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Numerical Model of the Spherical Elastic Wave Propagation in a Nonlinear Medium

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ABSTRACT

There are new experimental results showing that nonlinear effects are significant in seismic wave propagation through the upper part of the geological medium [1]. Nevertheless, no adequate models exist in seismology to describe theoretically such events. I describe here an attempt to derive a wave equation for the spherical nonlinear elastic wave and to solve it numerically.

1 The Numerical Model

The well-known model for describing nonlinear wave processes in solids is a five-constant elasticity theory [2]. There can be found the equations of motion in Cartesian coordinates.

The specific character of the seismic problems is, that unlike in acoustics there are always point sources, small compared with the wavelength. Let us consider the problem of the propagation of a nonlinear elastic wave from such a source in a five-constant medium. The problem will have full spherical symmetry.

The equation of motion proposed in [2] should be transformed to a spherical coordinate system. Having only a radial displacement component depending only on r, we obtain after manipulations with the coordinates

$$u_{tt} = u_{rr}\left(c^2 + \frac{\beta}{\rho}u_r + \frac{\gamma u}{\rho r}\right) + 2\left(\frac{u}{r}\right)_r \left(c^2 + \frac{\nu}{\rho}u_r + \frac{\eta u}{\rho r}\right), \qquad (1)$$

where u is a radial displacement, ρ is the density,

$$\begin{aligned}
c &= \sqrt{\frac{\lambda + 2\mu}{\rho}}, \\
\beta &= 2N_1 + N_2, \\
\gamma &= 2N_2, \\
\nu &= N_1 + N_2, \\
\eta &= N_1 + 2N_2, \\
N_1 &= \lambda + 3\mu + A + 2B, \\
N_2 &= \lambda + 2B + 2C.
\end{aligned}$$

λ and μ are Lamé parameters, A, B and C are third-order elastic moduli.

Equation (1) should be solved numerically. However, one difficulty exists. The second-order nonlinear wave equation (1) describes two identical waves propagating in opposite directions. For linear equations this is not important. But for the nonlinear case the interference of these waves causes their interaction, which has no physical sense. Consequently, eq. (1) should be factorized in order to reduce its order by one.

This can be done in a heuristic way by analogy with the linear wave equation. The factorization of eq. (1) results in the reduction in order of all derivatives and in taking the square root of all quantities between the parentheses having the dimension of velocity squared. We thus obtain the factorized version:

$$u_t = u_r\left(c+\sqrt{\frac{\beta}{\rho}}u_r+\sqrt{\frac{\gamma}{\rho}}\frac{u}{r}\right)+\frac{u}{r}\left(c+\sqrt{\frac{\nu}{\rho}}u_r+\sqrt{\frac{\eta}{\rho}}\frac{u}{r}\right),$$

or

$$u_t = u_r\left(c+\sqrt{\frac{\beta}{\rho}}u_r\right)+\frac{u}{r}\left(c+\frac{\sqrt{\gamma}+\sqrt{\nu}}{\sqrt{\rho}}u_r+\sqrt{\frac{\eta}{\rho}}\frac{u}{r}\right). \tag{2}$$

The second term in the r.h.s. describes the near-source region. Neglecting nonlinear terms we have the ordinary wave equation

$$u_t = c\left(u_r+\frac{u}{r}\right),$$

describing a spherically spreading wave

$$u = \frac{1}{r}e^{i(\omega t+kr)}.$$

Eq. (2) can be used for calculating the wave propagation from the source with a given motion. The boundary value problem with a zero initial condition can be formulated

$$u(r=r_0)=u_0(t),\quad u(t=0)=0.$$

For the effective solving of this problem the following numerical scheme can be used. Since the r-derivative in eq. 2 is squared, we do not succeed in obtaining a solution by stepping in the r-direction. That is why we should advance step by step in time and then take a cut at any needed $r=$ const. To advance one step in time, the fourth-order Adams predictor-corrector scheme was used, and the r-derivative was replaced by its second-order centered finite-difference approximation.

In the numerical experiment the step sizes $\Delta t = 2\times10^{-4}$ s and $\Delta r = 20$ m were used. A sine boundary condition of the form

$$u_0(t) = A\sin 2\pi f t \quad (0\le t\le t_{\max})$$

was chosen with a cosine time window, $f = 25$ Hz, $a = 10^{-3}$ m. Experiments have shown that the proposed scheme is stable when $\Delta t/\Delta r = 10^{-5}$ s/m.

The values of the elastic moduli appearing in eq. (1) were taken from [3]:

$$\begin{aligned}\rho &= 2.68\ \text{g/cm}^3;\ \lambda = 8.538\times10^5;\ \mu = 3.226\times10^5;\\ m &= 1.1\times10^9;\ l = -5.1\times10^9;\ n = 0.79\times10^9\ \text{kg/cm}^2.\end{aligned} \tag{3}$$

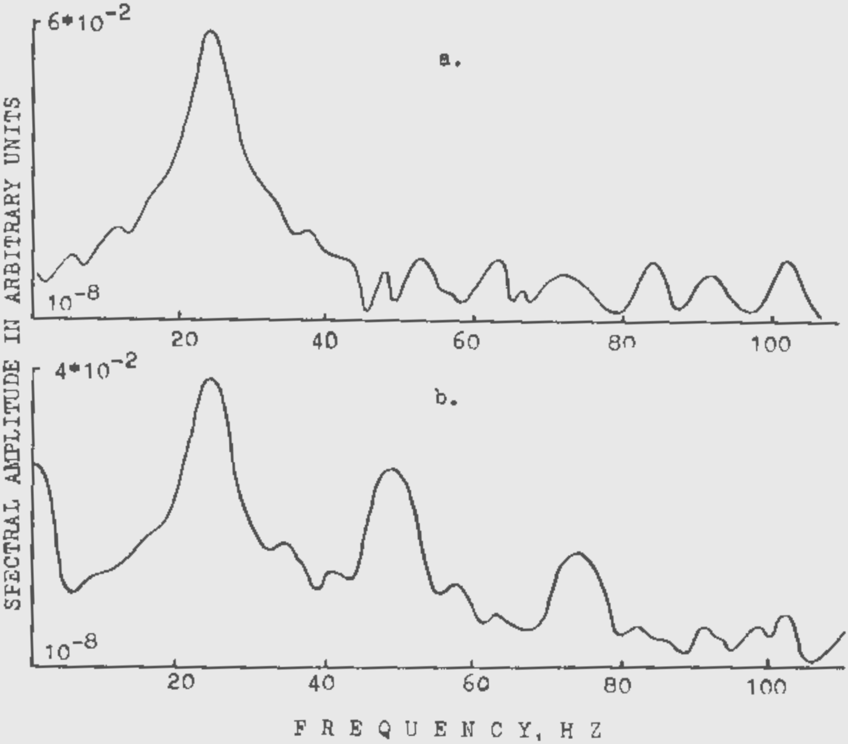

Figure 1; Amplitude spectra of the solutions of eq. 2: a - boundary condition, b - one wavelength from the source.

The moduli A, B and C are easily calculated from the Murnaghan moduli m, l and n. Some of the coefficients appearing in eq. (2) can be negative. When calculating the square roots, the moduli were taken.

2 Results

In Fig. 1 the amplitude spectra of the boundary condition and the solution at the distance of 300 m (one wavelength) are shown. The scale of curves is logarithmic. The harmonic ratios in Fig. 1 are $A_2/A_1 = 7.9 \times 10^{-3}$, $A_3/A_1 = 3.2 \times 10^{-5}$. For the component at zero frequency the ratio is $A_0/A_1 = 6.3 \times 10^{-3}$.

Despite the large values (3) of the nonlinear elastic moduli, the nonlinear parameter having the order of 10^4, the higher harmonics level obtained in the calculation is not very high. In our field experiments nonlinear effects are larger at the comparable distances. This discrepancy may indicate that the simple five-constant model does not work when applied to nonlinear properties of real soft grounds. The model of a variable-moduli medium which reacts differently upon compression and tension can be proposed as an alternative [4,5]. At the same time the five-constant model may describe the rheology of the denser crystalline rocks.

References

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