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Source Parameters Observable from the Corner Frequency of Earthquake Spectra

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Abstract The Brune (1970) classic theory's suggestion that the source radius be determined from the corner frequency of earthquake spectra is based on a number of insufficiently constrained assumptions that make the result virtually a guess. Viewing earthquakes as displacement-discontinuity sources radiating ω^{-2} spectra indicates that the two other parameters can be accurately resolved from the corner frequencies, without reliance on any additional assumptions. These parameters are the source duration (rise time) and the maximum slip velocity on the fault; their determination is rooted in the exact formulas of radiation from dislocation sources. These directly observable parameters can serve as important constraints on dynamic theories of friction and faulting.

Introduction

In an earlier article (Beresnev, 2001), I have argued against the use of corner frequencies of seismic spectra to determine the radius of the earthquake source, which has become common practice in earthquake studies. I showed that the classic formula of Brune (1970, equation 36), corrected by Brune (1971), is based on a series of insufficiently justified assumptions that make the result virtually a guess. It is unclear whether the source radii obtained in this way have any advantage over those simply derived from the empirical relations relating source dimensions to earthquake magnitude (e.g., Wells and Coppersmith, 1994). In this article, I go further to discuss which source parameters can realistically be obtained from the corner frequencies, without resorting to simplifying assumptions that make the results ambiguous.

Theoretical Background

It can be verified directly that the displacement-time history at the fault that radiates the far-field "ω^{-2}" spectrum has the form

$$u(t) = U[1 - (1 + t/\tau)\exp(-t/\tau)], \tag{1}$$

where τ is the parameter controlling the "rise time" by changing the speed with which the dislocation rises to its final (static) value, and U is the static displacement (e.g., Beresnev and Atkinson, 1997, equation 6). The modulus of the Fourier transform of displacement in the radiated field is then

$$\Omega(\omega) = \left| \int_{-\infty}^{\infty} \dot{u}(t)\exp(-i\omega t)dt \right| = U/[1 + (\omega\tau)^2], \tag{2}$$

where ω is the angular frequency and $\dot{u}(t)$ is the time derivative of equation (1) (slip velocity). The representation (2) of the spectrum is valid so long as the source dimensions can be considered small compared with the distance to the observation point and the distance is longer than the wavelengths of interest. A homogeneous space is also assumed (e.g., Aki and Richards, 1980, equation 14.7). It is more or less agreed upon in seismology that the observed earthquake spectra follow the shape of equation (2), at least for moderate earthquakes.

The quantity $\omega_c \equiv 1/\tau$ is the corner frequency of the spectrum. To clarify its meaning, we take the time derivative of equation (1) to obtain slip velocity and find that it reaches its maximum at $t = \tau$:

$$v_{\max} = U/e\tau = \omega_c U/e, \tag{3}$$

where $v_{\max}$ is the maximum slip velocity and e is the base of the natural logarithm. Consequently,

$$\omega_c = e v_{\max}/U, \tag{4}$$

which shows that the corner frequency carries information about the maximum slip velocity on the fault.

Note that because the far-field radiation of both S and P waves from a displacement-discontinuity source is controlled by the same displacement-time history (Aki and Richards, 1980, equation 14.7), both waves will have the same spectral shape and the same corner frequency. The following analysis will thus apply to both.

Using the aforementioned relationships, we can determine what type of information about the source can be gathered from the corner frequencies of observed spectra.

Source Duration (Rise Time)

The source rise time (the time it takes the dislocation to reach its static value *U*) is formally infinite in equation (1). However, it could be reasonably well defined as the time (*T*) over which 90% of static displacement is reached. From equation (1), we then have a simple equation

$$(1 + T/\tau)\ \exp(-T/\tau) = 1 - 0.9, \tag{5}$$

from which $T/\tau \approx 3.9$. The relation between the corner frequency (in Hz) and the source duration is then:

$$T \approx 0.6/f_c, \tag{6}$$

where $f_c = \omega_c/2\pi$. This formally deduced relation is in fact very close to $T = f_c^{-1}$ used by Boore (1983, equation 6) and serves as a good approximation for source duration.

Maximum Slip Velocity

Equation (3) provides the basis for estimating the maximum slip velocity on the fault from the value of the observed corner frequency and fault displacement. Although the fault displacement is often not a directly observable quantity, it can be related to the quantities that are routinely observed. Using the definition of the scalar seismic moment (M_0), $M_0 = \mu UA$, where μ is the shear modulus and A is the rupture area, and the relationship $V_s = (\mu/\rho)^{1/2}$, where V_s is the shear-wave velocity and ρ is the density, one can rewrite equation (3) as

$$v_{\max} = (2\pi/\mathrm{e})(M_0/\rho V_S^2 A)\ f_c. \tag{7}$$

Equation (7) allows one to estimate the maximum slip velocity from the directly observable data. The moment M_0 is determined routinely for all significant earthquakes, and A is, in many cases, estimated from aftershock distribution. Note that equation (7) is the *exact* relation, not involving any assumptions. The maximum slip velocities inferred in this way could serve as important constraints on the theories of dynamic faulting or shed light on the friction processes on faults, which control the velocity of slip.

In the application of equations (6) and (7) in determining the parameters of faulting for real earthquakes, the limits of applicability of this analysis should be kept in mind. The stations sufficiently distant from the fault plane must be chosen to satisfy the aforementioned distance requirements. Furthermore, since the validity of equation (2) assumes a homogeneous space without attenuation, corrections to the observed spectra for the path and site effects must also be applied.

Conclusions

Viewing an earthquake as a displacement-discontinuity source radiating the ω^{-2} spectrum suggests the two parameters that can be accurately resolved from an observed corner frequency of the spectrum, without the need for resorting to any further assumptions. The first parameter is the source rise time, which is directly obtainable from the corner frequency. The second parameter is the maximum slip velocity during fault rupture. Both can be determined from *P*- or *S*-wave spectra, which resolve the same quantities. The inferred rise time and maximum slip velocity could serve as observational constraints for the theories of dynamic faulting.

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