). The dimensional form of the equation and the periodic boundary conditions at the ends of a spatial interval were used.

Fig. 3a shows the initial disturbance in the form of white noise. Fig. 3b depicts a wave structure that appeared within a time period of $5/\mathrm{Re}(\sigma)$, obtained with the following set of coefficients:

$$\alpha = -29\ \mathrm{m}^2/\mathrm{s}, \quad \beta = 120\ \mathrm{m}^3/\mathrm{s}, \quad \tau = -240\ \mathrm{m}^4/\mathrm{s},$$
$$\delta = 0, \quad \varepsilon = -460\ \mathrm{m}^6/\mathrm{s}, \quad n = 2500. \tag{4}$$

Our experience showed that the addition of a fifth-order term did not produce significant changes in the behavior of the solution. The real part of eq. (2) and expression (3) give the following values for $k_{1,2}$ and k_{max} (see fig. 2): $k_1 = 0.44\ \mathrm{m}^{-1}$, $k_2 = 0.58\ \mathrm{m}^{-1}$, $k_{max} = 0.52\ \mathrm{m}^{-1}$.

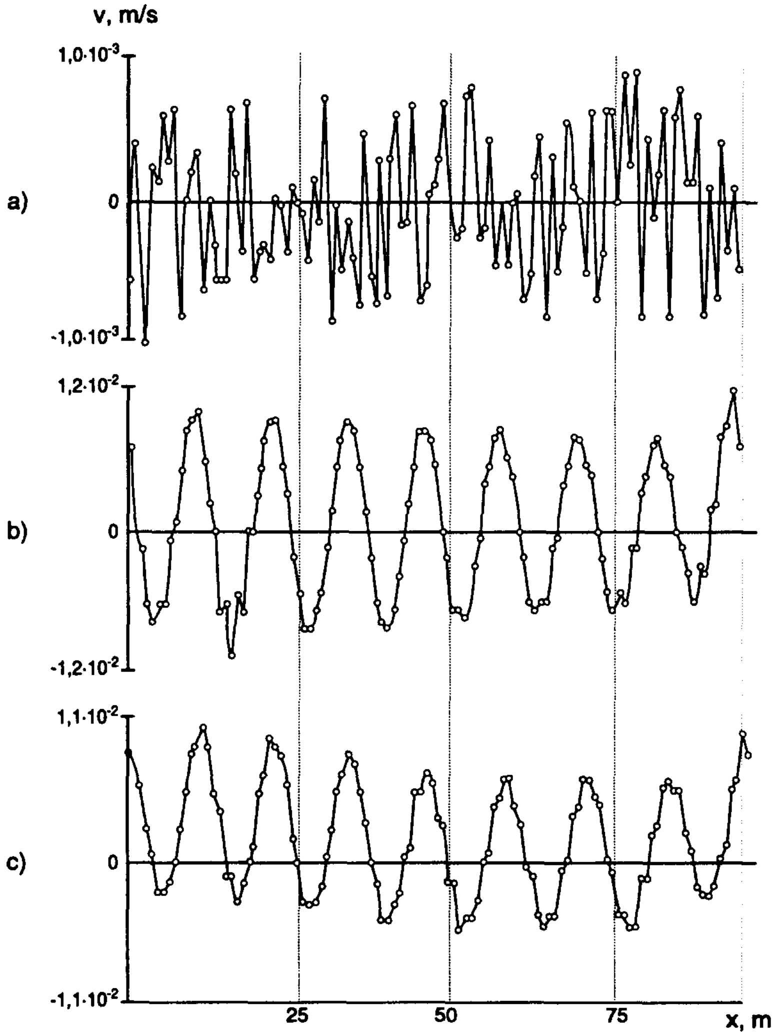

Fig. 3. Temporal evolution of the form of the initial disturbance: (a) initial disturbance in the form of white noise; (b), (c) resulting wave forms for different widths of the interval of dominant frequencies.

The real value of $\sigma(k_{max}) = 0.61$ (being the amplification coefficient at this dominant frequency). The solution shown in the fig. 3b has a form of a quasi-sinusoidal wave train having a dominant frequency. The wavelength calculated from k_{max} is $(2\pi/k_{max}) \approx 12$ m. The same value is obtained from fig. 3b. The small disturbances can be seen on the minima of this curve. We attribute them to the numerical errors.

In order to know how the width of the interval of dominant frequencies affects the form of a solution, we altered the coefficient τ slightly so that $\tau = -235$ m^4/s, keeping all other coefficients the same. It gives a narrower interval: $k_1 = 0.46$ m^{-1}, $k_2 = 0.55$ m^{-1}, and $k_{max} = 0.51$ m^{-1}. The amplification coefficient becomes much smaller in this case: $\text{Re}[\sigma(k_{max})] = 0.26$. The dominant wavelength remains unchanged.

Fig. 3c shows the result of the evolution of the initial condition up to the time of $7/\text{Re}(\sigma)$. The form of a solution has not changed significantly.

According to fig. 2, if the nonlinearity is neglected in eq. (1) then the amplitude of the dominant wave will grow infinitely in time. However, we see in fig. 3 that the steady-state solution exists. It is explained by the fact that the nonlinear term, if taken into account, plays a stabilizing rôle. It tends to shift the energy of the signal beyond the limits of the interval of dominant frequencies due to the generation of higher harmonics, and this energy decays rapidly. Two processes compete: the more the amplitude grows, the more there is shifting of energy beyond this interval. The steady-state solution arises as a result. Such a solution seems to not be a soliton in the traditional sense, which is formed as a result of a balance between nonlinearity and dispersion. In our case it is formed as the result of a balance between nonlinearity and instability.

This is confirmed by numerical results. In fig. 3c the form of the solution is shown calculated for a set of coefficients (4), except for τ which is $\tau = -235\ \mathrm{m}^4/\mathrm{s}$. The temporal variation of its amplitude is shown in fig. 4 up to the time of about $7/\mathrm{Re}(\sigma)$. To study the influence of nonlinearity and dispersion the calculation was performed for two different values of the nonlinear coefficient $n = 2500$ and 1000, and two different values of the coefficient β: $\beta = 120\ \mathrm{m}^3/\mathrm{s}$ (dashed line) and $480\ \mathrm{m}^3/\mathrm{s}$ (solid line). It is seen that nonlinearity in all cases compensates the growth of the amplitude due to instability. The increase of the nonlinearity coefficient leads to a decrease of the characteristic time over which the saturation of the amplitude is achieved, and to a decrease of the saturated amplitude value. The contrary effect is obtained by the increase of the dispersion coefficient β. Thus, nonlinearity works against instability, while dispersion enhances instability.

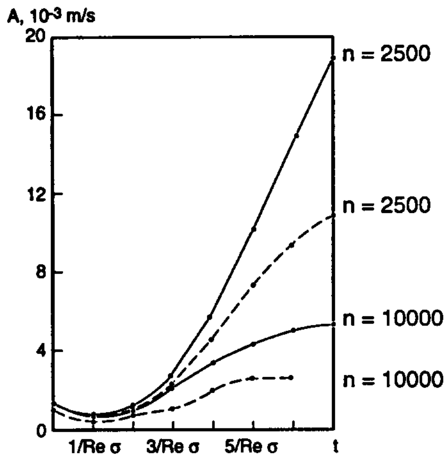

Fig. 4. Increase of the amplitude of the solution for different values of dispersion and nonlinearity. The numbers indicate the value of the nonlinear coefficient. The solid lines correspond to $\beta = 480\ \mathrm{m}^3/\mathrm{s}$, the dashed lines to $\beta = 120\ \mathrm{m}^3/\mathrm{s}$.

Let us estimate the values of saturated amplitudes for the case of $\beta = 120\ \mathrm{m}^3/\mathrm{s}$ (dashed lines in fig. 4). Their ratio is about $(12/3) = 4$. The same is the inverse ratio of the nonlinear coefficients corresponding to these curves (10000/2500). Hence, the relationship

$$\frac{A(n_1)}{A(n_2)} = \frac{n_2}{n_1}, \tag{5}$$

where $A(n)$ is a saturated amplitude corresponding to the nonlinear coefficient n, is satisfied very exactly.

5. Discussion

In the given calculations we tried to investigate the behavior of waves in a model of the geophysical medium which takes into account nonlinearity and instability, thus making an attempt to describe some experimentally observed phenomena. We proposed a phenomenological mechanism which may explain the appearance and rise of the stimulated seismic emission having narrow-band frequency content, as the medium is driven by a seismic source, as observed by Kuznetsov et al. [4] (fig. 1). In our model the driving frequency may be arbitrary, the frequency of the medium response being determined only by a set of elastic constants. The model also gives a possible physical mechanism for the existence of dominant frequency states that propagate almost without attenuation in a nonlinear active medium, whereas the signals with other frequencies decay strongly [7]. The key rôle here belongs to a combination of nonlinearity and instability, the former being an intrinsic property of rocks, as numerous investigations show, and the latter always existing as well because of the accumulation of elastic energy due to earth crust deformations. The conclusion which can be made is that the real geophysical medium can be described as an active medium with nonlinearity.

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