

# Subevent structure of large earthquakes – A ground-motion perspective

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**Abstract.** Our ability to predict ground motions from future earthquakes hinges on the accurate modeling of the radiation from the earthquake source. A successful approach to modeling earthquake radiation has been to represent faults as a series of discrete, independently-rupturing subfaults, although their physical interpretation has remained largely unclear. Our simulation of ground motions from twenty-six well-recorded moderate-to-large earthquakes substantiates the hypothesis that large ruptures are made up of a sequence of smaller subevents. The size of a subevent follows a simple linear relationship with the size of the earthquake fault in an apparently deterministic manner. The strength of the high-frequency radiation is controlled by the maximum slip velocity, which varies stochastically over a small range. Both the characteristic subevent size and the slip velocity appear to be region-independent, indicating a remarkable uniformity in earthquake source properties. This observation helps to reduce the uncertainty in ground-motion prediction by constraining its essential and previously ambiguous parameters.

## Introduction

In the absence of reliable techniques to predict the exact time and location of future earthquakes, efforts to reduce seismic hazard focus on the adequate design of engineered structures to withstand the expected ground motions. To estimate future motions, potential source zones are identified, and seismic waves are propagated to the site of interest. While the changes in wave characteristics along the propagation path can be predicted reasonably well, there is still much debate as to the nature of the seismic source.

Both point- and extended-source models are used in modern engineering seismology. Despite the popularity of the point-source model (introduced in a classic paper by Brune, 1970), the current emphasis in ground-motion prediction is shifting toward an extended-, or finite-fault, source representation. The point-source approximation is clearly unable to characterize key features of ground motions from large earthquakes, such as their long duration and dependence of amplitudes on the azimuth to the observation point (source directivity). These characteristics are naturally accounted for by a finite-fault representation.

Finite-fault effects not only contribute to the duration and directivity of ground motions, they also affect the shape of their spectra. The classic Fourier spectrum of ground acceleration near a point dislocation (an “ $\omega^2$ ” spectrum) is given by the function  $\omega^2/[1 + (\omega/\omega_0)^2]$ , where  $\omega$  is the angular frequency (Aki, 1967; Brune, 1970). At low frequencies (below  $\omega_0$ ), the spectrum rises with frequency, while at high frequencies (above  $\omega_0$ ) the spectrum is constant. The quantity  $\omega_0$  is the “corner frequency”, which is inversely proportional to the size of the event. The spectra from small to moderate earthquakes roughly follow the  $\omega^2$  model (e.g., Boore, 1983). However, the analysis of empirical databases suggests that large events (generally,  $M \geq 6$ , where  $M$  is the moment magnitude) do not obey this simple approximation, especially at low to intermediate frequencies ( $\sim 0.2$  to 2 Hz), where they radiate less energy than predicted by point-source models (e.g., Atkinson, 1993; Atkinson and Silva, 1997, 2000). This observation can be explained in terms of the finite spatial extent of large earthquake sources.

## Finite-Fault Models

A discrete finite-fault model of radiation from large events, which captures their salient features, has been popular over the past two decades. In this model, introduced by Hartzell (1978), the finite-fault plane is subdivided into elements (subfaults), and radiation from a large earthquake is obtained as the sum of contributions from all elements, each of which acts as a small independent (sub)source. In the typical implementation, the rupture starts at a hypocentral point on the fault and propagates radially, triggering the subfaults as it passes them. The fields from all subevents are geometrically delayed and added together at the observation point. Engineering simulations of ground motions from significant seismic events have been performed primarily through such kinematic models (Heaton and Hartzell, 1989; Somerville *et al.*, 1991; also see the review of recent work in Beresnev and Atkinson, 1997).

Despite the success of this method, its applicability has never been strictly justified and remained heuristic in nature. The only justification for the use of the discrete finite-fault model for ground-motion prediction has been that it appeared to work, and provided more realistic simulations than those obtained from point-source models. A critically-minded observer might ask, “What is the basis for the belief that a continuous earthquake rupture can be represented as a series of isolated, smaller events?” Should we consider this method a technical ploy, fortuitously leading to the right answer, or does it fundamentally reflect the way real earthquakes rupture? Answering these questions is important for engineering seismology and earthquake physics in general.

Apart from its conceptual weakness, another problem that discrete finite-fault models have faced is the lack of a practical “recipe” for the choice of the appropriate subfault size. Indeed, if the idea of discretization of a large earthquake rupture into smaller events can be taken for granted, what is the size of those “characteristic” earthquakes that make up a large event? Is the latter composed of a thousand small “patches” or no more than ten moderate events? This problem has not gone unnoticed, and the technical solutions proposed to date have generally been based on the postulate of “self-similarity”, or the assumption that the spectra of the largest events follow the same “ $\omega^2$ ” shape that is characteristic of smaller earthquakes. Simple summation rules, also prescribing the subfault size, have been developed to preserve the shape of the spectra through different modeling scales (e. g., Joyner and Boore, 1986). Although technically attractive, this solution cannot be considered satisfactory. There is no sound physical or empirical basis for the applicability of self-similar spectral behavior, which is strictly valid for point dislocations only, to the scales of large and giant earthquakes. Furthermore, the self-similarity postulate contradicts empirical data even for moderate events (e.g., Atkinson, 1993; Atkinson and Silva, 1997, 2000). In the absence of physically-justified or well-calibrated rules for the subfault-size selection, modelers have approached this aspect of ground-motion simulations on a virtually *ad hoc* basis, often basing their selection on a particular aftershock record available.

Intuitively, it appears quite natural that faults rupture as a sequence of breakage of small areas (called “asperities”), rather than in a smooth and continuous manner. If we accept that this is so, we can qualitatively explain why the discrete rupture process creates the observed deficit of energy at intermediate frequencies, relative to the “ $\omega^2$ ” shape of an equivalent point source. The acceleration spectrum of ground motions from each small event decays quickly at frequencies below its corner frequency, which is high due to the small size of the subevent. However, at very low frequencies, the signals sum up coherently, boosting low-frequency energy. The net result is the relatively high radiated energy at the high- and low-frequency ends of the spectrum, with a “sag” in between. Two parameters then govern the radiated spectral shape -- the slip velocity on the fault (controlling the amplitude level of high-frequency radiation) and the subfault (asperity) size, controlling the location and depth of the spectral sag.

We investigate these parameters through simulation of acceleration data from well-recorded moderate-to-large earthquakes in North America and the giant Michoacan, Mexico event. To avoid complications related to the effects of local soil conditions, only data recorded on rock sites have been used. The list of all twenty-six modeled events is given in Table 1 in order of increasing earthquake magnitude.

## Method

We use the stochastic finite-fault simulation method, which implements the concept of fault discretization wherein subevents are represented as stochastic point sources (Beresnev and Atkinson, 1997, 1998). Every subfault is assigned an average  $\omega^2$  spectrum with a stochastic component superimposed on it; this reproduces the realistic quasi-random shape of observed ground-acceleration time histories (Hanks and McGuire, 1981; Boore, 1983). The number of subsources

summed is prescribed by the total moment of the desired target event. Even though each elementary source radiates an  $\omega^2$  spectrum on average, the result of the summation of all radiated fields under the conservation-of-total-moment constraint does not lead to the same spectral shape; a “spectral sag” is created by the summation process, as described above.

The two free parameters of the simulations are the slip velocity on the fault and the subfault size ( $\Delta l$ ), controlling the amplitude and shape of the simulated finite-fault spectrum at high and intermediate frequencies, respectively. For each event, the error in the model -- defined by the ratio of the observed to simulated response spectrum, averaged over all stations -- is calculated for a given set of model parameters, over the frequency range from 0.2 – 20 Hz. The slip velocity and  $\Delta l$  are then iteratively adjusted to minimize the error. The subfault size was calculated as the average of its length and width. All simulations used our FORTRAN code FINSIM (Beresnev and Atkinson, 1998). All output and input parameter files used in this study, as well as a copy of the code, are freely available from the authors. The results of these simulations are summarized in Table 1.

## Results

### Slip Velocity and High-Frequency Radiation

A key factor affecting the damage potential of earthquake ground motions is the strength of the high-frequency radiation. High-frequency magnitude ( $\mathbf{m}$ , defined by Atkinson and Hanks, 1985) measures the level of the radiated acceleration spectrum at high frequencies. Earthquakes that are relatively rich in high-frequency energy will have  $\mathbf{m} - \mathbf{M} > 0$ , while earthquakes deficient in high-frequency energy will have  $\mathbf{m} - \mathbf{M} < 0$ . On average (for “typical” events),  $\mathbf{m} = \mathbf{M}$ . In our simulation, the high-frequency radiation strength is controlled by a scaling parameter ( $s$ ) that is proportional to the maximum slip velocity (Beresnev and Atkinson, 1998, equation 3). Slip velocities are typically of the order of  $\frac{1}{2}$  m/s. Relatively fast slips produce a spectrum that is enriched in high-frequency energy, while slow slips produce a weak high-frequency spectrum. The slip velocity is the physical factor that controls the value of  $\mathbf{m} - \mathbf{M}$ . In Table 1, we have listed the slip velocity ratio, where a value of 1.0 corresponds to the average slip velocity (standard deviation = 0.2).

The slip-velocity values in Table 1 may suggest a trend for them to decrease with increasing earthquake size; however, this effect could also be caused by insufficient representation of large-magnitude ( $\mathbf{M} > 7$ ) events. The reality of slip-velocity dependence on magnitude can only be resolved by collecting more data from large events. The current data suggest that slip velocities vary rather stochastically over a small range, independent of magnitude or tectonic region.

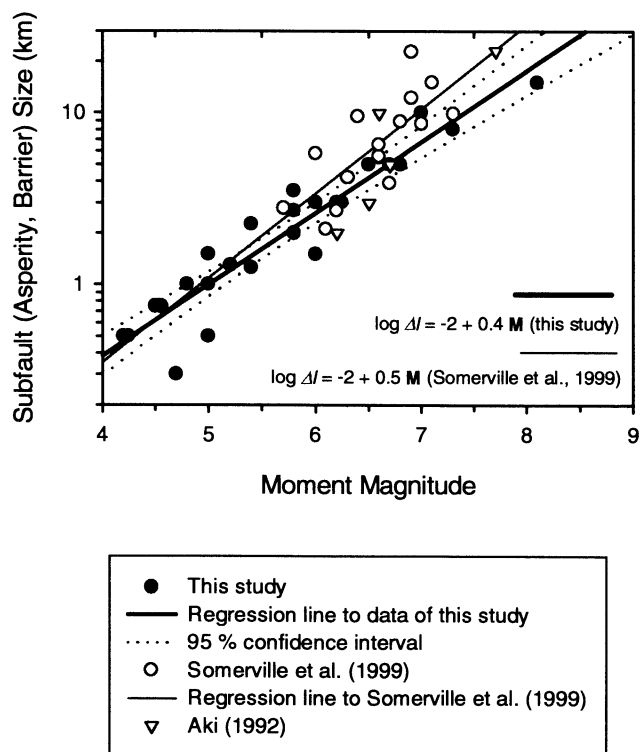
### Subfault Size and Spectral Shape

Figure 1 plots the subfault size determined for each simulated event, as a function of earthquake magnitude (solid circles). The total fault dimensions of the modeled earthquakes (an average of fault length and width) change from approximately 1 to 145 km over the range of magnitudes modeled. What comes as a surprise and cannot be explained by existing theories of rupture is that the best-fitting subevent size increases with magnitude, following a well-defined straight line in an apparently deterministic manner. The linear regression

**Table 1.** Simulated Earthquakes

Event Name	Date (m/y)	Moment Magnitude (M)	High-Frequency Magnitude (m)	Location	Slip Velocity Ratio*	Subfault Size (km)	Fault-to-Subfault-Size Ratio
Cap Rouge	11/97	4.2	4.2	Québec, Canada	1.1	0.5	2
Saguenay foreshock	11/88	4.2	4.2	Québec, Canada	0.7	0.5	2
Mont Laurier	10/90	4.5	4.4	Québec, Canada	0.9	0.75	2
St. Marys	07/86	4.5	4.8	Ohio	0.8	0.75	2
Oroville aftershock	08/75	4.7	4.9	California	1.2	0.3	7
Perry (Painsville)	01/86	4.8	4.8	Ohio	1.0	1.0	2
Coalinga aftershock	05/83	5.0	5.6	California	1.3	1.5	2
Goodnow	10/83	5.0	4.8	New York	0.8	1.0	2
Mammoth Lakes aftershock	06/80	5.0	4.7	California-Nevada	1.1	0.5	6
Coalinga aftershock	07/83	5.2	5.8	California	1.2	1.3	3
Livermore	01/80	5.4	5.7	California	1.1	2.25	2
Lytle Creek	09/70	5.4	5.6	California	1.2	1.25	4
Coalinga aftershock	07/83	5.8	6.3	California	0.9	2.7	3
Livermore	01/80	5.8	5.9	California	0.9	3.5	2
Saguenay	11/88	5.8	6.5	Québec, Canada	1.5	2.0	4
North Palm Springs	07/86	6.0	6.3	California	1.3	1.5	10
Whittier Narrows	10/87	6.1	6.3	California	1.2	3.0	3
Chalfant Valley	07/86	6.2	6.4	California-Nevada	0.7	3.0	4
Morgan Hill	04/84	6.2	6.3	California	0.8	3.0	6
Coalinga	05/83	6.4	6.7	California	0.9	5.0	3
San Fernando	02/71	6.6	6.3	California	0.9	5.5	4
Northridge	01/94	6.7	7.0	California	1.1	5.0	5
Nahanni	12/85	6.8	6.2	NWT, Canada	0.7	5.0	4
Loma Prieta	10/89	7.0	7.2	California	1.1	10	3
Landers	06/92	7.3	7.2	California	0.8	8.0	6
Michoacan	09/85	8.0	7.6	Mexico	0.7	15	10

\* The slip velocity for each event is given as the ratio to its average value. The average slip velocity corresponds to a value of the high-frequency strength factor (input to FINSIM) of  $s = 1.5 \pm 0.3$ .



**Figure 1.** Empirical dependencies of the size of characteristic rupture zone on an earthquake fault on earthquake magnitude.

line, drawn through the data, defines a simple relationship between the characteristic subfault size and the magnitude of the event:

$$\log \Delta l = -2 + 0.4 M \quad 4 \leq M \leq 8, \quad (1)$$

where  $\Delta l$  is the subfault size in km (bold solid line).

Equation 1 suggests that even relatively small events appear to rupture discretely. Since the size of the subevent increases with magnitude, the number of small events that “make up” the large event never grows too large. For example, the modeling suggests that the number of subevents that formed the giant Michoacan earthquake is only ten, while the  $M$  7.3 Landers, California earthquake was composed of six smaller events. Table 1 shows the ratio of the size of total ruptured zone to the size of a subevent for all earthquakes. Generally, for earthquakes varying in magnitude by nearly four units, the fault-to-subfault-size ratio varies in a narrow range, between two and ten, with the larger events ( $M > 6$ ) typically having higher ratios.

Two other studies have approached the problem of determining the characteristic size of the rupture zone on earthquake faults from different points of view. Somerville *et al.* (1999) recently summarized distributions of slip on the fault for fifteen significant crustal earthquakes, obtained through the inversion of long-period seismic data. The slip distributions were used to identify the size of patches that accommodated most of the slip. These zones, called “asperities” by Somerville *et al.* (1999), are functionally equivalent to the “subevents” in our investigation, in that these are the smaller

areas on the fault whose consecutive ruptures form the large event. The sizes of the asperities from Somerville *et al.* (1999) (derived from their Table 4) are shown in Figure 1 as open circles. These data nearly overlap the data from our study for earthquakes of magnitude 6 to 7; at larger magnitudes, Somerville *et al.*'s characteristic subevent sizes tend to exceed the sizes from our investigation. The linear-regression equation drawn through the data of Somerville *et al.* (1999) is  $\log \Delta l = -2 + 0.5 M$  (thin solid line), which shows that the two sets of modeling results define a nearly identical trend. Our database is additionally constrained by the significant number of smaller-magnitude events (in the range of 4 to 6), which were not considered in Somerville *et al.*'s investigation.

It is significant that the subevent sizes in these two investigations were obtained in fundamentally different ways. While Somerville *et al.* (1999) used slip distributions obtained from a deterministic inversion of low-frequency data, our simulation matched both the low-frequency (deterministic) and high-frequency (stochastic) part of the observed spectra using an entirely stochastic approach. The subevent size thus appears to be a stable characteristic of an earthquake of specified magnitude, regardless of the modeling methodology.

The other relevant study is that of Aki (1992), who determined the characteristic diameter of circular cracks responsible for the high-frequency radiation from five major California earthquakes. Aki (1992) uses the term "barrier interval" for crack diameter, which has the meaning of the characteristic rupture size on an earthquake fault. The crack sizes of Aki (1992) are indicated by inverted triangles in Figure 1. They agree remarkably well with the trends established by both our study and that of Somerville *et al.* (1999), being closer to our regression line in the magnitude range of approximately 5 to 7, and closer to that of Somerville *et al.* (1999) at higher magnitudes.

## Summary

The results obtained from finite-fault simulation of ground motions from well-recorded earthquakes indicate unambiguously that large earthquakes should be viewed as a sequence of smaller events that comprise the large rupture. The characteristic size of these constituent events is uniquely related to the size of the overall rupture. This conclusion is supported by other independent studies in which the characteristic size of the rupture patches on earthquake faults has been determined. Even though these subevents are labeled differently by different authors ("subfaults", "asperities", or "barrier intervals"), they reflect the same reality: large earthquakes rupture discretely on a series of subfaults and cannot be regarded as single sources.

This inference may have important implications for the prediction of strong ground motions. The shape of the frequency spectrum of seismic motions is affected by the slip velocity on the fault and the characteristic size of slip-generating zones. Significant uncertainty in ground-motion prediction follows if both parameters are unknown for future events. However, if these parameters are well predictable, the uncertainty is greatly reduced.

Both the slip velocity and the characteristic subevent size appear to be region-independent, indicating a remarkable uni-

formity in source properties across tectonic settings, and suggesting that a region-independent source model can be developed. Such a model has important engineering implications.

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